Distributed/Paralell computations using RecursivelyEnumeratedSet and Map-Reduce

There exists an efficient way to distribute computations when you have a set S of objects defined by RecursivelyEnumeratedSet() (see sage.sets.recursively_enumerated_set for more details) over which you would like to perform the following kind of operations:

- Compute the cardinality of a (very large) set defined recursively (through a call to RecursivelyEnumeratedSet of forest type)
- More generally, compute any kind of generating series over this set
- Test a conjecture: i.e. find an element of S satisfying a specific property; conversely, check that all of them do
- Count/list the elements of S having a specific property
- ullet Apply any map/reduce kind of operation over the elements of S

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How is this different from usual MapReduce?

This implementation is specific to RecursivelyEnumeratedSet of forest type, and uses its properties to do its job. Non only mapping and reducing is done on different processors but also **generating the elements of** S.

How can I use all that stuff?

First, you need the information necessary to describe a RecursivelyEnumeratedSet of forest type representing your set S (see sage.sets.recursively_enumerated_set). Then, you need to provide a Map function as well as a Reduce function. Here are some examples:

• Counting the number of elements: In this situation, the map function can be set to lambda x:1, and the reduce function just adds the elements together, i.e. lambda x,y:x+y.

Here's the Sage code for binary words of length ≤ 16

```
sage: S = RecursivelyEnumeratedSet(
....: [[]], lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
....: structure='forest', enumeration='depth')
sage: S.map_reduce(
....: map_function = lambda x: 1,
....: reduce_function = lambda x,y: x+y,
....: reduce_init = 0 )
131071</pre>
```

Note that the function mapped and reduced here are equivalent to the default values of the sage.combinat.backtrack.SearchForest.map_reduce() method so that to compute the number of element you only need to call:

```
sage: S.map_reduce()
131071
```

You don't need to use RecursivelyEnumeratedSet(), you can use directly RESetMapReduce. This is needed if you want to have fine control over the parallel execution (see *Advanced use* below):

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
....: roots = [[]],
....: children = lambda I: [I+[0], I+[1]] if len(I) <= 15 else [],</pre>
```

```
...: map_function = lambda x : 1,
...: reduce_function = lambda x,y: x+y,
...: reduce_init = 0)
sage: S.run()
131071
sage: factor(131071 + 1)
2^17
```

• Generating series: In this situation, the map function associates a monomial to each element of S, while the Reduce function is still equal to lambda x,y : x+y.

Here's the Sage code for binary words of length ≤ 16

```
sage: S = RecursivelyEnumeratedSet(
....: [[]], lambda l: [l+[0], l+[1]] if len(l) < 16 else [],
....: structure='forest', enumeration='depth')
sage: sp = S.map_reduce(
....: map_function = lambda z: x**len(z),
....: reduce_function = lambda x,y: x+y,
....: reduce_init = 0 )
sage: sp
65536*x*^16 + 32768*x*^15 + 16384*x*^14 + 8192*x*^13 + 4096*x*^12 + 2048*x*^11 + 1024*x*^10 + 512*x*^9 + 256*x*^8 + 128*x*^7 + 64*x*^6 + 32*x*^5 + 16*
</pre>
```

This is of course $\sum_{i=0}^{i=16} (2x)^i$:

```
sage: bool(sp == sum((2^*x)^i for i in range(17)))
True
```

Here is another example where we count permutations of size ≤ 8 (here we use the default values):

```
sage: S = RecursivelyEnumeratedSet( [[]],
....: lambda l: ([l[:i] + [len(l)] + l[i:] for i in range(len(l)+1)]
....: if len(l) < 8 else []),
....: structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1</pre>
```

This is of course $\sum_{i=0}^{i=8} i! x^i$:

```
sage: bool(sp == sum(factorial(i)*x^i for i in range(9)))
True
```

• **Post Processing**: We now demonstrate the use of post_process. We generate the permutation as precedingly by we only perform the map/reduce computation on those of even len. Of course we get the even part of the previous generating series:

