Theory Of Automata

- Example: The language **factorial**, of strings defined over $\Sigma=\{1,2,3,4,5,6,7,8,9\}$ *i.e.* $\{1,2,6,24,120,\ldots\}$
- Example: The language **FACTORIAL**, of strings defined over $\Sigma=\{a\}$, as $\{a^{n!} : n=1,2,3,\ldots\}$, can be written as $\{a, aa, aaaaaa, \ldots\}$. It is to be noted that the language FACTORIAL can be defined over any single letter alphabet.

- Example: The language **DOUBLEFACTORIAL**, of strings defined over $\Sigma=\{a, b\}$, as

  $\{a^{n!}b^{n!} : n=1,2,3,\ldots\}$, can be written as

  $\{ab, aabb, aaaaaabbbbbb,\ldots\}$
- Example: The language **SQUARE**, of strings defined over $\Sigma=\{a\}$, as

  $\{a^{n^2} : n=1,2,3,\ldots\}$, can be written as

  $\{a, aaaa, aaaaaaaaa,\ldots\}$

- Example: The language **DOUBLESQUARE**, of strings defined over $\Sigma=\{a,b\}$, as $\{a^{n^2} b^{n^2} : n=1,2,3,\ldots\}$, can be written as

  {ab, aaaabbbb, aaaaaaaaabbbbbbbbb,…}

- Example: The language **PRIME**, of strings defined over $\Sigma=\{a\}$, as

  $\{a^p : p \text{ is prime}\}$, can be written as

  {aa,aaa,aaaaa,aaaaaaa,aaaaaaaaaaa…}

Note

- Number of strings of length 'm' defined over alphabet of 'n' letters is n^m.
- Examples:
  - The language of strings of length 2, defined over $\Sigma=\{a,b\}$ is L={aa, ab, ba, bb} *i.e.* number of strings = 2^2
  - The language of strings of length 3, defined over $\Sigma=\{a,b\}$ is L={aaa, aab, aba, baa, abb, bab, bba, bbb} *i.e.* number of strings = 2^3

An Important language

- **PALINDROME**:

  The language consisting of Λ and the strings s defined over Σ such that Rev(s)=s.

  It is to be denoted that the words of PALINDROME are called palindromes.

- Example:For Σ={a,b},

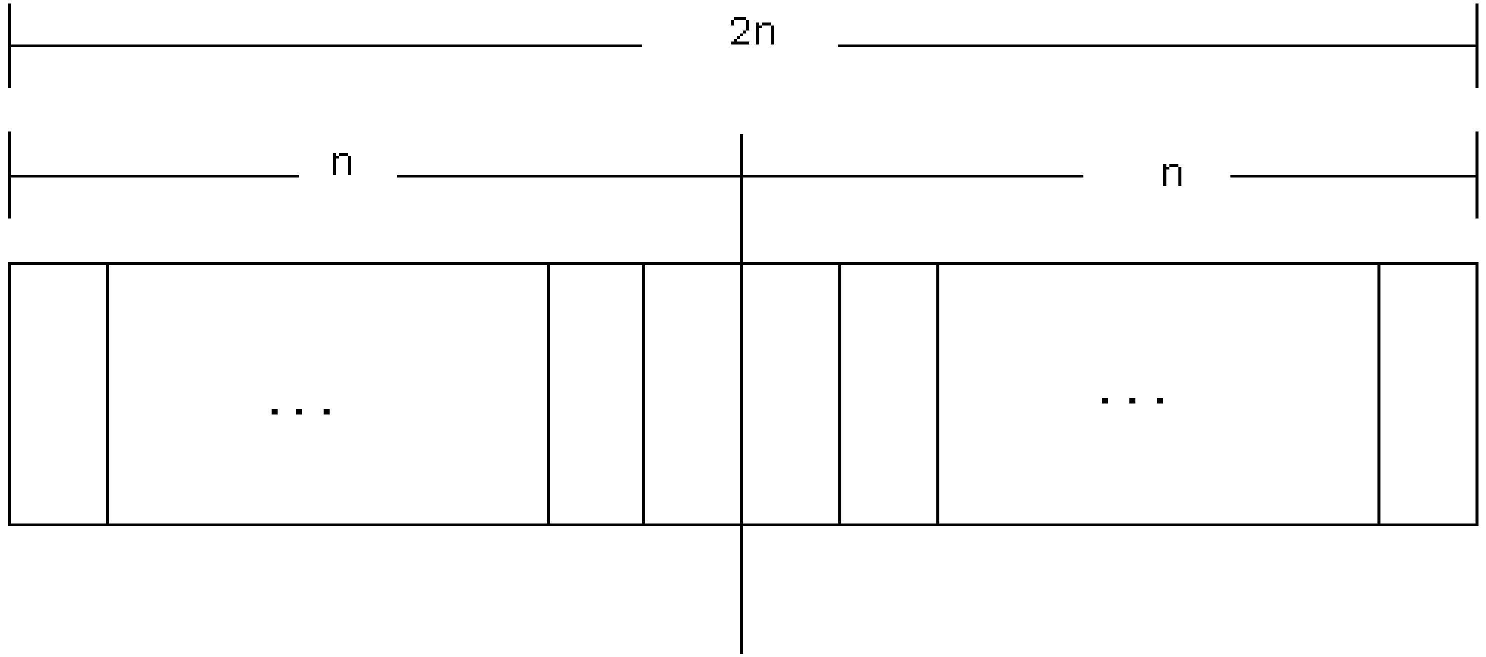
  PALINDROME={Λ , a, b, aa, bb, aaa, aba, bab, bbb, ...}

