Homework 3 Problem 3

Glen Madsen

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Problem 3.
1.Write a pseudocode algorithm for division. State the loop invariant for the main loop and prove partial correctness. State the decrementing function and prove that the main loop terminates.
Pseudo-Code

```
def div(RatPoly p):
    int deg = this.degree()-p.degree();
    if(deg < 0)
    {
        return RatPoly.ZERO;
    }
    RatNum[] q = new RatNum[deg+1];
    for(int i = 0; i < q.length; i++)
    {
        q[i] = RatNum.ZERO;
    }
    RatNum[] tempc = p.{.clone();
    RatPoly temp = new RatPoly(tempc);
    for(int i = 0; i < this.degree()+1; i++)
        {
        tempc = p.{.clone();
        temp = new RatPoly(tempc);
        if(deg < 0)
        {
            break;
        }
        if(deg == 0)
        {
            q[0] = this.{[this.degree()].div(temp.{[temp.degree()]);
            deg = -3;
            break;
```

```
            }
             q[deg] =
this.{[this.degree()].div(temp.{[temp.degree()]);

            RatPoly.scaleCoeff(tempc, q[deg]);
            RatPoly.increaseExp(tempc) (a function to convert 3x^2 * x to 3x^3, or other similar operations, meant to ensure the data representation scales as exponents grow);

            temp = new RatNum[tempc.length+deg]
             this = new RatPoly(this.sub(temp).{);
             temp = new RatPoly(p.{.clone());
             deg = this.degree() - temp.degree();
            }
      return new RatPoly(q);
      }
```

Loop Invariant: i < this.degree + 1. This holds true initially as i = 0 and the degree (minimum of zero) is increased by one, so it always holds true. The degree of this is then static even as it changes, the for loop implementation would not change, and since i is only iterated by one and is not changed, the loop invariant will hold until the loops terminates.

Decrementing Function: if deg ≤ 0, the loop halts. deg = this.degree()-temp.degree() every iteration. This clearly decreases as temp will be equal to the degree of p (we are assuming p is non zero in this case, otherwise there would be an if statement check for that case). Since the 'this' polynomial is being subtracted from by the 'temp' polynomial (another non zero polynomial), it also clearly decreases. Thus, as 'this' is decreasing in the loop and deg is determined by this.degree() -temp.degree() (which is a variable-a constant) it will eventually be $\leq$ to zero and the loop halts.