Math GR 5320: Financial Risk Management and Regulation

Assignment 6

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Due next Thursday by 1:00 pm.

For help. the preferred approach is to post questions on the Q&A tab in Piazza:

https://piazza.com/columbia/fall2016/mathg5320_001_2016_3/home

These will be quickly responded to and will be helpful to others in the class.

Otherwise, attend TA office hours, email a TA or the professor, or schedule a meeting.

1. **Portfolio mean and variance**

   As in prior homeworks, A is AMD stock (ticker AMD), and I is Intel stock (ticker INTC). Their historical values are in the spreadsheets `AMD-yahoo.csv` and `INTC-yahoo.csv`, respectively, which were downloaded using the script `getYahoo.sh`.

   Let S being a portfolio P consisting of \$5,000 invested in INTC and \$5,000 invested in AMD, where the investment was made 20 years ago. That would be using the 10/8/1996 prices of \$8.0625 for AMD, and \$8.860368 for INTC, which would be 620 shares of AMD, and 546 shares of INTC.

   Assuming S is GBM, compute the portfolio mean and variance with 2, 5, and 10 year windows, and with equivalent λs of 0.9972531953, 0.9989003714, and 0.9994500345, respectively.

2. **Portfolio VaR and ES**

   Compute the 99% 5 day VaR and 97.5% ES of the above portfolio two ways. One by assuming it follows GBM and using the above mean and variance estimates, and one by assuming it is normally distributed and using the formulas for the mean and variance of a portfolio of two stocks.

   Compare and contrast.

3. **Reduced form modeling 1**

   A reduced form model is used for the default time of a company. The hazard rate is $\lambda = 0.015$.

   (a) What is the probability that the firm will default within 5 years?

   (b) What is the probability that the firm will default in between 3 and 4 years?

4. **Reduced form modeling 2**

   Define the hazard rate $\lambda(t)$ by:

$$\lambda(t) = \begin{cases} 0.015 & t \leq 1 \\ 0.02 & 1 < t \leq 2 \\ 0.025 & t > 2 \end{cases}$$

Graph λ.

What is the survival probability function? Give its formula and graph it.

What is the default time probability density function? Give its formula and graph it.

Assuming a constant recovery rate of $R = 40\%$, and a constant risk free rate of $r = 5\%$, what is the spread for a zero coupon bond maturing at time t? Give its formula and graph it.