new color SolutionColorrgb0.1,0.1,0.1

Financial Risk Management&Regulation (M5320)

Fall 2016

https://courseworks.columbia.edu

Name : Duanhong(Gloria) Gao

UNI : DG2896

Homework 8

Due: 1:00 pm Thursday 3 Nov 2016

1. **Credit risk mitigation**

   Consider the following three credit risk mitigants:

   1. Netting
   2. Collateralization
   3. Use of a centralized clearing party

   Explain what each is, how it reduces credit risk and describe one way that each one increases risk.

Solution:

- **Netting:**

  A **netting agreement** is an optional part of the ISDA master agreement. It stip- ulates that in the event of a default, the recovery is on the net market value of all contracts covered.

  When one has more than one position with a counterparty and no netting agreement, in the event of default, one would incur losses on the assets and have to still pay the full value of the liabilities.

  With a netting agreement in place, one would only incur losses on the net value of the covered positions. Netting reduces risk when one has a combination of both assets and liabilities with a counterparty. It has no effect if all of the contracts are long or all are short.

  By stipulating that some assets will be credited to specific liabilities before other liabilities, a netting agreement essentially transfers risk from the covered derivative positions to the other creditors.

- **Collateralization:**

  Collateralization is where capital is held to cover losses in the event of default. In the event of a default, the loss will be on the value of the position less the collateral, thus reducing the losses in the event of default.

  Like with netting, by allocating assets (the collateral) to specific positions in advance of other positions, collateralization transfers risk to the other creditors.

Solution:

- **Centralized clearing party:**
  A **centralized clearing party** (CCP) is a third party that stands between a party and a counterparty that are entering into a contract. The party essentially enters into the contract with the CCP instead of counterparty, and the counterparty enters into the contract with the CCP instead of with the party.
  The CCP reduces risk by taking on the credit risk of the contract. Each trade is collateralized both with initial margin and variational margin, reducing the risk in the same way as collateralization. In addition, the CCP has capital of its own that can be used to cover any losses, and has backers that can be drawn on as well.
  CCPs can increase risk by reducing netting, if there is more than one. Also, if a default causes trouble for a clearing house, it could cause a default event to become a bigger problem.

2. **CDS**
   Assuming a constant hazard rate of 0.03, a risk free rate of 5%, a 40% recovery rate, and annual coupon payments, what is the par spread for a CDS contract maturing in 5 years? In 10 years?

Solution:
We will do a little more. We will solve for the spread of CDSs maturing in n years.
The discount function is:

$$D(t) = e^{-0.05t}$$

The hazard rate is given as:

$$\lambda(t) = 0.03$$

The survival probability function is then:

$$\begin{aligned} s(t) &= e^{-\int_0^t \lambda(u)du} \\ &= e^{-0.03t} \end{aligned}$$

The default time PDF is given by

$$p(t) = \lambda(t)s(t) = 0.03e^{-0.03t}$$

Let $T_i = i$, for $i = 0, 1, ..., 5$. The par spread for a CDS contract maturing in Tn years is:

$$C_n = \frac{\int_0^{T_n}(1-R)D(u)p(u)du}{\sum_{i=1}^{n}\alpha(T_i)D(T_i)s(T_i)+\int_0^{T_n}\alpha^* D(u)p(u)du}$$

The contract pays the spread annually. Each annual payment is the full spread payment, so

$$\lambda(t) = 1$$

The accrued interest factor is then just the part of the year within the period that has passed, so

$$\lambda^*(t) = t - T_i when T_i \leq t < T_{i+1}$$

The easy way to proceed is to now complete the calculation in Mathematica, or Matlab. The bare-knuckled approach is as follows:
The numerator in the expression for C_n is then:

$$\int_0^{T_n}(1-R)D(u)p(u)du = 0.225(1-e^{-0.08n})$$

The summation in the denominator simplifies as:

$$\sum_{i=1}^{n}\alpha(T_i)D(T_i)s(T_i = \sum_{i=1}^{n}e^{-0.08i}$$

That?s a geometric sum, so we can add it up, but we won?t bother.
The integral in the denominator is:

$$\int_0^{T_n}\alpha^* D(u)p(u)du = \sum_{i=1}^{n}\int_{T_{i-1}}^{T_i}(u-T_{i-1})e^{-0.05u}0.03e^{-0.03u}du$$

$$= 0.03\sum_{i=1}^{n}\int_{i-1}^{i}(u-i+1)e^{-0.08u}du$$

$$= \sum_{i=1}^{n}4.6875e^{-0.08(i-1)} - 5.0625e^{-0.08i}$$

Two more geometric sums that we won't bother to simplify. Thus, the formula for the CDS spreads at times T_n is:

$$C_n = \frac{0.225(1-e^{-0.08n})}{\sum_{i=1}^{n}e^{-0.08i}+\sum_{i=1}^{n}4.6875e^{-0.08(i-1)}-5.0625e^{-0.08i}}$$

$$= 0.2215854\frac{(1-e^{-0.08n})}{\sum_{i=1}^{n}e^{-0.08i}}$$

$$= 0.0184522$$

Thus, we see that in this case, a constant hazard rate yields a constant CDS spread. We could prove this in general with a little more work. This is done in the Mathematica notebook. This is very close to the rough estimate of the hazard rate times the LGD rate, which is 300 bp times 0.6, namely 180 bp.

