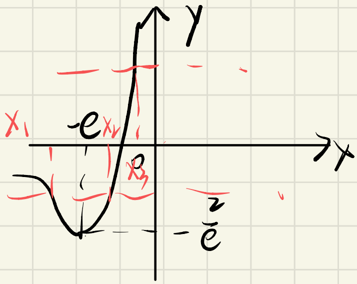
数形结合专题



令 $f(x)=t$

$g(t)+\frac{1}{m}=0$ 有三个解

$$\frac{3t^2+mt-2m^2}{3mt^2}=0$$

$$\Delta=m^2-4\times3\times(-2m^2)=25m^2$$

$t_1=\frac{2}{3}m$

$t_2=-m$

① 若 $m<0$ 　 $\triangle$ $\frac{4}{3}m-2m=-\frac{2}{3}m$

$-\frac{2}{e}<\frac{2}{3}m<0$

$0<-\frac{2}{3}m<\frac{2}{e}$

C

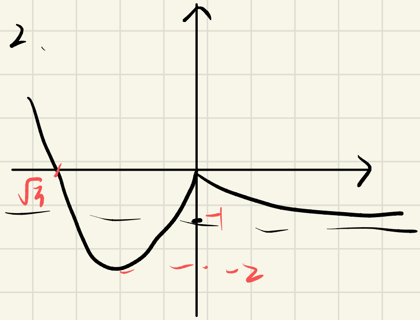
② 若 $m>0$ 　 $-\frac{2}{3}m$

$-\frac{2}{e}<-m<0$

$-\frac{4}{3e}<-\frac{2}{3}m<0$

D

2.



令 $f(x)=t$.

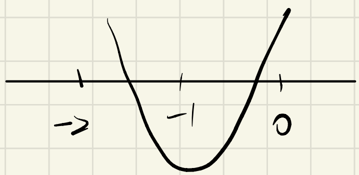
$\Delta>0$.

$\Delta = 16a^2-4\times4\times(2a+3)$

$= 16(a+1)(a-3)>0$

$a<-1$ 或 $a>3$. (舍去)

$t_1+t_2=a$.　　$(-1,0)$　　$[-2,1]$ 或 0



$$\begin{cases} g(0)\geq 0 \\ g(-2)\geq 0 \\ g(-1)\leq 0 \end{cases}$$

$f(x)=0$.

$2a+3\geq0$

$a\geq-\frac{3}{2}$

$a\geq-\frac{19}{10}$

$a\leq-\frac{7}{6}$

$-\frac{3}{2}\leq a\leq-\frac{7}{6}$

$a=-\frac{3}{2}$　　$4t^2+6t=0$.

$t_1=0$　　$t_2=-\frac{3}{2}$ (舍去)

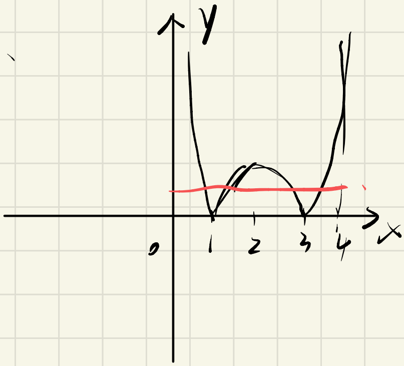
$a=-\frac{7}{6}$　　$4t^2+\frac{14}{3}t+\frac{2}{3}=0$

$$(6t+1)(t+1)=0$$

$$t_1=-\frac{1}{6}$$

$$t_2=-1$$

3、



$$x_1x_2=1$$

$$m=-\ln x_1=\ln x_2=\ln(4-x_3)=-\ln(4-x_4)$$

$$\ln(4-x_3)x_1=0$$

$$(4-x_3)x_1=1$$

$$x_3=4-\frac{1}{x_1}$$

$$x_4=4-\frac{1}{x_2}$$

$$kx_3x_4=k\left(4-\frac{1}{x_1}\right)\left(4-\frac{1}{x_2}\right)$$

$$\frac{x_1+x_2}{x_1x_2}$$

$$=k\left[16-4\left(\frac{1}{x_1}+\frac{1}{x_2}\right)+\frac{1}{x_1x_2}\right]$$

$$=k[16-4(x_1+x_2)+1]$$

$$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=(x_1+x_2)^2-2$$

$$k(16-4(x_1+x_2)+1)+(x_1+x_2)^2-2 \geq k+11$$

$$(x_1+x_2)^2-4k(x_1+x_2)+16k-13 \geq 0$$

$x_1x_2=1$

令 $x_1+x_2=t$. $\begin{cases} x_1+\frac{1}{x_1} \\ x_2+\frac{1}{x_2} \end{cases}$ $x_2\in(1,2)$

