CALCUL NUMERIC

TEMA 8

Maciuca Gloria - Ruxandra

Grupa 344

Exercitiul 1

Sa se afle polinomul de interpolare Lagrange $P_2(x)$ al functiei $f(x) = sin(x)$, relativ la diviziunea $(-\frac{\pi}{2},\ 0,\ \frac{\pi}{2})$, utilizand metodele Neville si Newton cu diferente divizate. Sa se evalueze eroarea $\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right|$.

<u>Metoda Neville</u>

$$x_1 = -\pi/2 \quad x_2 = 0 \quad x_3 = \pi/2$$

$$f(x_1) = -1 \quad f(x_2) = 0 \quad f(x_3) = 1$$

$$P_1(x) = f(x_1) = -1$$

$$P_2(x) = f(x_2) = 0$$

$$P_3(x) = f(x_3) = 1$$

$$\begin{aligned}
P_{1,2}(x) &= \frac{(x - x_1)P_2(x) - (x - x_2)P_1(x)}{x_2 - x_1} = \\
&= \frac{(x - (-\pi/2))0 - (x - 0)(-1)}{0 - (-\pi/2)} = \\
&= \frac{2x}{\pi} \\
P_{2,3}(x) &= \frac{(x - x_2)P_3(x) - (x - x_3)P_2(x)}{x_3 - x_2} = \\
&= \frac{(x - 0)1 - (x - \pi/2)0}{\pi/2 - 0} = \\
&= \frac{2x}{\pi}
\end{aligned}$$

$$\begin{aligned}
P_{1,2,3}(x) =& \frac{(x-x_1)P_{2,3}(x) - (x-x_3)P_{1,2}}{x_3 - x_1} = \\
=& \frac{(x-(-\pi/2))2x/\pi - (x-\pi/2)2x/\pi}{\pi/2 - (-\pi/2)} = \\
=& \frac{2x/\pi(\cancel{x} + \pi/2 - \cancel{x} + \pi/2)}{\cancel{2}\pi/\cancel{2}} = \\
=& \frac{2x}{\pi}
\end{aligned}$$

$$\Downarrow$$

$$\boldsymbol{P_2(x) = \frac{2}{\pi}x}$$

Exercitiul 1

Sa se afle polinomul de interpolare Lagrange $P_2(x)$ al functiei $f(x) = sin(x)$, relativ la diviziunea $(-\frac{\pi}{2},\ 0,\ \frac{\pi}{2})$, utilizand metodele Neville si Newton cu diferente divizate. Sa se evalueze eroarea $\left|P(\frac{\pi}{6}) - f(\frac{\pi}{6})\right|$.

Metoda Newton

$$x_1 = -\frac{\pi}{2} \quad x_2 = 0 \quad x_3 = \frac{\pi}{2}$$

$$f(x_1) = -1 \quad f(x_2) = 0 \quad f(x_3) = 1$$

$$P_n(x) = a_1 + a_2(x - x_1) + ... + a_n(x - x_1)...(x - x_{n-1})$$

$$\Rightarrow P_2(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$$

$$a_i = f[x_1, ..., x_i]$$

$$\Rightarrow a_1 = f[x_1]$$

$$\Rightarrow a_2 = f[x_1, x_2]$$

$$\Rightarrow a_3 = f[x_1, x_2, x_3]$$

$$f[x_1] = f(x_1)$$

$$f[x_2] = f(x_2)$$

$$f[x_3] = f(x_3)$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1} = \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1}$$

$$
\begin{aligned}
a_1 &= -1 \\
a_2 &= \frac{0-(-1)}{0-(-\pi/2)} = \frac{2}{\pi} \\
a_3 &= \frac{\dfrac{1-0}{\pi/2} - \dfrac{0-(-1)}{0-(-\pi/2)}}{\pi/2-(-\pi/2)} = 0
\end{aligned}
$$

$$\Downarrow$$

$$
\begin{aligned}
P_2(x) &= -1 + \frac{2}{\pi}(x-(-\frac{\pi}{2})) + 0(x-(-\frac{\pi}{2}))(x-0) \\
P_2(x) &= -1\frac{2}{\pi}x + 1
\end{aligned}
$$

$$\Rightarrow \boldsymbol{P_2(x) = \frac{2}{\pi}x}$$

Exercitiul 2

Fiind date functia $f(x) = 3^x$ si diviziunea $(-2, -1, 0, 1, 2)$, sa se aproximeze $\sqrt{3}$ folosind metoda Neville.

$$x_1 = -2 \quad x_2 = -1 \quad x_3 = 0 \quad x_4 = 1 \quad x_5 = 2$$
$$f(x_1) = 1/9 \quad f(x_2) = 1/3 \quad f(x_3) = 1 \quad f(x_4) = 3 \quad f(x_5) = 9$$

Cunoastem ca $1 < \sqrt{3} < 2$, asadar $\sqrt{3}$ apartine intervalului definit de punctele din diviziunea data.

