CALCUL NUMERIC

TEMA 9

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Grupa 344

Exercitiul 1

Fiind data functia $f(x) = 3xe^x - e^{2x}$, sa se aproximeze $f(1.03)$ folosind polinomul Hermite cu diferente divizate de gradul cel mult 3 si nodurile $x_1 = 1$, $x_2 = 1.05$.

Evaluati eroarea $|f(1.03) - H_3(1.03)|$.

$$\boldsymbol{H_{2n-1} = f[z_1] \; + \sum_{k=2}^{2n} f[z_1, .., z_k](x - z_1)\cdots(x - z_{k-1})}$$

$$\boldsymbol{z_{2i-1} = z_{2i} = x_i} \quad \Rightarrow \quad \begin{matrix} z_1 = z_2 = x_1 \\ z_3 = z_4 = x_2 \end{matrix}$$

$$f[z_1] = f[z_2] = f(x1) = 0.7658$$
$$f[z_3] = f[z_4] = 0.8354$$

$$f[z_1, z_2] = f'(x_1) = 1.5316$$
$$f[z_2, z_3] = \frac{f[z_3] - f[z_2]}{z_3 - z_2} = 1.3928$$
$$f[z_3, z_4] = f'(x_2) = 1.2422$$

$$f[z_1, z_2, z_3] = \frac{f[z_2, z_3] - f[z_1, z_2]}{z_3 - z_1} = -2.7749$$
$$f[z_2, z_3, z_4] = \frac{f[z_3, z_4] - f[z_2, z_3]}{z_4 - z_2} = -3.0124$$

$$f[z_1, z_2, z_3, z_4] = \frac{f[z_2, z_3, z_4] - f[z_1, z_2, z_3]}{z_4 - z_1} = -4.7502$$

$H_3(x) = 0.7658 + 1.5316(x-1) - 2.7749(x-1)(x-1) - 4.7502(x-1)(x-1)(x-1.05)$

$H_3(x) = 0.7658 + 1.5316(x-1) - 2.7749(x^2-2x+1) - 4.7502(x^3-3.05x^2+3.1x-1.05)$

$H_3(x) = 0.7658 + 1.5316x - 1.5316 - 2.7749x^2 + 5.5498x - 2.7749 - 4.7502x^3 + 14.4882x^2 - 14.7257x + 4.9877$

$\Rightarrow \boldsymbol{H_3(x) = 1.4470 - 7.6443x + 11.7133x^2 - 4.7502x^3}$

$H_3(1.03) = 1.4470 - 7.6443 \cdot 1.03 + 11.7133 \cdot 1.03^2 - 4.7502 \cdot 1.03^3$

$\Rightarrow \boldsymbol{H_3(1.03) = 0.8093}$

$E = |f(1.03) - H_3(1.03)|$

$E = |0.8093236189 - 0.8093248562|$

$\Rightarrow \boldsymbol{E = 0.000001237300423}$

Exercitiul 3

Sa se afle functia de interpolare spline liniara S asociata functiei $f(x) = sin(x)$ **relativ la diviziunea** $\left(-\frac{\pi}{2}, 0, \frac{\pi}{2}\right)$ **.**

$$\boldsymbol{S_L(x) = f(x_i) + \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}(x - x_i)}$$

$$x_1 = -\frac{\pi}{2} \qquad x_2 = 0 \qquad x_3 = \frac{\pi}{2}$$

$$f(x_1) = -1 \qquad f(x_2) = 0 \qquad f(x_3) = 1$$

$$S_L(x) = \begin{cases} S_1(x), & x \in [-\pi/2, 0] \\ S_2(x), & x \in [0, \pi/2] \end{cases}$$

$$S_L(x) = \begin{cases} f(x_1) + \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1), & x \in [-\pi/2, 0] \\ \\ f(x_2) + \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}(x - x_2), & x \in [0, \pi/2] \end{cases}$$

$$S_L(x) = \begin{cases} -1 + \dfrac{0 + 1}{0 + \pi/2}(x + \pi/2), & x \in [-\pi/2, 0] \\ \\ 0 + \dfrac{1 - 0}{\pi/2 - 0}(x - 0), & x \in [0, \pi/2] \end{cases}$$

$$S_L(x) = \begin{cases} \frac{2}{\pi}x, & x \in [-\pi/2, 0] \\ \\ \frac{2}{\pi}x, & x \in [0, \pi/2] \end{cases}$$

$$\Rightarrow \boldsymbol{S_L(x) = \frac{2}{\pi}x} \ , \quad x \in [-\pi/2, \pi/2]$$