Notational conventions

We use the following conventions for naming and denoting variables, parameters and function:

Notation	Description	Meaning
x_t	Lowercase latin	Model variables
$\hat{x}_t$	Dot	Gross rate of change, $\dot{x}_t = x_t / x_{t-1}$
x_{ss}	An ss subscript	Steady state of a model variable
x_t^a	Lowercase latin	Model variables with an explicit area reference
$\mathbf{x_t}$	Lowercase bold upright	Model variables externalized in the respective optimization problem
$\mathbf{log}$	Bold	Functions and function components
$\mathbf{E}_t$		Conditional model-consistent time-t expectations
α	Lowercase Greek	Parameters
Π_t	Uppercase Greek	Some of model nominal flows

We denote by A the set of all areas included in the model. Currently, $A = \{\mathrm{us}, \mathrm{ea}, \mathrm{ch}, \mathrm{rw}\}$. An additional code, GG, is used to index global common trends, such as the global productivity trend or the global population trend.

Area code	Description
us	United States (global reference area)
ea	Euro Area
ch	China
rw	Rest of world
gg	Common global trends

In most of the text, we do not explicitly include thee reference area in the names of variables for the ease of notation. Absent an explicit area reference, the variable or parameter belongs simply to the respective local area.

In several places, we use the concept of a so-called global reference area (GRA); for instance, the local nominal exchange rates are defined as the rates between the respective local area's currency and the GRA's currency. The convention is that the global reference area is always ordered first in the list of areas. In the baseline setup of the model, the United States is used as the GRA.

Households

Each area's household sector is modeled as a single representative household with an exogenous time-varying number of household members, nn_t. The household enteres a net position in debt instruments (e.g. loans, deposits, fixed-income securities, etc.) with the local financial sector, bh_t, and holds a portfolio of claims on production capital in all areas (including the local area), $\sum_a s_{a,t} \; ex_{a,t} \; pk_t^a \; k_t^a$; the latter is our way to mimic corporate equity holdings with cross-border exposures. During each period, the household purchases consumption goods, ch_t, supplies per-worker hours worked, nh_t, rents production capital, k_t^a, out to producers in the respective area, chooses the utilization rate of local production capital, u_t, invests in creating addition local capital, i_t, pays lump-sum taxes (or receives lump-sum transfers) of two types, $txls1_t$ and $txls2_t$, and collects period profits from local producers, local exporters, and the local financial sectors (of whom all the household is the ultimate owner).

The household chooses the following quantities

- total consumption, ch_t,
- per-capita hours worked, nh_t,
- shares of claims on production capital possibly from all areas, $s_{a,t} \in [0,\, 1]$, $a \in A$,
- the utilization rate of local production capital, u_t,
- investment in local production capital, i_t,
- net financial position with the local financial sector, bh_t,

to maximize its infinite lifetime utility function subject to a dynamic budget constaint. The household derives utility from consumption, disutility from work, and utility from its wealth (net worth).

Household preferences

The household preferences are described by a time-separable utility function over an infinite life horizon, $t = 0, \ldots, \infty$. The period utility function consists of a consumption utility component, $\mathbf{U}_t^{ch}$, a work disutility component, $\mathbf{U}_t^{nh}$, and a current wealth (net worth) utility component, $\mathbf{U}_t^{netw}$. The individual utility function components are each evaluated on a per-capita basis, and the overall period utility is multiplied by the total number of household members

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\mathbf{U}_t^{ch} - \mathbf{U}_t^{nh} + \mathbf{U}_t^{netw} \right) nn_t \tag{1}$$

The respective components of the utility function related to consumption, work and wealth, respectively, are given as follows

$$\mathbf{U}_t^{ch} \equiv \kappa_{ch} \, \mathbf{log} \, \frac{ch_t - \mathbf{ch}_t^{\mathrm{ref}}}{nn_t} \tag{2}$$

$$\mathbf{U}_t^{nh} \equiv \frac{1}{1+\eta} \, {nh_t}^{1+\eta} \tag{3}$$

$$\mathbf{U}_t^{netw} \equiv \nu_1 \left(\mathbf{log} \, \frac{netw_t}{pc_t \, \mathbf{ch}_t} - \nu_0 \, \frac{netw_t}{pc_t \, \mathbf{ch}_t} \right) \tag{4}$$

where

- ch_t^{ref} is the reference point in household consumption proportional to the level of real current labor income net of type 1 lump-sum taxes (or transfers) and externalized from the household optimization