$2<t<\frac{5}{2}$

$t^2-4kt+16k-13 \geq 0$. $2<t<\frac{5}{2}$

① 两侧 $\begin{cases} f(2)\geq 0 \\ f(\frac{5}{2})\geq 0 \end{cases} \Rightarrow k \geq \frac{9}{8}$

$\begin{cases} ~~2k\leq 2~~ \\ 2k \geq \frac{5}{2} \end{cases}$

$\frac{5}{4}$

② $\Delta \leq 0$

$k=2\pm\frac{\sqrt{3}}{2}$

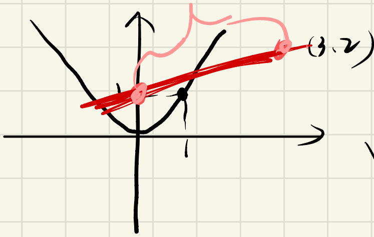
5. $f(x)=\sqrt{x^4-3x^2-6x+13}-\sqrt{x^4-x^2+1}$

$=\sqrt{(x^2-2)^2+(x-3)^2}-\sqrt{(x^2-1)^2+x^2}$

$y=x^2$ (x, x^2)

$(3,2)$

$(0,1)$



$\sqrt{(3-0)^2+(2-1)^2}$

$=\sqrt{10}$

代数 / 几何

$z = x + yi$　　(x, y)

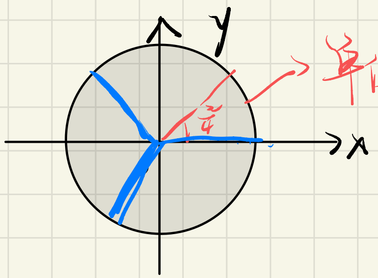
三角：$z = r(\cos\theta + i\sin\theta)$

指数　$z = re^{i\theta}$

$z = e^{i\frac{\pi}{4}}$

$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$



$w^3 = 1$　　2π

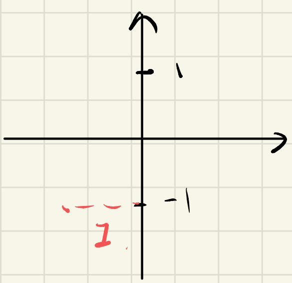
w_1

w_2

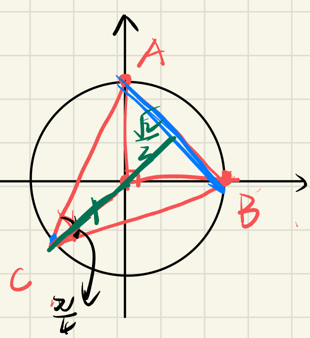
$z = (x, y)$　　$(0, -1)$

1. $|z+i|$

$(-1, -1)$



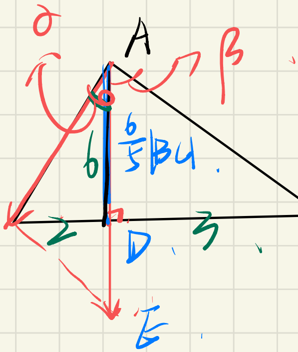
5.



$CA \cdot CB$

$\frac{1}{2} |CA|\,|CB| \cdot \sin\frac{\pi}{4} = \frac{1}{2}|AB|\,h$

2.



$(3\vec{AB}+2\vec{AC})\cdot\vec{BC}=0$

$\vec{AB}+\frac{2}{3}\vec{AC}$

$|\vec{BA}-t\vec{BC}|=|\vec{AB}+t\vec{BC}|$

$\vec{AD}=\lambda\vec{AE}$

$=\lambda(3\vec{AB}+2\vec{AC})$

$\tan\alpha=\frac{2}{6}=\frac{1}{3}$

$\tan\beta=\frac{3}{6}=\frac{1}{2}$

$\tan\angle BAC=\tan(\alpha+\beta)$

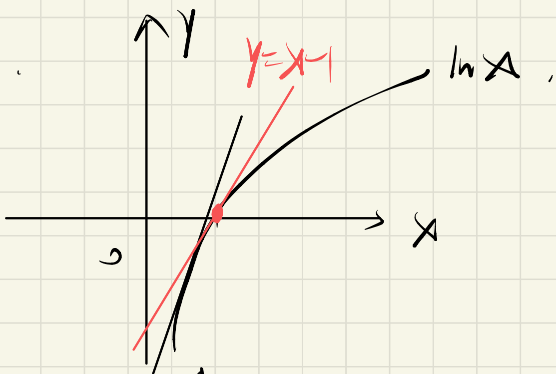
$3\lambda+2\lambda=1$ $\quad\lambda=\frac{1}{5}$

$\vec{AD}=\frac{3}{5}\vec{AB}+\frac{2}{5}\vec{AC}$

$\frac{3}{5}\vec{AD}+\frac{2}{5}\vec{AC}$

$\frac{|CD|}{|DB|}=\frac{3}{2}$

6.



$kx+b=0$

$x=-\frac{b}{k}$

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