```
sage: S = RecursivelyEnumeratedSet( [[]],
....: lambda l: ([l[:i] + [len(l)+1] + l[i:] for i in range(len(l)+1)]
....: if len(l) < 8 else []),
....: post_process = lambda l : 1 if len(l) % 2 == 0 else None,
....: structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 720*x^6 + 24*x^4 + 2*x^2 + 1</pre>
```

This is also useful for example to call a constructor on the generated elements:

```
sage: S = RecursivelyEnumeratedSet( [[]],
....: lambda l: ([l[:i] + [len(l)+1] + l[i:] for i in range(len(l)+1)]
....: if len(l) < 5 else []),
....: post_process = lambda l: Permutation(l) if len(l) == 5 else None,
....: structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**(len(z.inversions()))); sp
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x + 1</pre>
```

We get here a pollynomial called the x-factorial of 5 that is $\prod_{i=1}^{i=5} \frac{1-x^i}{1-x}$:

```
sage: (prod((1-x^i)/(1-x) for i in range(1,6))).normalize().expand() x^10 + 4^2x^9 + 9^2x^8 + 15^2x^7 + 20^2x^6 + 22^2x^5 + 20^2x^4 + 15^2x^3 + 9^2x^4 + 15^2x^5 + 20^2x^6 + 20^2x^6 + 15^2x^6 + 1
```

• Listing the objects: One can also compute the list of objects in a RecursivelyEnumeratedSet of forest type using RESetMapReduce.

As an example, we compute the set of number beetween 1 and 63, generated by their binary expansion:

```
sage: S = RecursivelyEnumeratedSet( [1],
....: lambda l: [(l<<1)|0, (l<<1)|1] if l < 1<<5 else [],
....: structure='forest', enumeration='depth')</pre>
```

Here is the list computed without RESetMapReduce:

```
sage: serial = list(S)
sage: serial
[1, 2, 4, 8, 16, 32, 33, 17, 34, 35, 9, 18, 36, 37, 19, 38, 39, 5, 10, 20, 40, 41, 21, 42, 43, 11, 22, 44, 45, 23, 46, 47, 3, 6, 12, 24, 48, 49, 25, 50, 51, 13,
```

Here is how to perform the parallel computation. The order of the lists depends on the synchronisation of the various computation processes and therefore should be considered as random:

```
sage: parall = S.map_reduce( lambda x: [x], lambda x,y: x+y, [] )
sage: parall # random
[1, 3, 7, 15, 31, 63, 62, 30, 61, 60, 14, 29, 59, 58, 28, 57, 56, 6, 13, 27, 55, 54, 26, 53, 52, 12, 25, 51, 50, 24, 49, 48, 2, 5, 11, 23, 47, 46, 22, 45, 44, sage: sorted(serial) == sorted(parall)
True
```

Advanced use

Fine control of the execution of a map/reduce computations is obtained by passing parameters to the RESetMapReduce.run() method. On can use the three following parameters:

- max_proc maximum number of process used. default: number of processor on the machine
- timeout a timeout on the computation (default: None)
- reduce_locally whether the workers should reduce locally their work or sends results to the master as soon as possible. See
 RESetMapReduceWorker for details.

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import RESetMPExample, AbortError
sage: EX = RESetMPExample(maxl = 8)
sage: try:
....: res = EX.run(timeout=0.1)
....: except AbortError:
....: print "Computation timeout"
....: else:
....: print "Computation normally finished"
....: res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: try:
...: res = EX.run(timeout=60)
...: except AbortError:
...: print "Computation Timeout"
...: else:
...: print "Computation normally finished"
...: res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

As for reduce_locally, one should not see any difference, except for speed during normal usage. Most of the time the user should leave it to True, unless he set up a mecanism to consume the partial results as soon as they arrive. See RESetParallellterator and in particular the __iter__ method for a example of consumer use.

Profiling

It is possible the profile a map/reduce computation. First we create a RESetMapReduce object:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
....: roots = [[]],
....: children = lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
....: map_function = lambda x : 1,
....: reduce_function = lambda x,y: x+y,
...: reduce_init = 0)</pre>
```