$$ch_t^{\mathrm{ref}} \equiv \chi \, \frac{curr_t}{pc_t} \tag{5}$$

- $\kappa_{ch} \equiv 1 - ch_{\mathrm{ss}}^{\mathrm{ref}} \, {ch_{\mathrm{ss}}}^{-1}$ is a steady-state correction constant ensuring that the marginal utility of consumption equals $nn_t \, {ch_{\mathrm{ss}}}^{-1}$ in steady state, a feature of modeling convenience,
- $curr_t$ is current labor income net of type 1 lump sum taxes (or transfers)

$$curr_t \equiv w_t \, nh_t \, nl_t - txls1_t \tag{6}$$

- $netw_t$ is the nominal net worth given by the sum of the value of the production capital portfolio, the net financial position of the household to the local financial sector, bh_t (a positive balance means net claims of the financial sector on the household), and the net worth of the local financial sector (whose ultimate owner the household is), bb_t,

$$netw_t \equiv \textstyle\sum_a s_{a,t} \, ex_{a,t} \, pk_{a,t} \, k_{a,t} - bh_t + bb_t \tag{7}$$

- $ex_{a,t}$ is the cross rate between local currency and area a's currency (with movements up meaning depreciation of local currency)

$$ex_{a,t} = \frac{e_{\text{local},t}}{e_{a,t}}, \quad ex_{\text{local},t} = 1 \tag{8}$$

Dynamic budget constraint

The dynamic budget constraint facing the household sector describes a stock-flow relationship between the household assets and liabilities (stocks) on the one hand, and current receipts and current outlays (flows) on the other hand. The household assets and liabilities consist of

- a net position with the local financial sector, $-bh_t$ (a positive balance means net lending by the household from the financial sector, a negative balance means net lending by the financial sector from the household),
- claims on production capital (local and ccross-border capital), $\sum_a s_{a,t}\ ex_{a,t}\ pk_t^a\ k_t^a$, and

The change in the household assets and liabilities is equal to the revaluation of capital claims, and the total amount of current receipts and outlays:

- revaluation of claims on production capital (both from the nominal exchange rate and the capital price), $\sum_a s_{a,t-1}\ \mathbf{\Delta}(ex_{a,t}\ pk_t^a)\ \ k_{t-1}^a$,
- interest receipts or outlays on the net position with the local financial sector, $(rh_{t-1} - 1)\, bh_{t-1}$
- current receipts from capital rentals net of capital utilization costs, $\sum_a s_{a,t}\ ex_{a,t}\ pu_t^a\ k_t^a - \Xi_{u,t}$,
- current receipts from labor income, $w_t\ nh_t\ nl_t$,
- current receipts from selling newly installed capital, $pk_t\ i_t$,
- profits from loccal producers, $\Pi_{y,t}$, exporters, $\Pi_{x,t}$, and the financial sector, $\Pi_{b,t}$,
- current outlays on consumption goods, $-pc_t\ ch_t$,
- current outlays on investment goods, $-pi_t\ i_t$,

$$\sum_a s_{a,t}\ ex_{a,t}\, pk_t^a\ k_t^a - bh_t\ \cdots$$

$$= \ \sum_a s_{a,t-1}\ ex_{a,t} \left[pu_t^a\ u_t^a + (1-\delta^a)\, pk_t^a \right] k_{t-1}^a - rh_{t-1} bh_{t-1}\ \cdots$$

$$+ \, w_t\ nh_t\ nl_t - pc_t\ ch_t + (pk_t - pi_t)\, i_t - txls1_t - txls2_t\ \cdots$$

$$+ \, \mathbf{\Pi}_{y,t} + \mathbf{\Pi}_{x,t} + \mathbf{\Pi}_{b,t} - \Xi_{i,t} - \Xi_{k,t} - \Xi_{u,t} + \mathbf{\Xi}_{h,t}$$

Lagrange multiplier associated with the budget constraint is denoted by vh_t (shadow value of nominal household wealth)

Real wage rigidities

The labor market exhibits real wage rigidities. These rigidities do not derive from explicit microfoundations in our model; they are introduced as an ad-hoc correction to the law of motion for the real wage rate in the following way. The household makes its choices as though the wage rate was fully flexible; we denote this hypothetical level of the nominal wage rate by ww_t, and use this hypothetical wage in the household Lagrangian, in place of the actual wage rate. Once the hypothetical flexible optimum wage rate is determined, the actual wage rate follows an autoregressive process with asymptotic convergence to the flexible optimum

$$\mathbf{log}\,\frac{w_t}{pc_t} = \rho_w\ \mathbf{log}\,\frac{\kappa_w\ w_{t-1}}{pc_{t-1}} + (1-\rho_w)\ \mathbf{log}\,\frac{ww_t}{pc_t} + \epsilon_{w,t} \tag{9}$$

where the past real wage is indexed by a steady-state adjustment constant, κ_w, given by the gross rate of change in the steady-state real wage rate

$$\kappa_w \equiv \hat{w}_{\mathrm{ss}}\ \hat{pc}_{\mathrm{ss}}{}^{-1} \tag{10}$$

and $\rho_w \in [0,\ 1)$ is an autoregression parameter.

