Application for a Dagstuhl Seminar
Integrating Systems for Mathematical Computation

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August 31, 2016

Abstract

[1] The last decades have witnessed the emergence of a full ecosystem of open source software for computational mathematical, developed by overlapping international communities of researchers (in both computer science and mathematics), teachers, engineers, and amateurs. This is part of the greater trend for open and reproducible science. Current systems range from specialized libraries (e.g. MPIR, LinBox) to thematic systems (e.g. GAP, PARI, Singular, xcas) to general purpose systems (e.g. Mathemagix, Sage). Moreover, many related systems have been developed such as online databases (e.g. OEIS, LMFDB), interactive computing environments (e.g., Jupyter), or knowledge management systems (e.g., MathHub). EdN:1

These very dynamic developments are making it more and more difficult to share computations across systems, develop generic support systems, and avoid the duplicate development of libraries. In this seminar, we bring together agents from all communities to develop tools, best practices, and open standards for sharing algorithms and data as well as for relating and combining computation systems.

[2] [3] EdN:2 EdN:3

Contents

[1] EdNote: @all revise

[2] EdNote: deadline is Nov 1

[3] EdNote: See http://www.dagstuhl.de/programm/dagstuhl-seminare/antrag/ for details about how to write proposal

1 Basic Information about the Seminar

1.1 Title

The title *Integrating Computational Mathematics* means to emphasize the vision of a universal environment for mathematical computation. We hope this will provide a joint goal for the currently disparate communities.

1.2 Organizers

[4] EdN:4

- PD Dr. Florian Rabe
  Jacobs University Bremen - Computer Science
  P.O. Box 750 561 28725 Bremen Germany
  Phone: + 49 421 200 3051
  Email: `f.rabe@jacobs-university.de`
  Homepage: http://kwarc.info/people/frabe/

1.3 Type of event, duration, and size

We propose a 5-day Dagstuhl Seminar with 45 participants.

1.4 Topics

[5] EdN:5

- data bases / information retrieval
- data structures / algorithms / complexity
- modelling / simulation
- semantics / formal methods
- sw-engineering
- verification / logic

1.5 Keywords

[6] EdN:6

- computer algebra
- computation
- reusability
- interoperability

1.6 Proposed Seminar Dates

[7] EdN:7

Block-out Dates:

-

Preferred Dates:

-

[4] EdNote: @all: add yourself
[5] EdNote: @all: we have to select 1-3 from a list of topics; strangely none fits very well
[6] EdNote: @all: revise/extend
[7] EdNote: @all: Dagstuhl will pick the date, typically early 2018, but it could be be several months earlier or later; please add your preferences

2 Description of the Seminar

BOP:8

The main goal of this conference, cofunded by the H2020 European E-Infrastructure project OpenDreamKit, is community building and training:

- Bringing together the various communities of users and developers of the ecosystem of (open source) (pure) mathematics software.
- Give newcomers as well as experts an overview of this ecosystem: what the existing software systems are, what they can compute or solve, how they are developed and by whom, what the success stories and difficulties are.
- Train newcomers as well as experts on using this ecosystem to solve their own problems.
- Share perspectives and best practices, build a joint vision, and seek venues for tighter cooperation.
- Encourage participants to get involved, especially women and more junior researchers, in particular by showcasing role models.

2.1 The ecosystem of open-source mathematical software

From their earliest days, computers have been used in pure mathematics, either to make tables, to prove theorems (famously the four colour theorem) or, as with the astronomer's telescope, to explore new theories. Computer-aided experiments, and the use of databases relying on computer calculations such as the Small Groups Library in GAP, the Modular Atlas in group and representation theory, or the L-functions and Modular Forms Database (LMFDB, see later), are part of the standard toolbox of the pure mathematician, and certain areas of mathematics completely depend on it. Computers are also increasingly used to support collaborative work and education.

The last decades witnessed the emergence of a wide ecosystem of open-source tools to support research in pure mathematics. This ranges from specialized to general purpose computational tools such as GAP, PARI/GP, LinBox, MPIR, Sage, or Singular, via online databases like the LMFDB and does not count online services like Wikipedia, arXiv, or MathOverflow. A great opportunity is the rapid emergence of key technologies, and in particular the Jupyter (previously IPython) platform for interactive and exploratory computing which targets all areas of science. This has proven the viability and power of collaborative open-source development models, by users and for users, even for delivering general purpose systems targeting a large public (researchers, teachers, engineers, amateurs, ...).