The profiling is activated by the profile parameter. The value provided should be a prefix (including a possible directory) for the profile dump:

```
sage: prof = tmp_dir('RESetMR_profile')+'profcomp'
sage: res = S.run(profile=prof) # random
```

```
[RESetMapReduceWorker-1:58] (20:00:41.444) Profiling in /home/user/.sage/temp/mymachine.mysite/32414/RESetMR_profilewRCRAx/profcomp1 ...
```

IRESetMapReduceWorker-1:57] (20:00:41.444) Profiling in /home/user/.sage/temp/mymachine.mysite/32414/RESetMR_profilewRCRAx/profcomp0 ... sage: res
131071

In this example, the profile have been dumped in files such as profcompo. One can then load and print them as follows. See profile.profile for more details:

```
sage: import cProfile, pstats
sage: st = pstats.Stats(prof+'0')
sage: st.strip_dirs().sort_stats('cumulative').print_stats() #random
...
Ordered by: cumulative time

ncalls tottime percall cumtime percall filename:lineno(function)
1 0.023 0.023 0.432 0.432 map_reduce.py:1211(run_myself)
11968 0.151 0.000 0.223 0.000 map_reduce.py:1292(walk_branch_locally)
...
<pstats.Stats instance at 0x7fedea40c6c8>
```

See also: The Python Profilers for more detail on profiling in python.

Logging

The computation progress is logged through a logging.Logger in sage.parallel.map_reduce.logger together with logging.StreamHandler and a logging.Formatter. They are currently configured to print warning message on the console.

See also: Logging facility for Python for more detail on logging and log system configuration.

How does it work?

The scheduling algorithm we use here is any adaptation of Wikipedia article Work_stealing:

In a work stealing scheduler, each processor in a computer system has a queue of work items (computational tasks, threads) to perform. [...]. Each work items are initially put on the queue of the processor executing the work item. When a processor runs out of work, it looks at the queues of other processors and "steals" their work items. In effect, work stealing distributes the scheduling work over idle processors, and as long as all processors have work to do, no scheduling overhead occurs.

For communication we use Python's basic multiprocessing module. We first describe the different actors and communications tools used by the system. The work is done under the coordination of a **master** object (an instance of RESetMapReduce) by a bunch of **worker** objects (instances of RESetMapReduceWorker).

Each running map reduce instance work on a RecursivelyEnumeratedSet of forest type called here C and is coordinated by a RESetMapReduce object called the *master*. The master is in charge of lauching the work, gathering the results and cleaning up at the end of the computation. It doesn't perform any computation associated to the generation of the element C nor the computation of the mapped function. It however occasionally perform a reduce, but most reducing is by default done by the workers. Also thanks to the work-stealing algorithm, the master is only involved in detecting the termination of the computation but all the load balancing is done at the level of the worker.

Workers are instance of RESetMapReduceWorker. They are responsible of doing the actual computations: elements generation, mapping and reducing. They are also responsible of the load balancing thanks to work-stealing.

Here is a description of the attribute of the *master* relevant to the map-reduce protocol:

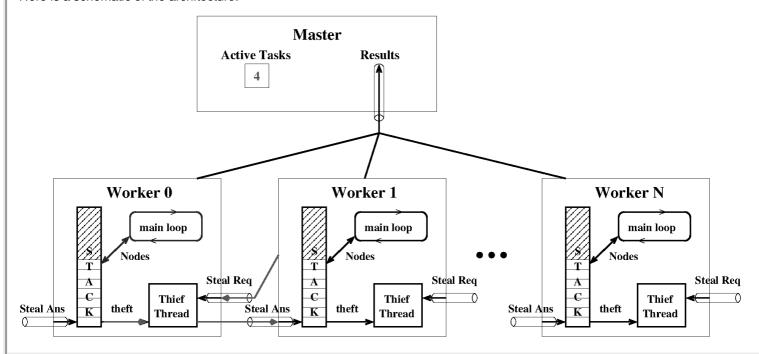
- master_results a **SimpleQueue** where the master gathers the results sent by the workers.
- master_active_tasks a Semaphore recording the number of active task. The work is done when it gets to 0.
- master._done a Lock which ensures that shutdown is done only once.
- master_abort a Value() storing a shared ctypes.c_bool which is True if the computation was aborted before all the worker runs
 out of work.
- master_workers a list of RESetMapReduceWorker objects. Each worker is identified by its position in this list.

Each worker is a process (RESetMapReduceWorker inherits from Process) which contains:

worker_iproc – the identifier of the worker that is its position in the master's list of workers

- worker_todo a collections.deque storing of nodes of the worker. It is used as a stack by the worker. Thiefs steal from the bottom
 of this queue.
- worker._request a SimpleQueue storing steal request submitted to worker.
- worker._read_task, worker._write_task a Pipe used to transfert node during steal.
- worker._thief a Thread which is in charge of stealing from worker._todo.

Here is a schematic of the architecture:



How thefts are performed

During normal time, that is when all worker are active) a worker w is iterating though a loop inside RESetMapReduceWorker.walk_branch_locally(). Work nodes are taken from and new nodes w_todo are appended to w_todo. When a worker w is running out of work, that is worker_todo is empty, then it tries to steal some word (ie: a node) from another worker. This is performed in the RESetMapReduceWorker.steal() method.

From the point of view of w here is what happens:

- w signals to the master that it is idle master. signal task done();
- w chose a victim v at random;
- w sends a request to v : it puts its identifier into v._request;
- w tries to read a node from w_read_task. Then three things may happen:
 - o a proper node is read. Then the theft was a success and w starts working locally on the received node.
 - O None is received. This means that v was idle. Then w tries another victim.
 - AbortError is received. This means either that the computation was aborted or that it simply succeded and that no more work is required by w. Therefore an AbortError exception is raised leading to w to shutdown.

We now describe the protocol on the victims side. Each worker process contains a Thread which we call T for thief which acts like some kinds of Troyan horse during theft. It is normally blocked waiting for a steal request.

From the point of view of v and τ , here is what happens:

- during normal time T is blocked waiting on V._request;
- upon steal request, T wakes up receiving the identification of w;
- T signal to the master that a new task is starting by master._signal_task_start();
- Two things may happen depending if the queue v_todo is empty or not. Remark that due to the GIL, there is no parallel execution between the victim v and its thief tread T.
 - o If v_todo is empty, then None is answered on w_write_task. The task is immediately signaled to end the master through master_signal_task_done().
 - Otherwise, a node is removed from the bottom of v_todo. The node is sent to w on w_write_task. The task will be ended by w, that is when finished working on the subtree rooted at the node, w will call master_signal_task_done().

The end of the computation

When a worker finishes working on a task, it calls master._signal_task_done(). This decrease the counter of the semaphore master._active_tasks. When it reaches 0, it means that there are no more nodes: the work is done. The worker executes master._shutdown() which sends AbortError on all worker._request() and worker._write_task(). Each worker or thief thread receiving such a message raise the corresponding exception, stoping therefore its work. A lock called master._done ensures that shutdown is only done once. Finally, it is also possible to interrupt the computation before its ends calling master.abort().

Are there examples of classes?

Yes! Here, there are:

- RESetMPExample a simple basic example
- RESetParallellterator a more advanced example using non standard communication configuration.

