**Name : Duanhong(Gloria) Gao**
UNI : DG2896

**Financial Risk Management&Regulation (M5320)**
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# Homework 3

Due: 1:00 pm Thursday 29 September 2016

1. **VaR**
   (a) What is VaR?
   (b) What desirable property or properties of risk measures does it fail to exhibit?
   (c) Give (an) example of the above property failure(s).

*Solution:* (a) The p percentile VaR is X if p percent of the time our losses are less than or equal to X:

$$VaR(V,T,p) = G^{-1}(p), where$$
$$G(X) = P[V_0 - V_T \leq X] = E^P[1_{V_0 - V_T \leq X}]$$

(b) VaR is not coherent, which fails subadditivity — diversification can increase risk.
(c) Consider the following horizon price distribution for stocks S1 and S2 with current value of 100 each:

| Time | S1 | S2 | S1+S2 |
|---|---|---|---|
| 1 | 100 | 50 | 150 |
| 2 | 100 | 100 | 200 |
| 3 | 100 | 100 | 200 |
| 4 | 100 | 100 | 200 |
| 5 | 100 | 100 | 200 |
| 6 | 100 | 100 | 200 |
| 7 | 100 | 100 | 200 |
| 8 | 100 | 100 | 200 |
| 9 | 100 | 100 | 200 |
| 10 | 50 | 100 | 150 |

Here we can see that the 80% VaR for Stock 1 is 0, so is stock 2. But the 80% $VaR(S1+S2) = 150$. Therefore, $VaR(S1)+VaR(S2) < VaR(S1+S2)$, violating the property of subadditivity.

2. **Expected Shortfall**
   (a) What is ES?
   (b) What desirable property or properties of risk measures does it fail to exhibit?

*Solution:* (a)The expected shortfall is the average loss in the percentile tail:

$$ES(V, T, p) = -E^p[V_T - V_0 | V_T - V_0 \leq -VaR(V_T, p)]$$

(b)Difficult to back test ? testable, but not elicitable. Not clear how actionable it is. Harder to compute.

3. **Scenario and sensitivity analysis**

   Intel is currently trading at \$37.55, interest rates are currently 2%, and 1 year call options on Intel with a strike of \$40 are trading at an implied vol of 22%.

   Suppose you have 1,000 shares of Intel.

   (a) What is the current value of the position and the sensitivity of your position to moves of Intel?

   (b) How many 1 year call options struck at \$40 should you write so that your sensitivity to moves of Intel becomes zero?

   (c) After putting on the position from the previous part, what would the loss be if the stock moves up 10% and what would be the loss if the stock moves down 10%

   (d) How does the risk of the portfolio with the call options compare to the risk of the portfolio without the call options?

*Solution:* (a) Current value $= 1000 * 37.55 = 37,550$, sensitivity of the position = 1000

(b) Price of option to buy 1 share (Black-Scholes formula) = \$2.60, delta = 0.46, should write $\frac{1000}{0.46} = 2173$ call options

(c) If the stock price moves up 10%, the loss is (stock value change + option value change)

$= 1000 * \$37.55 * 10\% - 2173 * (\$4.67\text{(new option price)}-\$2.60) = \$-743.11$ .

However, if the stock price moves down 10%, loss = (stock value change + option value change) $= 1000 * \$37.55 * 10\% + 2173 * (\$1.17\text{(new option price)}-\$2.60) = \$-647.61$ .

(d)The risk is smaller than the portfolio without the call options.

4. **Monte Carlo VaR calculation**

   A portfolio consisting of 3 shares of Apple and 2 share of IBM is purchased when the Apple price was \$106.74 and the price of IBM was \$163.60. Our empirical estimation of the Apple and IBM prices in 1 week gives the following 10 equally likely possibilities for the pair of prices.

| Apple | IBM |
|---|---|
| 116.52 | 162.56 |
| 108.6 | 163.17 |
| 101.21 | 160.95 |
| 112.11 | 162.39 |
| 111.25 | 171.11 |
| 105.08 | 163.36 |
| 109.67 | 159.67 |
| 109.43 | 168.89 |
| 105.81 | 162.05 |
| 116.58 | 173.98 |

What are the 1 week 90th, 80th, 70th and 60th percentile VaR and ES of the portfolio?

*Solution:* Here, we sort the price of the stocks first and calculate the portfolio value in a spreadsheet. We get:

| No | Apple | IBM | Portfolio | Loss | Sum | ES |
|---|---|---|---|---|---|---|
| 1 | 101.21 | 160.95 | 625.53 | 21.89 | 21.89 | 21.89 |
| 2 | 105.08 | 163.36 | 641.96 | 5.46 | 27.35 | 13.68 |
| 3 | 105.81 | 162.05 | 641.53 | 5.89 | 33.24 | 11.08 |
| 4 | 108.6 | 163.17 | 652.14 | -4.72 | 28.52 | 7.13 |
| 5 | 109.43 | 168.89 | 666.07 | -18.65 | 9.87 | 1.97 |
| 6 | 109.67 | 159.67 | 648.35 | -0.93 | 8.94 | 1.49 |
| 7 | 111.25 | 171.11 | 675.97 | -28.55 | -19.61 | -2.80 |
| 8 | 112.11 | 162.39 | 661.11 | -13.69 | -33.3 | -4.16 |
| 9 | 116.52 | 162.56 | 674.68 | -27.26 | -60.56 | -6.73 |
| 10 | 116.58 | 173.98 | 697.7 | -50.28 | -110.84 | -11.08 |

1 week 90th percentile VaR and ES is both 21.89, which is the worst loss in 1 week.
The 2nd worst is 5.89, so that's the 80th percentile VaR. The 80th percentile ES is the average of the two worst, namely 13.89.
The 3rd worst is the 70th percentile VaR, namely 5.46. The average of the 3 worst is the 70th percentile ES, namely 11.08.
The 60th percentile Var and ES are -0.93 and 8.08, respectively.

