MATH GR 5320 Financial Risk Management and Regulation

Lecture 4: Market Risk

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Outline

Review

Review

Risk measurement four questions:

1. What factors affect the value of the portfolio?
    - Identify risk factors.
2. What is the behavior of these dependencies?
    - Model behavior of risk factors:
        - Develop a model.
        - Fit model to data.
        - Use model to yield future distribution of risk factors.
3. What is the behavior of the P&L as a function of these factors?
    - Use structure of portfolio and pricing functions to compute distribution of V_T.
4. How do I summarize the behavior of the P&L?
    - Compute risk measures, which are statistics on V_T.

Review

Method varies depending on form of risk measures, factor models, and pricing models:

- Analytically.
- Semi-analytically.
- Using approximations.
- Making simplifying assumptions.
- Using Monte Carlo.

Review

Modeling examples:

- Direct modeling via fitted GBMs for stock portfolio.
- Factor model for stock portfolio.
- Option portfolio.

Risk measures:

- Common measures:
    - Variance
    - VaR
    - ES
    - Expectiles
- Compare based on:
    - Coherence
    - Elicitability/backtestability
    - Robustness
    - Ease of use
- Need large numbers of samples for accuracy!

Review

Computations:
$dS = \mu S dt + \sigma S dW$

- $E[S_T] = S_0 e^{\mu T}$
- $\mathrm{var}[S_T] = S_0^2 (e^{\sigma^2 T} - 1) e^{2\mu T}$
- $\mathrm{VaR}(S, T, p) = S_0 - S_0 e^{\sigma \sqrt{T} \Phi^{-1}(1-p) + (\mu - \frac{\sigma^2}{2}) T}$
- $\mathrm{ES}(S, T, p) =$ homework
- Mostly easy to compute using martingale techniques.

Market Risk

Market Risk

Market risk – The impact of market moves.

We will discuss:

- Estimating potential losses due to market moves - VaR.
- Practical issues of VaR computations.

References: Chen [Che13], Steele [Ste01]

Complications

For one stock, given the processes are known, the calculations are not too difficult.

But, there are complications:

- Intermediate events.
  - Dividends.
  - Option maturities.
  - Option exercises.
  - Cash flows.
  - Barrier crossings.
  - Trading.
- No formulas in general.
- Backtesting.
- Assumed that we know the stock processes.

Intermediate events

Nothing much can really be done about the intermediate events!

- Cannot model trading.
- What to do with cash flows?
  - Reinvest in what?
  - Carry cash?

Instead, compute impact on current portfolio!

- Consider changes.
- Apply to current portfolio.

Risks

- Risk of a frozen portfolio.
- Ignoring time decay.
- Ignoring trader behavior.

Formulas

Formulas are not bad for one stock following GBM.

Consider a position in 2 stocks:

$$V_T = aS_T + bS'_T$$

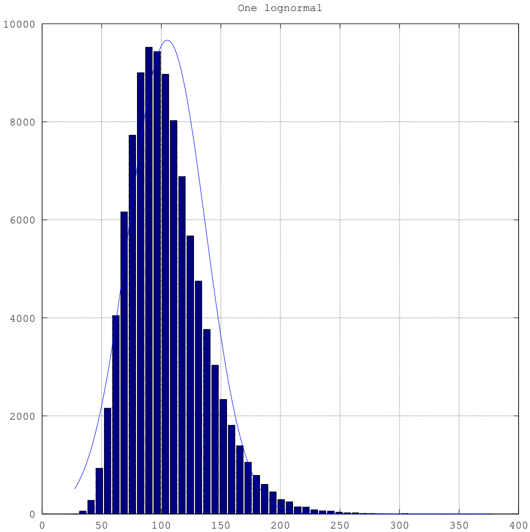
V_T is *not* lognormal.

It gets worse for a long-short portfolio:

- a is positive and b is negative.
- Portfolio can have negative values.