Tests

Generating series for sum of strictly decreassing list of integer smaller than 15:

```
sage: y = var('y')
sage: R = RESetMapReduce(
....: roots = [([], 0, 0)] + [([i], i, i) for i in range(1,15)],
....: children = lambda (list, sum, last):
....: [(list + [i], sum + i, i) for i in range(1,last)],
....: map_function = lambda (li, sum, unused): y**sum)
sage: sg = R.run()
sage: bool(sg == expand(prod((1+y^i) for i in range(1,15))))
True
```

Classes and methods

exception sage.parallel.map_reduce. AbortError

Bases: exceptions.Exception

Exception for aborting parallel computations

This is used both as exception or as abort message

TESTS:

```
sage: from sage.parallel.map_reduce import AbortError
sage: raise AbortError
Traceback (most recent call last):
...
AbortError
```

class sage.parallel.map_reduce. RESetMPExample (maxl=9)

Bases: sage.parallel.map_reduce.RESetMapReduce

An example of map reduce class

INPUT:

• maxl – the maximum size of permutations generated (default to 9).

This compute the generating series of permutations counted by their size upto size maxl.

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample()
sage: EX.run()
362880*x^9 + 40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

See also: This is an example of RESetMapReduce

children(/)

Return the chidren of the permutation l

INPUT:

I − a list containing a permutation

OUTPUT:

• the lists of len(I) inserted at all possible positions into I

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample sage: RESetMPExample().children([1,0]) [[2, 1, 0], [1, 2, 0], [1, 0, 2]]
```

map_function(/)

The monomial associated to the permutation l

INPUT:

• I – a list containing a permutation

OUTPUT: x^len(l).

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample sage: RESetMPExample().map_function([1,0]) x^2
```

roots()

Return the empty permutation

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample sage: RESetMPExample().roots()
[[]]
```

class sage.parallel.map_reduce. **RESetMapReduce** (roots=None, children=None, post_process=None, map_function=None, reduce_function=None, reduce_init=None, forest=None)

Bases: object

Map-Reduce on recursively enumerated sets

INPUT:

Description of the set:

- either forest=f where f is a RecursivelyEnumeratedSet of forest type
- or a triple roots, children, post_process as follows
 - o roots=r − The root of the enumeration
 - o children=c a function iterating through children node, given a parent nodes
 - post_process=p a post processing function

The option post_process allows for customizing the nodes that are actually produced. Furthermore, if $post_process(x)$ returns None, then x won't be output at all.

Decription of the map/reduce operation:

- map_function=f (default to None)
- reduce_function=red (default to None)
- reduce_init=init (default to None)

See also: the Map/Reduce module for details and examples.

abort()

Abort the current parallel computation

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....: lambda l: [I+[0], I+[1]] if len(l) < 17 else [])
sage: it = iter(S)
sage: it.next()
[]
sage: S.abort()
sage: hasattr(S, 'work_queue')
False</pre>
```

Cleanups:

```
sage: S.finish()
```

finish()

Destroys the worker and all the communication objects.

Also gathers the communication statistics before destroying the workers.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=5)
sage: S.setup_workers(2) # indirect doctest
sage: S._workers[0]._todo.append([])
sage: for w in S._workers: w.start()
sage: _ = S.get_results()
sage: S__shutdown()
sage: S._print_communication_statistics()
Traceback (most recent call last):
...
AttributeError: 'RESetMPExample' object has no attribute '_stats'
sage: S._frinish()
sage: S._print_communication_statistics()
#proc: ...
...
sage: _ = S.run() # Cleanup
```

```
See also: print_communication_statistics()
```

get_results()

Get the results from the queue

OUTPUT:

• the reduction of the results of all the workers, that is the result of the map/reduce computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: for v in [1, 2, None, 3, None]: S._results.put(v)
sage: S.get_results()
6
```

Cleanups:

```
sage: del S._results, S._active_tasks, S._done, S._workers
```

map_function(o)

Return the function mapped by self

INPUT: o – a node

OUTPUT: By default 1.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce sage: S = RESetMapReduce()
```

