



科学哲学

归纳和概率

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THE PROBLEM OF INDUCTION 归纳问题



Empiricism

- epistemology that has tried to make sense of **the role of observation** in the justification of scientific knowledge.



THE PROBLEM OF INDUCTION 归纳问题



John Locke

- realism
 - 理论 (例如 Newton mechanism) 描述实在
- The **real properties of matter** are just the ones that Newtonian mechanics tells us it has—mass, extension in space, velocity, etc.



THE PROBLEM OF INDUCTION 归纳问题



George Berkeley

- makes doubtful our beliefs about things that we do not directly observe.
- radical instrumentalism 激进的工具主义
 - we employ that including Newtonian mechanics, as a body of **heuristic devices**, **calculating rules**, and **convenient fictions** to organize our experiences.



Study Question p188



Why would it be correct to call Locke the father of modern scientific realism and Berkeley the originator of instrumentalism? How would Berkeley respond to the argument for realism as an inference to the best explanation of science' s success?



为什么说洛克是现代科学实在论之父而贝克莱是工具主义的创始人呢？对于实在论作为科学的成功最好的解释的推论，贝克莱怎样回应？

THE PROBLEM OF INDUCTION

归纳问题



David Hume

- Realistic interpretation of science
- Problem of induction 归纳问题
 - The conflict of **realism** and **empiricism**
 - Given our current **sensory experience**, how can we **justify inferences** from them and from our **records** of the past, to the **future** and to the sorts of **scientific laws** and theories that we seek?



THE PROBLEM OF INDUCTION 归纳问题



解决Problem of induction?

- deductive argument 演绎论证
 - 1. If a practice has been reliable in the past, it will be reliable in the future.
 - 2. In the past inductive arguments have been reliable.
 - Therefore,
 - 3. Inductive arguments will be reliable in the future.
- 无效, (1) 乞题



THE PROBLEM OF INDUCTION 归纳问题



Mill

- Inductive inferences were grounded on a **commitment to the uniformity of nature** 自然界齐一性承诺
 - the future will be like the past.



THE PROBLEM OF INDUCTION 归纳问题



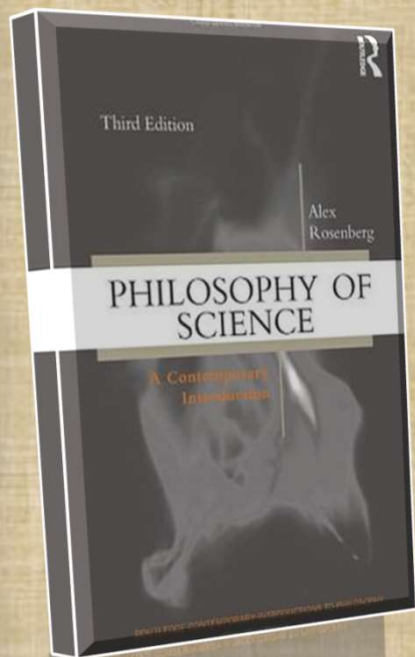
The logical positivists 逻辑经验主义

- provide a “confirmation theory (确证理论)” that would formalize and explicate the notion of inductive inference and solve Hume’s problem too.
- 第11章我们会看到这里头的麻烦比归纳问题本身还大



STATISTICS AND PROBABILITY TO THE RESCUE?

用统计和概率去救？



e is positive evidence for hypothesis h if and only if $p(h, e \text{ and } b) > p(h, b)$

