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Financial Risk Management&Regulation (M5320)
Fall 2016
https://courseworks.columbia.edu

Homework 6

Due: 1:00 pm Thursday 20 Oct 2016

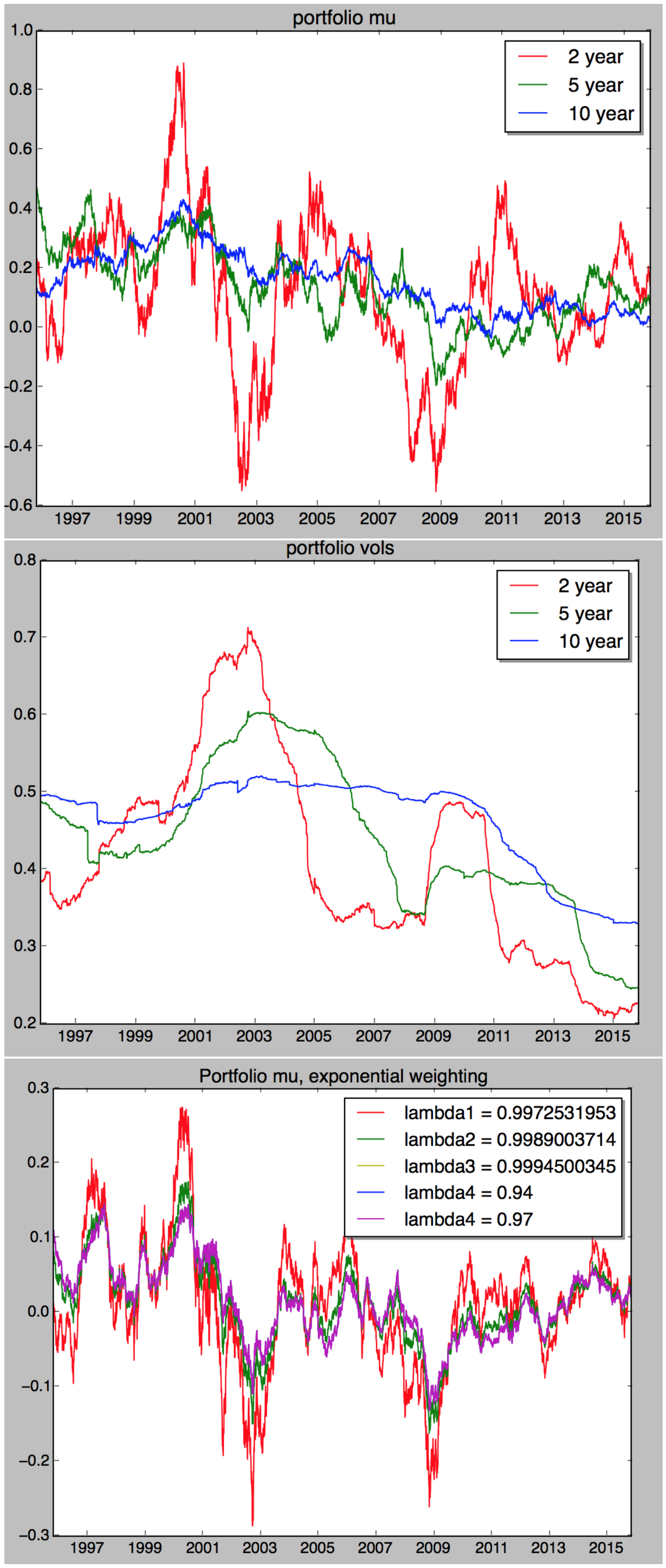
1. **Portfolio mean and variance**
   As in prior homeworks, A is AMD stock (ticker AMD), and I is Intel stock (ticker INTC). Their historical values are in the spreadsheets AMD-yahoo.csv and INTC-yahoo.csv, respectively, which were downloaded using the script getYahoo.sh.
   Let S being a portfolio P consisting of $5,000 invested in INTC and $5,000 invested in AMD, where the investment was made 20 years ago. That would be using the 10/8/1996 prices of $8.0625 for AMD, and $8.860368 for INTC, which would be 620 shares of AMD, and 546 shares of INTC.
   Assuming S is GBM, compute the portfolio mean and variance with 2, 5, and 10 year windows, and with equivalent λs of 0.9972531953, 0.9989003714, and 0.9994500345, respectively.