```
sage: S.map_function(7)
1
sage: S = RESetMapReduce(map_function = lambda x: 3*x + 5)
sage: S.map_function(7)
26
```

post_process(a)

Return the post-processing function for self

INPUT: a - a node

By default, returns a itself

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.post_process(4)
4
sage: S = RESetMapReduce(post_process=lambda x: x*x)
sage: S.post_process(4)
16
```

print_communication_statistics(blocksize=16)

Print the communication statistics in a nice way

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=6)
sage: S.run()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

sage: S.print_communication_statistics() # random
#proc: 0 1 2 3 4 5 6 7
reqs sent: 5 2 3 11 21 19 1 0
reqs rcvs: 10 10 9 5 1 11 9 2
- thefs: 1 0 0 0 0 0 0 0 0
+ thefs: 0 0 1 0 0 0 0 0 0
```

random_worker()

Returns a random workers

OUTPUT: A worker for self chosed at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker sage: from threading import Thread sage: EX = RESetMPExample(maxl=6) sage: EX.setup_workers(2) sage: EX.random_worker() < RESetMapReduceWorker(RESetMapReduceWorker-..., initial)> sage: EX.random_worker() in EX._workers
True
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

reduce_function(a, b)

Return the reducer function for self

INPUT: a, b - two value to be reduced

OUTPUT: by default the sum of a and b.

Note: This should be overloaded in applications.

EXAMPLES:

sage: from sage.parallel.map_reduce import RESetMapReduce

```
sage: S = RESetMapReduce()
sage: S.reduce_function(4, 3)
7
sage: S = RESetMapReduce(reduce_function=lambda x,y: x*y)
sage: S.reduce_function(4, 3)
12
```

reduce init()

Return the initial element for a reduction

Note: This should be overloaded in applications.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.reduce_init()
0
sage: S = RESetMapReduce(reduce_init = 2)
sage: S.reduce_init()
2
```

roots()

Return the roots of self

OUTPUT: an iterable of nodes

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(42)
sage: S.roots()
42
```

run(max_proc=None, reduce_locally=True, timeout=None, profile=None)

Run the computations

INPUT:

- max_proc maximum number of process used. default: number of processor on the machine
- reduce locally See RESetMapReduceWorker (default: True)
- timeout a timeout on the computation (default: None)
- profile directory/filename prefix for profiling, or None for no profiling (default: None)

OUTPUT:

the result of the map/reduce computation or an exception AbortError if the computation was interrupted or timeout.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 8)
sage: EX.run()
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import AbortError
sage: try:
....: res = EX.run(timeout=0.1)
....: except AbortError:
.....: print "Computation timeout"
....: else:
....: print "Computation normally finished"
....: res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: from sage.parallel.map_reduce import AbortError
sage: try:
....: res = EX.run(timeout=60)
....: except AbortError:
....: print "Computation Timeout"
```

```
....: else:
....: print "Computation normally finished"
....: res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

run_serial()

Serial run of the computation (mostly for tests)

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample sage: EX = RESetMPExample(maxl = 4) sage: EX.run_serial() 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

setup_workers(max_proc=None, reduce_locally=True)

Setup the communication channels

INPUT:

- mac_proc an integer: the maximum number of workers
- reduce_locally whether the workers should reduce locally their work or sends results to the master as soon as possible. See RESetMapReduceWorker for details.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: S._results
<multiprocessing.queues.SimpleQueue object at 0x...>
sage: len(S._workers)
2
```

start_workers()

Lauch the workers

The worker should have been created using setup_workers().