An exemplary success in the last decade is that of Sage, a free general purpose open-source mathematics software system licensed under the GPL whose mission is to create a viable free open source alternative to Magma, Maple, Mathematica and Matlab. It has been developed since 2005 by a growing worldwide community of about 150 researchers and teachers. It builds on top of many existing open-source packages, including NumPy, SciPy, matplotlib, Sympy, Maxima, and the aforementioned ones, all accessible from a Python-based library containing itself many unique mathematical features. Sage can also be used as batch program, through the Jupyter interactive computing interface (command-line or graphical), or on the cloud.

Thanks to this, Sage is regularly used in universities, both for research and education pruposes. An indicator of its success in the French system is that Sage has been since 2014 in the shortlist of official software for the oral examinations of the Agrgation de Mathmatiques (the nation-wide recruiting competition for high school teachers), and that since 2015 this list has been containing only open source software.

[8] Old Part: This is from a thematically but not formally related workshoop proposal. Some text may be reusable.

2.2 Upcoming challenges

Some of the upcoming major challenges are:

- Lower the entry barrier, in particular via **unified user interfaces**, and **Virtual Research Environments** that groups of users can setup to collaborate on data, software, computations, or knowledge;
- Further enable **computations involving multiple systems**, as transparently as possible;
- Keep the development efforts manageable as the size and complexity of software systems increase;
- Train a new generation of users and developers.

A key step is to strengthen collaborations *between* the various communities, in order to:

- Seek for opportunities for collaboration or outsourcing of components to save on development efforts;
- Share expertise and best practices;
- Improve cross-systems development workflows.

2.3 Goals: community building and training

The conference goals can be summarized as follows:

- Bringing together the community of users and developers of the ecosystem of (open source) (pure) mathematics software;
- Give newcomers as well as specialists an overview of this ecosystem: what are the existing software systems, what sorts of problems can be solved thanks to them, how they are developped and by whom;
- Train a new generation of users (and the current generations as well) on using this ecosystem to solve their own problems;
- Encourage people to get involved and contribute, especially women and junior researchers, in particular by exposing them with role models;
- Share perspectives and best practices;
- Build a joint vision and seek venues for tighter cooperation.

2.4 Public and prerequisites

According to the above objectives, the public is meant to be diverse:

- Newcomers, and especially **graduate students and young researchers**, that need to train themselves with computational tools for their teaching or research;
- More advanced users that want to get an overview on the ecosystem or benefit from expert advice;
- Contributors of the different systems of various levels of expertise that want to share their knowledge, learn more, and participate to coding sprints, in particular on cross-system features.

It's to be expected that most participants will have a strong background in pure mathematics or computer science, although engineers, teachers, or researchers in neighboring fields (e.g. physics) having a need for computational mathematics tools will be welcome.

Generally speaking there will be little prerequisites besides a strong desire to learn, get one's hands dirty, work in team, and possibly get involved. It's expected also that the participants will come with their own laptops. The software will be available either for direct installation or for remote usage through the web.

2.5 A cofunding opportunity: OpenDreamKit

OpenDreamKit is a Horizon 2020 European Research Infrastructure project (#676541) funded under the H2020-EINFRA-2015-1 call, that is running for four years, starting in September 2015. It will provide substantial funding to the open source computational mathematics ecosystem, and in particular popular tools such as LinBox, MPIR, Sage, GAP, PARI/GP, LMFDB, Singular, MathHub, and the IPython/Jupyter interactive computing environment.

From this ecosystem, OpenDreamKit will deliver a flexible toolkit enabling research groups to set up Virtual Research Environments, customised to meet the varied needs of research projects in pure mathematics and applications, and supporting the full research life-cycle from exploration, through proof and publication, to archival and sharing of data and code.

The OpenDreamKit consortium consists of core European developers of the aforementioned systems for pure mathematics, and extending toward the numerical community, and in particular the Jupyter community, to work together on joint needs.

Community building, dissemination and training is at the heart of OpenDreamKit. From the beginning of the design of the proposal, back in 2014 (during ALEA 2014 at CIRM!), it was planned to fund yearly large meetings to reach out to the larger community, disseminate the outcome, and train new users and developers. Besides CIRM in 2018, we are aiming for meetings in Dagstuhl, and ICMS (Edimburgh).

