taller 2 met lineales 1

sección 2.2.4

6.

a.

$$|a\rangle + |b\rangle = |b\rangle + |a\rangle$$

$$|b\rangle + |a\rangle = (b_0 + b_1x + b_2x^2 + b_3x^3) + (a_0 + a_1x + a_2x^2 + a_3x^3)$$

$$= ((b_0+a_0) + (b_1+a_1)x + (b_2+a_2)x^2 + (b_3+a_3)x^3) \rightarrow c_i$$

$$= c_i x^i \qquad c_i = b_i + a_i = a_i + b_i$$

b.

$$|a\rangle = a_i x^i, \; |b\rangle = b_i x^i, \; |c\rangle = c_i x^i$$

$$(|a\rangle + |b\rangle) + |c\rangle = ((a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2) + (c_0 + c_1x + c_2x^2)$$

$$= (a_0+b_0+c_0) + (a_1+b_1+c_1)x + (a_2+b_2+c_2)x^2$$

$$(|c\rangle + |a\rangle) + |b\rangle = ((c_0+a_0) + (c_1+a_1)x + (c_2+a_2)x^2) + (b_0 + b_1x + b_2x^2)$$

$$= (c_0+a_0+b_0) + (c_1+a_1+b_1)x + (c_2+a_2+b_2)x^2$$

$$d_i x^i$$

$$|a\rangle = a_i x^i, \; |b\rangle = b_i x^i, \; m_0 = m_1 = m_2 = m_i = 0$$

$$|a\rangle + |b\rangle = |a\rangle = (a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2$$

$$= (a_0+0) + (a_1+0)x + (a_2+0)x^2$$

$$= a_i x^i$$

$$|a\rangle = a_i x^i, \; |-a\rangle = -a_i x^i$$

$$|a\rangle + |-a\rangle = 0 = (a_0 + (-a_0)) + (a_1 + (-a_1))x + (a_2 + (-a_2))x^2$$

$$= (0) + (0)x + (0)x^2 = 0$$

$|a\rangle = a_i x^i$, b = esc

$$b|a\rangle = b(a_i x^i) = b(a_0) + b(a_1 x) + b(a_2 x^2)$$

$$= ba_0 + (ba_1)x + (ba_2)x^2 \qquad ba_i = j_i$$

$$= j_i x^i$$

$$b(|a\rangle c) = c(|a\rangle b)$$
$$= (ba_0 + ba_1x + ba_2x^2)c \rightarrow (ca_0 + ca_1x + ca_2x^2)b$$
$$= cba_0 + cba_1x + cba_2x^2 = (cba_0 + cba_1x + cba_2x^2)$$

$$1 \cdot |a\rangle = |a\rangle$$
$$1(a_0) + 1(a_1x) + 1(a_2x^2) \rightarrow 1(a_ix^i) = |a\rangle$$

$$c(|a\rangle + |b\rangle) = c((a_0+b_0) + (a_1+b_1)x + (a_2+b_2)x^2)$$
$$= ca_0 + cb_0 + (ca_1 + cb_1)x + (ca_2 + cb_2)x^2$$
$$= (ca_0 + ca_1x + ca_2x^2) + (cb_0 + cb_1x + cb_2x^2)$$
$$= c|a\rangle + c|b\rangle$$

$$(b+c)|a\rangle = (b+c)(a_0 + a_1x + a_2x^2)$$
$$= (b+c)a_0 + (b+c)a_1x + (b+c)a_2x^2$$
$$= ba_0 + ca_0 + ba_1x + ca_1x + ba_2x^2 + ca_2x^2$$
$$= ba_ix^i + ca_ix^i = b|a\rangle + c|a\rangle$$

b.

$|b\rangle \equiv (b^0, b)$ y $|n\rangle \equiv (n^0, n)$ $\quad q = x$

$$|b\rangle \odot |n\rangle = (b^0, b) \odot (n^0, n)$$
$$= (b^0 + bq) \odot (n^0 + nq)$$
$$= b^0n^0 + b^0(n \cdot q) + n^0(b \cdot q) + (b \cdot q)(n \cdot q)$$
$$= b^0n^0 + b^0n^jq_j + n^0b^iq_i + b^in^jq_iq_j$$
$$= b^0n^0 + b^0n^jq_j + n^0b^iq_i + n^jb^i(-\delta_{ij}1 + \epsilon_{ijk}q_k)$$
$$= b^0n^0 + b^0n^jq_j + n^0b^iq_i + n^jb^i(-\delta_{ij}1) + n^jb^i(\epsilon_{ijk}q_k)$$
$$= b^0n^0 + b^0n^jq_j + n^0b^iq_i - n \cdot b + n \times b$$
$$= b^0n^0 + b^0n + n^0b - n \cdot b + n \times b$$

c.

