Stackelberg Leader Strategy

--Group 27

Project Overview

What we have done in this project:

- **1. Linear model:**
- Linear regression model with imperfect information
- Updating of linear reaction function
  - Moving window approach
  - Weighted least square with forgetting factor
- **2. Non-linear model:**
- Support Vector Machine
- Long Short-Term Memory

0. Stackelberg problem with imperfect information

- Estimate the follower’s reaction function

- Find the best leader strategy base on the estimated function

Part 1: Linear Model

1.1 Linear regression model

Estimate followers's reaction function $R(u_L)$

- Design $R(u_L)$ as a linear function :

$$R'(u_L) = a' + b'u_L$$

- Calculated the parameters a' and b' using historical data and used derivative to confirm it is the minimum point.

1.1 Maximisation by differentiation

Find the best leader strategy by maximiuming the function:

$$J_L[u_L, R(u_L)] = (u_L - c_L)\, S_L(u_L, u_F)$$

where

$$S_L(u_L, u_F) = 2 - u_L + 0.3R'(u_L)$$

$$R'(u_L) = a' + b'u_L$$

which can be solving by using first order derivative:

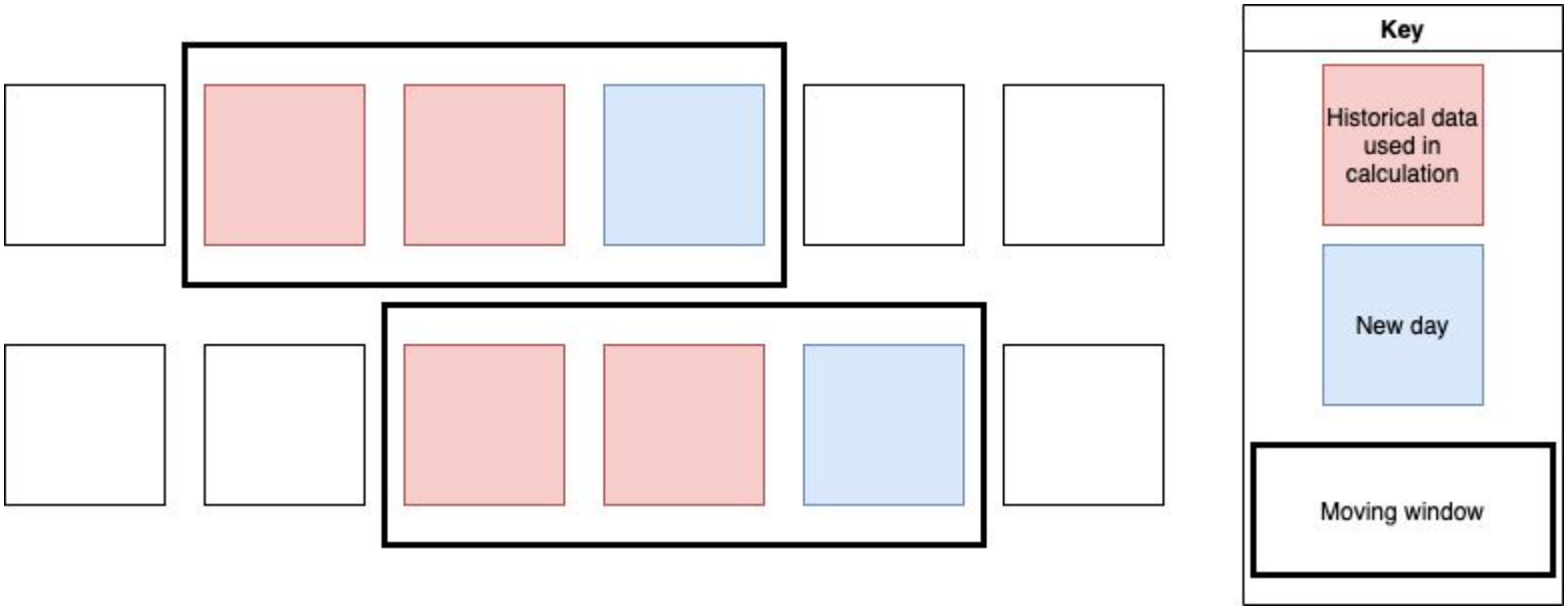
$$u_L = (0.3b' - 0.3a' - 3) / (0.6b' - 2)$$