5. **Formula VaR calculation**

Our portfolio is currently worth \$10,000. We believe our portfolio follows GBM with drift $\mu = 0.05$ and volatility $\sigma = 30\%$. What is the 1 day, 1 week, and 1 year expected value of the portfolio value, standard deviation of the portfolio value and 90% VaR of the portfolio?
Note - 1 week is 5 trading days out of 252 trading days per year.

*Solution:*
$dV = \mu W dt + \sigma V dW$

- $E[VT] = V_0 e^{\mu T}$
- $var[V_T] = V_0^2(e^{\sigma^2 T} - 1)e^{2\mu T}$
- $VaR(V, T, p) = V_0 - V_0 e^{\sigma\sqrt{T}\Phi^{-1}(1-p)+(\mu-\sigma^2/2)T}$
- $V_0 = 10000$
- $\mu = 0.05$
- $\sigma = 0.03$
- 1 day horizon $T = 1/252$
- 5 day horizon $T = 5/252$
- 1year horizon $T = 1$

1 day horizon:

- t = 1/252
- Expected future value = 10001.98
- Standard deviation = 189.04
- VaR at 90% = 239.09

5 day horizon:

- t = 5/252
- Expected future value = 10009.93
- Standard deviation = 423.19
- VaR at 90% = 526.21

1 year horizon:

- t=1
- Expected future value = 10512.71
- Standard deviation = 3226.12
- VaR at 90% = 3157.73

# VaR_Calculate

September 26, 2016

```
In [1]: #####Problem 4
        import scipy.stats as ss
        import numpy as np
        def VaRCalculate(V0, mu, sigma, T, p):
            ans = []
            Expect = V0 * np.exp(mu * T)
            ans.append(Expect)
            Var = pow(V0,2) * ( np.exp(pow(sigma,2)*T) - 1) * np.exp(2*mu*T)
            ans.append(Var**(0.5))
            VaR = V0 - V0 * np.exp( sigma * T**(0.5)* ss.norm.ppf(1-p) + (mu - pow(sigma,2)/2)*T )
            ans.append(VaR)
            return ans

        V0 = 10000
        mu = 0.05
        sigma = 0.3
        T = 1/252
        p = 0.9
        VaRCalculate(V0, mu, sigma, T, p)

Out[1]: [10001.98432383514, 189.03661471820388, 239.08753658932801]

In [2]: V0 = 10000
        mu = 0.05
        sigma = 0.3
        T = 5/252
        p = 0.9
        VaRCalculate(V0, mu, sigma, T, p)

Out[2]: [10009.925557498198, 423.18546674214952, 526.21167589945435]

In [3]: V0 = 10000
        mu = 0.05
        sigma = 0.3
        T = 1
        p = 0.9
        VaRCalculate(V0, mu, sigma, T, p)

Out[3]: [10512.710963760241, 3226.1227438149945, 3157.7294840360928]

In [ ]:
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6. **ES formula**
If $dS = \mu S dt + \sigma S dW$, for constants $\mu$ and $\sigma$, what is $ES(S, T, p)$? Please provide formula and proof.

*Solution:* Here, we need to compute the

$$ES(S, T, p) = E[S_0 - S_T | S_0 - S_T > VaR(S, T, p)]$$
$$= S_0 - E[S_T | S_T < S0 - VaR(S, T, p)]$$
$$= S_0 - \frac{E[S_T 1_{S_T < S_0 - VaR(S,T,p)}]}{1-p}$$
$$= S_0 - \int_0^X S_T P(S_T) dS_T$$
where $X = S_0 - VaR(S, T, p)$

We can use BS model to derive it.

$$C = e^{-rT} E^+[max(S_T - K), 0]$$
$$= e^{-rT} \int_K^\infty (S_T - K) P(S_T) dS_T$$
$$= e^{-rT} (\int_K^\infty S_T P(S_T) dS_T - \int_K^\infty K P(S_T) dS_T)$$
$$= \Phi(d_1) S_0 - \Phi(d_2) K e^{-rT}$$
where $d_1 = \frac{1}{\sigma\sqrt{T}}(log(S_0/K) + (r + \sigma^2/2)T)$ and,

$d_2 = d_1 - \sigma\sqrt{T}$ Also, we have E[S$_T$] $= \int_0^\infty S_T P(S_T) dS_T$

$$= \int_0^K S_T P(S_T) dS_T + \int_K^\infty S_T P(S_T) dS_T$$
$$= \int_0^K S_T P(S_T) dS_T + e^{rT} \Phi(d_1) S_0$$
Plus, $S_0 e^{rT} = E[S_T]$

Therefore,
$\int_0^K S_T P(S_T) dS_T =$S$_0 e^{rT} - e^{rT} \Phi(d_1) S_0$

Now replace r by $\mu$ and K by X and result can be derived.