Math GR 5320: Financial Risk Management and Regulation

Assignment 3

Department of Mathematics
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Due next Thursday by 1:00 pm.

For help. the preferred approach is to post questions on the Q&A tab in Piazza:

https://piazza.com/columbia/fall2016/mathg5320_001_2016_3/home

These will be quickly responded to and will be helpful to others in the class.

Otherwise, attend TA office hours, email a TA or the professor, or schedule a meeting.

1. **VaR**

   (a) What is VaR?

   (b) What desirable property or properties of risk measures does it fail to exhibit?

   (c) Give (an) example of the above property failure(s).

2. **Expected Shortfall**

   (a) What is ES?

   (b) What desirable property or properties of risk measures does it fail to exhibit?

3. **Scenario and sensitivity analysis**

   Intel is currently trading at \$37.55, interest rates are currently 2%, and 1 year call options on Intel with a strike of \$40 are trading at an implied vol of 22%.

   Suppose you have 1,000 shares of Intel.

   (a) What is the current value of the position and the sensitivity of your position to moves of Intel?

   (b) How many 1 year call options struck at \$40 should you write so that your sensitivity to moves of Intel becomes zero?

   (c) After putting on the position from the previous part, what would the loss be if the stock moves up 10% and what would be the loss if the stock moves down 10%

   (d) How does the risk of the portfolio with the call options compare to the risk of the portfolio without the call options?

4. **Monte Carlo VaR calculation**

   A portfolio consisting of 3 shares of Apple and 2 share of IBM is purchased when the Apple price was \$106.74 and the price of IBM was \$163.60. Our empirical estimation of the Apple and IBM prices in 1 week gives the following 10 equally likely possibilities for the pair of prices.

Apple	IBM
116.52	162.56
108.60	163.17
101.21	160.95
112.11	162.39
111.25	171.11
105.08	163.36
109.67	159.67
109.43	168.89
105.81	162.05
116.58	173.98

   What are the 1 week 90th, 80th, 70th and 60th percentile VaR and ES of the portfolio?

5. **Formula VaR calculation**

   Our portfolio is currently worth \$10,000. We believe our portfolio follows GBM with drift $\mu = 0.05$ and volatility $\sigma = 30\%$. What is the 1 day, 1 week, and 1 year expected value of the portfolio value, standard deviation of the portfolio value and 90% VaR of the portfolio?

   Note - 1 week is 5 trading days out of 252 trading days per year.

6. **ES formula**

   If $dS = \mu S dt + \sigma S dW$, for constants μ and σ, what is $\mathrm{ES}(S, T, p)$? Please provide formula and proof.