$$|d\rangle = |b\rangle \odot |n\rangle = b^i x^i \odot n^i x^i$$
$$= b^i n^j (x^i x^j)$$
$$= b^0 n^0 + b^0 n^j x^j + n^0 b^i x^i + n^j b^i x_i x_j$$
$$= b^0 n^0 + b^0 n^j x^j + n^0 b^i x^i + n^j b^i (-\delta_{ij} + \varepsilon_{ijk} X_k)$$

$a = b^0 n^0 - b^j n^j$; $d^i = b^0 n^i + n^0 b^i + \varepsilon^i{}_{jk} b^j n^k$

$S^{(0j)} = b^0 n^j + n^0 b^j$; $A^{[jk]i}{}_{bj\,nk} = \varepsilon^i_{jk} b^j n^k$

$|d\rangle = a + S^{(ij)} \delta^0_i x_j + A^{[jk]i}{}_{b_j n_k} x_i$

d.

$a = b^0 n^0 - b^j n_j \rightarrow$ escalar $S^{(ij)} = b^0 n^j + n^0 b^j \rightarrow$ parte simetrica

$A^{(jk)i} = \varepsilon^i_{jk} b^j n^k \rightarrow$ parte antisimetrica

$|d\rangle = |b\rangle \odot |n\rangle$ es un pseudovector por culpa de la asimetria del producto entre cuaterniones

e.

$$\alpha \sigma_1 + \beta \sigma_2 + \gamma \sigma_3 + \lambda \sigma_4 = 0$$

$$\begin{bmatrix} 0 & \alpha \\ \alpha & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\beta i \\ \beta i & 0 \end{bmatrix} + \begin{bmatrix} \gamma & 0 \\ 0 & -\gamma \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} \gamma + \lambda & \alpha - \beta i \\ \alpha + \beta i & \lambda - \gamma \end{bmatrix} = 0$$

$\gamma + \lambda = 0$ $\quad \lambda = -\gamma$ $\quad \alpha = -\beta i$ $\quad \lambda = \gamma = \alpha = \beta = 0$

$\alpha - \beta i = 0$ $\quad \lambda = \gamma$ $\quad \alpha = \beta i$

$\alpha + \beta i = 0$

$\lambda - \gamma = 0$

para la 2x2

$$|b\rangle = \begin{bmatrix} z & w \\ -w^* & z^* \end{bmatrix} = \begin{bmatrix} \gamma + \lambda & \alpha - \beta i \\ \alpha + \beta i & \lambda - \gamma \end{bmatrix}$$

$z = \gamma + \lambda \rightarrow$ $\quad z^* = \lambda - (z - \lambda)$ $\quad -w^* = \alpha + (\alpha - w)$

$w = \alpha - \beta i$ $\quad \lambda = \frac{z^* - z}{}$ $\quad \alpha = \frac{w - w^*}{2}$

$-w^* = \alpha + \beta i$

$z^* = \lambda - \gamma$ $\quad z = \gamma + (z^* + \gamma)$ $\quad w = (-w^* - \beta i) - \beta i$

$\gamma = \frac{z - z^*}{2}$ $\quad \beta = \frac{i(w + w^*)}{2}$

f. para la matriz 4x4 toca analizar la independencia lineal

$$\alpha|q_1\rangle + \beta|q_2\rangle + \gamma|q_3\rangle + \lambda|q_4\rangle = 0$$

↳ I

$$\alpha\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} + \beta\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} + \gamma\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} + \lambda\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & \alpha & -\gamma & -\beta \\ -\alpha & \lambda & -\beta & \gamma \\ \gamma & \beta & \lambda & \alpha \\ \beta & -\gamma & -\alpha & \lambda \end{bmatrix} = 0 \quad ; \quad \alpha = \beta = \gamma = \lambda = 0$$

g.