2.6 The Sage Days tradition

The success of any research software or service is strongly linked to its ability to attract and retain a large number of users. The different communities (Sage, GAP, PARI, Singular, Jupyter, ...) have each developed sustainable networks. For example, Sage has accumulated thousands of users in under 10 years.

This has been achieved thanks to a very strong community building philosophy, especially through the organization of Sage Days all over the world. The first Sage Days was held in 2006 with 10 participants; to date there has been at least 77 of them, including about ten in 2015. Those workshops are typically week-long and cover a wide variety of topics, ranging from focused workshops gathering a dozen developers for coding sprints on specific mathematical or technical features to training workshops with more than 80 people. To cite just a few of the recent or upcoming ones:

- Sage Education Days 6 (June 16-18, 2014, University of Washington, Seattle)
- Sage Days 69 (September 4-9, 2015, San Diego): Women in Sage 6
- Sage Days 73 (May 04-07, 2016, Oaxaca (Mexico)): Translation surfaces
- Sage Days 77 (April 04-08, 2016. Cernay, Paris area, France): Packaging, portability, documentation tools
- Sage Days 79 (November 21-25, Jerusalem, Israel): Combinatorics

All those workshops have in common to be highly hands on, with lots of room for tutorials and collaborative work, and a strong dynamic (project sessions, status reports, ...) to get everyone involved.

2.7 Previous Sage Days at CIRM

One particularly successful workshop, Sage Days 20, was organized at CIRM in 2010, at the occasion of the thematic month Math-Info. This workshop had a double focus on training and research. It was one of the largest Sage Days, with about 80 participants, many of which became at this occasion regular Sage users, if not contributors. One of the many outcome was a strong initial impetus to what was to become the first book about Sage: Calcul Mathmatique avec Sage, in French and under an open source license. Also a lot of training material was written during the workshop and reused extensively in followup workshops; a large chunk of this material was integrated in the Sage official documentation in the form of thematic tutorials.

The commodities at CIRM turned out just right for the purpose, and since then the proposers have wished to organize another one.

2.8 Composition of the committees, invited speakers, and gender issues

All the members of the Organization and Scientific committees have a strong experience with Sage Days or similar events. For example, Nicolas Thiry attended 20+ Sage Days, was main organizer or coorganizer of more than a dozen of them, and invited speaker in many of the others, including that at CIRM. They are also involved in the OpenDreamKit project and present a variety of perspectives.

Alas, our field suffers very badly from the lack of gender parity (5 to 10% of female researchers). Despite our best efforts this is reflected by the composition of the committees.

We have focused on showcasing female role models in the keynote speakers (Marie Franoise Roy and Anne Schilling and we are seeking for another one), and for maintaining a proper ratio among tutorial leaders. We will also take measures to attract and promote female participants, and in particular PhD students and Postdocs.

2.9 Joint dynamics with JNCF 2018 (or ALEA 2018)

The communities involved in open source mathematical software are strong in Europe. There is in particular a strong overlap with the French Computer Algebra community. We are therefore coordinating the organization of this conference with the "Journes Nationales du Calcul Formel 2018", in the hope to organize them one right after the other. This will encourage participants to attend both events, reduce travel costs, and strongly increase the international attractiveness.

In case this turns out to not be possible, there is also a good overlap with the ALEA community, and it would be fruitful to organize this event next to ALEA 2018.

2.10 Hands-on tutorials

We will reserve 6-8 slots for hands-on tutorial on the various software of the ecosystem. Each tutorial will be delivered by experts (typically one leader and several helpers). After a brief overview, the participants will be guided through a collection of exercise worksheets, designed to accommodate various levels of expertise. The participants will be able to keep working on them after the tutorials, with informal help from the experts whenever needed.

Many such worksheets have been crafted over time at the occasion of previous such meetings (including several of Sage's thematic tutorials). This workshop will be the occasion to polish them, design new ones, and possibly submit some to become Software Carpentry lessons.

Here are some tentative titles:

- Software installation
- Jupyter notebooks and reproducible research
- Introduction to GAP
- Introduction to Linbox
- Introduction to Mathemagix
- Introduction to Pari/GP
- Introduction to Sage
- Introduction to Singular
- Introduction to XCas
- Programming in Python
- Collaborative software development (git, ...)