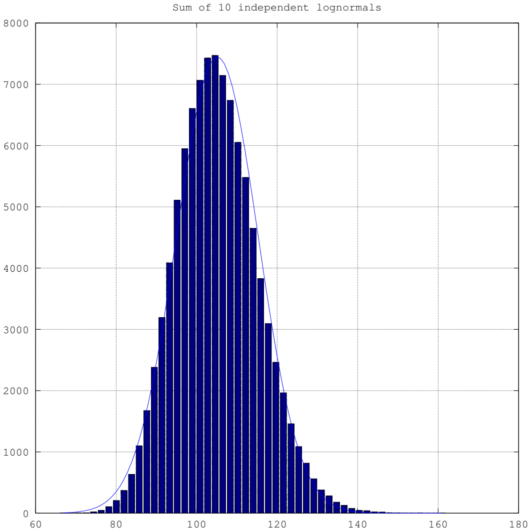
Formulas example

Consider 1 lognormal vs sum of 10 lognormals ($S_0 = 100$, $\mu = 0.05$, $\sigma = 0.3$):



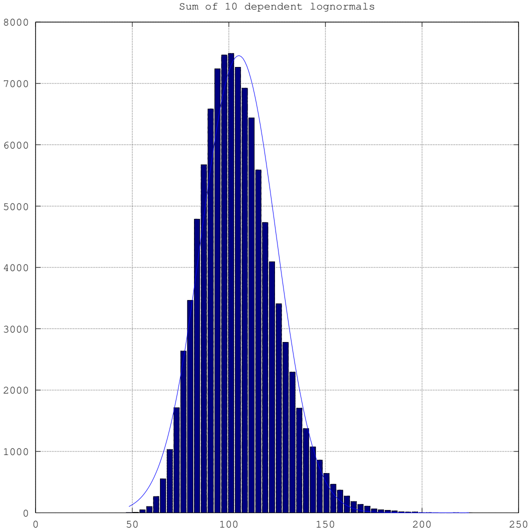
Formulas example

Sum of 10 independent lognormals:



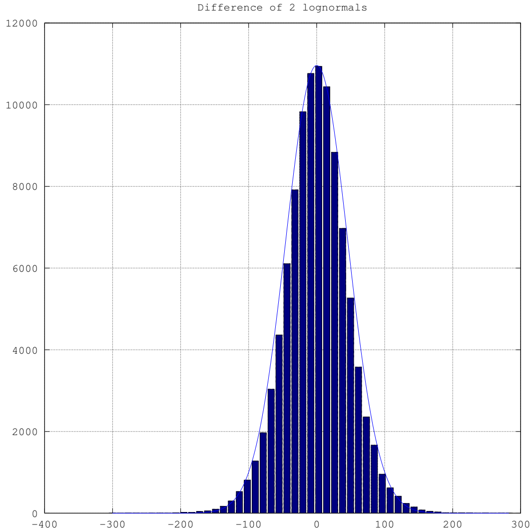
Formulas example

Sum of 10 dependent lognormals ($\rho = 30\%$) :



Formulas example

Difference of 2 lognormals:



Formulas

What about a call option with strike K maturing at time T'?

$$
\begin{aligned}
V_T &= \Phi(d_1)S_T - \Phi(d_2)Ke^{-r(T'-T)} \\
d_1 &= \frac{1}{\sigma'\sqrt{T'-T}}(\log(S_T/K) + (r + \sigma'^2/2)(T'-T)) \\
d_2 &= \frac{1}{\sigma'\sqrt{T'-T}}(\log(S_T/K) + (r - \sigma'^2/2)(T'-T))
\end{aligned}
$$

For a call option:

- Value is highly nonlinear.
- Need to consider joint distribution at time T of:
  - Implied volatility σ'.
  - Risk free rate from T to T'.
  - Stock price S_T.

Formulas

We don't have formulas for the distribution of V_T for

- Portfolios of log-normally distributed stocks
- Options
- Bonds
- ...

Approaches:

- Grossly oversimplify — Assume normality everywhere!
  - Everything has normal or lognormal returns – Parametric VaR.
  - Original Riskmetrics.
- Use numerical methods – Monte Carlo:
  - Fit the complicated model and simulate – Monte Carlo VaR.
  - Apply historical changes to current portfolio – Historical VaR.

Backtesting

Standard deviation:

- If computed from history, then it should automatically pass back tests.

VaR:

- If 99% VaR is X, 1% of the time we should see losses exceeding X.

Simple VaR backtest:

- Each year, should have 2.5 VaR exceptions for the 1 day 99% VaR.
- Not a strong test — What's the variance on the number of VaR exceptions?

Expected shortfall:

- Test VaR at a variety of levels?
- Harder to validate.

Types of VaR

Types of VaR

Three types of VaR:

- Parametric VaR.
- Historical VaR.
- Monte Carlo VaR.

Parametric VaR

Parametric VaR – make simplifying assumptions so as to yield a formula for the VaR based on approximate mean and variance calculations.

- Not really a model or a type – an approximation methodology applied to the problem.
- Accurate when positions and payoffs are linear, but then no different from a simple variance calculation.
- Inaccurate otherwise – requires hacks (like quadratic approximation) to improve it.