Vom construi polinomul P_4 de grad patru astfel:

$$\begin{aligned}
P_4(\sqrt{3}) =& \frac{(\sqrt{3}-x_1)(\sqrt{3}-x_2)(\sqrt{3}-x_3)(\sqrt{3}-x_4)}{(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)} f(x_5) + \frac{(\sqrt{3}-x_1)(\sqrt{3}-x_2)(\sqrt{3}-x_3)(\sqrt{3}-x_5)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)(x_4-x_5)} f(x_4) + \\
&+ \frac{(\sqrt{3}-x_1)(\sqrt{3}-x_2)(\sqrt{3}-x_4)(\sqrt{3}-x_5)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)} f(x_3) + \frac{(\sqrt{3}-x_1)(\sqrt{3}-x_3)(\sqrt{3}-x_4)(\sqrt{3}-x_5)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)(x_2-x_5)} f(x_2) + \\
&+ \frac{(\sqrt{3}-x_2)(\sqrt{3}-x_3)(\sqrt{3}-x_4)(\sqrt{3}-x_5)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} f(x_1) \\
P_4(\sqrt{3}) =& \frac{(\sqrt{3}-(-2))(\sqrt{3}-(-1))(\sqrt{3}-0)(\sqrt{3}-1)}{(2-(-2))(2-(-1))(2-0)(2-1)} 9 + \frac{(\sqrt{3}-(-2))(\sqrt{3}-(-1))(\sqrt{3}-0)(\sqrt{3}-2)}{(1-(-2))(1-(-1))(1-0)(1-2)} 3 + \\
&+ \frac{(\sqrt{3}-(-2))(\sqrt{3}-(-1))(\sqrt{3}-1)(\sqrt{3}-2)}{(0-(-2))(0-(-1))(0-1)(0-2)} 1 + \frac{(\sqrt{3}-(-2))(\sqrt{3}-0)(\sqrt{3}-1)(\sqrt{3}-2)}{(-1-(-2))(-1-0)(-1-1)(-1-2)} 1/3 + \\
&+ \frac{(\sqrt{3}-(-1))(\sqrt{3}-0)(\sqrt{3}-1)(\sqrt{3}-2)}{(-2-(-1))(-2-0)(-2-1)(-2-2)} 1/9
\end{aligned}$$

$\boldsymbol{P_4(\sqrt{3}) = 6.7802}$

$$\Downarrow$$

$$\boldsymbol{f(\sqrt{3}) \approx 6.7802}$$

Exercitiul 3

Fiind date $x_j = j,\ j = \overline{1,4}$**,** $P_{1,2}(x) = x + 1$**,** $P_{2,3}(x) = 3x - 1$**,** $P_{2,3,4}\left(\frac{3}{2}\right) = 4$**, sa se calculeze** $P_{1,2,3,4}\left(\frac{3}{2}\right)$**.**

$$x_1 = 1 \quad x_2 = 2 \quad x_3 = 3 \quad x_4 = 4$$

$$P_{1,2,3}(x) = \frac{(x - x_1)P_{2,3}(x) - (x - x_3)P_{1,2}(x)}{x_3 - x_1}$$

$$\Rightarrow P_{1,2,3}\left(\frac{3}{2}\right) = \frac{\left(\frac{3}{2} - 1\right)\left(3\frac{3}{2} - 1\right) - \left(\frac{3}{2} - 3\right)\left(\frac{3}{2} + 1\right)}{3 - 1} = \frac{1}{2}$$

$$P_{2,3,4}\left(\frac{3}{2}\right) = 4$$

$$P_{1,2,3,4}(x) = \frac{(x - x_1)P_{2,3,4}(x) - (x - x_4)P_{1,2,3}(x)}{x_4 - x_1}$$

$$\Rightarrow P_{1,2,3,4}\left(\frac{3}{2}\right) = \frac{\left(\frac{3}{2} - 1\right)4 - \left(\frac{3}{2} - 4\right)\frac{1}{2}}{4 - 1}$$

$$\boldsymbol{\Rightarrow P_{1,2,3,4}\left(\frac{3}{2}\right) = \frac{13}{12}}$$

Exercitiul 4

Fie polinomul $P_2(x) = f[x_1] + f[x_1, x_2](x - x_1) + a_3(x - x_1)(x - x_2)$**. Folosind** $P_2(x_3)$**, aratati ca** $a_3 = f[x_1, x_2, x_3]$**.**

Pornim de la urmatoarea forma a polinomului P de interpolare:

$$P_n(x) = a_1 + a_2(x - x_1) + ... + a_n(x - x_1)...(x - x_{n-1})$$
$$\Rightarrow P_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$$

$$P_1(x_1) = f(x_1)$$
$$\Rightarrow \boldsymbol{a_1 = f(x_1)}$$

$$P_2(x_2) = f(x_2)$$
$$P_1(x_2) + a_2(x_2 - x_1) = f(x_2)$$
$$f(x_1) + a_2(x_2 - x_1) = f(x_2)$$
$$\Rightarrow \boldsymbol{a_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

$$P_3(x_3) = f(x_3)$$
$$P_2(x_3) + a_3(x_3 - x_1)(x_3 - x_2) = f(x_3)$$
$$f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2) = f(x_3)$$
$$f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x_3 - x_2 + x_2 - x_1) + a_3(x_3 - x_1)(x_3 - x_2) = f(x_3)$$
$$\cancel{f(x_1)} + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x_3 - x_2) + f(x_2) - \cancel{f(x_1)} + a_3(x_3 - x_1)(x_3 - x_2) = f(x_3)$$
$$\Rightarrow \boldsymbol{a_3 = \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1}}$$

Calculam diferentele divizate:

$$f[x_1] = f(x_1)$$

$$f[x_2] = f(x_2)$$

$$f[x_3] = f(x_3)$$

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$f[x_1, x_2, x_3] = \frac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$$
$$\boldsymbol{\Rightarrow f[x_1, x_2, x_3] = \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1}}$$

$$\left.\begin{aligned} a_3 &= \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1} \\ f[x_1, x_2, x_3] &= \frac{\frac{f(x_3) - f(x_2)}{x_3 - x_2} - \frac{f(x_2) - f(x_1)}{x_2 - x_1}}{x_3 - x_1} \end{aligned}\right\} \Rightarrow \underline{\boldsymbol{a_3 = f[x_1, x_2, x_3]}}$$