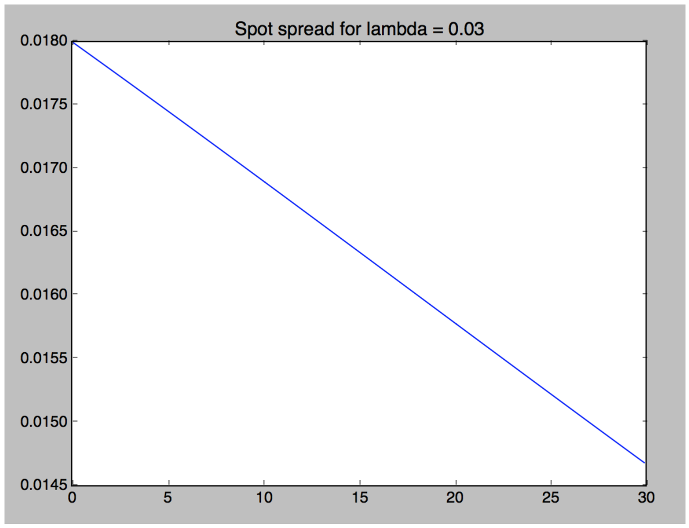
The spot spread starts off close at the above estimate, but drops off over time. It is given by:

$$sprd(t) = (-1/t)log(1 - (1 - R)(1 - s(t)))$$

$$= (-1/t)log(1 - 0.6(1 - e^{-0.03t})$$

$$= (-1/t)log(0.4 + 0.6e^{-0.03t})$$

Plot of spot spread,



3. **Monte Carlo VaR I**

Same setting as in previous VaR calculation problems.

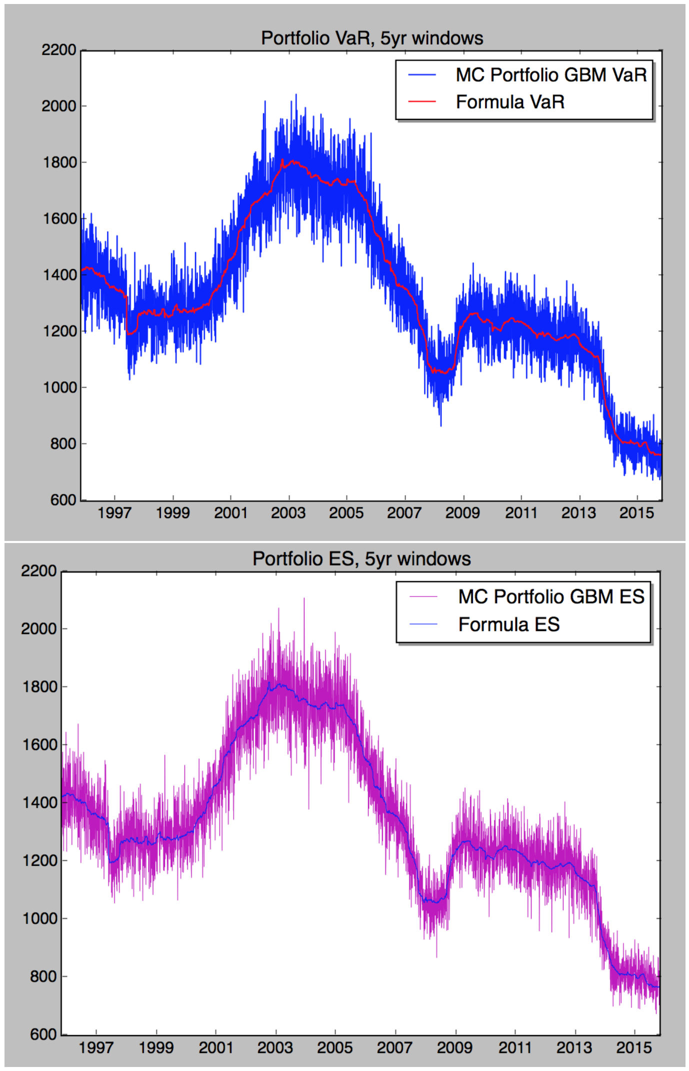
Compute the 99% 5 day VaR and 97.5% 5 day ES of P by Monte Carlo. Simulate it based on the volatility estimates for P from previous problems using the parameter estimates based on 5 year windows and assuming that P follows GBM. Again, compute it each day assuming a $10,000 position in P each day.

