Small open economy basics

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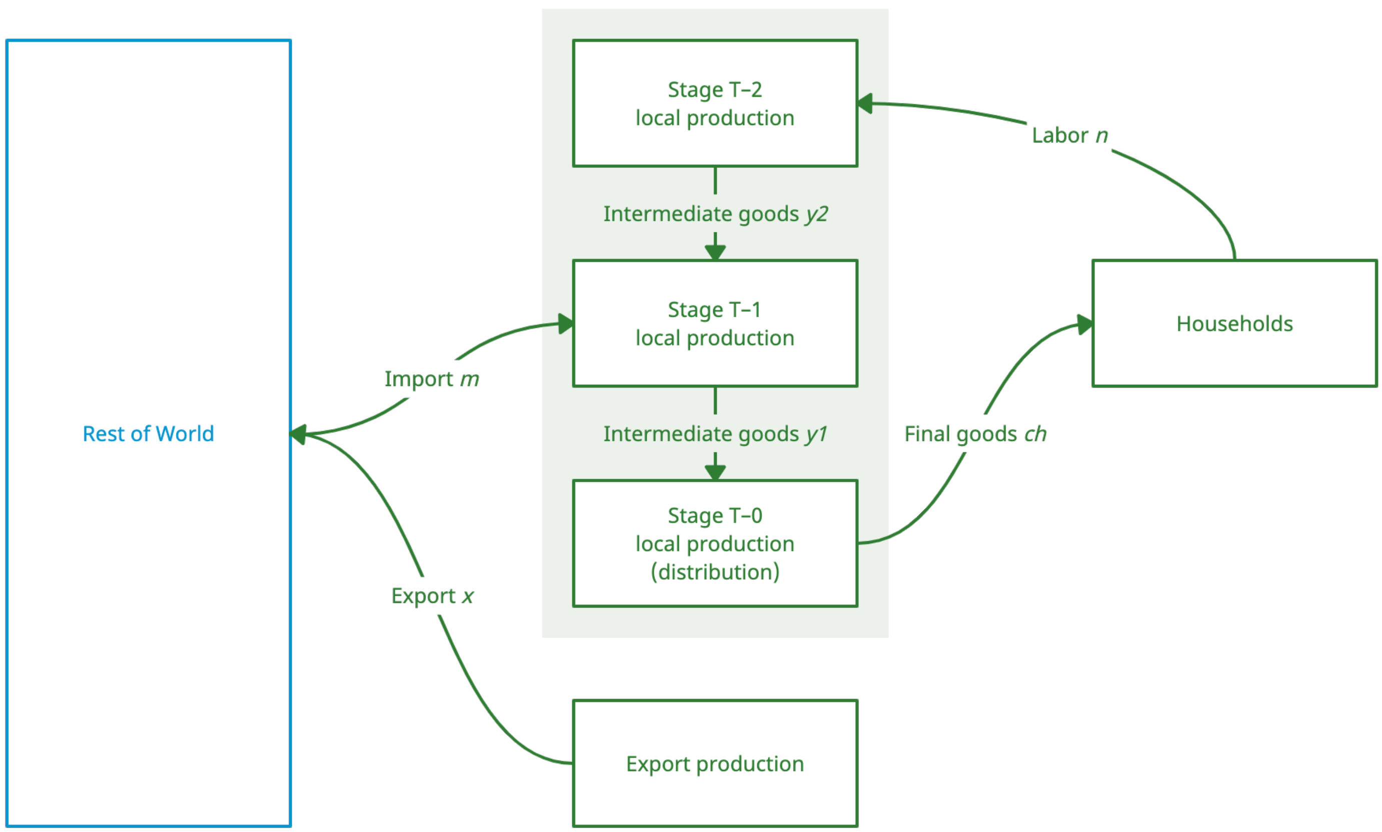
Model building topics

- Top-down design
- Well-behaved steady state, comparative static analysis
- Determination of interest rates and foreign positions in SOE models
- Stationary model $\longrightarrow$ balanced growth model
- IrisT model source file

Flow of goods and input factors



Households

- Representative, infitely lived household
- Choose consumption, hours worked, net financial assets (LCY, FCY)
- Maximize lifetime utility function subject to budget constraint

Utility function

Choose

- consumptions, ch_t
- hours worked, n_t
- net position in home currency, bh_t^{lcy}
- net position in foreign currency, bh_t^{fcy}

to maximize the expected lifetime utility function

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log\!\left(ch_t - ch_t^{\mathrm{ref}} \right) - \eta_0 \, \tfrac{1}{1+\eta} \, {n_t}^{\eta} + \beta_0 \, \frac{bh_t}{pch_t \; ch_t} \right] \right\} \tag{2}$$

with the reference level of consumption given by (external habit)

$$ch_t^{\mathrm{ref}} = \chi \, \overline{ch}_{t-1} \; \exp \varepsilon_{ch,t} \tag{3}$$

subject to

- a sequence of dynamic budget constraints
- labor market (real wage) rigidities

Budget constraint

A sequence of constraints $t = 0,\ 1,\ 2, \dots$

$$
\begin{aligned}
& bh_t^{\text{lcy}} + bh_t^{\text{fcy}} \\
& = rh_{t-1}^{\text{lcy}}\ bh_{t-1}^{\text{lcy}} + rh_{t-1}^{\text{fcy}}\ \frac{e_t}{e_{t-1}}\ bh_{t-1}^{\text{fcy}} + \mathit{off}_t \\
& + w_t\ n_t + \Pi_{y,t} + \Pi_{x,t} \\
& -\ pc_t\ ch_t
\end{aligned}
$$

where

- off_t is the sum of other payments (on the financial account of the BOP) received by households from the rest of the world (e.g. equity investment income, remittances, etc.)
- $\Pi_{y,t}$ is the sum of period profits received from the local production sector
- $\Pi_{x,t}$ is the sum of period profits received from exporters

Optimal behavior

- Constrained optimization $\Rightarrow$ Lagrangian
- In the Lagrangia, we replace the actual interest payments with hypothetical interest payments based on a measure of overall credit conditions
- Choose ch_t, n_t, bh_t^{lcy}, bh_t^{fcy} , vh_t to maximize

$$
\begin{aligned}
&\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log ch_t - \tfrac{1}{1+\eta}\, {n_t}^{\eta} \right) \ \cdots \\
&\quad + \ \mathrm{E}_0 \sum_{t=0}^{\infty} \ \beta^t \ vh_t \ \Bigg(- \, bh_t^{\mathrm{lcy}} - bh_t^{\mathrm{fcy}} + \ rh_{t-1}^{\mathrm{cond,lcy}} \ bh_{t-1}^{\mathrm{lcy}} + rh_{t-1}^{\mathrm{cond,fcy}} \ \frac{e_t}{e_{t-1}} \ bh_{t-1}^{\mathrm{fcy}} \ + \mathit{off}_t \cdots \\
&\qquad\qquad + w_t \ n_t + \Pi_{y,t} + \Pi_{x,t} - pc_t \ ch_t \ \Bigg)
\end{aligned}
\tag{4}
$$

- All prices (including factor prices) are taken as given: pc_t, w_t
- Profits are taken as given: $\Pi_{Y,t}$, $\Pi_{X,t}$

Demand for consumption and financial positions

- Consumption

$$vh_t \; pc_t = \frac{1}{ch_t - ch_t^{\text{ref}}} \tag{5}$$

- Financial position in local currency

$$vh_t + \frac{\beta_0}{pch_t \; ch_t} = \beta \;\; \mathrm{E}_t[vh_{t+1}] \;\; rh_t^{\text{cond,lcy}} \tag{6}$$

- Financial position in foreign currency

$$vh_t + \frac{\beta_0}{pch_t \; ch_t} = \beta \;\; \mathrm{E}_t\left[vh_{t+1} \; \frac{e_{t+1}}{e_t}\right] \;\; rh_t^{\text{cond,fcy}} \tag{7}$$

- Combining the conditions for local and foreign currency positions $\Rightarrow$ uncovered interest parity

$$r_t \approx r_t^{\text{fcy}} \;\; \mathrm{E}_t\left[\frac{e_{t+1}}{e_t}\right] \tag{8}$$

Denomination of household financial assets

- The first-order conditions only determine the interest parity between local and foreign currency interest rates (a no-arbitrage condition)
- Currency composition of household financial positions is left undetermined
- Need to impose exogenous assumptions about

$$\frac{bh_t^{\mathrm{fcy}}}{bh_t^{\mathrm{lcy}} + bh_t^{\mathrm{fcy}}} \tag{9}$$

Labor supply

- Use the household optimization problem to determine the optimal wage rate, w_t^{flex}, that would prevail with no labor market rigidities

- Upward sloping labor supply curve with η being the inverse wage elasticity

$$vh_t \; w_t^{\text{flex}} = \eta_0 \; n_t^{\eta} \tag{10}$$

- Limit case for $\eta = 0$ (infinitely elastic labor supply) used as a proxy for an *indivisible* labor assumptions

$$vh_t \; w_t^{\text{flex}} = \eta_0 \tag{11}$$

- Parameter η_0 is only a scaling factor (e.g. determining the labor and wage units) and has absolutely no impact on the properties of the model

Labor market rigidities

- Labor market theory identifies many sources of real wage rigidities
- No explicit microfoundations at the moment
- Actual real wage, rate is rigid in response to optimal (fully flexible) wage rate determined by household optimal choice

$$\log\left[\frac{w}{pc}\right]_t = \rho_w \ \log\left[\frac{w}{pc}\right]_{t-1} + \ (1-\rho_w) \ \log\left[\frac{w^{\text{flex}}}{pc}\right]_t + \ \varepsilon_{w,t} \tag{12}$$

Local production

Three production stages

- Stage $T-2$: Local labor
- Stage $T-1$: Combine with imports
- Stage $T-0$ (distribution): Resell domestically as consumption goods

Stage *T*–2 local production

Production function

$$y_{2,t} = an_t \; n_t \tag{13}$$

Period profits

$$\Pi_{y2,t} = py_{2,t} \; y_{2,t} - w_t \; n_t \tag{14}$$

Productivity in local production

Exogenous productivity process

Stage *T*–1 local production

Leontief production function (no elasticity of substitution)

$$y_{1,t} = \min\left\{\frac{y_{2,t}}{1-\gamma_M}\,,\,\frac{m_t}{\gamma_M}\right\} \tag{15}$$

Optimal choice of inputs

$$y_{2,t} = (1-\gamma_M)\, y_{1,t}$$

$$m_t = \gamma_M\; y_{1,t}$$

Period profits

$$\Pi_{y1,t} = py_{1,t}\; y_{1,t} - py_{2,t}\; y_{2,t} - pm_t\; m_t \tag{16}$$

Stage *T*–0 (Distribution)

Resell as consumption goods

$$y_{0,t} = y_{1,t} \tag{17}$$

Downward sloping demand curve faced by the representative distributor (seller)

$$y_{0,t} = \overline{y}_{0,t} \cdot \left(\frac{py_{0,t}}{\overline{py}_{0,t}} \right)^{\mu/(\mu-1)} \tag{18}$$

Period profits including a price adjustment cost

$$\Pi_{y0,t} = \left(py_{0,t} - py_{1,t} \right) \, y_{0,t} \; - \tfrac{1}{2} \, \xi_{py} \left(\Delta \log py_{0,t} - j_t \right)^2 \, \overline{py_{0,t} \; y_{0,t}} \tag{19}$$

where

- $py_{0,t}$ and $y_{0,t}$ are prices and quantities selected by an individual (representative) distributor (seller)
- $\overline{py}_{0,t}$ and $\overline{y}_{0,t}$ are aggregate (market-wide) prices and quantities whose movements are not internalized by an individual distributor (i.e. taken as given)
- j_t is a price indexation variable such that in steady state $j_{\mathrm{ss}} = \Delta \log py_{0,\mathrm{ss}}$
- ξ_{py} is the adjustment cost parameter ($\xi_{py} = 0$ means fully flexible prices)

Optimal price setting

In steady state, plain vanilla markup pricing (adjustment cost disappears)

$$py_{0,t} = \mu_{py} \cdot \ py_{1,t} \tag{20}$$

In dynamic simulations, markup pricing with an adjustment cost

$$py_{0,t} \left(\cdots \right) = \mu_{py} \cdot py_{1,t} \tag{21}$$

where $\left(\cdots \right)$ is

$$1 \ + \ \left(\mu_{py} - 1 \right) \xi_{py} \ \Big[\left(\Delta \log py_{0,t} - j_t \right) - \beta \left(\Delta \log py_{0,t+1} - j_{t+1} \right) \Big] \tag{22}$$

Total profits of local production sector

Sum up the periods profits across the individual prodution stages

$$\Pi_{y,t} = \Pi_{y0,t} + \Pi_{y1,t} + \Pi_{y2,t} \tag{23}$$

After substituting for the profits at individual production stages:

$$\Pi y, t = pc_t \; ch_t - pm_t \; m_t - w_t \; n_t \tag{24}$$

Exports

Real exports are an exogenous endowment (with no cost of production involved)

$$x_t = \cdots \tag{25}$$

Export prices are linked to te general world price level

$$px_t = \cdots \tag{26}$$

Exporter periods revenues and profits

$$\Pi_{x,t} = px_t \; x_t \tag{27}$$

Monetary policy

- Primary long-term objective: price stability expressed in an inflation target, $targ$
- Secondary short-term considerations: exchange rate fluctuations

Monetary policy reaction function

Response in short-term money rate

- Autoregression (conservatism, uncertainty)
- Steady state (long-run level)
- Reaction term

$$rm_t^{\mathrm{lcy}} = \rho_{rm} \, rm_{t-1}^{\mathrm{lcy}} + (1 - \rho_{rm}) \left(rm_{\mathrm{ss}} + react_t \right) + \epsilon_{rm,t} \tag{28}$$

Monetary policy reaction term

- Response to deviations in consumer price inflation from the target
- Reponse to fluctuations in the nominal exchange rate

$$react_t = \kappa_{pc} \left(\hat{pch}_{t+1} - targ\right) + \kappa_e \, (\hat{e}_t - \hat{e}_{\mathrm{ss}}) \tag{29}$$

Fiscal policy

- Government makes purchases of consumption goods cg_t
- Government purchases financed by a combination of levying lump-sum taxes tx_t^{ls} and issuing net fiscal debt (government bonds), dg_t
- Fiscal debt is stabilized at a given level in the long run

Fiscal finance in steady state

Dynamic fiscal budget equation

$$dg_t = rg_{t-1}\ dg_{t-1} + pc_t\ cg_t - tx_t^{\text{ls}} \tag{30}$$

Government consumption

$$pc_t\ cg_t = \sigma_{cg}\ ngdp_t \tag{31}$$

Fiscal debt in steady state

$$dg_t = \sigma_{dg}\ ngdp_t \tag{32}$$

The steady-state path of the lump-sum taxes is implicitly determined by these equations

Denomination and remuneration of government bonds

- Who holds the government bonds?
- What currency are the government bonds denominated in?
- How is the interest rate determined?

Ricardian equivalence

What it is?

- Financing
- The way of financing govt expenditures (debt financing vs tax financing, or the timing of either) is irrelevant for the private sector
- Holds in standard DSGE models without distortionary taxes and without current income or current wealth effects

What it is not?

- Expenditures
- Fiscal multipliers - connection between govt consumption and private consumption (so called crowding in)

International linkages

- Import and export prices
- Balance of payments
- Country credit risk
- Denomination of net foreign assets

Import and export prices

Import and export prices taken as given; linked to an underlying world price index, pw_t^{fcy}

$$pm_t = e_t \; pm_t^{\mathrm{fcy}}$$

$$px_t = e_t \; px_t^{\mathrm{fcy}}$$

$$pm_t^{\mathrm{fcy}} = \left[\frac{pm}{pw}\right]_t pw_t^{\mathrm{fcy}}$$

$$px_t^{\mathrm{fcy}} = \left[\frac{px}{pw}\right]_t pw_t^{\mathrm{fcy}}$$

Balance of payments

Country credit risk

Local interest rate for foreign currency denominations is marked up over world foreign-currency interest rate

$$r_t^{\text{fcy}} = rw_t^{\text{fcy}} \; prem_t \tag{33}$$

Country credit risk premium

$$prem_t = \exp\left(\phi_0 - \phi_1 \left[\frac{nfa}{ngdp}\right]_t\right) \tag{34}$$

Net foreign assets to GDP ratio

$$\left[\frac{nfa}{ngdp}\right]_t = \frac{b_t + b_t^{\text{fcy}}}{ngdp_t} \tag{35}$$

Definitions and Identities

Nominal GDP

$$gdp_t = pch_t \; ch_t + px_t \; x_t - pm_t \; m_t \tag{36}$$