$$|a\rangle^{\dagger} = a^0|q_0\rangle - a^j|q_j\rangle \qquad j = 1,2,3$$

$$\begin{aligned} \langle a|b\rangle &= |a\rangle^{\dagger} \odot |b\rangle \\ &= (a^0 - a^i|q_i\rangle)(b^0 + b^j|q_j\rangle) \\ &= a^0b^0 + a^0b^j|q_j\rangle - b^0a^i|q_i\rangle - a^ib^j|q_i\rangle|q_j\rangle \end{aligned}$$

$$\begin{aligned} \langle b|a\rangle &= |b\rangle \odot |a\rangle^{\dagger} \\ &= (b^0 + b^j|q_j\rangle)(a^0 - a^i|q_i\rangle) \\ &= b^0a^0 - b^0a^i|q_i\rangle + a^0b^j|q_j\rangle - b^ja^i|q_i\rangle|q_j\rangle \end{aligned}$$

$$\begin{aligned} \langle a|a\rangle &= |a\rangle^{\dagger} \odot |a\rangle \\ &= (a^0 - a^i|q_i\rangle)(a^0 + a^j|q_j\rangle) \\ &= a^0a^0 + \cancel{a^0a^j|q_j\rangle} - \cancel{a^0a^i|q_i\rangle} - a^ia^j|q_i\rangle|q_j\rangle \\ &= a^0a^0 - a^ia^j(-1) \\ &= a^0a^0 + a^ia^j \end{aligned}$$

$$\langle a | \alpha b + \beta c \rangle = \alpha \langle a|b\rangle + \beta \langle a | c\rangle$$

$$= |a\rangle^{*} \odot |\alpha b + \beta c\rangle = (a^0 - a^i|q_i\rangle)(\alpha b + \beta c) = \alpha(b^0 + b^j|q_j\rangle) + \beta(c^0 + c^m|q_m\rangle)$$

$$= a^0\alpha(b^0 + b^j|q_j\rangle) + a^0\beta(c^0 + c^m|q_m\rangle) - a^i|q_i\rangle\alpha(b^0 + b^j|q_j\rangle) - a^i|q_i\rangle\beta(c^0 + c^m \ldots$$

$$= \alpha(a^0b^0 + a^0b^j|q_j\rangle) + \beta(a^0c^0 + a^0c^m|q_m\rangle) - \alpha(b^0a^i|q_i\rangle + a^ib^j|q_i\rangle|q_j\rangle) -$$
$$\beta(c^0a^i|q_i\rangle + c^ma^i|q_i\rangle|q_m\rangle)$$

$$= \alpha(a^0b^0 + a^0b^j|q_j\rangle - b^0a^i|q_i\rangle + a^ib^j|q_i\rangle|q_j\rangle) + \beta(a^0c^0 + a^0c^m|q_m\rangle - c^0a^i|q_i\rangle$$
$$- c^ma^i|q_i\rangle|q_m\rangle)$$

$$= \alpha\langle a|b\rangle + \beta\langle a|c\rangle$$

H.

$$\langle a|b\rangle = \frac{1}{2}[\langle \tilde{a}|b\rangle - |q_1\rangle \odot \langle \tilde{a}|b\rangle \odot |q_1\rangle]$$

$$= \frac{1}{2}[\,|a\rangle^{*} \odot |b\rangle - |q_1\rangle \odot |a\rangle^{*} \odot |b\rangle \odot |q_1\rangle\,]$$

$$= \frac{1}{2}[(a^0 - a^1|q_1\rangle)(b^0 + b^2|q_2\rangle) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(b^0 + b^2|q_2\rangle) \odot |q_1\rangle]$$

$$= \frac{1}{2}[a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle - |q_1\rangle(a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle)$$
$$|q_1\rangle]$$

$$= \frac{1}{2}[2a^0b^0 + 2a^0b^2|q_2\rangle - 2b^0a^1|q_1\rangle - 2a^1b_2|q_1\rangle|q_2\rangle]$$

$$= a^0b^0 + a^0b^2|q_2\rangle - b^0a^1|q_1\rangle - a^1b^2|q_1\rangle|q_2\rangle$$

$$\langle a|a\rangle = \||a\rangle\|^2$$

$$= \frac{1}{2}[\langle \tilde{a}|a\rangle - |q_1\rangle \odot \langle \tilde{a}|a\rangle \odot |q_1\rangle]$$

$$= \frac{1}{2}[(a^0 - a^1|q_1\rangle)(a^0 + a^1|q_1\rangle) - |q_1\rangle \odot (a^0 - a^1|q_1\rangle)(a^0 + a^1|q_1\rangle) \odot |q_1\rangle]$$

$$= \frac{1}{2}[a^0a^0 + a^1a^1 - |q_1\rangle \odot (a^0a^0 + a^1a^1) \odot |q_1\rangle]$$

$$= \frac{1}{2}[a^0a^0 + a^1a^1 + a^0a^0 + a^1a^1]$$

$$= \frac{1}{2}[2a^0a^0 + 2a^1a^1] = a^0a^0 + a^1a^1 = (a^0)^2 + (a^1)^2$$

i.