Costs of short-term adjustment processes

The optimizing behavior of the representative household is subjected to two types of costly short-term adjustment processes:

- an investment adjustment/installation cost
- a capital utilization cost.

The investment adjustment/installation cost comprises two components: departures from a steady-state investment-to-capital ratio, and departures from a steady-state rate of change in investment

$$\Xi_{i,t} \equiv \tfrac{1}{2}\, \xi_{i1}\, pi_t\, \mathbf{i}_t \left(\mathbf{log}\, i_t - \mathbf{log}\, \mathbf{i}_t^{\mathrm{ref}} \right)^2 + \tfrac{1}{2}\, \xi_{i2}\, pi_t\, \mathbf{i}_t \left(\boldsymbol{\Delta}\mathbf{log}\, i_t - \mathbf{log}\, \kappa_i \right)^2 \tag{11}$$

where i_t^{ref} is a point of reference derived from the steady-state investment-to-capital ratio applied to the stock of capital last period,

$$i_t^{\mathrm{ref}} \equiv \frac{i_{\mathrm{ss}}}{k_{\mathrm{ss}}}\, k_{t-1}\, \hat{\imath}_{\mathrm{ss}} \tag{12}$$

and $\kappa_i \equiv \hat{\imath}_{\mathrm{ss}}$ is a steady-state adjustment constant ensuring that the cost term disappears in steady-state.

The cost of capital utilization give rise to a cyclical response in the rate of utilization of the existing stock of capital. The cost function is given by

$$\Xi_{u,t} \equiv s_{\mathrm{local},t}\, py_t\, k_t\, \frac{\upsilon_0}{1+\upsilon_1}\, {u_t}^{1+\upsilon_1} \tag{13}$$

Capital accumulation

The household purchases investment goods, converts them to newly installed production capital (paying the adjustment/installation cost in the process) and adds these to the existing stock of capital

$$k_t = (1-\delta)\, k_{t-1} + i_t \tag{14}$$

Lagragian for the household optimization problem

The Lagrangian for the contrained optimization problem facing the representative household consists of the lifetime utility function and a sequence of dynamic budget constraints for each time from now until infinity, $t = 0, \ldots, \infty$. Note that we use ww_t in place of w_t in the Lagrangian.

$$
\begin{multline}
\mathbf{max}_{\{ch_t, bh_t, s_{a,t}, i_t, nh_t, u_t\}} \\
\sum_t \beta^t \left[\kappa_{ch} \,\mathbf{log}\, \frac{ch_t - \mathbf{ch}_t^{\mathrm{ref}}}{nn_t} + \frac{1}{1+\eta}\, {nh_t}^{1+\eta} + \nu_1 \left(\mathbf{log}\, \frac{netw_t}{pc_t\, \mathbf{ch}_t} - \nu_0 \frac{netw_t}{pc_t\, \mathbf{ch}_t} \right) \right] nn_t \;\cdots \\
+ \; \sum_t \beta^t vh_t \left\{ - \sum_a s_{a,t}\, ex_{a,t}\, pk_t^a\, k_t^a + bh_t \;\cdots \right. \\
+ \; \sum_a s_{a,t-1}\, ex_{a,t} \left[pu_t^a\, u_t^a + (1 - \delta^a)\, pk_t^a \right] k_{t-1}^a - rh_{t-1} bh_{t-1} \;\cdots \\
+ \; ww_t\, nh_t\, nl_t - pc_t\, ch_t + (pk_t - pi_t)\, i_t - txls1_t - txls2_t \;\cdots \\
\left. + \mathbf{\Pi}_{y,t} + \mathbf{\Pi}_{x,t} + \mathbf{\Pi}_{b,t} - \; \Xi_{i,t} - \Xi_{k,t} - \Xi_{u,t} + \mathbf{\Xi}_{h,t} \right\} \tag{15}
\end{multline}
$$

where vh_t is the Lagrange multiplier on time-t budget constraint.

Optimality conditions

The optimal (utility maximizing) choices of the representative household are described by the following first-order conditions.