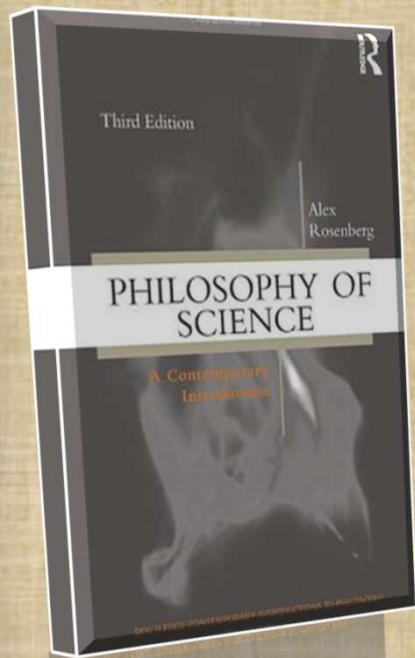
- **【反例】**

- 1000张彩票，A选10张，B选1张。
- h: B会获奖。e: 已开出9989张空白彩票，A、B所选彩票均不在其中。

- **【补救】 $p(h, e \text{ and } b) > 0.5?$**

- **【反例】** h: 安迪（男）没有怀孕。e: 安迪吃维他麦早餐麦片。

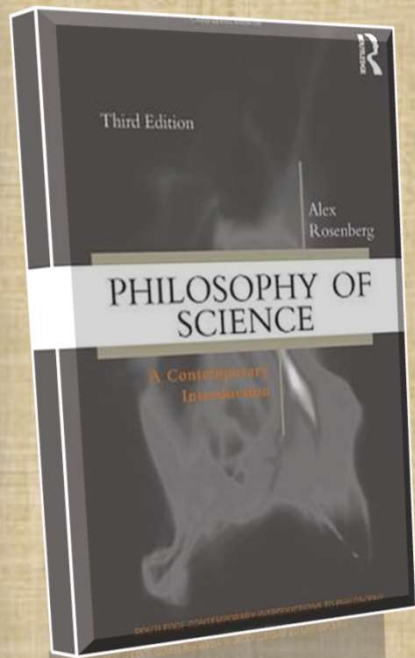
Study Question p188



What should the relationship be between the ordinary concept of evidence, as for example employed in law courts, and the scientist's use of the term in the testing of general theories? 在法庭上使用的普通证据概念与科学家在一般理论的检验中使用该术语之间有什么关系?

STATISTICS AND PROBABILITY TO THE RESCUE?

用统计和概率去救？



how data test hypotheses will respond to these problems that they reflect the **misfit** between **probability** and our **common-sense notions of evidence**.



Hypothesis testing in science is always a comparative affair:

- H_1 is more well confirmed by evidence e than is H_2 .

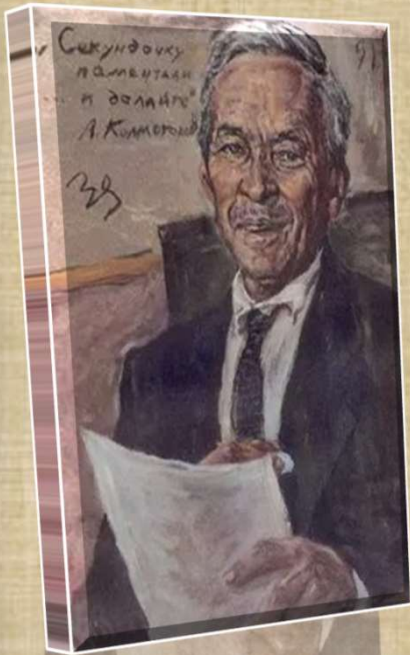
STATISTICS AND PROBABILITY TO THE RESCUE?

用统计和概率去救？



科尔莫格罗夫公理 p177

- 非负性 Probabilities are measured in numbers from 0 to 1.
- 规范性 The probability of a necessary truth (like “4 is an even number”) is 1.
- 可列可加性（完全可加性） If hypothesis h and j are incompatible, then $p(h \text{ or } j) = p(h) + p(j)$.
 - [等可能概型（古典概型和几何概型）是其特殊情形（加入一条样本空间中基本事件等可能）]



插播一段概率简史

“如果我们能把一切事件永恒地观察下去，则我们终将发现：世间的一切事物都受到因果律的支配，而我们也注定会在种种极其纷纭杂乱的现象中认识到某种必然。”