$n(|b\rangle) = \||a\rangle\| = \sqrt{\langle a|a\rangle} = \sqrt{|a\rangle^{+} \odot |a\rangle}$

$= \sqrt{a^0a^0 + a^ia^i}$ $\qquad \alpha\||a\rangle\| = \sqrt{\alpha(a^0)^2 + \alpha(a^i)^2}$

$= \sqrt{(a^0)^2 + (a^i)^2} \geq 0$ $\qquad = |\alpha|\|a\|$

j.

$\overline{|a\rangle} = \dfrac{|a\rangle^{+}}{\||a\rangle\|^2}$ $\qquad |a\rangle \odot \overline{|a\rangle} = I$

$(a^0 + a^i|q_i\rangle)\left(\dfrac{(a^0 - a^j|q_j\rangle)}{(a^0)^2 + (a^i)^2}\right)$

$= \dfrac{(a^0 - a^j|q_j\rangle)}{(a^0)^2 + (a^i)^2}$

$= \dfrac{a^0a^0 - a^0a^j|q_j\rangle + a^0a^i|q_i\rangle - a^ia^j|q_i\rangle|q_j\rangle}{(a^0)^2 + (a^i)^2}$

$= \dfrac{(a^0)^2}{(a^0)^2 + (a^i)^2} - \cancel{\dfrac{a^0a^j|q_j\rangle}{(a^0)^2 + (a^i)^2}} + \cancel{\dfrac{a^0a^i|q_i\rangle}{(a^0)^2 + (a^i)^2}} - \dfrac{a^ia^j|q_i\rangle|q_j\rangle}{(a^0)^2 + (a^i)^2}$

$= \dfrac{(a^0)^2}{(a^0)^2 + (a^i)^2} + \dfrac{(a^i)^2}{(a^0)^2 + (a^i)^2} = \dfrac{(a^0)^2 + (a^i)^2}{(a^0)^2 + (a^i)^2} = I$

K.

$|a\rangle = a_0 + a \;,\; |b\rangle = b_0 + b$

$|a\rangle \odot |b\rangle = (a_0b_0 - a \cdot b,\; a^0b + b^0a + a \times b) = (c_0, c) = |c\rangle \in H$

$|a\rangle = \underbrace{\begin{bmatrix} a_0 + a_1 i & a_2 + a_3 i \\ -a_2 + a_3 i & a_0 - a_1 i \end{bmatrix}}_{m_1}$ $\quad |b\rangle = \underbrace{\begin{bmatrix} b_0 + b_1 i & b_2 + b_3 i \\ -b_2 + b_3 i & b_2 - b_1 i \end{bmatrix}}_{m_2}$

$|d\rangle = \underbrace{\begin{bmatrix} d_0 + d_1 i & d_2 + d_3 i \\ -d_2 + d_3 i & d_0 - d_1 i \end{bmatrix}}_{m_3}$

$(|a\rangle \odot (|b\rangle \odot |d\rangle)) = (|d\rangle \odot (|a\rangle \odot |b\rangle))$

$M_1(M_2M_3) = M_3(M_1M_2)$

I.

$$|v'\rangle = |\bar{a}\rangle \odot |v\rangle \odot |q\rangle$$

$$= \frac{|a\rangle^{+}}{\||a\rangle\|^{2}} \odot |v\rangle \odot |a\rangle$$

$$= \left(\frac{(a^0 - a^i|q_i\rangle)}{(a^0)^2 + (a^i)^2}\right)(v^j|q_j\rangle) \odot |q\rangle$$

$$= \left(\frac{(a^0 v^j|q_j\rangle - a^i v^j|q_i\rangle|q_j\rangle}{(a^0)^2 + (a^i)^2}\right)(a^0 + a^k|q_k\rangle)$$

$$= \frac{(a^0)^2 v^j|q_j\rangle}{(a^0)^2 + (a^i)^2} + \frac{(a^i)^2 v^j|q_j\rangle}{(a^0)^2 + (a^i)^2}$$

$$= \frac{((a^0)^2 + (a^i)^2)}{(a^0)^2 + (a^i)^2}(v^j|q_j\rangle) = v^j|q_j\rangle$$

$$\|v'\|^2 = ((v^1)^2 + (v^2)^2 + (v^3)^2) \equiv ((v^1)^2 + (v^2)^2 + (v^3)^2) = \|v\|^2$$