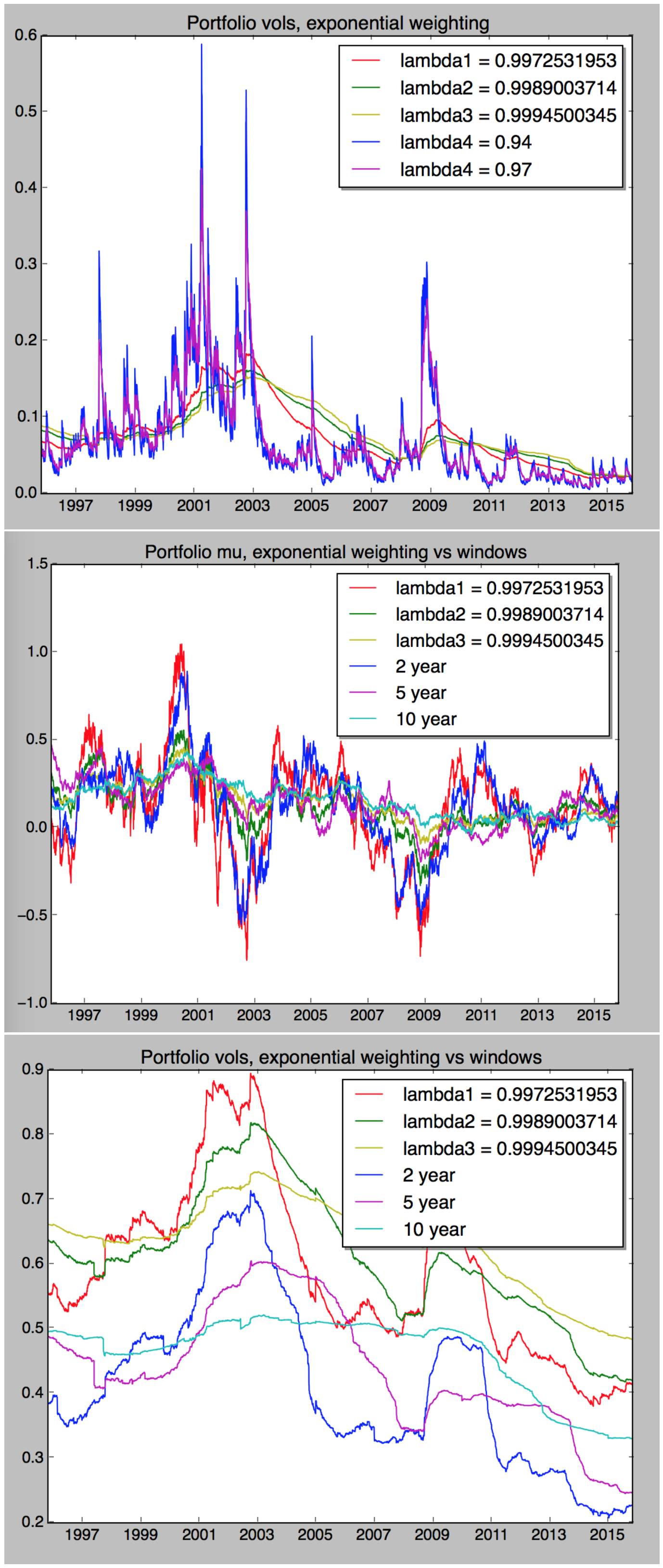
Solution:
See HW6-problem1.ipynotebook
1. Make a list with only the dates and prices from AMD and INTC.
2. Make a check column to confirm that the dates line up. Return 1 if they don?t match, else 0.
3 Chop off INTC at end so that both data sets extend to the same date.
4. If the sum of the check column is 0, we?re good to go.
5. Create a column for the value of 620 shares of AMD.
6. Create a column for the value of 546 shares of INTC.
7. Adding them yields P.
8. Copy the calculation columns for INTC (or for AMD) from previous homework solutions and apply them to the portfolio (P).
Observations (from plots below):

- Portfolio Vols are similar to INTC vols, maybe a little more stable.

