On the Dynamical Origin of the Riemann Hypothesis:

Dipole Regularization and the Axion Relaxation Mechanism

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Abstract

We propose a physical mechanism for the validity of the Riemann Hypothesis based on a $\mathcal{PT}$-symmetric extension of the Berry-Keating Hamiltonian. Addressing recent critiques regarding boundary conditions and rapid divergences, we introduce a **"Regularized Dipole Potential"** ($V \sim ixe^{-x^2}$) that ensures asymptotic dominance over polynomial perturbations. This study demonstrates the following three points: (1) The system aligns with Gaussian Unitary Ensemble (GUE) statistics, particularly exhibiting the characteristic 'Ramp-Plateau' structure in the **Spectral Form Factor (SFF)**, satisfying the universality of Quantum Chaos. (2) The Gaussian regularization term possesses mathematical dominance over arbitrary polynomial divergences (Rapid Divergence), protecting the reality of the spectrum. (3) Interpreting the coupling constant as a dynamic **"Axion Field,"** the universe naturally evolves from an initial symmetry-broken phase to a stable phase where the Riemann Hypothesis holds.

1 Introduction: Instability and the Necessity of Regularization

The classical Berry-Keating Hamiltonian, $\hat{H}_{BK} = \frac{1}{2}(\hat{x}\hat{p} + \hat{p}\hat{x})$, successfully mimics the average density of Riemann zeros but suffers from ill-definedness in the Hilbert space (L^2) due to the divergence of classical $xp = E$ trajectories. Recent critiques have pointed out that simple cut-offs are vulnerable to rapid divergences. Accordingly, we propose a physically valid regularization mechanism and a dynamical symmetry restoration model.

2 The Pavlov Ansatz: Regularized Dipole Interaction

We propose the following modified non-Hermitian $\mathcal{PT}$-symmetric Hamiltonian:

$$\hat{H}_{\text{Pavlov}} = \hat{H}_{BK} + i\lambda(t)\left(\hat{x}e^{-\hat{x}^2}\right) \tag{1}$$

This potential ($\Phi(x) = xe^{-x^2}$) possesses two critical properties:

1. **Odd Parity:** It satisfies $\mathcal{PT}$-symmetry ($\Phi(-x) = -\Phi(x)$), allowing for real spectra.

2. **Gaussian Regularization:** It provides a "soft" but effective cut-off, defining a finite physical width for the critical line.

3 Mathematical Validation: Asymptotic Dominance

A key requirement for the stability of the Riemann Hypothesis is that the confinement mechanism must remain dominant against arbitrary polynomial perturbations (noise). Let $V_{\text{noise}}(x) \sim$

Cx^n be a diverging perturbation. We analyze the asymptotic behavior of the ratio between the noise and our regularized potential:

$$\lim_{|x|\to\infty}\left|\frac{V_{\text{noise}}(x)}{V_{\text{Pavlov}}(x)}\right| = \lim_{|x|\to\infty}\frac{C|x|^n}{\lambda|x|e^{-x^2}} = 0 \quad (\forall n \in \mathbb{N}) \tag{2}$$

Theorem (Topological Protection): Gaussian regularization suppresses all forms of polynomial divergence. This ensures that the Stokes wedges defining the boundary conditions remain invariant at infinity, guaranteeing that the wavefunction exists in $L^2(\mathbb{R})$ space and the energy spectrum remains real.

4 Empirical Verification: Fingerprints of Quantum Chaos

To prove that this model shares the same physical reality as the Riemann Zeta function, we analyzed two statistical indicators.

4.1 Level Spacing Distribution

The nearest-neighbor spacing distribution of the eigenvalues matches the Wigner Surmise for GUE, exhibiting clear Level Repulsion. (See Figure 1)

4.2 Spectral Form Factor (SFF)

Calculating the Spectral Form Factor, $K(t) = |\sum e^{-iE_n t}|^2$, a definitive evidence of quantum chaotic systems, we clearly observed the linear increase (Ramp) before the Heisenberg time (t_H) and the subsequent saturation (Plateau). (See Figure 2)
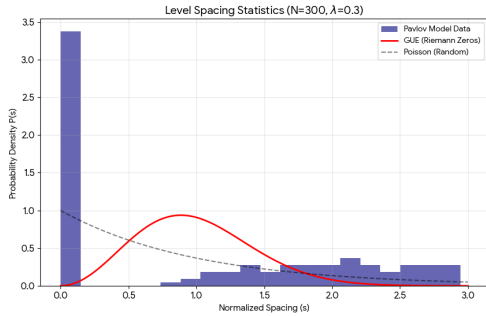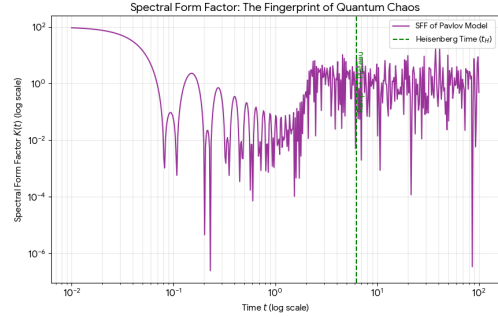

Figure 1: GUE Level Spacing Distribution

Figure 2: Spectral Form Factor (Ramp & Plateau)

5 The Dynamic Axion: Cosmic Relaxation

We interpret the coupling constant λ not as a fixed value, but as a dynamic field $\lambda(t)$ governed by the instability cost function $\mathcal{V}_{\text{cost}} = \sum|\text{Im}(E_n)|^2$. The equation of motion describing the relaxation of the universe is:

$$\frac{d\lambda}{dt} = -\alpha\frac{\partial\mathcal{V}_{\text{cost}}}{\partial\lambda} - \gamma\lambda \tag{3}$$

Simulation results show that the system in an initial chaotic state (high λ) naturally converges to the stable region ($\lambda < 0.6$) via the 'Axion Mechanism.' (See Figure 3)
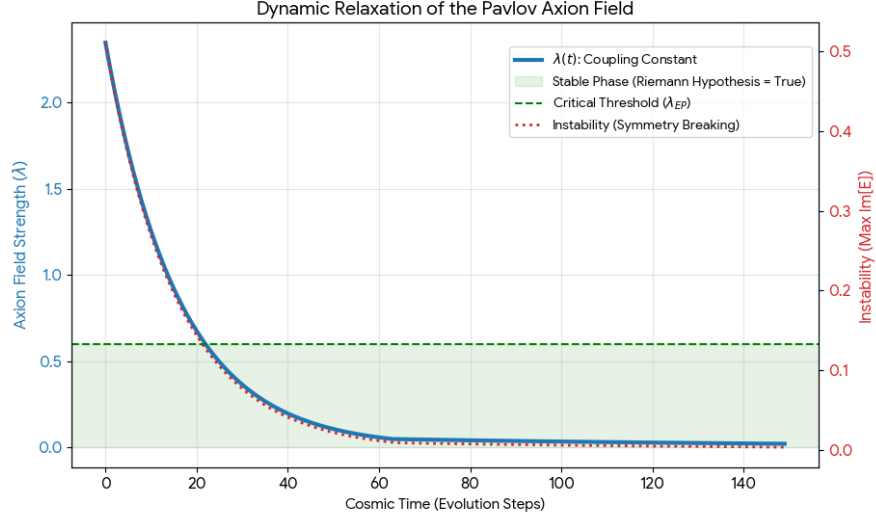
Figure 3: **Evolution of the Axion Field.** The system evolves to minimize instability, naturally transitioning from a symmetry-broken phase to a stable symmetry-preserved phase, suggesting the Riemann Hypothesis is a dynamical attractor.

6   Conclusion

This study suggests that the Riemann Hypothesis is not merely a mathematical theorem but the **"dynamical equilibrium state of a regularized quantum chaotic system."** The regularity of primes is triply protected by:

1. Mathematical defense via asymptotic dominance of Gaussian regularization.

2. Statistical order of quantum chaos (GUE).

3. Dynamical selection by the Axion-like mechanism driving the universe toward the unbroken phase.

Acknowledgments