Remark

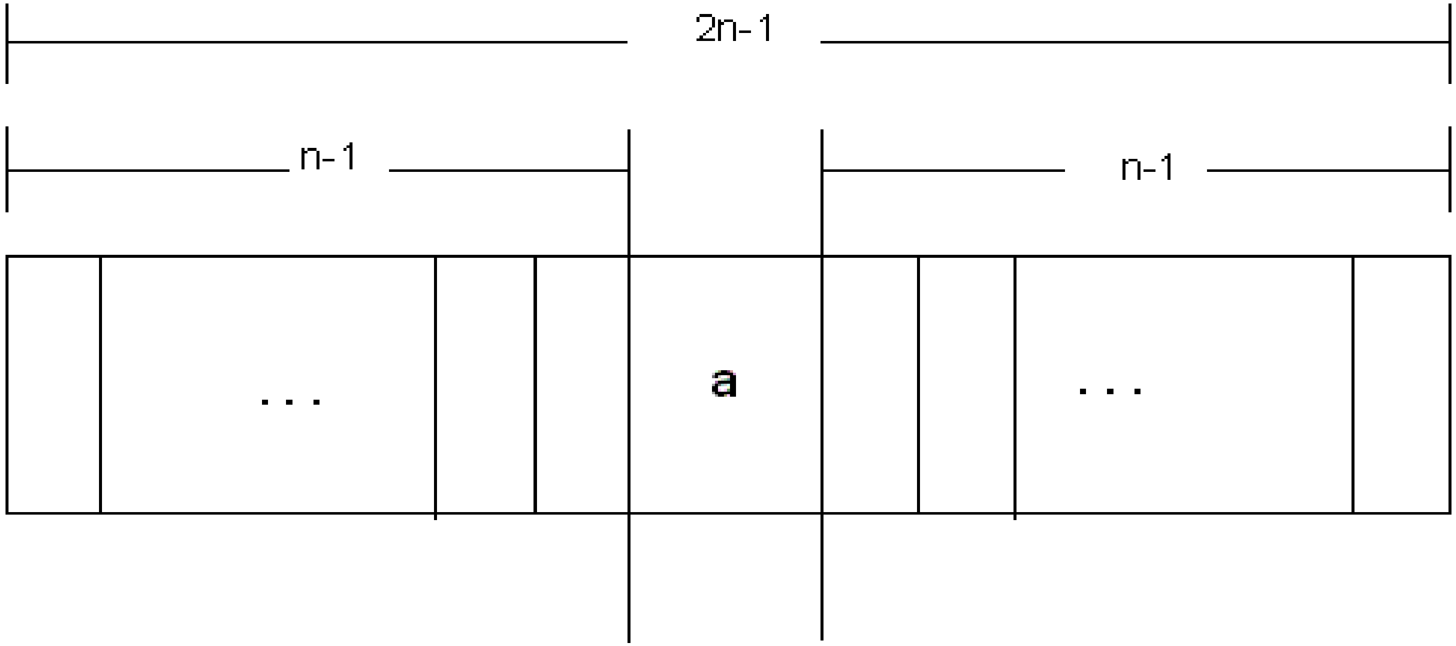
- There are as many palindromes of length 2n as there are of length 2n-1.

  To prove the above remark, the following is to be noted:

- To calculate the number of palindromes of length(2n), consider the following diagram,



- To calculate the number of palindromes of length (2n-1) with ‘a’ as the middle letter, consider the following diagram,



Kleene Star Closure

- Given Σ, then the Kleene Star Closure of the alphabet Σ, denoted by Σ^*, is the collection of all strings defined over Σ, including Λ.
- It is to be noted that Kleene Star Closure can be defined over any set of strings.

Examples

- If $\Sigma = \{x\}$

  Then $\Sigma^* = \{\Lambda, x, xx, xxx, xxxx, \ldots.\}$
- If $\Sigma = \{0,1\}$

  Then $\Sigma^* = \{\Lambda, 0, 1, 00, 01, 10, 11, \ldots.\}$
- If $\Sigma = \{aaB, c\}$

  Then $\Sigma^* = \{\Lambda, aaB, c, aaBaaB, aaBc, caaB, cc, \ldots.\}$

Note

- Languages generated by Kleene Star Closure of set of strings, are infinite languages. (By infinite language, it is supposed that the language contains infinite many words, each of finite length).

PLUS Operation ($^+$)

- Plus Operation is same as Kleene Star Closure except that it does not generate Λ (null string), automatically.

Example:

- If $\Sigma = \{0,1\}$

  Then $\Sigma^+ = \{0, 1, 00, 01, 10, 11, \ldots.\}$
- If $\Sigma = \{aab, c\}$

  Then $\Sigma^+ = \{aab, c, aabaab, aabc, caab, cc, \ldots.\}$

Remark

- It is to be noted that Kleene Star can also be operated on any string *i.e.* a^* can be considered to be all possible strings defined over {a}, which shows that a^* generates

  Λ, a, aa, aaa, …

It may also be noted that a^+ can be considered to be all possible non empty strings defined over {a}, which shows that a^+ generates

a, aa, aaa, aaaa, …