- Estimates are not as jumpy as they are for INTC and AMD.

Results:


portfolio mu
2 year
5 year
10 year
portfolio vols
2 year
5 year
10 year
Portfolio mu, exponential weighting
lambda1 = 0.9972531953
lambda2 = 0.9989003714
lambda3 = 0.9994500345
lambda4 = 0.94
lambda4 = 0.97



Portfolio vols, exponential weighting
lambda1 = 0.9972531953
lambda2 = 0.9989003714
lambda3 = 0.9994500345
lambda4 = 0.94
lambda4 = 0.97
0.0
0.1
0.2
0.3
0.4
0.5
0.6
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
Portfolio mu, exponential weighting vs windows
lambda1 = 0.9972531953
lambda2 = 0.9989003714
lambda3 = 0.9994500345
2 year
5 year
10 year
−1.0
−0.5
0.0
0.5
1.0
1.5
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
Portfolio vols, exponential weighting vs windows
lambda1 = 0.9972531953
lambda2 = 0.9989003714
lambda3 = 0.9994500345
2 year
5 year
10 year
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015

2. **Portfolio VaR and ES**

Compute the 99% 5 day VaR and 97.5% ES of the above portfolio two ways. One by assuming it follows GBM and using the above mean and variance estimates, and one by assuming it is normally distributed and using the formulas for the mean and variance of a portfolio of two stocks. Compare and contrast.

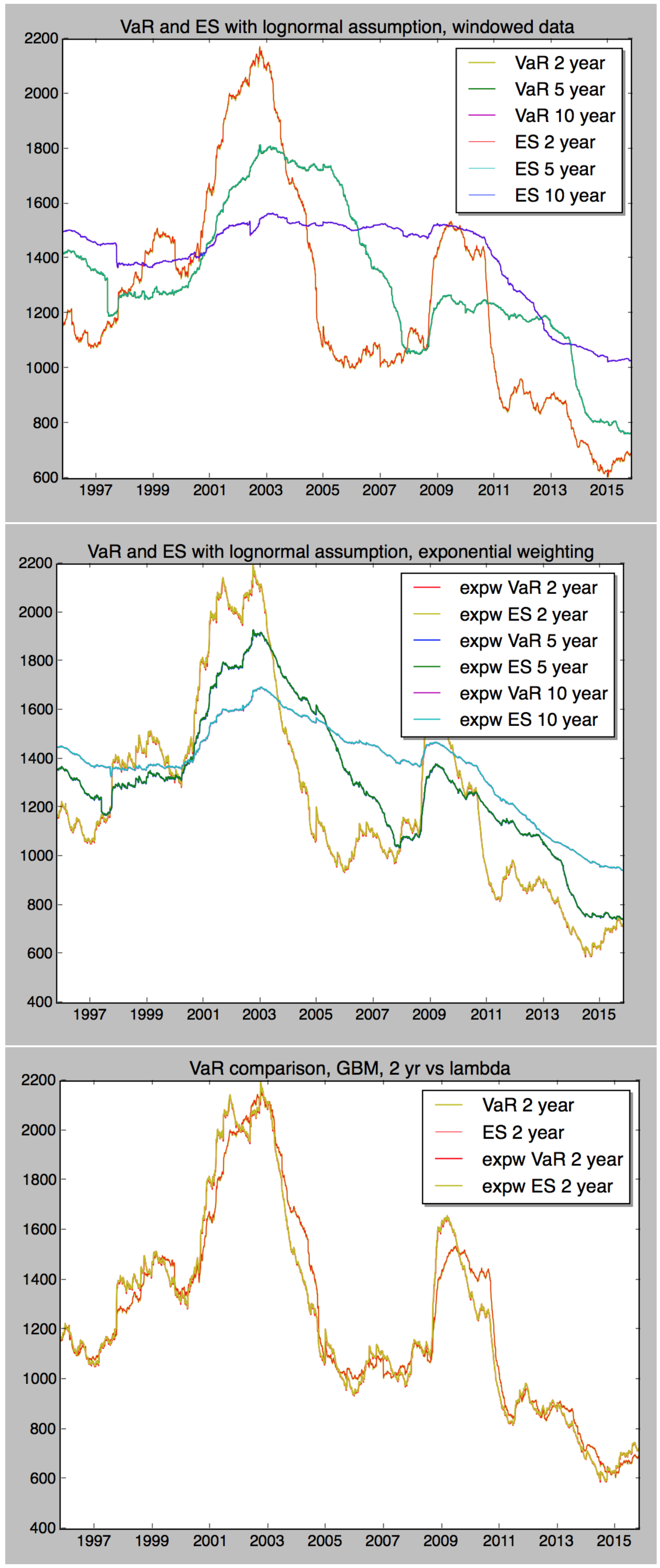
Solution:
See HW6-problem2.ipynotebook.
First, we calculate the VaR and ES for the portfolio based on fitting GBM parameters for the portfolio itself. This is the same as problem 5.2 applied to the index time series instead of the Apple time series (for example).
For each date, we have the GBM for the portfolio based on the drifts and volatilities computed above. For each case, we compute the VaR using the formulas for the VaR and ES of a GBM. Details are in the code.
Next, we calculate the VaR and ES for the portfolio based on assuming A and I follow GBM. Using the GBM parameters from problem 5.2, we compute the VaR assuming normally distributed future values using the formulas from lecture 4. Details are in the code.
To apply the VaR formula, we need to estimate the correlation of the Brownian motion drivers. The model is:

$$
\begin{aligned}
dS_1 &= \mu_1 S_1 dt + \sigma_1 S_1 dW_1 \\
\mathrm{dS}_2 &= \mu_2 S_2 dt + \sigma_2 S_2 dW2 \\
\mathrm{dW}_1 dW_2 &= \rho dt
\end{aligned}
$$

In other words, for each date, proceed as when computing the drift and volatility of A and I. First compute the log returns of the last n years of daily returns of A and I, call the R1 and R2. Then we have,for i=1and i=2:

$$
\begin{aligned}
\sigma_i &\approx \sigma_{i0} * \sqrt{252} \\
\mu_i &\approx 252\mu_{i0} + \sigma_i^2/2 \\
\rho &\approx 252Cov(R_1, R_2)/\sigma_1\sigma_2
\end{aligned}
$$

Observations (from plots below): 1). 97.5% ES is virtually identical to 99% VaR for both the lognormal case and the normal case.
2). Equivalent exponential weighting rolls in changes in vol faster than windowing does.
3). Exponential is also smoother, presumably because of the impact of still having some weight beyond the corresponding windows.
4). Portfolio VaRs are lower than the individual stock VaRs, showing that diver- sification reduces risk.
5). The GBM and normal based VaR calculations with parameter estimations based on 2 year windows are quite similar. The diverge substantially in the 5 year window and 10 year window cases. Either the correlation calculated over the long periods of time are not reflective of the behavior, or there?s a bug in the code.