Graph the results and compare to all of the previous VaR calculations that used 5 year windows.

Which VaR calculations does the Monte Carlo VaR agree with, and which does it differ from? Why?

Solution:

See Code: RISK-MNGM-Homework8.ipynotebook



4. **Monte Carlo VaR II**
Repeat the last problem, but this time compute the VaR by simulating the underlying stocks, which are assumed to follow GBM, and estimated using 5 year windows. Graph the results and compare to the previous VaR calculations using 5 year windows.
Which VaR calculations does the Monte Carlo VaR agree with, and which does it differ from? Why?

Solution:
See Code: RISK-MNGM-Homework8.ipynotebook
First, we calculate the VaR and ES for the portfolio based on fitting GBM parameters for the portfolio itself.
For each date, we have the GBM for the portfolio based on the drift and volatility computed above. So, we can simulate the 5 day portfolio values, subtract from the initial value, and take the 1% worst case to get the VaR, and average the 2.5% worst cases for the ES.

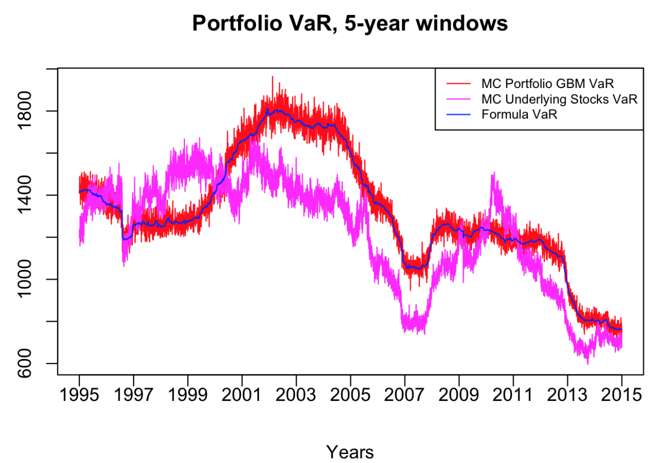
Next, we calculate the VaR and ES for the portfolio based on simulating A and I and taking their weighted average to get the portfolio value. For this we use the formulas from last week?s homework relating the historical samples to the two stock model. Recall:

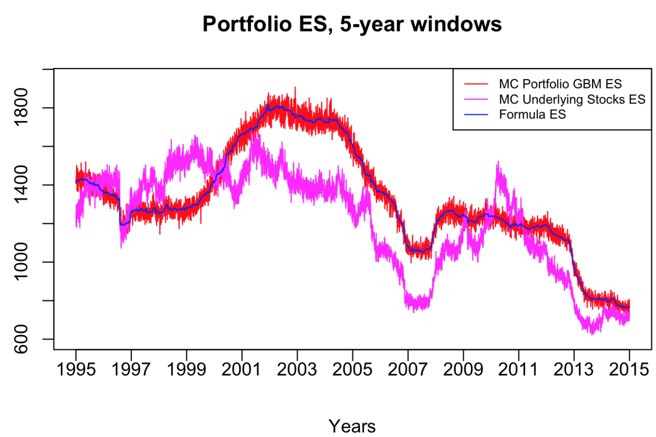
$$dS_1 = \mu_1 S_1 dt + \sigma_1 S_1 dW_1$$
$$\mathrm{dS}_2 = \mu_2 S_2 dt + \sigma_2 S_2 dW2$$
$$\mathrm{dW}_1 dW_2 = \rho dt$$

In other words, for each date, proceed as when computing the drift and volatility of A and I. First compute the log returns of the last n years of daily returns of A and I, call the R1 and R2. Then we have,for i=1and i=2:

$$\sigma_i \approx \sigma_{i0} * \sqrt{252}$$
$$\mu_i \approx 252\mu_{i0} + \sigma_i^2/2$$
$$\rho \approx 252Cov(R_1, R_2)/\sigma_1\sigma_2$$

Results:





Observations:

- The Monte Carlo VaR and ES using the portfolio GBM parameters matches the results from the formulas, but is noisier and takes longer to run. This is as expected, because the Monte Carlo and the formula are two different implementations of the same model. This serves as model validation of the previous results.
- The Monte Carlo VaR and ES do not match the formulas when the Monte Carlo is run by simulating the underlying AMD and INTC prices. This is also as expected, as it cannot be the case that AMD prices, INTC prices, and the portfolio of the two all follow GBM. We see that they are fairly close, but often differ by as much as 20%.
- As the MC VaR using the underlying stock prices is so close to the portfolio GBM case, it?s hard to say that the MC VaR on the underlying stock prices is consistently more or less in line with the historical VaR than the VaR assuming the portfolio prices themselves follow GBM.