——伯努利



1713 伯努利 伯努利定理



1777 蒲丰 几何概型



1812 拉普拉斯 《概率的分析理论》

- 将分析引入概率，概率的古典定义（归纳概率）



1866 切比雪夫大数定理



1899 贝特朗悖论

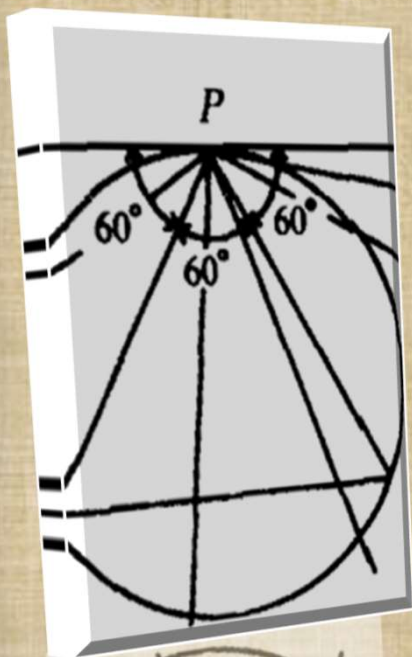
- 几何概型中的“等可能”意义不明



1933 科尔莫格罗夫公理

- 使得古典概型成为特例

插播一段概率简史



古典概型

- 样本空间内样本点有限且等可能



几何概型 (蒲丰, 1777)

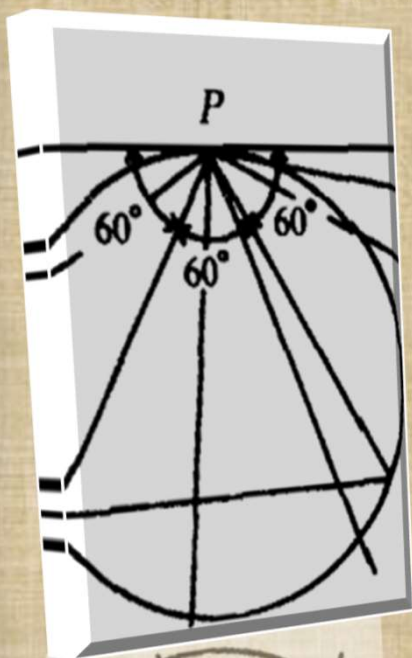
- 将古典概型拓展样本空间内样本点无限且等可能



贝特朗悖论 (1899)

- 在半径为 r 的圆内随机选择弦, 计算弦长超过圆内接正三角形边长的概率. 根据“随机选择”的不同意义, 可以得到不同的答案.
 - (1) 考虑与某确定方向平行的弦, 则所求概率为 $1/2$
 - (2) 考虑从圆上某固定点 P 引出的弦, 则所求概率为 $1/3$
 - (3) 弦的中点落在圆的某个部分的概率与该部分的面积成正比, 则所求概率为 $1/4$
- 【焦点】样本点 (圆内每个点) 无限且等可能, 但圆内每个弦?

插播一段概率简史



伯雷尔 (1905)

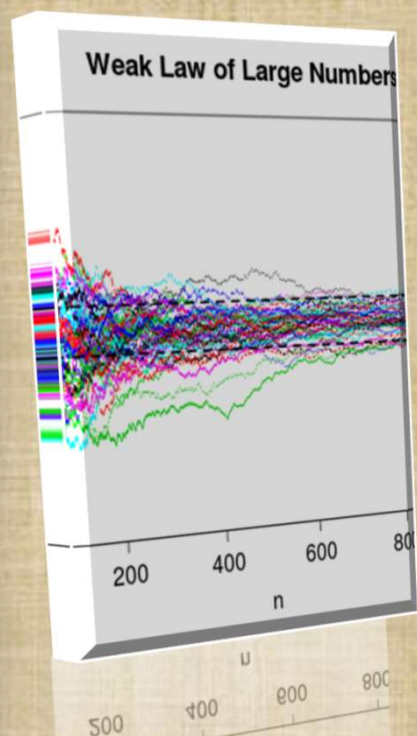
- 测度论方法引入概率论



科尔莫格罗夫公理(1933)

- 样本空间的子集组成的集类，如果是 σ 域，那么 σ 域中的某个集合就是随机事件
 - 如果这个集合是个单点集，那么它是一个基本事件
- 只有随机事件才有概率。每一个随机事件，都有个确定的小于1的非负实数与之对应，这个数就叫做该事件的概率
 - 概率定义在了事件而非样本点上（而且并非等可能的），所以不会有贝特朗悖论