VaR and ES with lognormal assumption, windowed data
VaR 2 year
VaR 5 year
VaR 10 year
ES 2 year
ES 5 year
ES 10 year
2200
2000
1800
1600
1400
1200
1000
800
600
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
VaR and ES with lognormal assumption, exponential weighting
expw VaR 2 year
expw ES 2 year
expw VaR 5 year
expw ES 5 year
expw VaR 10 year
expw ES 10 year
2200
2000
1800
1600
1400
1200
1000
800
600
400
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
VaR comparison, GBM, 2 yr vs lambda
VaR 2 year
ES 2 year
expw VaR 2 year
expw ES 2 year
2200
2000
1800
1600
1400
1200
1000
800
600
400
1997
1999
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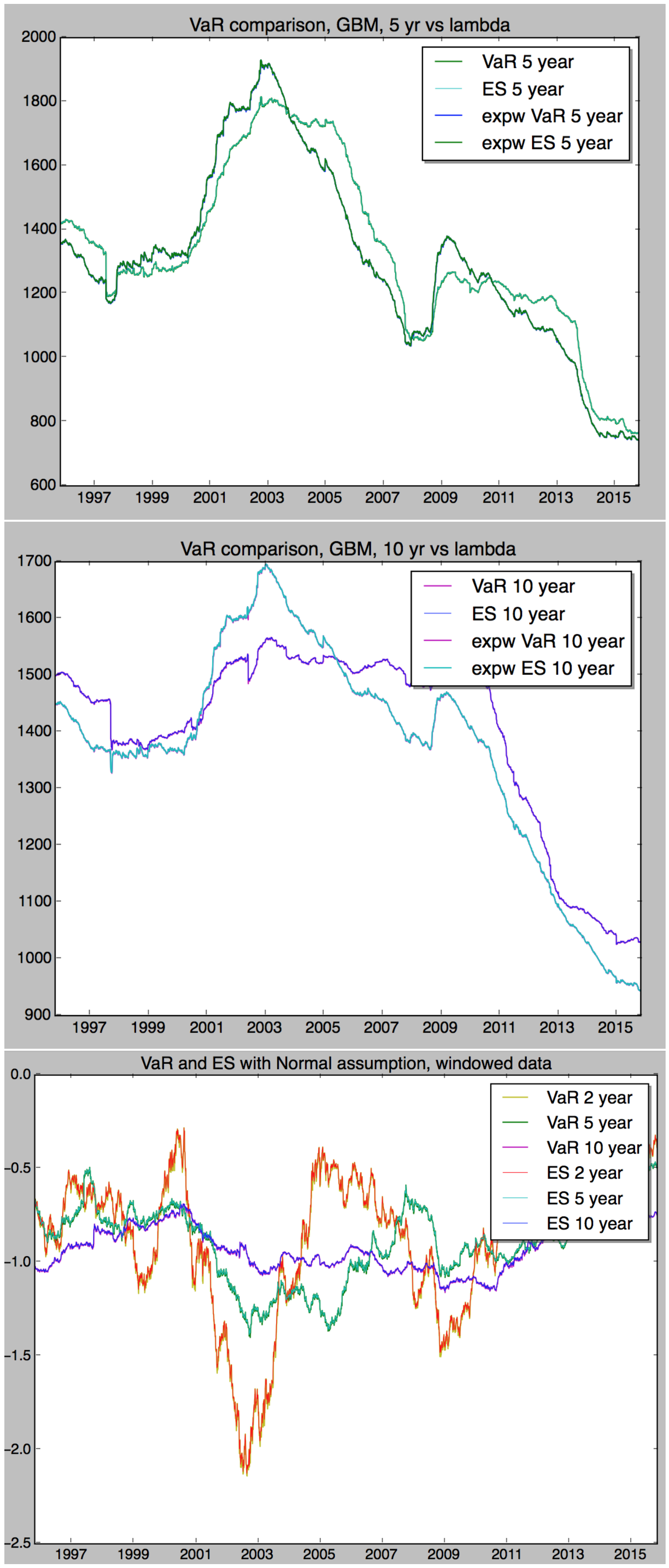
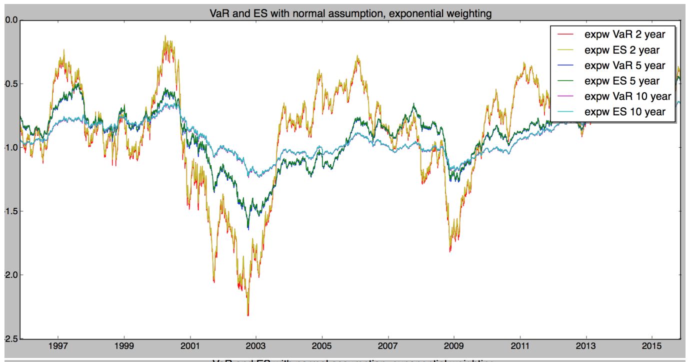
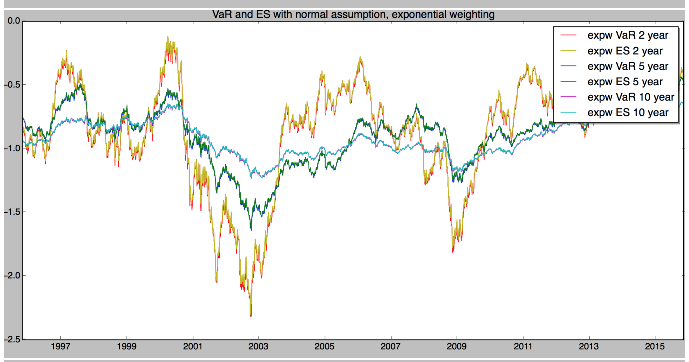
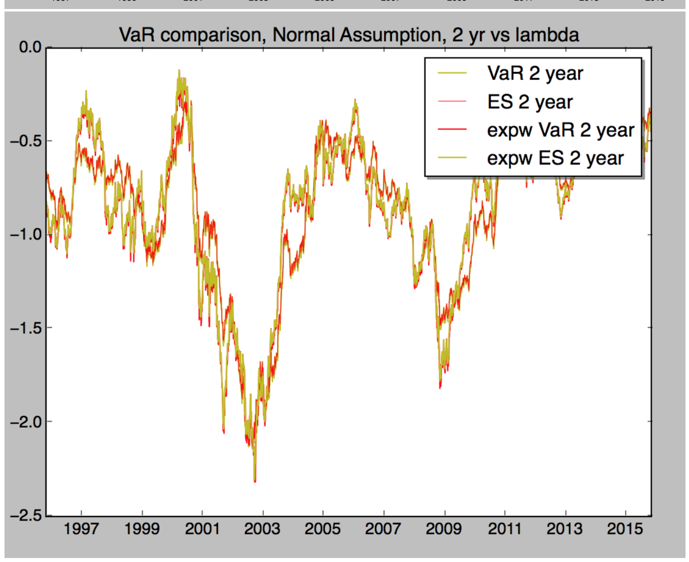
2007
2009
2011
2013
2015