2.11 Round tables and special event for teaching

We are considering the organization of a few round tables to run debates and discussions on topics such as:

- Women in open source mathematics software (tentatively led by Marie-Franoise Roy);
- Development models, best practices, and funding for open source mathematics software development;
- Software for teaching;
- ...

We will also reach toward local high-school, *classes prparatoires*, and university teachers and, pending enough interest, organize one half-day event focused on teaching (as in the previous Sage Days at CIRM).

2.12 Collaborative work and coding sprints

A lot of free time will be reserved each day, especially at the end of the week, for collaborative work. To support the self-organization of this time, there will be regular plenary discussions where participants will briefly expose the projects they want to work on and call for collaborators. Typical projects will be of the form:

- I would like to train myself further on XXX by going through worksheet YYY; who would like to join?
- I would like to learn more about XXX; who else would be interested? Who could deliver a tutorial?
- For my research, I am writing a program that XXX; I would need expert help on using YYY for this.
- We want to add feature XXX to YYY; who wants to join?

A list of projects, and related progress report, will be maintained on the conference web page (typically through a wiki). Participants will be able to start suggesting projects in the months before the conference, and keep adding more during the conference. EOP:8

This application proposes a meeting bringing together researchers working in computational logics and automated formal proof. In Section 6.1, we give an overview on the seminar topics and on the different current approaches. Then, Section 6.2 describes the goals of the seminar. BOP:9

2.13 Topics of the seminar

Proof systems are programs that allow a user to build formal proofs either interactively or automatically. Building such a formal proof is always difficult. For instance the Feit-Thompson odd order theorem, the CompCert verified C compiler, the seL4 verified operating system micro-kernel and the proof of the Kepler conjecture, required several years of medium to large teams to be completed.

A formal proof is defined as a package (disseminated in different files or modules) containing one or more theories defining the objects or concepts required to express the statement to be proved, the statement and a proof presented as a proof term, a list of proof commands or a structured text according to the proof system. Generally the proof requires intermediate lemmas or sub-proofs. In this seminar we focus on proof systems that provide means to verify the produced proofs (e.g. by type-checking proof terms or applying correct proof rules).

Usually a formal proof is developed in one logical formalism, with one specific proof systems, it may even be specific to one particular version of a proof system. The choice of a proof system can be guided by a deep scientific reason (e.g. the underlying logics), a development strategy (e.g. extraction of code or not) or the available surrounding management tools (e.g. conjecture checkers, automatic decision procedures). Some sub-proofs can be delegated to automatic provers like SAT/SMT solvers, this is largely used in Isabelle or proof platforms such as Why3. In that case, the question of the validity of the whole proof quickly arises. Different solutions are then proposed to recover confidence over all parts of the proofs: the automatic prover produces a proof that can be checked or it provides some information that allow another tool to reconstruct a proof in the main host proof system.

This use of automatic provers can be considered as a first step to multi-formalisms formal proofs guided by the purpose of obtaining more automation. Another important objective to go through multi-formalisms formal proofs is reuse: reusing a theory, a proof, a proof schema done in another context (logics or proof system) in a formal proof, thus allowing *off-the-shelf* formal components, *proving in the large* and *interoperability* of theories and proofs, quoting software engineering techniques. It is a real challenge the formal proof community is currently working on with different objectives, approaches and efforts. Many partial answers in several directions exist.

Porting formal proofs between theorem provers can be carried out manually or with the help of a (semi-)automatic translation using for example an intermediate XML language (Isabelle/HOL to ACL2 - Aransay et al. 2012) or an external tool (HOL to Coq - Denney 2000). Different point to point translations have been proposed allowing porting of specifications, proof statements and/or proofs, with a deep or shallow embedding, filling the gap between two proof systems. Relying on the Coq reflection, Keller and Werner proposed a translation both able to restore theorems meanings in Coq and give a small proof of them. However these translations, in particular shallow embeddings, can be difficult if not impossible if the two proof systems use very different foundations. A particular effort has been brought on the interoperability of higher order logic based proof systems with the design of the OpenTheory format (Hurd - 2011). It allows HOL Light, HOL4 and ProofPower to share specifications and proofs. In this context,

[9] Old Part: This is from a successful Dagstuhl proposal on integrating proof systems. The structure may be reusable.

interoperability is achieved as soon as a theory in one of these proof systems uses only the standard theory library in OpenTheory format. A large research effort over the last two decades concerns logical frameworks and meta-languages that provide a solid basis for representing, implementing, and reasoning about a wide variety of deductive systems. LF (Harper, Honsell and Plotkin 1993), Twelf (Pfenning and Schuermann 1999), Beluga (Pientka and Dunfield 2010), Dedukti (Cousineau and Dowek 2007), MMT (Rabe 2013) are such logical frameworks in where many formalisms can be defined. When systems are defined in a given logical framework, they are embedded in a same meta-formalism, e.g. the lambda-calculus with dependent types, and thus proofs in those systems can be studied or assembled. Frameworks like MMT feature transformation operators to go from a logic to another. Dedukti is not only a logical framework but also a checker for proofs coming from some automated theorem provers (e.g. Zenon, iProver) and some proof assistants (Coq, Matita, HOL and FoCaliZe). But in practice, current frameworks do not scale well and usually interact poorly with proof systems developed outside the logical framework. Furthermore as for point to point translations, logical frameworks and mea-languages have been designed with different purposes and often guided by application domains and special circumstances.

Achieving a better interoperability between proof systems also relies on techniques that adapt and package together existing formalizations, inside a given proof system. Most of proof systems provide modularity, inheritance, parametrization or refinement to help reusing a formal development. However it is not always sufficient because we may need to change a representation, pick some elements here and there or extend an open inductive types. Solutions have been proposed to provide some flexibility, e.g., features *à la carte* (Delaware 2013) for formal proofs about language meta-theory. However there is no overarching, general foundation and methodology.

The previous mentioned approaches clearly bring altogether some progress to the problem of making proof assistants interoperate. We believe that bringing together experts in the above topics, to exchange experiences and insights, will be fruitful.

2.14 Goals of the seminar

Conference, seminars, etc. often focus on the exchange of ideas. It is of course one of the goals of our proposed semainar, in particular brushing a precise state-of-the-art of the above topics. However our goals go beyond that: we wold like to develop a common objective of universality that can unify the communities. In the long run, this requires designing uniform conceptualizations, interchange formats, and interoperability layers. Therefore, **the goal of this seminar is to systematically identify the current obstacles to universality, to collect requirements for universality, and to sketch out future solutions**.

Research Questions Participants will be asked to give short talks that specifically address the following research questions from the perspective of their field:

- Why are current systems not more interoperable? What design changes are necessary to increase interoperability in the future?
- What are the current approaches towards interoperability? How successful or promising are they?
- How can correctness be guaranteed in a distributed setting? Should there be a single universal checker (which would be hard to agree on) or many decentral ones (which may preclude interoperability)?
- How can we design interchange languages that naturally subsume existing (and future!) formal systems?

- Should a logical framework permit the definition of any logical system? Or do the logics currently implemented have points in common that could be hard wired into the framework itself?
- How reasonnable is it to propose a single universal proof format? Or do we need different formats for different families of proof systems and a partial interoperability between the formats? How should a proof format be evaluated (generality, conciseness, efficiency of proof-checking, ...)?
- How should universal proof library be exchanged? Is Web technology sufficient or do we need specific tools to organize data bases of proofs?
- How can we practically and reliably relate individual systems with their representation in an interchange format or a logical framework? How can two systems agree on the meaning of an exchanged theorem and thus trust each other?

Impact on the Research Community By challenging participants to address research questions concerning universality, we do not only raise awareness of the importance of these issues. We also help identify the key steps towards *proving in the large* and *universality* of proofs. This will allow the development of a common objective and framework for interoperable and reusable proof development that is crucial for realizing the full potential of formal mechanizations.

This seminar with the associated Dagstuhl proceedings will provide an overview of the problem, the state of the art of current solutions and the active researchers pursuing them, and the most promising ideas for future solutions. It will collect and strengthen the small, often-disparate communities that currently work towards universality, e.g., in the very different PxTP (Proof eXchange for Theorem Proving) and LFMTP (dedicated to logical frameworks and meta-languages) workshops.

