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Financial Risk Management&Regulation (M5320)

Fall 2016

https://courseworks.columbia.edu

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Homework 7

Due: 1:00 pm Thursday 27 Oct 2016

1. **Merton**

Assume a Merton model for default with a constant risk free rate of r and an initial firm value of $1,000,000. The firm issued a zero coupon bond with face value B that matures at time T:

$$dV = \mu V dt + \lambda V dW$$
$$\mu = 0.1$$
$$\sigma = 0.2$$
$$\text{r} = .05$$
$$\text{V}_0 = 1,000,000$$
$$\text{T}=5$$
$$\text{B} = 700,\ 000$$

(a) What is the probability that the firm will default within 5 years?

(b) What is the probability that the firm will default in between 3 and 4 years?

Solution:

(a) The probability of defaulting within 5 years is the probability that ST $<$ B. This is given by:

$$p = \Phi\left(\frac{log(B/V_0)-(r-\sigma^2/2)T}{\sigma\sqrt{T}}\right)$$

$$= \Phi\left(\frac{log(0.7)-(0.05-0.02)5}{0.2\sqrt{5}}\right)$$

$$= \Phi(-1.13296)$$

$$= 0.12861553$$

The probability of defaulting within 5 years is 12.86%.

(b) In the Merton model, default only happens at the bond redemption time, which is T = 5, so the probability of defaulting in between 3 and 4 years is zero.

2. **Historical VaR, relative changes**
Continuing with the settings in the previous homework, namely A being AMD, I being INTC, and P being the portfolio consisting of 620 shares of AMD, and 546 shares of INTC.
Compute the 99% 5 day historical VaR and the 97.5% 5 day historical ES for A, I, and P for each day in the last 20 years. Do this for each date d by applying the previous 5 years of daily log returns to the position on that date.
How do the historical VaR and ES compare to the previous results?
Would it matter if the VaR for P was computed from the historical time series changes for P or from the historical time series changes for A and I applied to the underlying stocks in the portfolio? Why or why not?

Solution:
The solution to the next problem includes the solution to this one.

3. **Historical VaR, absolute changes**
Repeat the previous problem applying absolute changes instead of log changes. How do the VaR and ES change? In this case, would it matter if the calculation for P is done using the historical changes for P or using the historical changes for A and I applied to the underlying stocks in the portfolio? Why or why not?

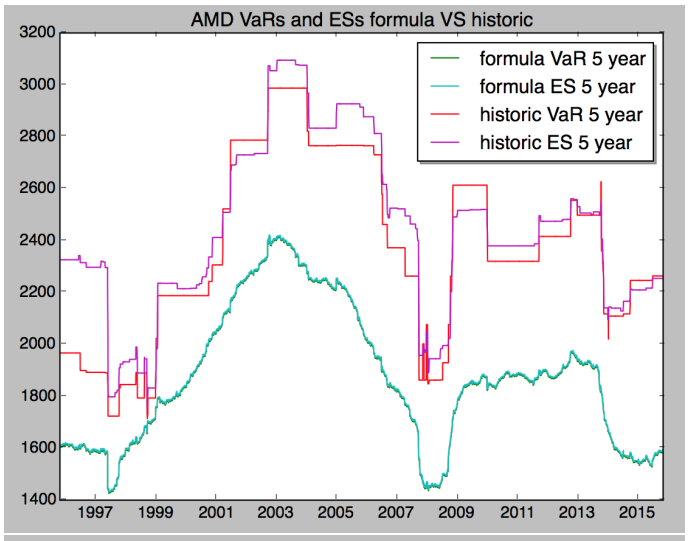
Solution:
Solution to this problem and the previous one combined.

Problems in previous homework compute 5 day VaRs and ESs by making assumptions about the processes, estimating parameters, and computing the VaR and ES directly.

To compute 5 day VaRs historically, we need to directly apply historically observed changes over 5 day periods to today's portfolio. This means applying 5 successive daily returns to generate each scenario. Equivalently, we can apply the 5 day returns.

