REPORT ON OpenDreamKit DELIVERABLE D5.1

Turn the Python prototypes for tree exploration into production code, integrate to SAGE.

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Progress on and finalization of this deliverable has been tracked publicly at: `https://github.com/OpenDreamKit/OpenDreamKit/issues/107`	

DELIVERABLE DESCRIPTION, AS TAKEN FROM GITHUB ISSUE #107 ON 2016-09-07

- **WP5:** High Performance Mathematical Computing
- **Lead Institution:** Université Paris-Sud
- **Due:** 2015-11-30 (month 3)
- **Submitted to Sage:** 2015-11-30
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- **Task**: T5.6 (#104)
- **Nature:** Demonstrator
- **Proposal:** p.51
- **Final report**, **slides**

MapReduce is a classical programming model for distributed computations where one maps a function on a large data set and use a reduce function to summarize all the produced information. A use case that occurs often e.g. in combinatorics is to have a data sets that is described by a recursion tree, and is too big to be expanded in memory. Instances include counting the number of elements in the data set, or collecting some statistics on them.

A prototype distributed implementation of this programming model had been written in 2010-2014 for SageMath, using multiple processes on a single machine and work-stealing for load balancing. In this deliverable, we have turned this prototype into production code and integrated it into the SageMath distribution.

See Trac Ticket 13580 for the source code and the discussion about the integration into Sage, as well as this snapshot of the documentation.

This work was presented at the journée du groupe de travail LaMHA at Université Pierre et Marie Curie on November the 26th of 2016. The slides give an overview of the motivations, algorithm, and implementation.

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Appendix A. Snapshot of the documentation at the time of delivery

Distributed/Paralell computations using RecursivelyEnumeratedSet and Map-Reduce

There exists an efficient way to distribute computations when you have a set S of objects defined by **RecursivelyEnumeratedSet()** (see **sage.sets.recursively_enumerated_set** for more details) over which you would like to perform the following kind of operations :

- Compute the cardinality of a (very large) set defined recursively (through a call to **RecursivelyEnumeratedSet of forest type**)
- More generally, compute any kind of generating series over this set
- Test a conjecture : i.e. find an element of S satisfying a specific property; conversely, check that all of them do
- Count/list the elements of S having a specific property
- Apply any map/reduce kind of operation over the elements of S

AUTHORS :

- Florent Hivert – code, documentation (2012-2015)
- Jean Baptiste Priez – prototype (2011, June)
- Nathann Cohen – Some doc (2012)

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How is this different from usual MapReduce ?

This implementation is specific to **RecursivelyEnumeratedSet of forest type**, and uses its properties to do its job. Non only mapping and reducing is done on different processors but also **generating the elements of S**.

How can I use all that stuff?

First, you need the information necessary to describe a **RecursivelyEnumeratedSet of forest type** representing your set S (see **sage.sets.recursively_enumerated_set**). Then, you need to provide a Map function as well as a Reduce function. Here are some examples :

- **Counting the number of elements**: In this situation, the map function can be set to `lambda x : 1`, and the reduce function just adds the elements together, i.e. `lambda x,y : x+y`.

  Here's the Sage code for binary words of length ≤ 16

  ```
  sage: S = RecursivelyEnumeratedSet(
  ....:   [[]], lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
  ....:   structure='forest', enumeration='depth')
  sage: S.map_reduce(
  ....:   map_function = lambda x: 1,
  ....:   reduce_function = lambda x,y: x+y,
  ....:   reduce_init = 0 )
  131071
  ```

  Note that the function mapped and reduced here are equivalent to the default values of the **sage.combinat.backtrack.SearchForest.map_reduce()** method so that to compute the number of element you only need to call:

  ```
  sage: S.map_reduce()
  131071
  ```

  You don't need to use **RecursivelyEnumeratedSet()**, you can use directly **RESetMapReduce**. This is needed if you want to have fine control over the parallel execution (see *Advanced use* below):

  ```
  sage: from sage.parallel.map_reduce import RESetMapReduce
  sage: S = RESetMapReduce(
  ....:   roots = [[]],
  ....:   children = lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
  ```

```
....:    map_function = lambda x : 1,
....:    reduce_function = lambda x,y: x+y,
....:    reduce_init = 0 )
sage: S.run()
131071
sage: factor(131071 + 1)
2^17
```

- **Generating series**: In this situation, the map function associates a monomial to each element of S, while the Reduce function is still equal to `lambda x,y : x+y`.

  Here's the Sage code for binary words of length ≤ 16

```
sage: S = RecursivelyEnumeratedSet(
....:    [[]], lambda l: [l+[0], l+[1]] if len(l) < 16 else [],
....:    structure='forest', enumeration='depth')
sage: sp = S.map_reduce(
....:    map_function = lambda z: x**len(z),
....:    reduce_function = lambda x,y: x+y,
....:    reduce_init = 0 )
sage: sp
65536*x^16 + 32768*x^15 + 16384*x^14 + 8192*x^13 + 4096*x^12 + 2048*x^11 + 1024*x^10 + 512*x^9 + 256*x^8 + 128*x^7 + 64*x^6 + 32*x^5 + 16*
```

  This is of course $\sum_{i=0}^{i=16}(2x)^i$:

```
sage: bool(sp == sum((2*x)^i for i in range(17)))
True
```

  Here is another example where we count permutations of size ≤ 8 (here we use the default values):

```
sage: S = RecursivelyEnumeratedSet( [[]],
....:    lambda l: ([l[:i] + [len(l)] + l[i:] for i in range(len(l)+1)]
....:              if len(l) < 8 else []),
....:    structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

  This is of course $\sum_{i=0}^{i=8} i!x^i$:

```
sage: bool(sp == sum(factorial(i)*x^i for i in range(9)))
True
```

- **Post Processing**: We now demonstrate the use of `post_process`. We generate the permutation as precedingly by we only perform the map/reduce computation on those of even `len`. Of course we get the even part of the previous generating series:

