3.2. $\frac{e^x - 1}{x} = \frac{\cancel{1} + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots - \cancel{1}}{x} =$

$$= 1 + \frac{x}{2} + \frac{x^2}{6} + \dots + \frac{x^n}{n!}$$

$$R_T = \left|\cancel{1} + \frac{x}{2} + \underbrace{\frac{x}{6} + \dots + \frac{x^2}{n!}}_{0} - \cancel{1}\right| \approx \frac{x}{2}$$

3.3. $f(x) = \frac{e^x - 1}{x} = \frac{a - 1}{x} = \frac{b}{x} = c$

$$\left|\frac{\Delta a}{a}\right|, \left|\frac{\Delta b}{b}\right|, \left|\frac{\Delta c}{c}\right| \le \mu$$

$$|\Delta f| \lessapprox \left|\frac{\partial f}{\partial a}\right| |\Delta a| + \left|\frac{\partial f}{\partial b}\right| + \left|\frac{\partial f}{\partial c}\right| |\Delta c| =$$

$$= \left|\frac{1}{x}\right| |\Delta a| + \left|\frac{1}{x}\right| |\Delta b| + |1| \, |\Delta c| \le$$

$$\le \left|\frac{a}{x}\right| \left|\frac{\Delta a}{a}\right| + \left|\frac{b}{x}\right| \left|\frac{\Delta b}{b}\right| + |c| \left|\frac{\Delta c}{c}\right| \le$$

$$\le \mu\left(\left|\frac{a}{x}\right| + \left|\frac{b}{x}\right| + |c|\right) =$$

$$= \mu\left(\underbrace{\left|\frac{e^x}{x}\right|}_{\to 1, x \to 0} + \underbrace{\left|\frac{e^x - 1}{x}\right|}_{\to 1, x \to 0} + \underbrace{\left|\frac{e^x - 1}{x}\right|}_{\to 1, x \to 0}\right) =$$

$$= \mu\left(\left|\frac{1}{x}\right| + 2\right) \approx \frac{\mu}{|x|} \quad (R_x)$$

$$3.5 \quad f(x) = \frac{a^x - 1}{\ln e^x} = \frac{a-1}{\ln a} = \frac{b}{\ln a} = \frac{b}{c} = d$$

$$\left|\frac{\Delta a}{a}\right|, \left|\frac{\Delta b}{b}\right|, \left|\frac{\Delta c}{c}\right|, \left|\frac{\Delta d}{d}\right| \leq \mu$$

$$|\Delta f| \lessapprox \left|\frac{\partial f}{\partial a}\right||\Delta a| + \left|\frac{\partial f}{\partial b}\right||\Delta b| + \left|\frac{\partial f}{\partial c}\right||\Delta c| + \left|\frac{\partial f}{\partial d}\right||\Delta d| =$$

// förlänger och deriverar //

$$\left|a\frac{\ln a - \frac{(a-1)}{a}}{(\ln a)^2}\right|\left|\frac{\Delta a}{a}\right| + \left|\frac{b}{\ln a}\right|\left|\frac{\Delta b}{b}\right| + \left|\frac{b}{c}\right|\left|\frac{\Delta c}{c}\right| + |d|\left|\frac{\Delta d}{d}\right| \leq$$

$$\leq \mu\left(\left|a\frac{\ln a - \frac{(a-1)}{a}}{(\ln a)^2}\right| + \left|\frac{b}{\ln a}\right| + \left|\frac{b}{c}\right| + |d|\right)$$

$$= // \begin{matrix} a = e^x \to 1, x \to 0 \\ \ln a = x \end{matrix} \quad \frac{e^x - 1}{x} \to 1, x \to 0 // =$$

$$= \mu\left(|e^x|\left|\frac{x - (e^x - 1)e^{-x}}{x^2}\right| + |1| + |0| + |1|\right) =$$

$$= \mu\left(1\left|\frac{x + e^{-x} - 1}{x^2}\right| + 2\right) =$$

Taylorutveckling.

$$= \mu\left(\frac{x + 1 - x + \frac{x^2}{2} \dots - 1}{x^2} + 2\right) =$$

$$= \mu\left(\frac{1 + \frac{1}{x} + \frac{x}{2} + \dots}{x} + 2\right) =$$

$$= \mu \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{2} + \cdots + 2 \right)$$

$$\approx \mu \left| \frac{1}{2} \right| = \frac{\mu}{2} \quad (R_+)$$