Task

- Determine the RE of the language, defined over $\Sigma=\{a, b\}$ of **words beginning with a**.

Solution:

The required RE may be $a(a+b)^*$

- Determine the RE of the language, defined over $\Sigma=\{a, b\}$ of **words beginning with and ending in same letter.**

Solution:

The required RE may be $(a+b)+a(a+b)^*a+b(a+b)^*b$

Task Continued ...

- Determine the RE of the language, defined over $\Sigma=\{a, b\}$ of **words ending in b.**

<u>Solution:</u>

The required RE may be

$(a+b)^*b$.

- Determine the RE of the language, defined over $\Sigma=\{a, b\}$ of **words not ending in a.**

<u>Solution:</u> The required RE may be

$(a+b)^*b + \Lambda$ Or $((a+b)^*b)^*$

An important example

The Language EVEN-EVEN :

Language of strings, defined over $\Sigma=\{a, b\}$ having **even number of a's and even number of b's**. *i.e.*

EVEN-EVEN = {Λ, aa, bb, aaaa,aabb,abab, abba, baab, baba, bbaa, bbbb,…} ,

its regular expression can be written as

$(aa+bb+(ab+ba)(aa+bb)^*(ab+ba))^*$

Note

- It is important to be clear about the difference of the following regular expressions

  $r_1=a^*+b^*$

  $r_2=(a+b)^*$

  Here r_1 does not generate any string of concatenation of a and b, while r_2 generates such strings.

Equivalent Regular Expressions

- **Definition:**

  Two regular expressions are said to be equivalent if they generate the same language.

  Example:

  Consider the following regular expressions

  $r_1 = (a + b)^* (aa + bb)$

  $r_2 = (a + b)^* aa + (a + b)^* bb$ then

  both regular expressions define the language of strings **ending in aa or bb**.

Note

⌘ If $r_1 = (aa + bb)$ and $r_2 = (a + b)$ then

1. $r_1 + r_2 = (aa + bb) + (a + b)$
2. $r_1 r_2 = (aa + bb)\ (a + b)$
   $= (aaa + aab + bba + bbb)$
3. $(r_1)^* = (aa + bb)^*$

Regular Languages

- **Definition**:

  The language generated by any regular expression is called a **regular language**.

  It is to be noted that if r_1, r_2 are regular expressions, corresponding to the languages L_1 and L_2 then the languages generated by $r_1 + r_2$, r_1r_2(or r_2r_1) and r_1^*(or r_2^*) are also regular languages.

Note

- It is to be noted that if L_1 and L_2 are expressed by $r_1$and r_2, respectively then the language expressed by
  1) $r_1 + r_2$, is the language $L_1 + L_2$ or $L_1 \cup L_2$
  2) $r_1 r_2$, , is the language $L_1 L_2$, of strings obtained by prefixing every string of L_1 with every string of L_2
  3) r_1^*, is the language L_1^*, of strings obtained by concatenating the strings of L, including the null string.

Example

- If r_1=(aa+bb) and r_2=(a+b) then the language of strings generated by r_1+r_2, is also a regular language, expressed by (aa+bb)+(a+b)
- If r_1=(aa+bb) and r_2=(a+b) then the language of strings generated by r_1r_2, is also a regular language, expressed by (aa+bb)(a+b)
- If r=(aa+bb) then the language of strings generated by r^*, is also a regular language, expressed by $(aa+bb)^*$

All finite languages are regular.

Example:

Consider the language L, defined over $\Sigma=\{a,b\}$, of strings of length 2, **starting with a**, then

L={aa, ab}, may be expressed by the regular expression aa+ab. Hence L, by definition, is a regular language.

Note

It may be noted that if a language contains even thousand words, its RE may be expressed, placing ‘ + ’ between all the words.

Here the special structure of RE is not important.

Consider the language L={aaa, aab, aba, abb, baa, bab, bba, bbb}, that may be expressed by a RE aaa+aab+aba+abb+baa+bab+bba+bbb, which is equivalent to (a+b)(a+b)(a+b).