1.2 Updating linear regression model

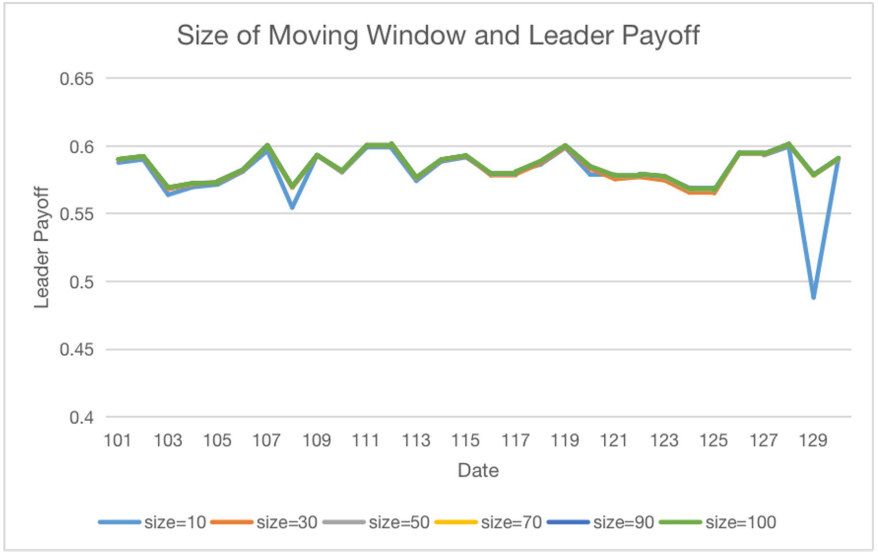
As the game procresses, new data became available.

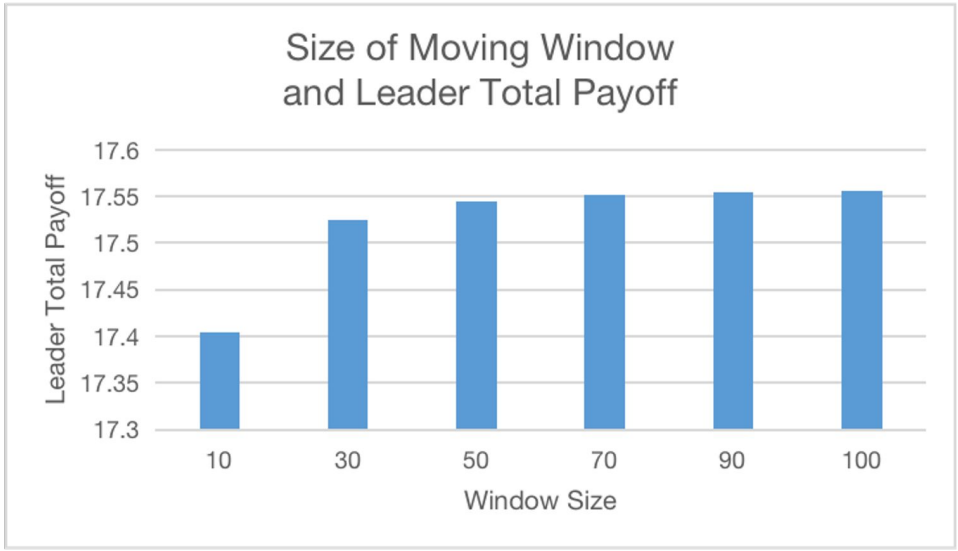
To integrate the data into the reaction function calculation, the linear regression method needs to be updated.