```
sage: S = RecursivelyEnumeratedSet( [[]],
....:    lambda l: ([l[:i] + [len(l)+1] + l[i:] for i in range(len(l)+1)]
....:              if len(l) < 8 else []),
....:    post_process = lambda l : l if len(l) % 2 == 0 else None,
....:    structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**len(z)); sp
40320*x^8 + 720*x^6 + 24*x^4 + 2*x^2 + 1
```

  This is also useful for example to call a constructor on the generated elements:

```
sage: S = RecursivelyEnumeratedSet( [[]],
....:    lambda l: ([l[:i] + [len(l)+1] + l[i:] for i in range(len(l)+1)]
....:              if len(l) < 5 else []),
....:    post_process = lambda l : Permutation(l) if len(l) == 5 else None,
....:    structure='forest', enumeration='depth')
sage: sp = S.map_reduce(lambda z: x**(len(z.inversions()))); sp
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x + 1
```

  We get here a pollynomial called the x-factorial of 5 that is $\prod_{i=1}^{i=5}\frac{1-x^i}{1-x}$:

```
sage: (prod((1-x^i)/(1-x) for i in range(1,6))).normalize().expand()
x^10 + 4*x^9 + 9*x^8 + 15*x^7 + 20*x^6 + 22*x^5 + 20*x^4 + 15*x^3 + 9*x^2 + 4*x + 1
```

- **Listing the objects**: One can also compute the list of objects in a **RecursivelyEnumeratedSet of forest type** using **RESetMapReduce**. As an example, we compute the set of number beetween 1 and 63, generated by their binary expansion:

```
sage: S = RecursivelyEnumeratedSet( [1],
....:     lambda l: [(l<<1)|0, (l<<1)|1] if l < 1<<5 else [],
....:     structure='forest', enumeration='depth')
```

Here is the list computed without **RESetMapReduce**:

```
sage: serial = list(S)
sage: serial
[1, 2, 4, 8, 16, 32, 33, 17, 34, 35, 9, 18, 36, 37, 19, 38, 39, 5, 10, 20, 40, 41, 21, 42, 43, 11, 22, 44, 45, 23, 46, 47, 3, 6, 12, 24, 48, 49, 25, 50, 51, 13,
```

Here is how to perform the parallel computation. The order of the lists depends on the synchronisation of the various computation processes and therefore should be considered as random:

```
sage: parall = S.map_reduce( lambda x: [x], lambda x,y: x+y, [] )
sage: parall   # random
[1, 3, 7, 15, 31, 63, 62, 30, 61, 60, 14, 29, 59, 58, 28, 57, 56, 6, 13, 27, 55, 54, 26, 53, 52, 12, 25, 51, 50, 24, 49, 48, 2, 5, 11, 23, 47, 46, 22, 45, 44,
sage: sorted(serial) == sorted(parall)
True
```

Advanced use

Fine control of the execution of a map/reduce computations is obtained by passing parameters tu the **RESetMapReduce.run()** method. On can use the three following parameters:

- `max_proc` – maximum number of process used. default: number of processor on the machine
- `timeout` – a timeout on the computation (default: `None`)
- `reduce_locally` – whether the workers should reduce locally their work or sends results to the master as soon as possible. See **RESetMapReduceWorker** for details.

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import RESetMPExample, AbortError
sage: EX = RESetMPExample(maxl = 8)
sage: try:
....:     res = EX.run(timeout=0.1)
....: except AbortError:
....:     print "Computation timeout"
....: else:
....:     print "Computation normally finished"
....:     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: try:
....:     res = EX.run(timeout=60)
....: except AbortError:
....:     print "Computation Timeout"
....: else:
....:     print "Computation normally finished"
....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

As for `reduce_locally`, one should not see any difference, except for speed during normal usage. Most of the time the user should leave it to `True`, unless he set up a mecanism to consume the partial results as soon as they arrive. See **RESetParallelIterator** and in particular the `__iter__` method for a example of consumer use.

Profiling

It is possible the profile a map/reduce computation. First we create a **RESetMapReduce** object:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(
....:     roots = [[]],
....:     children = lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
....:     map_function = lambda x : 1,
....:     reduce_function = lambda x,y: x+y,
....:     reduce_init = 0 )
```

The profiling is activated by the `profile` parameter. The value provided should be a prefix (including a possible directory) for the profile dump:

```
sage: prof = tmp_dir('RESetMR_profile')+'profcomp'
sage: res = S.run(profile=prof) # random
```

```
[RESetMapReduceWorker-1:58] (20:00:41.444) Profiling in /home/user/.sage/temp/mymachine.mysite/32414/RESetMR_profilewRCRAx/profcomp1 ...
...
[RESetMapReduceWorker-1:57] (20:00:41.444) Profiling in /home/user/.sage/temp/mymachine.mysite/32414/RESetMR_profilewRCRAx/profcomp0 ...
sage: res
131071
```

In this example, the profile have been dumped in files such as `profcomp0`. One can then load and print them as follows. See **profile.profile** for more details:

```
sage: import cProfile, pstats
sage: st = pstats.Stats(prof+'0')
sage: st.strip_dirs().sort_stats('cumulative').print_stats() #random
...
   Ordered by: cumulative time

   ncalls  tottime  percall  cumtime  percall filename:lineno(function)
        1    0.023    0.023    0.432    0.432 map_reduce.py:1211(run_myself)
    11968    0.151    0.000    0.223    0.000 map_reduce.py:1292(walk_branch_locally)
...
<pstats.Stats instance at 0x7fedea40c6c8>
```

See also: The Python Profilers for more detail on profiling in python.

Logging

The computation progress is logged through a **logging.Logger** in **sage.parallel.map_reduce.logger** together with **logging.StreamHandler** and a **logging.Formatter**. They are currently configured to print warning message on the console.

See also: Logging facility for Python for more detail on logging and log system configuration.

How does it work ?

The scheduling algorithm we use here is any adaptation of Wikipedia article Work_stealing:

> In a work stealing scheduler, each processor in a computer system has a queue of work items (computational tasks, threads) to perform. [...]. Each work items are initially put on the queue of the processor executing the work item. When a processor runs out of work, it looks at the queues of other processors and “steals” their work items. In effect, work stealing distributes the scheduling work over idle processors, and as long as all processors have work to do, no scheduling overhead occurs.