插播一段概率简史



伯努利（大数）定理（1713）



切比雪夫（弱）大数定理
(1887)

- 条件：evidence独立，方差存在（有限？）
- 收敛方式：依概率收敛
- 一个关于概率的收敛命题：
设想每一个可能世界有一个 data stream，所有可能世界以概率越来越接近1的方式收敛（随着n增大，收敛的世界总是越来越多）



辛钦（弱）大数定理（1929）

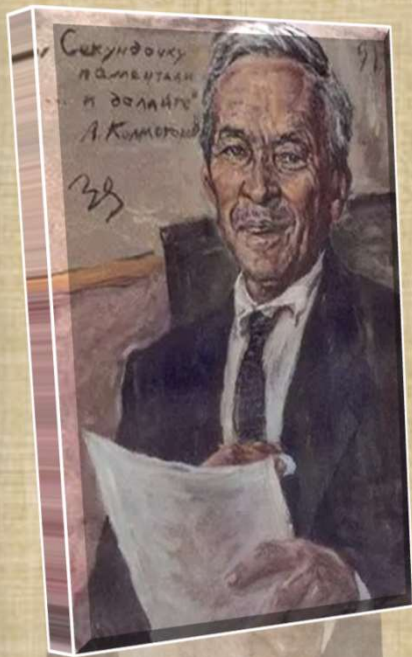
- 条件：evidence独立同分布（收敛方式同切比雪夫）



科尔莫格罗夫强大数定理
(1929?)

- 条件：evidence独立同分布
- 收敛方式：almost everywhere 几乎处处
- 一个关于收敛的概率命题：
设想每个可能世界有一个 data stream，任选一个世界，这个世界收敛的概率是1（随着n增大，几乎任何世界都收敛，就算仍有不能收敛的世界测度也可忽略）

STATISTICS AND PROBABILITY TO THE RESCUE? 用统计和概率去救?



条件概率定义

- $P(h|e) = P(eh)/P(e)$



贝叶斯公式 (1763)

- $$\frac{P(h|e)}{\text{后验概率}} = \frac{P(e|h)}{\text{先验概率}} \frac{P(h)}{P(e)}$$

- [*书上把 $P(e)$ 也称为 prior probability, p181]

- 由科尔莫格罗夫公理和条件概率定义演绎

- 【例】

- 水星近日点移动 (e) , p

(e) = 0.2, 因为这里是先验概率

- 【疑问】 p (e) 太低吧, 它不应是主观的, 因为我们并不更新它

- 牛顿定律 (h) , p (h) = 0.95

- 假设 $P(e|h) = 0.1$

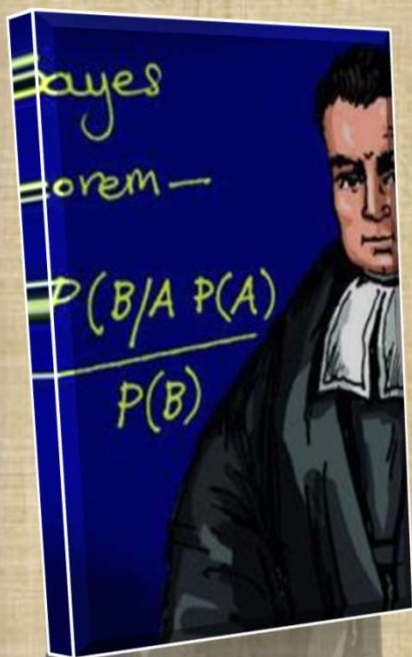
- $P(e|h) < P(h)$

HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?