1.2.1 Moving window approach



1.2.1 Moving window approach result





1.2.2 Weighted least square with forgetting factor

- A Forgetting Factor is introduced to discount old data.
- $\hat{R}(X, \theta_T^*)$ is the weighted least square estimator based on:

$$\theta_T^* = \arg\ \min_{\theta} \sum_{t=1}^{T} \lambda^{T-t} \left\{ y(t) - \hat{R}[X(t), \theta] \right\}^2$$

- Lambda is the forgetting factor, normally choosing between 0.95 and 0.99. Changing lambda doesn't make much difference.
- So the more recent data will be fitted better than the older data; otherwise they will lead more errors than old data.

Part 2: Non-Linear Model

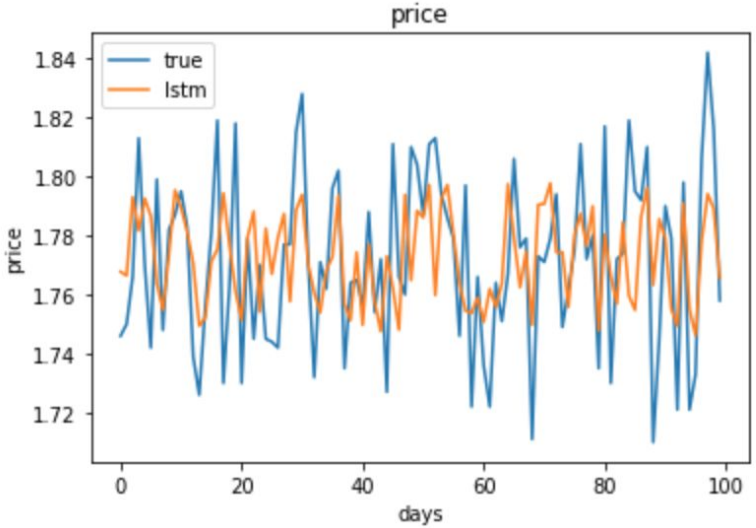
2.1 Support Vector Machine (SVM)

- Support vector machines (SVMs) are a set of supervised learning methods used for classification, regression and outliers detection.
- kernel= ['linear','poly','rbf','sigmoid']
  - For example (mk1):
  - Best leader price for linearkernel is:**1.79** and prediected profit is: **0.5852510500281196**
  - Best leader price for polykernel is:**1.77** and prediected profit is: **0.5871807026978179**
  - Best leader price for rbfkernel is:**1.81** and prediected profit is: **0.5868159530304073**
  - Best leader price for sigmoidkernel is:**1.55** and prediected profit is: **9.298510378119301**

- To conclude: 'rbf' kernel performs the best.

2.2 Long Short-Term Memory (LSTM)

- LSTM networks are well-suited to classifying, processing and making predictions based on time series data
- try different training simulations
- maximser: simple maxiser right now, considering analytical in the future



3 Result Evaluation

Agent/Total Profit	Linear(moving window)	Linear(forgetting factor)	SVM	LSTM
MK1	17.55217171	17.58903658	16.96745345	16.96745345
MK2	16.953722	16.83769237	15.98498108	16.04428459
MK3	19.48818398	19.50389562	18.05962759	18.78504287

Table 1: Evaluation

3 Conclusions

- We have implemented a few methods to solve Stackelberg pricing game problems, including linear models and non-linear models.
- We learned the concept of off-line learning and on-line updating. Moving Window Approach and Weighted Least Square with Forgetting Factor is used for on-line updating.
- Although changing the window size and forgetting factor does not significantly affect the leader's payoff for provided data in the linear approach, it will be extremely useful if the follower's strategy or environment changes.

3 Conclusions

- We have implemented a few methods to solve Stackelberg pricing game problems, including linear models and non-linear models.
- As linear models are quite limited, we also dig into some non-linear models, Support Vector Machine and Long Short-Term Memory to do regression.
- The simulations fit the original curve better, but the predicted profit doesn't have significant differences with the linear ones.