Code: RISK-MNGM-HW7-Problem23.ipynotebook

Results:


AMD VaRs and ESs formula VS historic
formula VaR 5 year
formula ES 5 year
historic VaR 5 year
historic ES 5 year
3200
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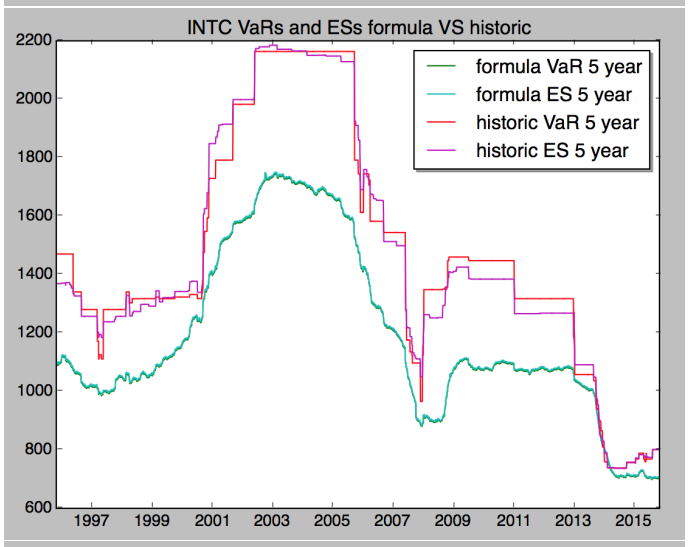
INTC VaRs and ESs formula VS historic
formula VaR 5 year
formula ES 5 year
historic VaR 5 year
historic ES 5 year
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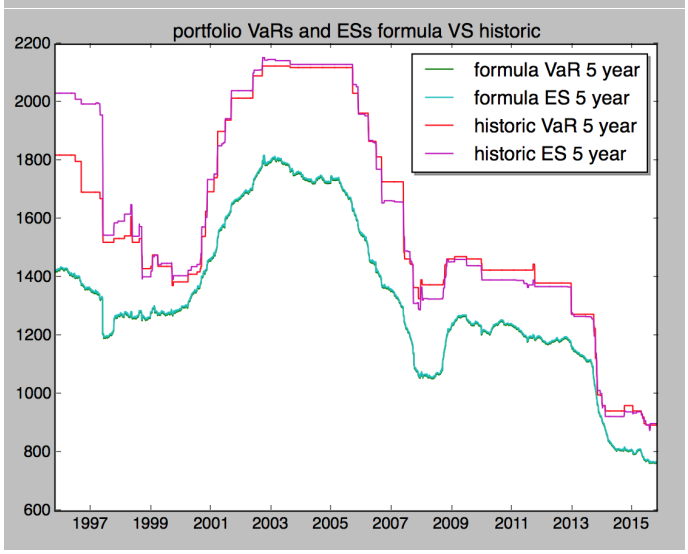
portfolio VaRs and ESs formula VS historic
formula VaR 5 year
formula ES 5 year
historic VaR 5 year
historic ES 5 year
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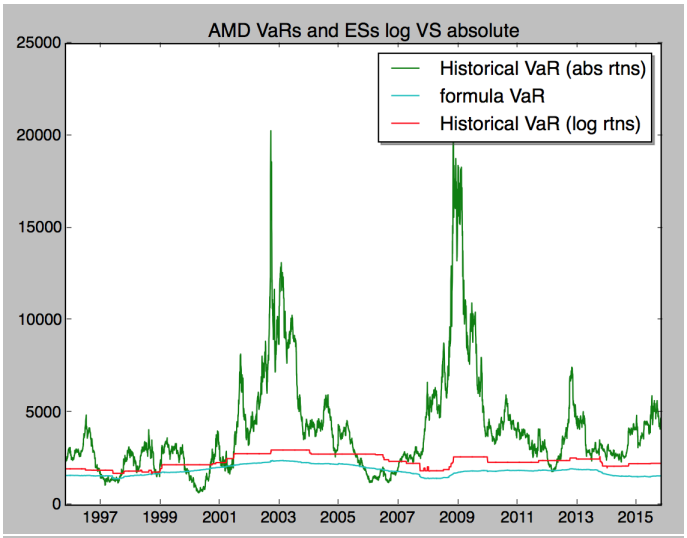

