CALCUL NUMERIC

TEMA 7

Maciuca Gloria - Ruxandra

Grupa 344

Exercitiul 3

Sa se afle polinomul de interpolare Lagrange $P_2(x)$ al functiei $f(x) = sin(x)$, relativ la diviziunea $(-\frac{\pi}{2},\ 0,\ \frac{\pi}{2})$. Sa se evalueze eroarea $\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right|$.

__Metoda Directa__

$$x_0 = -\frac{\pi}{2} \Rightarrow f(x_0) = sin\left(-\frac{\pi}{2}\right) = -1$$
$$x_1 = 0 \Rightarrow f(x_1) = sin(0) = 0$$
$$x_2 = \frac{\pi}{2} \Rightarrow f(x_0) = sin\left(\frac{\pi}{2}\right) = 1$$

$$P(x_0) = f(x_0) \Rightarrow a_0 + a_1 x_0 + a_2 x_0^2 = f(x_0) \Rightarrow a_0 - a_1\frac{\pi}{2} + a_2 - \frac{\pi^2}{4} = -1$$
$$P(x_1) = f(x_1) \Rightarrow a_0 + a_1 x_1 + a_2 x_1^2 = f(x_1) \Rightarrow \boldsymbol{a_0 = 0}$$
$$P(x_2) = f(x_2) \Rightarrow a_0 + a_1 x_2 + a_2 x_2^2 = f(x_2) \Rightarrow a_0 + a_1\frac{\pi}{2} + a_2 - \frac{\pi^2}{4} = 1$$

$$\left.\begin{array}{l} -a_1\frac{\pi}{2} + a_2 - \frac{\pi^2}{4} = -1 \\ a_1\frac{\pi}{2} + a_2 - \frac{\pi^2}{4} = 1 \end{array}\right\} \Rightarrow \begin{array}{r} a_2\frac{\pi^2}{2} = 0 \Rightarrow \boldsymbol{a_2 = 0} \\ -a_1\frac{\pi}{2} = -1 \Rightarrow \boldsymbol{a_1 = \frac{2}{\pi}} \end{array}$$

$$\Downarrow$$

$$\boldsymbol{P_2(x) = \frac{2}{\pi}x}$$

$$\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right| = \left|\frac{2}{\pi}\frac{\pi}{6} - sin(\frac{\pi}{6})\right| = \left|\frac{1}{1} - \frac{1}{2}\right| = \frac{1}{6} \approx \mathbf{0.1667}$$

Exercitiul 3

Sa se afle polinomul de interpolare Lagrange $P_2(x)$ al functiei $f(x) = sin(x)$, relativ la diviziunea $(-\frac{\pi}{2},\ 0,\ \frac{\pi}{2})$. Sa se evalueze eroarea $\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right|$.

<u>**Metoda Lagrange**</u>

$$x_0 = -\frac{\pi}{2} \;\Rightarrow\; f(x_0) = sin\left(-\frac{\pi}{2}\right) = -1$$
$$x_1 = 0 \;\Rightarrow\; f(x_1) = sin(0) = 0$$
$$x_2 = \frac{\pi}{2} \;\Rightarrow\; f(x_0) = sin\left(\frac{\pi}{2}\right) = 1$$

$$P_2(x) = L_{2,0}f(x_0) + L_{2,1}f(x_1) + L_{2,2}f(x_2)$$

$$L_{2,0} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \;\Rightarrow\; L_{2,0} = \frac{2(x^2 - \frac{\pi}{2}x)}{\pi^2} \;\Rightarrow\; \boldsymbol{L_{2,0} = \frac{2x^2 - \pi x}{\pi^2}}$$
$$L_{2,1} = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_0-x_2)} \;\Rightarrow\; L_{2,1} = \frac{4(x^2 - \frac{\pi^2}{4})}{-\pi^2} \;\Rightarrow\; \boldsymbol{L_{2,1} = \frac{4x^2 - \pi^2}{-\pi^2}}$$
$$L_{2,2} = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \;\Rightarrow\; L_{2,2} = \frac{2(x^2 + \frac{\pi}{2}x)}{\pi^2} \;\Rightarrow\; \boldsymbol{L_{2,2} = \frac{2x^2 + \pi x}{\pi^2}}$$

$$P_2(x) = \frac{4x^2-\pi^2}{-\pi^2}(-1) + \frac{4x^2-\pi^2}{-\pi^2}0 + \frac{2x^2+\pi x}{\pi^2}$$
$$\Rightarrow P_2(x) = \frac{2\pi x}{\pi^2}$$

$$\Rightarrow \boldsymbol{P_2(x) = \frac{2}{\pi}x}$$

Exercitiul 3

Sa se afle polinomul de interpolare Lagrange $P_2(x)$ al functiei $f(x) = sin(x)$, relativ la diviziunea $(-\frac{\pi}{2},\ 0,\ \frac{\pi}{2})$. Sa se evalueze eroarea $\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right|$.

<u>**Metoda Newton**</u>

$$x_0 = -\frac{\pi}{2} \ \Rightarrow \ f(x_0) = sin\left(-\frac{\pi}{2}\right) = -1$$
$$x_1 = 0 \ \Rightarrow \ f(x_1) = sin(0) = 0$$
$$x_2 = \frac{\pi}{2} \ \Rightarrow \ f(x_0) = sin\left(\frac{\pi}{2}\right) = 1$$

$$P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$P_0(x_0) = f(x_0)$$
$$\Rightarrow \boldsymbol{a_0 = f(x_0)}$$

$$P_1(x_1) = f(x_1)$$
$$P_0(x_1) + a_1(x_1 - x_0) = f(x_1)$$
$$f(x_0) + a_1(x_1 - x_0) = f(x_1)$$
$$\Rightarrow \boldsymbol{a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

$$P_2(x_2) = f(x_2)$$
$$P_1(x_2) + a_2(x_2 - x_0)(x_2 - x_1) = f(x_2)$$
$$f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) = f(x_2)$$
$$f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_1 + x_1 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) = f(x_2)$$

$$f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_1) + f(x_1) - f(x_0) + a_2(x_2 - x_0)(x_2 - x_1) = f(x_2)$$

$$\Rightarrow \boldsymbol{a_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}}$$

$$a_0 = -1$$

$$a_1 = \frac{0 - (-1)}{0 - (-\frac{\pi}{2})} = \frac{2}{\pi}$$

$$a_2 = \frac{\frac{1 - 0}{\frac{\pi}{2} - 0} - \frac{0 - (-1)}{0 - \left(-\frac{\pi}{2}\right)}}{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)} = 0$$

$$\Downarrow$$

$$P_2(x) = -1 + \frac{2}{\pi}(x - x_0) + 0(x - x_0)(x - x_1)$$

$$\Rightarrow P_2(x) = -1 + \frac{2}{\pi}\left(x - \left(-\frac{\pi}{2}\right)\right)$$

$$\Rightarrow P_2(x) = -1\frac{2}{\pi}x + 1$$

$$\Rightarrow \boldsymbol{P_2(x) = \frac{2}{\pi}x}$$