- Consumption, ch_t

$$vh_t \, ph_t = \kappa_{ch} \, \frac{1}{ch_t - \mathbf{ch}_t^{\mathrm{ref}}} \, nn_t \tag{16}$$

- Per-worker hours worked depending on the hypothetical flexible wage rate, ww_t

$$vh_t \, ww_t = {nh_t}^{\eta} \tag{17}$$

- Net position with the financial sector, bh_t (an intertemporal no-arbitrage condition)

$$vh_t = \beta \, vh_{t+1} \, rh_t + \nu_1 \, \frac{1}{pc_t \, \mathbf{ch}_t} \left(\frac{pc_t \, \mathbf{ch}_t}{netw_t} - \nu_0 \right) \tag{18}$$

- Utilization rate of production capital, u_t

$$\upsilon_0 \, {u_t}^{\upsilon_1} \, py_t = pu_t \tag{19}$$

- Investment in local production capital, i_t

$$pk_t = pi_t \left[1 + \xi_{i1} \left(\mathbf{log} \, i_t - \mathbf{log} \, \mathbf{i}_t^{\mathrm{ref}} \right) + \xi_{i2} \left(\mathbf{\Delta log} \, i_t - \kappa_i \right) - \xi_{i2} \, \beta \left(\mathbf{\Delta log} \, i_{t+1} - \kappa_i \right) \right] \tag{20}$$

- Claims on area a's production capital, $s_{a,t}$

$$vh_t \, pk_t^a = vh_t \, pu_t^a \, u_t^a + \beta \, vh_{t+1} \, (1 - \delta^a) \, pk_{t+1}, \quad \forall a \in A \tag{21}$$

The last set of equations defines arbitrage-free conditions (AFCs) for a corporate equity portfolio choice. We need to further address the following two characteristics of these NACs:

1. As is common in macro models, the AFCs themselves do not determine the actual portfolio shares, $s_{a,t}$, only the relationship between the price of production capital, the cash flows it generates, and the household discount factor. The actual shares are then calibrated and kept fixed in the baseline version of the model.
2. Since we allow for cross-border holdings, each area's capital is subject to muliple AFCs, each relating to the household residing in a different area and exhibiting, in general, different preferences. We therefore create aggregate AFCs by taking the weighted average with the weights equal the portfolio shares. The aggregate AFCs for the capital markets are described in the Global equilibrium section.

Glossary of model quantities

Glossary of variables

Variable	Source code name	Description
bh_t	`xxx`	Net claims of the financial sector on the household
ch_t	`xxx`	Household consumption
ch_t^{ref}	`xxx`	Point of reference in household consumption
$curr_t$	`xxx`	Nominal current income of households
i_t	`xxx`	Investment in private production capital
k_t	`xxx`	Private production capital
$netw_t$	`xxx`	Nominal net worth of households
nh_t	`xxx`	Per-worker labor supply (e.g. per-worker hours worked)
nl_t	`xxx`	Labor force
pc_t	`xxx`	Price of consumption goods
pi_t	`xxx`	Price of investment goods
pu_t	`xxx`	Rental price (user cost) of capital services
u_t	`u`	Rate of production capital utilization
$s_{a,t}$	`xxx`	Share of claims on private production capital in area a
$txls1_t$	`xxx`	Type 1 net lump-sum taxes+/transfers–
$txls2_t$	`xxx`	Type 2 net lump-sum taxes+/transfers–
w_t	`xxx`	Nominal wage rate
ww_t	`xxx`	Hypothetical nominal wage rate absent labor market ridigities
e_t	`e`	Nominal exchange rate against the global reference area's currency
$ex_{a,t}$	`xxx`	Nominal cross rate between local currency and area a's currency
rh_t	`rh`	Gross rate of interest on financial claims on the household sector
rg_t	`rg`	Gross rate of interest on financial claims on the government
$\Pi_{y,t}$	`xxx`	Profits from local producers

$\Pi_{x,t}$	`xxx`	Profits from local exporters
$\Pi_{b,t}$	`xxx`	Profits from the local financial sector
$\Xi_{i,t}$	`xxx`	Investment adjustment cost
$\Xi_{k,t}$	`xxx`	Cost of deviations from capital reference point
$\Xi_{u,t}$	`xxx`	Capital utilization cost
$\Xi_{h,t}$	`xxx`	All private costs paid to the household

Glossary of steady-state parameters

Parameter	Source code name	Description
β	`beta`	Household discount factor
δ	`delta`	Depreciation rate of production capital
ν_0	`nu_0`	Level parameter in utility from net worth
ν_1	`nu_1`	Elasticity parameter in utility from net worth
υ_0	`upsilon_0`	Level parameter in capital utilization cost function
υ_1	`upsilon_1`	Elasticity parameter in capital utilization cost function

Glossary of transitory parameters

Parameter	Source code name	Description
χ	`chi`	Parameter in point of reference in consumption
ρ_w	`rho_w`	Autoregression in real wage