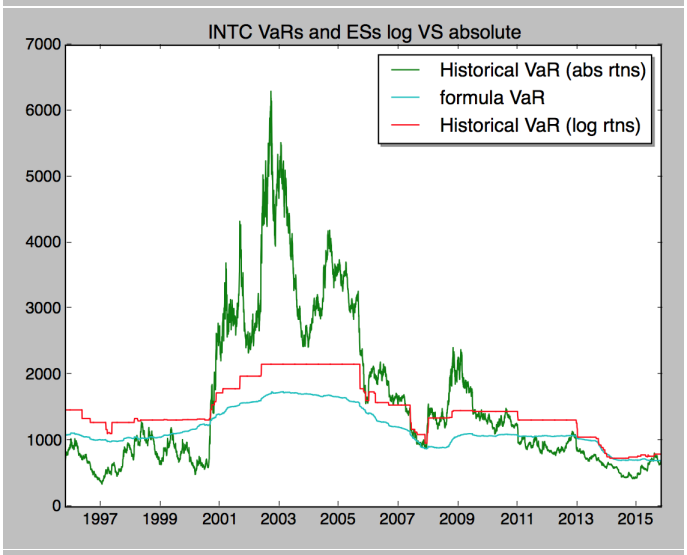

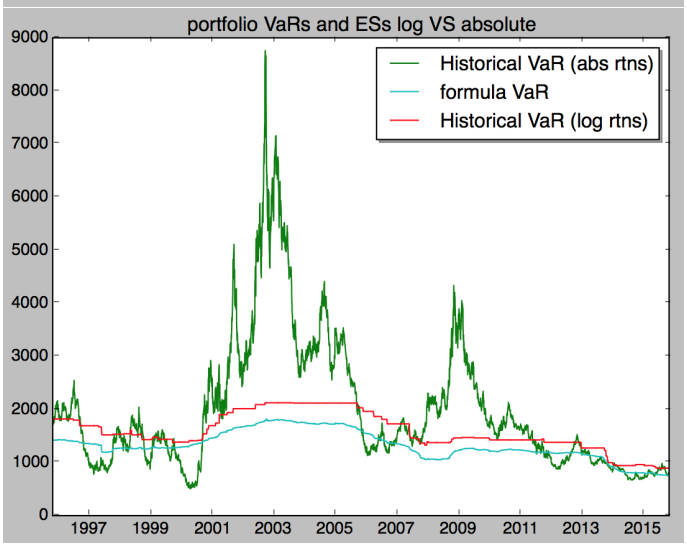

Observations:

1. The historical VaRs and ESs have long flat periods. What is happening is that there?s a jump in the VaR whenever one of the extreme scenarios impacting the VaR at that percentile enters or exits the sliding window.

2. The historical VaRs are similar to the formula VaRs, but also have periods where they are substantially higher. The historical VaR is rarely lower. This indicates that using GBM for the VaR calculations underestimates the VaR.

3. The historical VaR differs the most from the historical ES for Apple, indicating that the approximation of Apple by a GBM is worse than for IBM and for the portfolio.

4. With lognormal assumptions, we saw that the 99% VaR and the 97.5% ES were pretty much identical for these stocks and this portfolio, and the same holds for normal assumptions. Here, we see that the historical VaR and ES are often close, but also often differ. This indicates the history is neither normal nor lognormal.

5. The historical VaR and ES for the portfolio when using the relative changes of the portfolio itself will be different from computing the portfolio change based on the corresponding changes of AMD and INTC. This is because the value of the AMD and INTC components varies over time. For example, consider a portfolio consisting of 1 share of A and 1 share of I. If on one day $A = 1$ and $I = 1$, and 5 days later $A = 2$ and $I = 1$, then between those dates, A had 100% growth, I had 0% growth and the portfolio had 50% growth. However, if today $A = 10$ and $I = 5$, then applying the portfolio change to today?s portfolio value of 15 brings it to 22.5. Applying the corresponding Apple and IBM changes would bring it to 25.

6. The absolute changes applied to the positions rapidly jump around because we are looking at the VaR of a fixed dollar amount of the stock. Since the stocks are rising over time, as you roll back in time, \$10,000 of stock is more and more shares. This causes the historical VaR with absolute changes to vary day by day as the absolute stock price change is applied to a different number of shares each day. Since the stocks are rising over time, this causes the VaR to increase as we roll back in time, until the relevant scenarios roll into or out of the 5 year window. Absolute changes do not appear to make a lot of sense for historical VaR on stocks.

7. The portfolio has a fixed number of shares of AMD and of INTC. The absolute changes applied to AMD and INTC is the same as the absolute change to the corresponding portfolio. So the historical VaR in this case would be the same whether we apply the changes to the portfolio price directly, or compute it based on the changes to AMD and INTC.