

Homework 10

Due: 1:00 pm Thursday 17 Nov 2016

1. Capital

For each type of position at a bank, tell whether it's an asset or a liability.

- (a) Cash
- (b) Savings accounts
- (c) Long bond positions
- (d) Loans from another bank
- (e) Mortgages made by the bank
- (f) Long position in mortgage backed securities
- (g) Office leases
- (h) Payroll

Solution:

- (a) Asset
- (b) Liability
- (c) Asset
- (d) Liability
- (e) Asset
- (f) Asset
- (g) Liability
- (h) Liability

2. Risk weighted assets

A bank has the following assets and liabilities:

- 100,000 in cash
- 50,000 in residential mortgages 50,000 in corporate bonds
- 195,000 in deposits

Given risk weights of:

- 0% - Home country sovereign debt
- 10% - Certain public debt
- 20% - Interbank loans
- 50% - Residential mortgages
- 100% - Private sector debt

- (a) What are the assets and liabilities?
- (b) What is the capital?
- (c) What are the risk weighted assets?
- (d) What is the capital ratio?
- (e) What is the leverage ratio?

Solution:

- (a) The cash, residential mortgages and corporate bonds are assets. The deposits are liabilities.
- (b) The capital is the $assets - liabilities = 200,000 - 195,000 = 5,000$
- (c) The risk weighted assets are $100,000 \times 0 + 50,000 \times .5 + 50,000 \times 1 = 75,000$
- (d) The capital ratio is the capital divided by the risk weighted assets $= 5,000 / 75,000 = 6.66\%$
- (e) The leverage ratio is the capital divided by the assets $= 5,000 / 200,000 = 2.5\%$

3. Option portfolio VaR

A portfolio consists of a \$10,000,000 position in the SP. Risk management says that the 99% 5 day VaR has to be reduced by 20%. Instead of liquidating 20% of the portfolio to do this, the portfolio manager would like to liquidate a portion of the portfolio and use the money to buy 1 year maturity ATM puts to bring his VaR down the required amount.

Use the data from the spreadsheet hw10.csv, which contains approximately the last 10 years of SP prices and the corresponding implied volatilities for at the money 1 year puts. The spreadsheet was generated by hw10.xls, which downloaded the data from the Bloomberg. Assume the implied volatility surface is flat (i.e. that all options on the SP on a given date trade at the same implied volatility, regardless of strike and maturity).

Assume the 1 year risk free rate is 0.5%

Assume the only risk factor is the stock price (i.e. ? ignore implied volatility risk).

(a) What are the GBM parameters and the current VaR of the portfolio(on 11/4/2016)?

Solution:

See RISK-MNGT-HW10.ipynb

Calculating the VaR parametrically, we get:

Part 1: GBM parameters for SPX on 11/4/2016: $\mu = 0.0659$, $\sigma = 0.197$, VaR on \$10000000 position = 6154283

(b) What is the VaR of the portfolio if 1% is liquidated and used to buy at the money 1 year maturity puts?

Solution:

We reduce our stock position by 1%, price the ATM put, use the 1% to buy the appropriate number of puts, and compute the VaR by Monte Carlo. All in the above octave script. Results:

Part 2: Liquidating 1% and buying ATM puts.

SPX price = 2085.18, Put price on 1 share = 157.77

SPX shares = 4747.79, Put shares = 633.384

SPX position = 9900000.00, Put position = 100000.00

VaR = 5292683, percentage reduction = 14.00.

We see that liquidating 1% and purchasing puts reduces the VaR by over 10%. We would have to liquidate 9% of the stock to have the same effect. This is because the options have a delta of -1/2, but cost about 1/20th of the cost of a share. So, for \$214.09, you can buy a put option on two shares of stock, and its price will go up by \$1 when the stock price drops by \$1. So, for about 1/10th of the price of a share of stock, you can roughly eliminate the losses on 1 share of stock. Hence, putting 1% of the stock into options eliminates about 10% of the risk.

(c) What percentage of the portfolio should be liquidated to purchase ATM 1 year puts so as to reduce the portfolio VaR by 20%?

Solution:

In general, we cannot just compute the put VaR and scale because the VaR of the portfolio is not the sum of the VaRs of the positions. However, in this case, since a) the option price is

monotonic in the stock price, and b) our ATM options will not have huge gammas compared to the underlying price movements, we could get a fairly accurate estimate by assuming linearity. None the less, we will just apply a solver to the Monte Carlo VaR calculation. The quick and accurate way to do this would be to first do 1 Monte Carlo, and save the horizon stock and option price vectors S_T and O_T . Then, the VaR for a given percentile sale of p would be

$$\text{quantile}((1 - p)10,000,000/S_0(S_0 - S_T) + p \times 10,000,000/V_0(V_0 - V_T), .99)$$

which is a much faster and more accurate calculation than repeating the Monte Carlo for each iteration.

Iterating using the Monte Carlo subjects us to the Monte Carlo noise, which is significant even with 1,000,000 paths. As a result, it is difficult to reduce the VaR exactly 20%. One could improve the accuracy by repeating the root finding with a larger number of paths.

Results:

Part 3: Percentage liquidation.

Percentage to use = 1.52

New VaR = 4923426 , percentage reduction in VaR = 20.15.

(d) How would the results change if volatility risk were taken into account?

Solution:

If volatility risk were taken into account, and the volatility is sufficiently positively correlated with the stock, then on average when the stock moves down, the option vols go down, so the value of the option position would not grow as much and would not reduce the VaR as much. If the volatility is sufficiently negatively correlated with the stock, then on average when the stock moves down, the volatility moves up, so the same option position would reduce the VaR more.

If the correlation is close enough to zero, then it won't have much impact because the volga (2nd derivative of price with respect to vol) is zero for an at the money option. It could be give a slight increase of the VaR or a slight decrease, depending on shape of the distribution of the implied volatility.