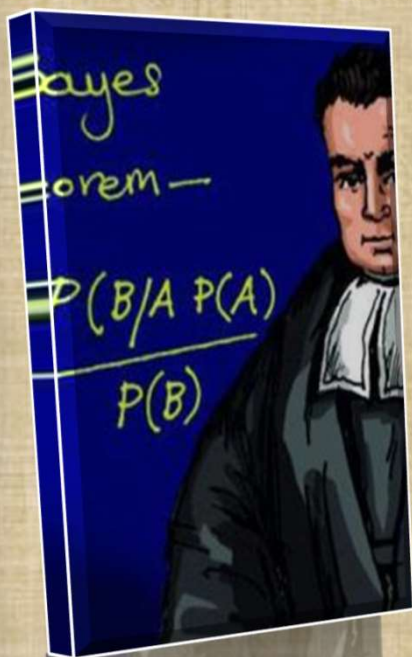
We need to make sense of $p(h)$

- First, there is the “metaphysical” question of **what fact** it is about the world that makes a particular probability value the true or correct one.
- Second, there is the epistemological question of **justifying** our estimate of this probability value.



$p(B)$

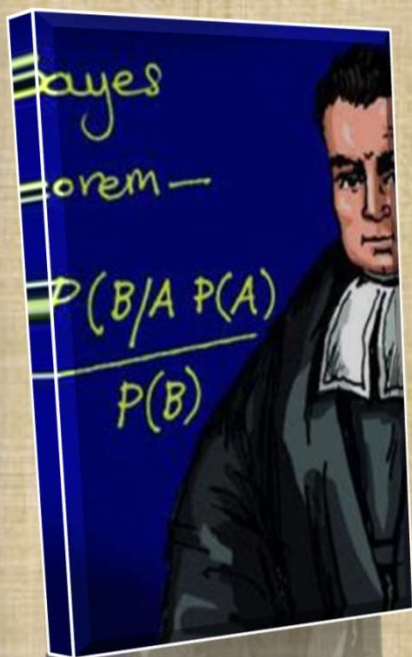
HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?



Long-run relative frequency 长程相对频率

- 【例】 we calculate for finite numbers of coin flips converge on one value no matter how many times we continue flipping.
- 【科学哲学】 科学史中的数据
- 【问题】 p183
 - (a) justifying the inference from a finite actual sequence to a long-run relative frequency
 - (b) justifying the inference from a long-run relative frequency to the next case, the new hypothesis.

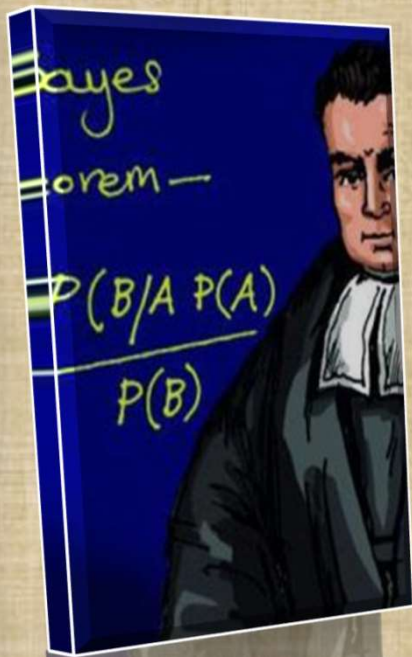
HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?



subjective probability 主观概率

- 【例】 kind of probability that theory testing invokes is the **gambler's kind**, what has come to be called "subjective probability."
 - "probability" because the bets the gambler makes should honor the axioms of probability
- 【科学哲学】 gambler's odds, which is the distinctive mark of the Bayesian.
- 【优势】 in the long run it doesn't matter what values they start with.*
 - [疑惑?如果有客观probability, 长程相对频率似乎也不管]
- 【问题×3】 problem of old evidence; 收敛不到真实概率; how to reconcile empiricism and explanation in theoretical science

Study Question p188

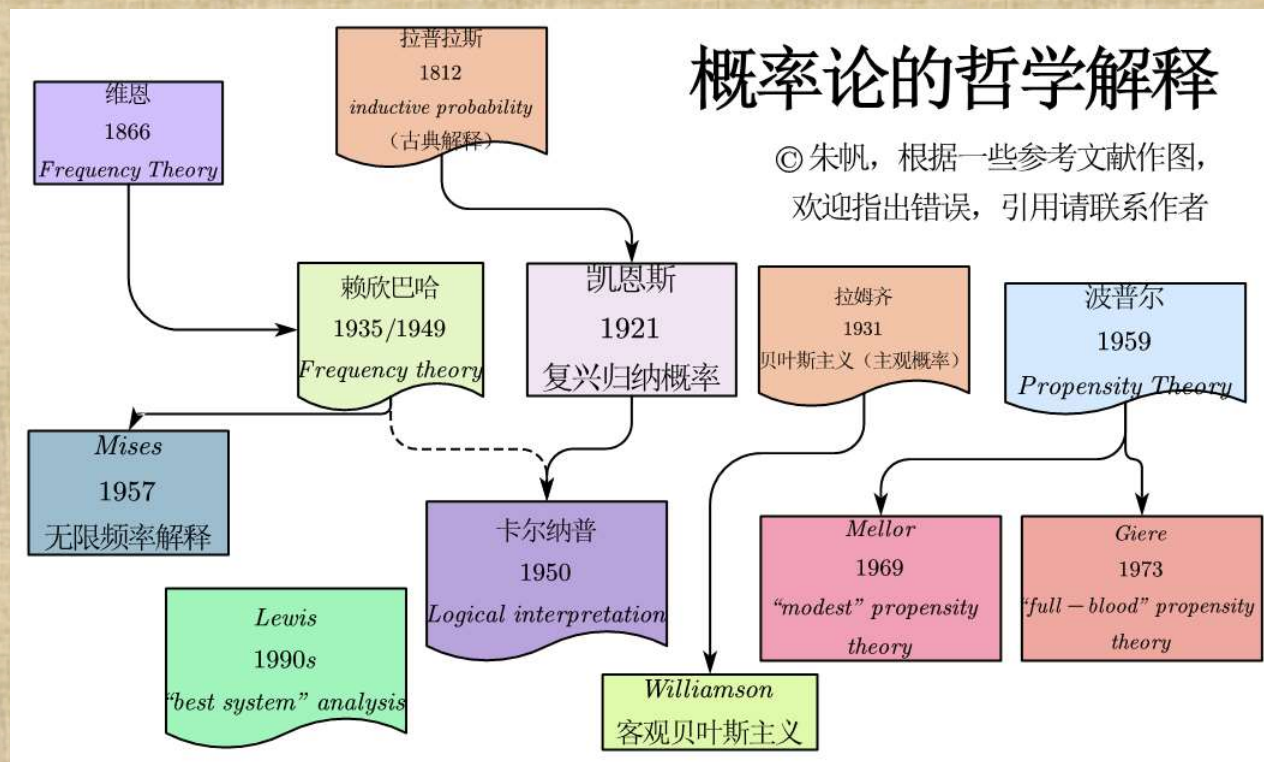


Defend the claim that there are several different but compatible meanings of the word “probability” in science. Is one of them more fundamental than the others?



为“概率”一词在科学中有几种不同但又相互兼容的含义的说法进行辩护。其中一个比其他的更基本吗？

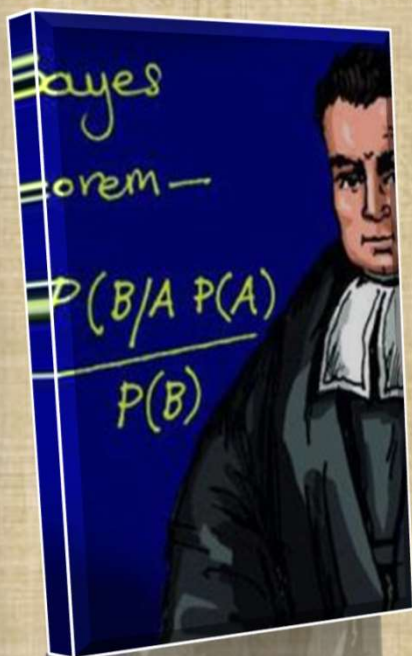
插播一段概率论哲学



HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?



problem of old evidence 旧证据难题



$$\bullet P\left(h \mid \underbrace{e}_{\text{旧证据}}\right) = \frac{1}{\underbrace{P(e|h)}} \cdot P(h)$$

- 【反例】Newton's theory was **confirmed** by its ability to explain the data on which Galileo's and Kepler's theories were based.