For communication we use Python’s basic **multiprocessing** module. We first describe the different actors and communications tools used by the system. The work is done under the coordination of a **master** object (an instance of **RESetMapReduce**) by a bunch of **worker** objects (instances of **RESetMapReduceWorker**).

Each running map reduce instance work on a **RecursivelyEnumeratedSet of forest type** called here C and is coordinated by a **RESetMapReduce** object called the *master*. The master is in charge of lauching the work, gathering the results and cleaning up at the end of the computation. It doesn’t perform any computation associated to the generation of the element C nor the computation of the mapped function. It however occasionally perform a reduce, but most reducing is by default done by the workers. Also thanks to the work-stealing algorithm, the master is only involved in detecting the termination of the computation but all the load balancing is done at the level of the worker.

Workers are instance of **RESetMapReduceWorker**. They are responsible of doing the actual computations: elements generation, mapping and reducing. They are also responsible of the load balancing thanks to work-stealing.

Here is a description of the attribute of the *master* relevant to the map-reduce protocol:

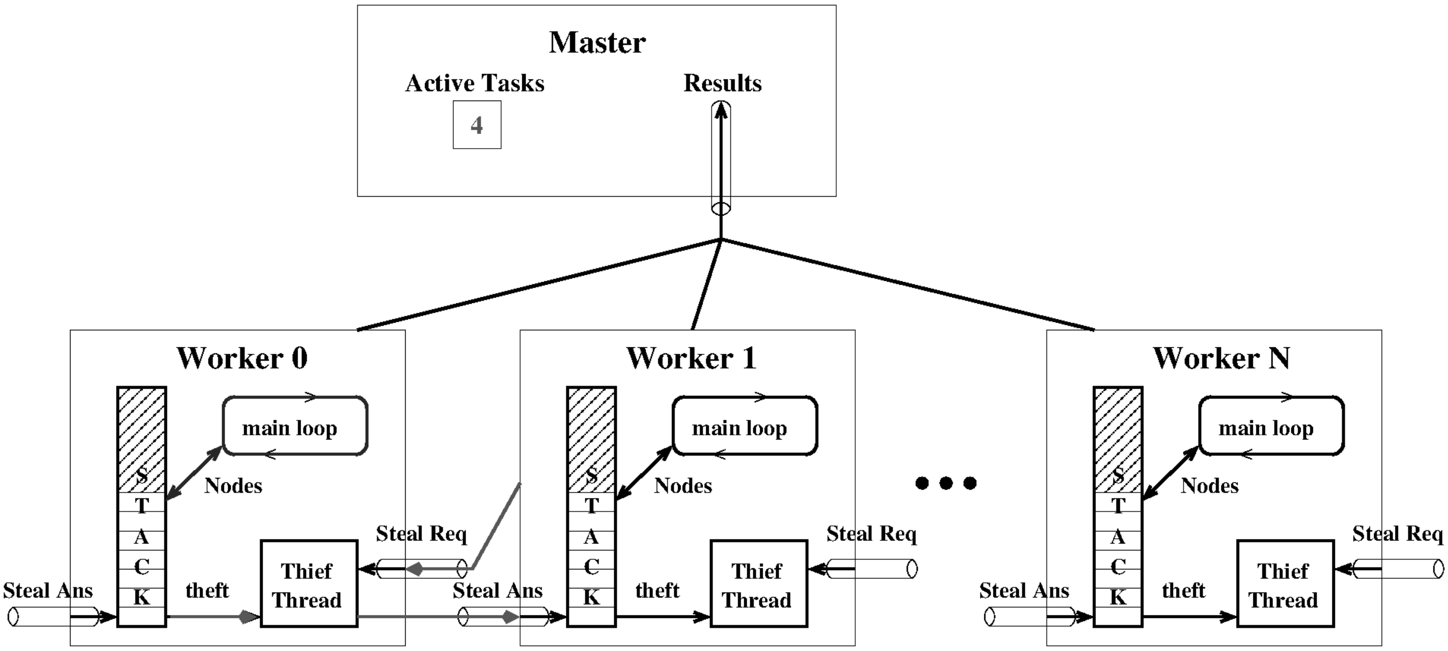
- `master._results` – a **SimpleQueue** where the master gathers the results sent by the workers.
- `master._active_tasks` – a **Semaphore** recording the number of active task. The work is done when it gets to 0.
- `master._done` – a **Lock** which ensures that shutdown is done only once.
- `master._abort` – a **Value()** storing a shared **ctypes.c_bool** which is `True` if the computation was aborted before all the worker runs out of work.
- `master._workers` – a list of **RESetMapReduceWorker** objects. Each worker is identified by its position in this list.

Each worker is a process (**RESetMapReduceWorker** inherits from **Process**) which contains:

- `worker._iproc` – the identifier of the worker that is its position in the master’s list of workers

- worker._todo – a collections.deque storing of nodes of the worker. It is used as a stack by the worker. Thiefs steal from the bottom of this queue.
- worker._request – a **SimpleQueue** storing steal request submitted to worker.
- worker._read_task, worker._write_task – a **Pipe** used to transfert node during steal.
- worker._thief – a Thread which is in charge of stealing from worker._todo.

Here is a schematic of the architecture:



How thefts are performed

During normal time, that is when all worker are active) a worker W is iterating though a loop inside RESetMapReduceWorker.walk_branch_locally(). Work nodes are taken from and new nodes W._todo are appended to W._todo. When a worker W is running out of work, that is worker._todo is empty, then it tries to steal some word (ie: a node) from another worker. This is performed in the RESetMapReduceWorker.steal() method.