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(roots=[])
sage: S.setup_workers(2)
sage: S.start_workers()
sage: all(w.is_alive() for w in S._workers)
True

sage: sleep(1)
sage: all(not w.is_alive() for w in S._workers)
True
```

Cleanups:

```
sage: S.finish()
```

class sage.parallel.map_reduce. RESetMapReduceWorker(mapred, iproc, reduce_locally)

 $Bases: {\it multiprocessing.process.Process}$

Worker for generate-map-reduce

This shouldn't be called directly, but instead created by RESetMapReduce.setup_workers().

INPUT:

- mapred the instance of RESetMapReduce for which this process is working.
- iproc the id of this worker.
- reduce_locally when reducing the results. Three possible values are supported:
 - True means the reducing work is done all locally, the result is only sent back at the end of the work. This ensure the lowest level of communication.
 - False results are sent back after each finished branches, when the process is asking for more work.

run()

The main function executed by the worker

Calls run_myself() after possibly setting up parallel profiling.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker sage: EX = RESetMPExample(maxl=6) sage: EX.setup_workers(1)

sage: w = EX._workers[0] sage: w._todo.append(EX.roots()[0])

sage: w.run() sage: sleep(1) sage: w._todo.append(None)

sage: EX.get_results() 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

run_myself()

The main function executed by the worker

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker sage: EX = RESetMPExample(maxl=6) sage: EX.setup_workers(1)

sage: w = EX._workers[0] sage: w._todo.append(EX.roots()[0]) sage: w.run_myself()

sage: sleep(1) sage: w._todo.append(None)

sage: EX.get_results() 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

send partial result()

Send results to the MapReduce process

Send the result stored in self_res to the master an reinitialize it to master.reduce_init.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: EX.setup_workers(1)
sage: w = EX._workers[0]
sage: w _res = 4
sage: w.send_partial_result()
sage: w._res
0
sage: EX._results.get()
```

steal()

Steal some node from another worker

OUTPUT: a node stolen from another worker choosed at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker sage: from threading import Thread sage: EX = RESetMPExample(maxl=6) sage: EX.setup_workers(2)
```

```
sage: w0, w1 = EX._workers
sage: w0._todo.append(42)
sage: thief0 = Thread(target = w0._thief, name="Thief")
sage: thief0.start()
sage: w1.steal()
42
```

walk_branch_locally(node)

Work locally

Performs the map/reduce computation on the subtrees rooted at node.

INPUT: node – the root of the subtree explored.

OUTPUT: nothing, the result are stored in self._res

This is where the actual work is performed.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: w = RESetMapReduceWorker(EX, 0, True)
sage: def sync(): pass
sage: w.synchronize = sync
sage: w._res = 0
sage: w.walk_branch_locally([])
sage: w._res
x^4 + x^3 + x^2 + x + 1
sage: w.walk_branch_locally(w._todo.pop())
sage: w._res
2*x^4 + x^3 + x^2 + x + 1
sage: while True: w.walk_branch_locally(w._todo.pop())
Traceback (most recent call last):
IndexError: pop from an empty deque
sage: w. res
24^{*}x^{4} + 6^{*}x^{3} + 2^{*}x^{2} + x + 1
```

class sage.parallel.map_reduce. **RESetParallelIterator**(roots=None, children=None, post_process=None, map_function=None, reduce init=None, forest=None)

Bases: sage.parallel.map_reduce.RESetMapReduce

A parallel iterator for recursively enumerated sets

This demonstrate how to use RESetMapReduce to get an iterator on a recursively enumerated sets for which the computation are done in parallel.

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....: lambda l: [l+[0], l+[1]] if len(l) < 15 else [])
sage: sum(1 for _ in S)
65535</pre>
```

$map_function(z)$

Return a singleton tuple

INPUT: z - a node

OUPUT: (z,)

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....: lambda l: [l+[0], l+[1]] if len(l) < 15 else [])
sage: S.map_function([1, 0])
([1, 0],)</pre>
```

sage.parallel.map_reduce.proc_number(max_proc=None)

Computing the number of process used

INPUT:

• max_proc - the maximum number of process used

EXAMPLE:

```
sage: from sage.parallel.map_reduce import proc_number
sage: proc_number() # random
8
sage: proc_number(max_proc=1)
1
sage: proc_number(max_proc=2) in (1, 2)
True
```