• [*我的疑惑：但牛顿定律可以解释更多问题啊?]

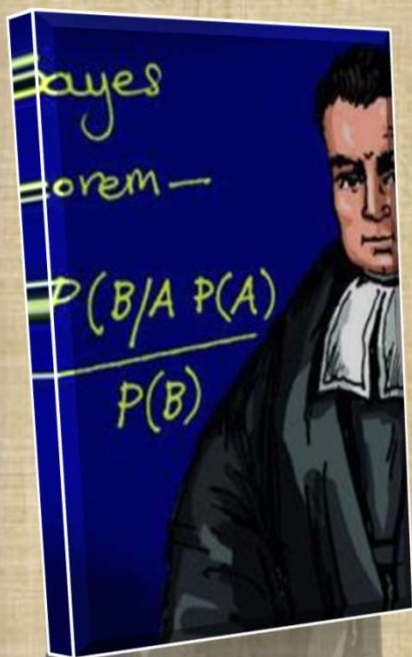
- 【可能解决】贝叶斯主义者拒绝给出 $P(h) = 1$

HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?

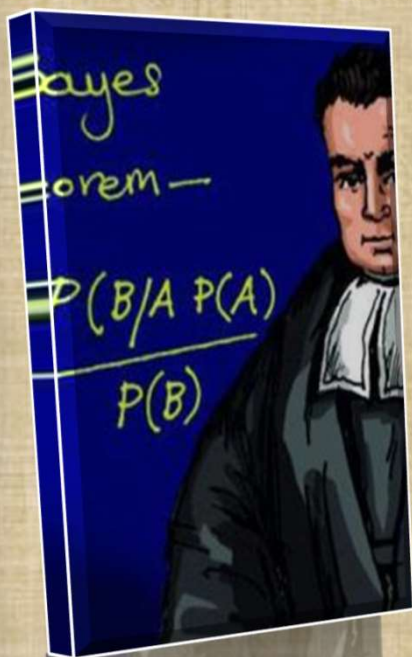


收敛不到真实概率

- 贝叶斯主义太主观
- 那换用频率主义行不?
 - 预设 nature is uniform, 乞题。



HOW MUCH CAN BAYES' THEOREM REALLY HELP? 贝叶斯定理能帮多少?



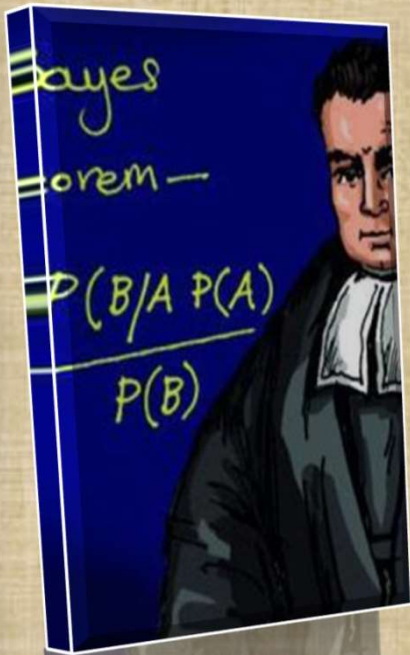
how to reconcile *'rek(ə)nsaɪl/* **empiricism** and **explanation** in theoretical science.

- [empiricism] must attach the **highest probability** to statements that describe **observations**, and **lower probability** to those that make claims about **theoretical entities**.
- **t** is the **theory**, which explains observations.
- $t \rightarrow h$ reflects the **explanatory relation** between the theoretical claims of the theory
- 贝叶斯公式不能解释: $p(h) < p(t \text{ and } t \rightarrow h)$
- antecedent of the $t \rightarrow h$ is a statement about what cannot be observed whose only consequence for observation is **h**.

Study Question p188



What do you need to add to Bayes' theorem to solve the problem of induction?



$P(B)$