VaR comparison, GBM, 5 yr vs lambda
VaR 5 year
ES 5 year
expw VaR 5 year
expw ES 5 year
VaR comparison, GBM, 10 yr vs lambda
VaR 10 year
ES 10 year
expw VaR 10 year
expw ES 10 year
VaR and ES with Normal assumption, windowed data
VaR 2 year
VaR 5 year
VaR 10 year
ES 2 year
ES 5 year
ES 10 year


VaR and ES with normal assumption, exponential weighting
expw VaR 2 year
expw ES 2 year
expw VaR 5 year
expw ES 5 year
expw VaR 10 year
expw ES 10 year
0.0
0.5
1.0
1.5
2.0
2.5
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


VaR and ES with normal assumption, exponential weighting
expw VaR 2 year
expw ES 2 year
expw VaR 5 year
expw ES 5 year
expw VaR 10 year
expw ES 10 year
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


VaR comparison, Normal Assumption, 2 yr vs lambda
VaR 2 year
ES 2 year
expw VaR 2 year
expw ES 2 year
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
1997
1999
2001
2003
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2007
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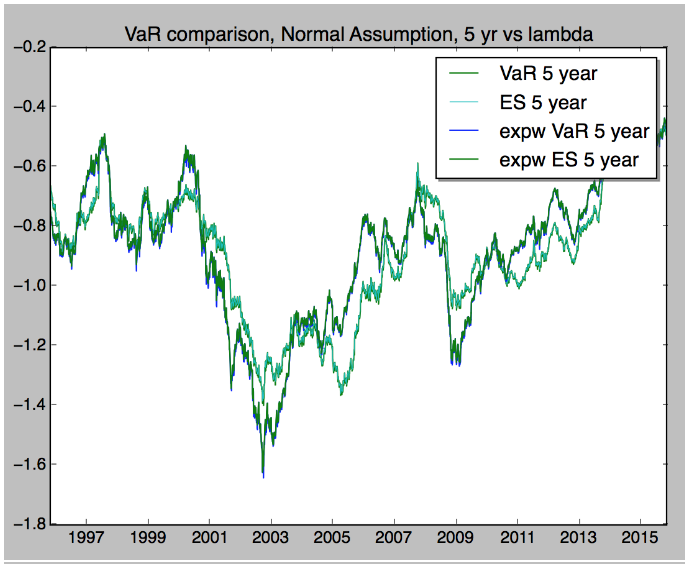
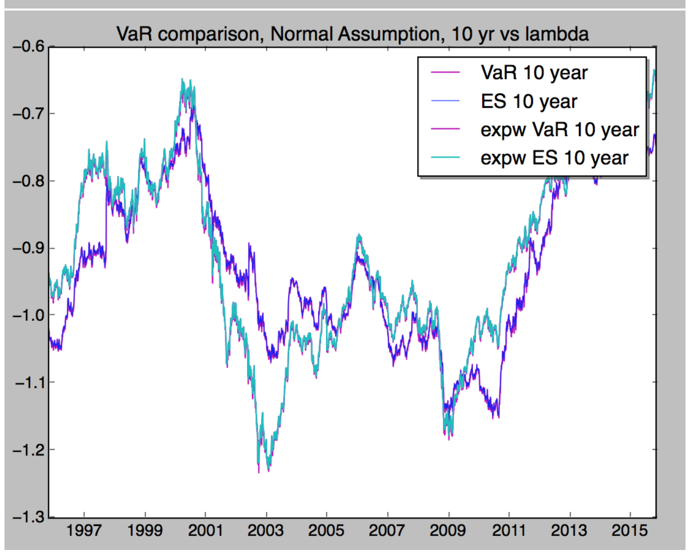
2015





3. **Reduced form modeling 1**

A reduced form model is used for the default time of a company. The hazard rate is $\lambda = 0.015$.
(a) What is the probability that the firm will default within 5 years?
(b) What is the probability that the firm will default in between 3 and 4 years?

Solution:
A reduced form model is used for the default time of a company. The hazard rate is $\lambda = 0.015$.
The survival probability for time t is $s(t) = e\text{lambda}t = e^{-0.015t}$
The probability of defaulting within 5 years is then:
$1 - s(5) = 1 - e^{-0.015*5} = 0.072256$
There is a 7.2256% chance of default within 5 years.
The probability that the firm will default in between 3 and 4 years is the prob- ability of defaulting within 4 years minus the probability of defaulting within 3 years. This is:

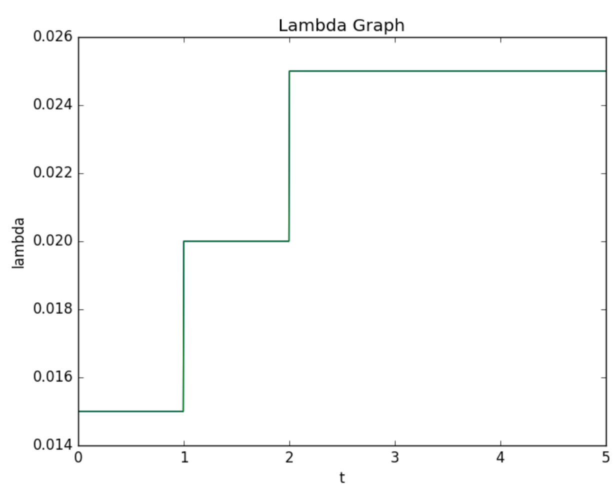
$(1-s(4))-(1-s(3)) = s(3)-s(4) = e^{-0.015*3} - e^{-0.015*4} = 0.0142329$
There is a 1.423% chance of defaulting in between 3 and 4 years.