The seminar will not only allow for cross-fertilization between

- research on logical frameworks, proof formats, logics, proof engineering, mathematics formalization, and program verification,
- foundational research on these topics and application or system-oriented approaches.

It will also structure and streamline future collaboration, e.g., by kicking off new workshops or large international grant proposals. EOP:9

2.15 Relation to Previous Dagstuhl Seminars

3 List of Potential Participants

We have identified [10] relevant researchers with a particular focus on EdN:10

- including leading experts from all involved communities,
- bringing together researchers with particular interest in universality and interoperability.

[11] EdN:11

We carefully selected participants to cover the following areas:

- computation systems
  - symbolic computation
  - exact computation
  - numerical computation
- related systems
  - user interfaces

[10] EdNote: add number

[11] EdNote: @all: revise areas and subareas depending on what people we want; the topics listed here will be referenced in the spreadsheet containing the suggested participants

    - databases
    - knowledge bases
- applications
    - mathematical databases
    - scientific computation
    - industrial applications
- system integration
    - integration frameworks
    - individual system connections

Moreover, the participants include[12] EdN:12

- XXX from Germany, XXX from France, XXX from UK, XXX from the rest of Europe, XXX from North America
- XXX junior, XXX female (one of them co-organizer), and XXX industrial researchers.

4 Information on the Organizers

[13] EdN:13

4.1 Brief presentation of the organizers

Florian Rabe has developed the MMT framework and the LF module system and is the main contributor of the LATIN atlas. He has extensive expertise in individual deduction systems including major case studies regarding Mizar, HOL Light, PVS, and TPTP.

[12] EdNote: add numbers at the end

[13] EdNote: add a 0.5-1 page research CV of yourself: overview of an organizer's academic career, especially points out community services and recognitions, list the five most relevant papers

4.2 Florian Rabe - CV

Name	Dr. habil. Florian Rabe, born 1979-09-28
Diploma	Computer science 2004, Universitity of Karlsruhe (distinction)
PhD	Computer sciecne 2008, Jacobs University Bremen (distinction)
Habilitation	Computer sciecne 2014, Jacobs U.
Employment	2008–2014, post-doctoral fellow, Jacobs U.
	2014– DFG-Eigene Stelle, Jacobs U.
Awards and Scholarships	
2005	Best diploma thesis, Computer science faculty
2006	PhD scholarship (1 year), DAAD
2006	Winner Modal Logic $100 Challenge
2007–2008	PhD scholarship, German Merit Foundation
2010	Best Paper Award, MKM conference
2015	Contest Winner "The Future of Logic", UniLog Congress
Membership in Academic Self-governance Committees	
2008 – 2010	Staff Council (Jacobs U.)
2010 - 2012	Provost search committee (Jacobs U.)
2011 - 2012	Constitution committee (Jacobs U.)
2010 – 2013	Board of trustees of MKM interest group
2012 –	Steering committee of CICM conference
Student Advising	14 BSc., 6 MSc., 3 PhD. (some in progress)
Organization	2 conferences, 4 workshops
PC Membership	8 conferences (2 as track chair), 11 workshops (4 as chair)
Third Party Funding	
2009–2012	LATIN (DFG), de-facto PI
2014–2017	OAF (DFG), lead PI
2015–2019	OpenDreamKit (EU Horizon 2020), PI

5 **Important Publications**

- M. Kohlhase and F. Rabe. QED Reloaded: Towards a Pluralistic Formal Library of Mathematical Knowledge. *Journal of Formalized Reasoning*, 2015. accepted pending minor revisions; see `http://kwarc.info/frabe/Research/KR_qed_14.pdf`.
- F. Rabe. The Future of Logic: Foundation-Independence. *Logica Universalis*, 2015. Winner of the Contest "The Future of Logic" at the World Congress on Universal Logic; to appear; see `http://kwarc.info/frabe/Research/rabe_future_15.pdf`.
- F. Rabe. How to Identify, Translate, and Combine Logics? *Journal of Logic and Computation*, 2014. doi:10.1093/logcom/exu079.
- F. Rabe and M. Kohlhase. A Scalable Module System. *Information and Computation*, 230(1):1–54, 2013.
- F. Horozal and F. Rabe. Representing Model Theory in a Type-Theoretical Logical Framework. *Theoretical Computer Science*, 412(37):4919–4945, 2011.