From the point of view of W here is what happens:

- W signals to the master that it is idle **master._signal_task_done()**;
- W chose a victim V at random;
- W sends a request to V : it puts its identifier into V._request;
- W tries to read a node from W._read_task. Then three things may happen:
  - a proper node is read. Then the theft was a success and W starts working locally on the received node.
  - None is received. This means that V was idle. Then W tries another victim.
  - AbortError is received. This means either that the computation was aborted or that it simply succeded and that no more work is required by W. Therefore an AbortError exception is raised leading to W to shutdown.

We now describe the protocol on the victims side. Each worker process contains a **Thread** which we call T for thief which acts like some kinds of Troyan horse during theft. It is normally blocked waiting for a steal request.

From the point of view of V and T, here is what happens:

- during normal time T is blocked waiting on V._request;
- upon steal request, T wakes up receiving the identification of W;
- T signal to the master that a new task is starting by **master._signal_task_start()**;
- Two things may happen depending if the queue V._todo is empty or not. Remark that due to the GIL, there is no parallel execution between the victim V and its thief tread T.
  - If V._todo is empty, then None is answered on W._write_task. The task is immediately signaled to end the the master through **master._signal_task_done()**.
  - Otherwise, a node is removed from the bottom of V._todo. The node is sent to W on W._write_task. The task will be ended by W, that is when finished working on the subtree rooted at the node, W will call **master._signal_task_done()**.

The end of the computation

When a worker finishes working on a task, it calls **master._signal_task_done()**. This decrease the counter of the semaphore `master._active_tasks`. When it reaches 0, it means that there are no more nodes: the work is done. The worker executes **master._shutdown()** which sends `AbortError` on all **worker._request()** and **worker._write_task()**. Each worker or thief thread receiving such a message raise the corresponding exception, stoping therefore its work. A lock called `master._done` ensures that shutdown is only done once. Finally, it is also possible to interrupt the computation before its ends calling **master.abort()**.

Are there examples of classes ?

Yes ! Here, there are:

- **RESetMPExample** – a simple basic example
- **RESetParallelIterator** – a more advanced example using non standard communication configuration.

Tests

Generating series for sum of strictly decreassing list of integer smaller than 15:

```
sage: y = var('y')
sage: R = RESetMapReduce(
....:  roots = [([], 0, 0)] +[([i], i, i) for i in range(1,15)],
....:  children = lambda (list, sum, last):
....:      [(list + [i], sum + i, i) for i in range(1,last)],
....:  map_function = lambda (li, sum, unused): y**sum)
sage: sg = R.run()
sage: bool(sg == expand(prod((1+y^i) for i in range(1,15))))
True
```

Classes and methods

exception sage.parallel.map_reduce.**AbortError**

Bases: **exceptions.Exception**

Exception for aborting parallel computations

This is used both as exception or as abort message

TESTS:

```
sage: from sage.parallel.map_reduce import AbortError
sage: raise AbortError
Traceback (most recent call last):
...
AbortError
```

class sage.parallel.map_reduce.**RESetMPExample**(*maxl=9*)

Bases: **sage.parallel.map_reduce.RESetMapReduce**

An example of map reduce class

INPUT:

- `maxl` – the maximum size of permutations generated (default to 9).

This compute the generating series of permutations counted by their size upto size `maxl`.

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample()
sage: EX.run()
362880*x^9 + 40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

See also: This is an example of **RESetMapReduce**

children(*l*)

Return the chidren of the permutation l

INPUT:

- l – a list containing a permutation

OUTPUT:

- the lists of len(l) inserted at all possible positions into l

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().children([1,0])
[[2, 1, 0], [1, 2, 0], [1, 0, 2]]
```

map_function(*l*)

The monomial associated to the permutation l

INPUT:

- l – a list containing a permutation

OUTPUT: x^len(l).

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().map_function([1,0])
x^2
```

roots()

Return the empty permutation

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: RESetMPExample().roots()
[[]]
```

class sage.parallel.map_reduce.**RESetMapReduce**(*roots=None*, *children=None*, *post_process=None*, *map_function=None*, *reduce_function=None*, *reduce_init=None*, *forest=None*)

Bases: **object**

Map-Reduce on recursively enumerated sets

INPUT:

Description of the set:

- either forest=f – where f is a **RecursivelyEnumeratedSet of forest type**
- or a triple roots, children, post_process as follows
  - roots=r – The root of the enumeration
  - children=c – a function iterating through children node, given a parent nodes
  - post_process=p – a post processing function

The option post_process allows for customizing the nodes that are actually produced. Furthermore, if post_process(x) returns None, then x won't be output at all.

Decription of the map/reduce operation:

- map_function=f – (default to None)
- reduce_function=red – (default to None)
- reduce_init=init – (default to None)

See also: **the Map/Reduce module** for details and examples.

abort()

Abort the current parallel computation

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....:   lambda l: [l+[0], l+[1]] if len(l) < 17 else [])
sage: it = iter(S)
sage: it.next()
[]
sage: S.abort()
sage: hasattr(S, 'work_queue')
False
```

Cleanups:

```
sage: S.finish()
```

finish()

Destroys the worker and all the communication objects.

Also gathers the communication statistics before destroying the workers.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=5)
sage: S.setup_workers(2) # indirect doctest
sage: S._workers[0]._todo.append([])
sage: for w in S._workers: w.start()
sage: _ = S.get_results()
sage: S._shutdown()
sage: S.print_communication_statistics()
Traceback (most recent call last):
...
AttributeError: 'RESetMPExample' object has no attribute '_stats'

sage: S.finish()

sage: S.print_communication_statistics()
#proc: ...
...

sage: _ = S.run() # Cleanup
```

See also: print_communication_statistics()

get_results()

Get the results from the queue

OUTPUT:

- the reduction of the results of all the workers, that is the result of the map/reduce computation.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: for v in [1, 2, None, 3, None]: S._results.put(v)
sage: S.get_results()
6
```

Cleanups:

```
sage: del S._results, S._active_tasks, S._done, S._workers
```

map_function(*o*)

Return the function mapped by `self`

INPUT: `o` – a node

OUTPUT: By default `1`.

Note: This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
```

```
sage: S.map_function(7)
1
sage: S = RESetMapReduce(map_function = lambda x: 3*x + 5)
sage: S.map_function(7)
26
```

post_process(*a*)

Return the post-processing function for `self`

INPUT: `a` – a node

By default, returns `a` itself

> **Note:** This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.post_process(4)
4
sage: S = RESetMapReduce(post_process=lambda x: x*x)
sage: S.post_process(4)
16
```

print_communication_statistics(*blocksize=16*)

Print the communication statistics in a nice way

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: S = RESetMPExample(maxl=6)
sage: S.run()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1

sage: S.print_communication_statistics()    # random
#proc:        0    1    2    3    4    5    6    7
reqs sent:    5    2    3   11   21   19    1    0
reqs rcvs:   10   10    9    5    1   11    9    2
- thefs:      1    0    0    0    0    0    0    0
+ thefs:      0    0    1    0    0    0    0    0
```

random_worker()

Returns a random workers

OUTPUT: A worker for `self` chosed at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(2)
sage: EX.random_worker()
<RESetMapReduceWorker(RESetMapReduceWorker-..., initial)>
sage: EX.random_worker() in EX._workers
True
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

reduce_function(*a*, *b*)

Return the reducer function for `self`

INPUT: `a`, `b` – two value to be reduced

OUTPUT: by default the sum of `a` and `b`.

> **Note:** This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
```

```
sage: S = RESetMapReduce()
sage: S.reduce_function(4, 3)
7
sage: S = RESetMapReduce(reduce_function=lambda x,y: x*y)
sage: S.reduce_function(4, 3)
12
```

reduce_init()

Return the initial element for a reduction

> **Note:** This should be overloaded in applications.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.reduce_init()
0
sage: S = RESetMapReduce(reduce_init = 2)
sage: S.reduce_init()
2
```

roots()

Return the roots of `self`

OUTPUT: an iterable of nodes

> **Note:** This should be overloaded in applications.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(42)
sage: S.roots()
42
```

run(*max_proc=None*, *reduce_locally=True*, *timeout=None*, *profile=None*)

Run the computations

INPUT:

- `max_proc` – maximum number of process used. default: number of processor on the machine
- `reduce_locally` – See **RESetMapReduceWorker** (default: `True`)
- `timeout` – a timeout on the computation (default: `None`)
- `profile` – directory/filename prefix for profiling, or `None` for no profiling (default: `None`)

OUTPUT:

- the result of the map/reduce computation or an exception **AbortError** if the computation was interrupted or timeout.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 8)
sage: EX.run()
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Here is an example or how to deal with timeout:

```
sage: from sage.parallel.map_reduce import AbortError
sage: try:
....:     res = EX.run(timeout=0.1)
....: except AbortError:
....:     print "Computation timeout"
....: else:
....:     print "Computation normally finished"
....:     res
Computation timeout
```

The following should not timeout even on a very slow machine:

```
sage: from sage.parallel.map_reduce import AbortError
sage: try:
....:     res = EX.run(timeout=60)
....: except AbortError:
....:     print "Computation Timeout"
```

```
....: else:
....:     print "Computation normally finished"
....:     res
Computation normally finished
40320*x^8 + 5040*x^7 + 720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

run_serial()

Serial run of the computation (mostly for tests)

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample
sage: EX = RESetMPExample(maxl = 4)
sage: EX.run_serial()
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

setup_workers(*max_proc=None*, *reduce_locally=True*)

Setup the communication channels

INPUT:

- `mac_proc` – an integer: the maximum number of workers
- `reduce_locally` – whether the workers should reduce locally their work or sends results to the master as soon as possible. See `RESetMapReduceWorker` for details.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce()
sage: S.setup_workers(2)
sage: S._results
<multiprocessing.queues.SimpleQueue object at 0x...>
sage: len(S._workers)
2
```

start_workers()

Lauch the workers

The worker should have been created using `setup_workers()`.

TESTS:

```
sage: from sage.parallel.map_reduce import RESetMapReduce
sage: S = RESetMapReduce(roots=[])
sage: S.setup_workers(2)
sage: S.start_workers()
sage: all(w.is_alive() for w in S._workers)
True

sage: sleep(1)
sage: all(not w.is_alive() for w in S._workers)
True
```

Cleanups:

```
sage: S.finish()
```

class sage.parallel.map_reduce.**RESetMapReduceWorker**(*mapred*, *iproc*, *reduce_locally*)

Bases: **multiprocessing.process.Process**

Worker for generate-map-reduce

This shouldn't be called directly, but instead created by `RESetMapReduce.setup_workers()`.

INPUT:

- `mapred` – the instance of `RESetMapReduce` for which this process is working.
- `iproc` – the id of this worker.
- `reduce_locally` – when reducing the results. Three possible values are supported:
    - `True` – means the reducing work is done all locally, the result is only sent back at the end of the work. This ensure the lowest level of communication.
    - `False` – results are sent back after each finished branches, when the process is asking for more work.

run()

The main function executed by the worker

Calls run_myself() after possibly setting up parallel profiling.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])

sage: w.run()
sage: sleep(1)
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

run_myself()

The main function executed by the worker

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(1)

sage: w = EX._workers[0]
sage: w._todo.append(EX.roots()[0])
sage: w.run_myself()

sage: sleep(1)
sage: w._todo.append(None)

sage: EX.get_results()
720*x^6 + 120*x^5 + 24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

Cleanups:

```
sage: del EX._results, EX._active_tasks, EX._done, EX._workers
```

send_partial_result()

Send results to the MapReduce process

Send the result stored in `self._res` to the master an reinitialize it to `master.reduce_init`.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: EX.setup_workers(1)
sage: w = EX._workers[0]
sage: w._res = 4
sage: w.send_partial_result()
sage: w._res
0
sage: EX._results.get()
4
```

steal()

Steal some node from another worker

OUTPUT: a node stolen from another worker choosed at random

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: from threading import Thread
sage: EX = RESetMPExample(maxl=6)
sage: EX.setup_workers(2)
```

```
sage: w0, w1 = EX._workers
sage: w0._todo.append(42)
sage: thief0 = Thread(target = w0._thief, name="Thief")
sage: thief0.start()

sage: w1.steal()
42
```

walk_branch_locally(*node*)

Work locally

Performs the map/reduce computation on the subtrees rooted at `node`.

INPUT: `node` – the root of the subtree explored.

OUTPUT: nothing, the result are stored in `self._res`

This is where the actual work is performed.

EXAMPLES:

```
sage: from sage.parallel.map_reduce import RESetMPExample, RESetMapReduceWorker
sage: EX = RESetMPExample(maxl=4)
sage: w = RESetMapReduceWorker(EX, 0, True)
sage: def sync(): pass
sage: w.synchronize = sync
sage: w._res = 0

sage: w.walk_branch_locally([])
sage: w._res
x^4 + x^3 + x^2 + x + 1

sage: w.walk_branch_locally(w._todo.pop())
sage: w._res
2*x^4 + x^3 + x^2 + x + 1

sage: while True: w.walk_branch_locally(w._todo.pop())
Traceback (most recent call last):
...
IndexError: pop from an empty deque
sage: w._res
24*x^4 + 6*x^3 + 2*x^2 + x + 1
```

class sage.parallel.map_reduce.**RESetParallelIterator**(*roots=None*, *children=None*, *post_process=None*, *map_function=None*, *reduce_function=None*, *reduce_init=None*, *forest=None*)

Bases: `sage.parallel.map_reduce.RESetMapReduce`

A parallel iterator for recursively enumerated sets

This demonstrate how to use `RESetMapReduce` to get an iterator on a recursively enumerated sets for which the computation are done in parallel.

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....:   lambda l: [l+[0], l+[1]] if len(l) < 15 else [])
sage: sum(1 for _ in S)
65535
```

map_function(*z*)

Return a singleton tuple

INPUT: `z` – a node

OUPUT: `(z, )`

EXAMPLE:

```
sage: from sage.parallel.map_reduce import RESetParallelIterator
sage: S = RESetParallelIterator( [[]],
....:   lambda l: [l+[0], l+[1]] if len(l) < 15 else [])
sage: S.map_function([1, 0])
([1, 0],)
```

sage.parallel.map_reduce.**proc_number**(*max_proc=None*)

Computing the number of process used

INPUT:

- `max_proc` – the maximum number of process used

EXAMPLE:

```
sage: from sage.parallel.map_reduce import proc_number
sage: proc_number() # random
8
sage: proc_number(max_proc=1)
1
sage: proc_number(max_proc=2) in (1, 2)
True
```

APPENDIX B. SLIDES OF THE PRESENTATION AT THE JOURNÉE DU GROUPE DE TRAVAIL LAMHA

HPC in Combinatorics : Application of Work-Stealing

Florent Hivert

LRI / Université Paris Sud 11 / CNRS

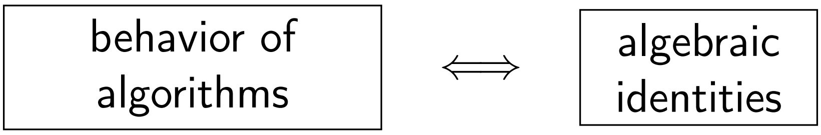
Mars 2015

Outline

1. Background: Sage/OpenDreamKit/HPC in Combinatorics
2. Recursively enumerated sets (RESets)
3. The problem: Map/Reduce on RESets
4. A Python Implementation of Map/Reduce on RESets
5. HPC with Cilk/SIMD

Algebraic combinatorics

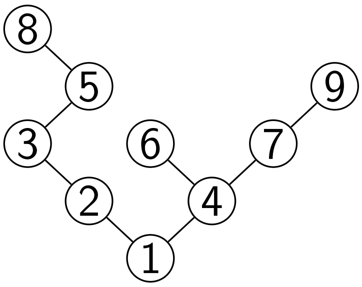
Deep relations



behavior of algorithms ⟺ algebraic identities

Algebraic computation **using** / **on** / **combinatorial objects**

(()(()())(()))(()())



5				
2	6	9		
1	3	4	7	8

Algebraic combinatorics : experimental mathematics

A very large range of tools mathematical tools are needed:

- Manipulation of combinatorial objects:
  - integer partitions, set partitions, permutations, trees, . . .
  - words, languages, automatons, . . .
  - relations, graphs, partial orders, . . .
- fast exact linear algebra over various rings
- commutative or not algebra (polynomials, series, . . .)
- advanced algebraic computations (groups, group algebra, modules . . .).

Together with very good language support for:

- advanced programming concept (objects, aspects, closures . . .).
- basic persistent data structures, databases
- multicore, distributed computation

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Algebraic combinatorics : experimental mathematics

We code primarily for research

- rapid prototyping
- 90% of the code is thrown away

- need high level, expressive language and libraries
- the code should be as close as possible to maths
- mathematical modeling

Combinatorial explosion

- We need the code to be reasonably efficient
- Everything which allows to speed-up high level code is good !

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Today: Map/Reduce of RESets

Perform a map/reduce operation on a very large set described recursively.

- Typically the sets doesn't fit in the computer memory.

- Compute the cardinality
- Compute any kind of generating series
- Test a conjecture : i.e. find an element of S satisfying a specific property, or check that all of them do
- Count/list the elements of S having this property

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Today: Map/Reduce of RESets

Inputs:

A **recursively enumerated set**

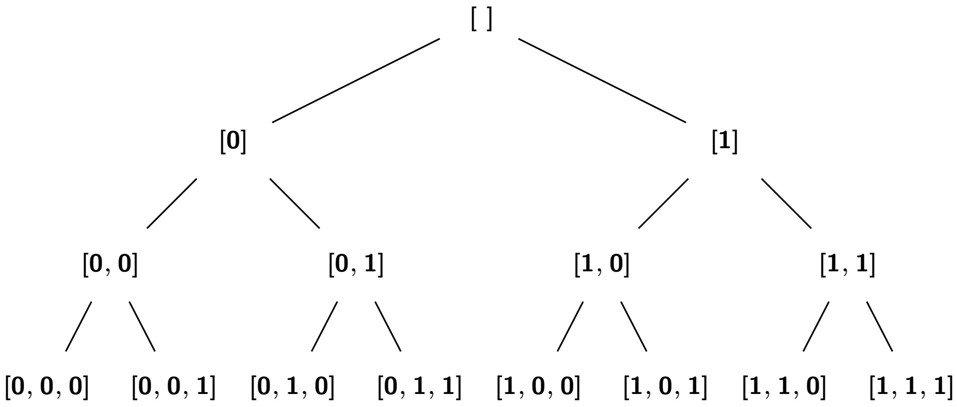
- the `roots` of the recursion
- the `children` function
- the `postprocessing` function

A **Map/Reduce problem**

- the `mapped` function
- the `reduce_init` function
- the `reduce` function

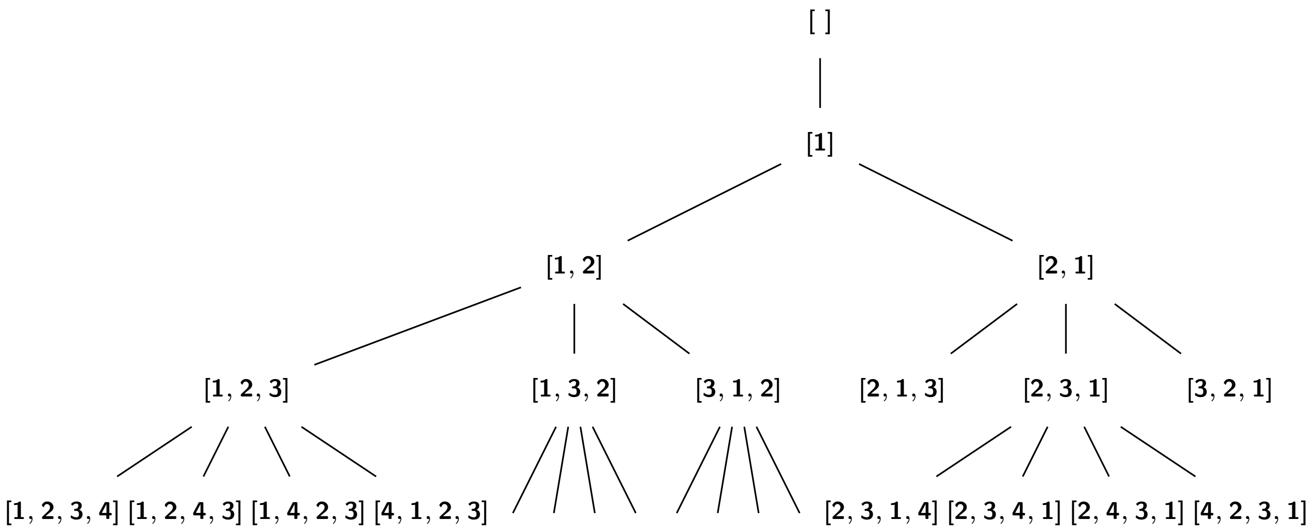
Examples of recursively enumerated sets (RESets)

Binary words: generation tree



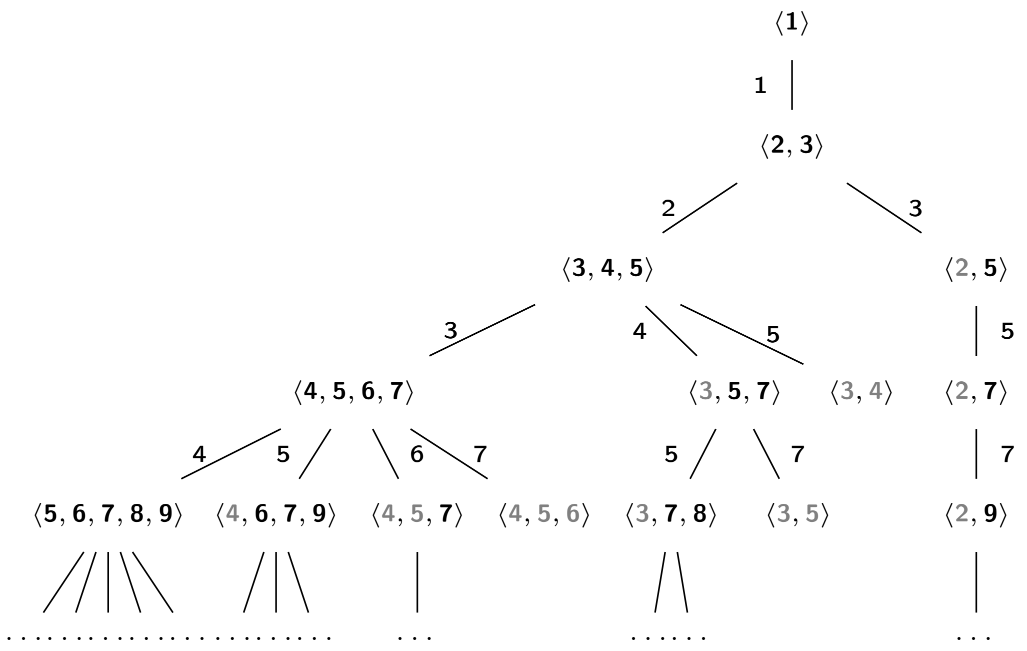
Examples of recursively enumerated sets (RESets)

Permutations: generation tree



Examples of recursively enumerated sets (RESets)

The tree of numerical semigroups



Map/Reduce on RESets

```
sage: S = RecursivelyEnumeratedSet(
....:    [[]],
....:    lambda l: [l+[0], l+[1]] if len(l) <= 15 else [],
....:    structure='forest', enumeration='depth')
sage: S.map_reduce(
....:    map_function = lambda x: 1,
....:    reduce_function = lambda x,y: x+y,
....:    reduce_init = 0 )
131071
```

Parallelism in Python

CPython has a Global interpreter lock (GIL) !

- No Parallel thread execution
  - Note: Python's GC uses reference counting, therefore the destructor `__del__` isn't thread-safe
  - Note: it is possible to release the GIL in C modules

Solution:

- multiprocess with several Python interpreters with IPC
- serialization (pickling in Python's dialect)
- Uses the `multiprocessing` module

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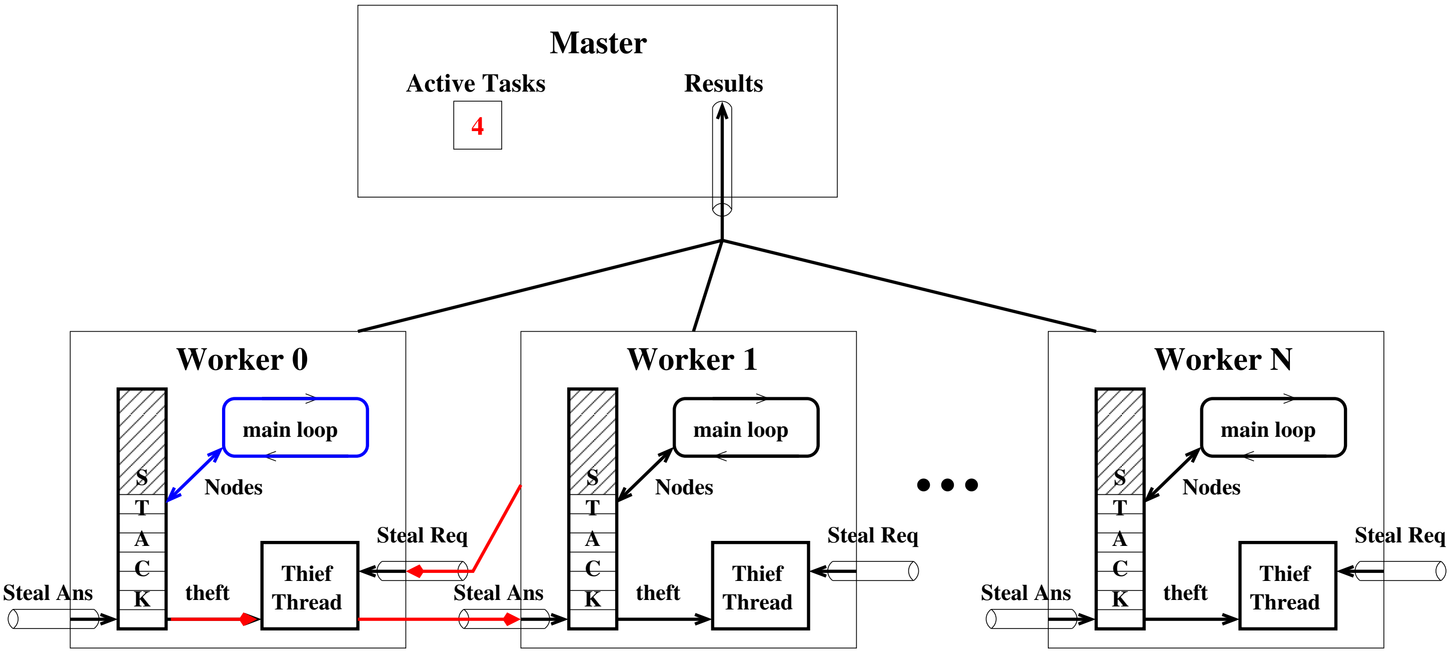
Implantation en Python utilisant multiprocessing

- work stealing algorithm (Leiserson-Blumofe / Cilk)
- one process by worker
- communication by pipes and serialization
- thread used for thief

Work-Stealing System Architecture



When we really need speed !

Cython: optimising static compiler for both **Python** and extended Cython programming language.

- write Python code that calls back and forth from and to C or C++ code natively at any point.
- easily tune readable Python code into plain C performance by adding **static type declarations**.

Cilk: multithreaded parallel computing

- based on the C and C++ programming languages
- constructs to express parallel loops and the fork–join idiom.

Cilk extensions module for Python/Sage

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Cilk extensions module for Python/Sage

Some results !

Computation of 16 days on a AMD Opteron(TM) Processor 6276, 2.3Gz using 32 cores. Generation of 80$Gbytes/s$ of combinatorial objects.

g	n_g	g	n_g	g	n_g
0	1	23	170 963	46	14 463 633 648
1	1	24	282 828	47	23 527 845 502
2	2	25	467 224	48	38 260 496 374
3	4	26	770 832	49	62 200 036 752
4	7	27	1 270 267	50	101 090 300 128
5	12	28	2 091 030	51	164 253 200 784
6	23	29	3 437 839	52	266 815 155 103
7	39	30	5 646 773	53	433 317 458 741
8	67	31	9 266 788	54	703 569 992 121
9	118	32	15 195 070	55	1 142 140 736 859
10	204	33	24 896 206	56	1 853 737 832 107
11	343	34	40 761 087	57	3 008 140 981 820
12	592	35	66 687 201	58	4 880 606 790 010
13	1 001	36	109 032 500	59	7 917 344 087 695
14	1 693	37	178 158 289	60	12 841 603 251 351
15	2 857	38	290 939 807	61	20 825 558 002 053
16	4 806	39	474 851 445	62	33 768 763 536 686
17	8 045	40	774 614 284	63	54 749 244 915 730
18	13 467	41	1 262 992 840	64	88 754 191 073 328
19	22 464	42	2 058 356 522	65	143 863 484 925 550
20	37 396	43	3 353 191 846	66	233 166 577 125 714
21	62 194	44	5 460 401 576	67	377 866 907 506 273
22	103 246	45	8 888 486 816		

The future

- Better integration Sage/Python/Cython/Cilk
- Generation graph (not tree !) in parallel ?
- Trac Ticket 13580
  `http://trac.sagemath.org/ticket/13580`
- *Exploring the Tree of Numerical Semigroups* Jean Fromentin and Florent Hivert
  `https://hal.inria.fr/UNIV-ROUEN/hal-00823339v3`

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