4. **Reduced form modeling 2**
Define the hazard rate λ(t) by:

$$\lambda\text{(t)}= \begin{cases} 0.015, & t \leq 1 \\ 0.02, & 1 < t \leq 2 \\ 0.025, & t > 2 \end{cases} \quad (1)$$

Graph λ.
What is the survival probability function? Give its formula and graph it.
What is the default time probability density function? Give its formula and graph it.
Assuming a constant recovery rate of R = 40%, and a constant risk free rate of r = 5%, what is the spread for a zero coupon bond maturing at time t? Give its formula and graph it.

Solution:

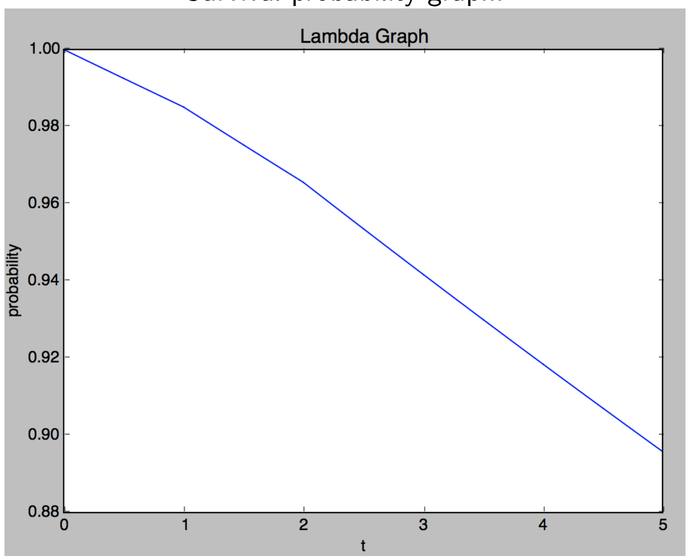


The survival probability function is:

$s(t) = e^{-\int_0^t \lambda(\mu)d\mu}$, and

$$\int_0^t \lambda(\mu)d\mu = \begin{cases} 0.015t, & t \leq 1 \\ 0.02t - 0.005, & 1 < t \leq 2 \\ 0.025t - 0.015, & t > 2 \end{cases} \quad (2)$$

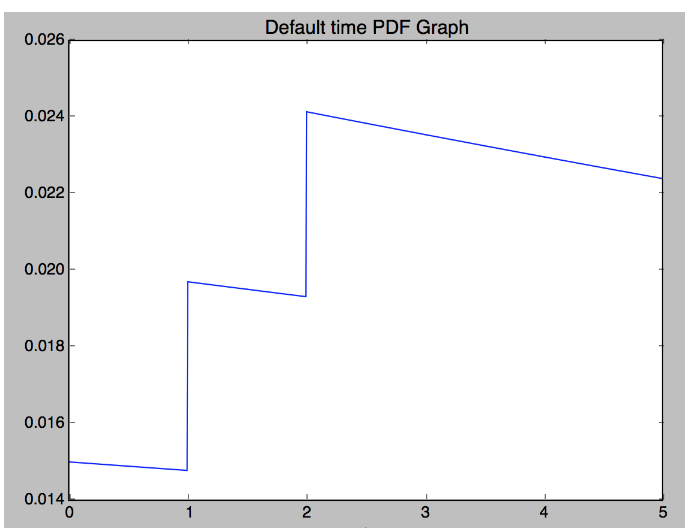
Survival probability graph:



Default time PDF is given by

$$p(t) = \lambda(t)s(t)$$

Graph of default time PDF:



Formula for spread for a risky zero coupon bond with recovery R = 40%, and risk free rate r = 5%:

$sprd(t) = 1/t * log(1 - (1 - R) * (1 - s(t)))$ and graph of spread: