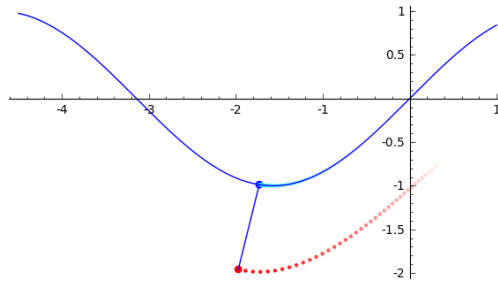


Mechanics with SageMath

(with SageMath)



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https://github.com/marcinofulus/Mechanics_with_SageMath

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1 Preface

Computer algebra system provides usefull tool for analyzing mechanical systems. Classical problems which require an hour or more of *paper and pencil* algebra can be solved within minutes. Derivation of Euler-Lagrange equation in arbitrary system of coordinates or solveing problems with constraints using d'Alembert principle are prominent examples.

In this set of notebooks we include set of classical problems in mechanics, which are solved with help of SageMath computer algebra system. Moreover, the resulting euqation of motion are analyzed numerically and reader can create visually appealing computer animations.