Parametric VaR example 1

Consider a portfolio V:

- Current value: 1,000,000
- Expected value in 1 week: 1,000,600
- 1 week std dev of V: 40,000

If V_t is assumed normal, what is the 1 week 99% VaR?

Parametric VaR example 1

Solution:

Normal distribution:

- 1% tail is 2.326 standard deviations out.

$$
\begin{aligned}
\mathrm{VaR}(V) &= V_0 - (E[V_t] - 2.326\,\mathrm{sd}[V_t]) \\
&= 1,000,000 - (1,000,600 - 2.326 \times 40,000) \\
&= 92,454.
\end{aligned}
$$

Parametric VaR formulas

More commonly, our portfolio is expressed as a sum of positions:

$$V_t = aS_{1,t} + bS_{2,t}$$
$$dS_i = \mu_i S_i dt + \sigma_i S_i dW_i$$
$$dW_1 dW_2 = \rho dt$$

Assume V_t is normal (can be a big assumption).

- Then distribution is determined by mean and variance.

At a horizon t:

$$E[V_t] = aE[S_{1,t}] + bE[S_{2,t}]$$
$$E[V_t^2] = E[a^2 S_{1,t}^2 + b^2 S_{2,t}^2 + 2abS_{1,t}S_{2,t}]$$
$$\mathrm{var}[V_t] = E[V_t^2] - E[V_t]^2$$
$$\mathrm{sd}[V_t] = \sqrt{\mathrm{var}[V_t]}$$

Know $E[S_{1,t}^2]$ and $E[S_{2,t}^2]$. Just need $E[S_{1,t}S_{2,t}]$.

Parametric VaR formulas

Computing $E[S_{1,t}S_{2,t}]$. We know its formula:

$$S_{1,t}S_{2,t} = S_{1,0}S_{2,0}\exp\left((\mu_1+\mu_2-(\sigma_1^2+\sigma_2^2)/2)t+\sigma_1 W_{1,t}+\sigma_2 W_{2,t}\right)$$

$\sigma_1 W_{1,t}+\sigma_2 W_{2,t}$ is normal with mean zero, so variance is given by:

$$\begin{aligned} E[(\sigma_1 W_{1,t}+\sigma_2 W_{2,t})^2] &= E[\sigma_1^2 W_{1,t}^2+\sigma_2^2 W_{2,t}^2+2\sigma_1\sigma_2 W_{1,t}W_{2,t}] \\ &= \sigma_1^2 t+\sigma_2^2 t+2\sigma_1\sigma_2 E[W_{1,t}W_{2,t}] \end{aligned}$$

We compute $E[W_{1,t}W_{2,t}]$ using Ito's formula:

$$\begin{aligned} dW_1W_2 &= W_1dW_2+W_2dW_1+dW_1dW_2 \\ &= W_1dW_2+W_2dW_1+\rho dt \end{aligned}$$

so

$$\begin{aligned} W_{1,t}W_{2,t} &= \int_0^t W_1dW_2+\int_0^t W_2dW_1+\int_0^t \rho ds \\ &= \text{martingale} \ + \ \text{martingale} \ + \rho t \end{aligned}$$

so

$$\begin{aligned} E[W_{1,t}W_{2,t}] &= \rho t \\ E[(\sigma_1 W_{1,t}+\sigma_2 W_{2,t})^2] &= (\sigma_1^2+\sigma_2^2+2\sigma_1\sigma_2\rho)t \end{aligned}$$

Parametric VaR formulas

We have:

$$S_{1,t}S_{2,t} = S_{1,0}S_{2,0}e^{(\mu_1+\mu_2-(\sigma_1^2+\sigma_2^2)/2)t+\sigma_1 W_{1,t}+\sigma_2 W_{2,t}}$$
$$\sigma_1 W_{1,t} + \sigma_2 W_{2,t} \sim \mathcal{N}\left(0, (\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2)t\right)$$

So, $S_1 S_2$ is lognormal and we know the variance of the Brownian motion part, so we can use our formulas for the mean of a lognormal (or just factor out the martingale part):

$$\begin{aligned}
E[S_{1,t}S_{2,t}] &= S_{1,0}S_{2,0}e^{(\mu_1+\mu_2+\rho\sigma_1\sigma_2)t} \\
E[V_t] &= aS_{1,0}e^{\mu_1 t} + bS_{2,0}e^{\mu_2 t} \\
E[V_t^2] &= a^2 S_{1,0}^2 e^{(2\mu_1+\sigma_1^2)t} + b^2 S_{2,0}^2 e^{(2\mu_2+\sigma_2^2)t} \\
&\quad + 2abS_{1,0}S_{2,0}e^{(\mu_1+\mu_2+\rho\sigma_1\sigma_2)t} \\
\mathrm{var}[V_t] &= E[V_t^2] - E[V_t]^2
\end{aligned}$$

Parametric VaR example 2

Example:

$$
\begin{aligned}
V_t &= 300S_{1,t} + 200S_{2,t} \\
S_{1,0} &= 95 \\
S_{2,0} &= 105 \\
dS_1 &= 0.05S_1 dt + 0.3dW_1 \\
dS_2 &= 0.03S_2 dt + 0.2dW_2 \\
dW_1 dW_2 &= 0.25dt
\end{aligned}
$$

What is 99% 1 week VaR of V?

Parametric VaR example 2

Solution:

Let $t = 5/252$. We can compute the mean and variance of V_t:

$$
\begin{aligned}
V_0 &= 300 \times 95 + 200 \times 105 = 49,500 \\
E[aS_{1,t}] &= 300 \times 95 \times e^{0.05t} = 28,528.29 \\
E[bS_{2,t}] &= 200 \times 105 \times e^{0.03t} = 21,012.50 \\
E[V_t] &= E[a \times S_{1,t}] + E[b \times S_{2,t}] = 49,540.79 \\
E[a^2 S_{1,t}^2] &= 300^2 \times 95^2 \times e^{(2\times 0.05+0.3^2)t} = 815,317,833 \\
E[b^2 S_{2,t}^2] &= 441,875,869 \\
E[S_{1,t} S_{2,t}] &= 95 \times 105 \times e^{(0.05+0.03+0.25\times 0.3\times 0.2)t} \\
&= 9994 \\
E[V_t^2] &= E[a^2 S_{1,t}^2] + E[b^2 S_{2,t}^2] + 2abE[S_{1,t} S_{2,t}] = 2,456,452,079 \\
\mathrm{var}[V_t] &= E[V_t^2] - E[V_t]^2 = 2,162,051 \\
\mathrm{sd}[V_t] &= 1,470.39
\end{aligned}
$$

Parametric VaR example

Summarizing:

$$\begin{aligned} V_0 &= 300 \times 95 + 200 \times 105 = 49,500 \\ E[V_t] &= 49,540.79 \\ \mathrm{sd}[V_t] &= 1,470.39 \end{aligned}$$

Normal distribution:

- The 1% tail is 2.326 standard deviations out.

Assuming V_t is normal:

$$\begin{aligned} \mathrm{VaR}[V] &= V_0 - (E[V_t] - 2.326\,\mathrm{sd}[V_t]) \\ &= 3,380 \end{aligned}$$

Monte Carlo VaR

Monte Carlo VaR – simulate the risk factors and use the pricers to directly compute the VaR:

- Not really a model or type – a numerical method applied to the problem.
- Straight forward to compute.
- Correct for nonlinear payoffs.
- Very slow to get high accuracy.
- If positions and payoffs are linear and normally distributed, then the same as parametric VaR.

Monte Carlo VaR Example

Same parameters as before.

Generate 10,000,000 samples of $W_{1,t}$ and $W_{2,t}$ (e.g. with MVNRND() in matlab or octave).

Compute:

$$\begin{aligned} V_t &= aS_{1,t} + bS_{2,t} \\ &= aS_{1,0}e^{(\mu_1-\sigma_1^2/2)t+\sigma_1 W_{1,t}} + bS_{2,0}e^{(\mu_2-\sigma_2^2/2)t+\sigma_2 W_{2,t}} \end{aligned}$$

Compute losses:

$$V_0 - V_t$$

Select (1-p)th biggest loss.

Get a var of approximately 3,270 – quite close to the normal approximation of 3,380.

Historical VaR

Historical VaR – assume risk factors follow actual historical distributions:

- Don't make assumptions about distribution of historical changes (i.e. – no fitting of historical changes to a model).
- Assume: Today's distribution of market changes = historical distribution of market changes.
- For each day of history, apply that day's change to today.

Simple, but

- Apply the absolute change or relative change?

HVaR change choice

Absolute changes or relative changes for HVaR?

- Low history and low values – irrelevant.
- High history and high values – irrelevant.
- Low history and high current values – absolute would yield tiny changes. Relative would yield huge changes.
- High history and low current values – absolute causes huge changes. Relative would yield tiny changes.
- Absolute crossing zero – another problem.

Still need to pick a model!

- If we think a risk factor has constant absolute volatility (ABM), then absolute changes would be most representative.
- If we think a risk factor has constant relative volatility (GBM), then relative changes would be most representative.

In practice:

- Rates and prices – relative changes.
- Spreads – absolute changes.
- Be careful about crossing zero.

HVaR Example

Current Apple price: 108.29. Historical daily prices:

ID	Apple
1	116.52
2	108.60
3	101.21
4	112.11
5	111.25
6	105.81
7	109.67
8	109.43
9	105.08
10	116.58
11	108.29

What is 1 day historical 90% VaR?

HVaR Example

Current Apple price: 108.29.

Apply 1 day log returns to current price and find 10% worst loss:

ID	Apple	log rtn	historical samples
1	116.52		
2	108.60	-0.07039	100.93
3	101.21	-0.07047	100.92
4	112.11	0.10228	119.95
5	111.25	-0.00770	107.46
6	105.81	-0.05013	102.99
7	109.67	0.03583	112.24
8	109.43	-0.00219	108.05
9	105.08	-0.04056	103.99
10	116.58	0.10386	120.14
11	108.29	-0.07376	100.59

- Current price $= S_{t_{11}} = 108.29$
- Log rtn $= l_i = \log(S_{t_i}/S_{t_{i-1}})$.
- Historical sample $= S_{t_{11}} \exp(l_i) = S_{t_{11}}(S_{t_i}/S_{t_{i-1}})$.

90% VaR = worst loss out of 10 = 108.29 - 100.59 = 7.70

VaR example statistics

Example VaR

For one stock currently priced at 100 following GBM, 30% volatility (σ), 7% return (μ), 1 month horizon

10 trials, 100 paths, daily steps. Sample mean and SD have substantial variance when using 100 samples, as do the VaRs.

mean	SD	99% VaR	98% VaR	97% VaR	96% VaR
99.6109	9.1659	21.1383	20.8379	16.7244	14.7811
99.9691	8.8731	22.2433	17.3749	16.9490	15.3527
100.5667	8.8222	17.4592	16.0440	15.1173	14.1633
100.4893	8.3405	19.9404	14.1496	13.7077	11.6792
100.5707	9.2033	16.3801	15.8131	13.0964	13.0946
100.5740	8.1757	16.8400	15.6773	14.4420	12.8950
99.6381	9.1630	19.9573	19.3267	17.7674	16.6342
100.2372	9.3863	18.4765	17.0683	16.9515	16.8807
100.7004	7.7344	17.7944	16.6895	14.2739	13.3346
100.4341	9.0694	21.6419	21.4432	18.1027	17.3739

Example ES

For one stock currently priced at 100 following GBM, 30% volatility (σ), 7% return (μ), 1 month horizon

10 trials, 100 paths, daily steps. ESs also have substantial variance.

mean	SD	99% ES	98% ES	97% ES	96% ES
99.6109	9.1659	21.1383	20.9881	19.5668	18.3704
99.9691	8.8731	22.2433	19.8091	18.8557	17.9800
100.5667	8.8222	17.4592	16.7516	16.2068	15.6959
100.4893	8.3405	19.9404	17.0450	15.9326	14.8692
100.5707	9.2033	16.3801	16.0966	15.0965	14.5960
100.5740	8.1757	16.8400	16.2586	15.6531	14.9636
99.6381	9.1630	19.9573	19.6420	19.0171	18.4214
100.2372	9.3863	18.4765	17.7724	17.4988	17.3443
100.7004	7.7344	17.7944	17.2420	16.2526	15.5231
100.4341	9.0694	21.6419	21.5426	20.3960	19.6404

Estimation

What's the model?

The biggest issue with what we've done so far:

- Where does the model come from?
- How do we know the parameters are right?

We need to estimate the model.

Stock paths

History:

- One historical path for each risk factor.
- How do we determine overall behavior?

Statistics:

- Compute mean and variance.
- Of prices, or returns or what?
- What do they mean and how do we use them to project?
- Is past performance indicative of future returns?

We need a model.

Mean

Mean estimation is hard:

- Mean return is essentially difference of end points.
- High uncertainty on a small number.
- Often assume mean $= 0$ when simulating.
- Assuming mean $= 0$ will throw off correlation calculations.

Consider:

- N daily samples.
- S_i, $i = N$ is today, $i = N - 1$ is yesterday, etc.
- Arithmetic returns: $a_i = S_{i+1} - S_i$.
- Mean daily return $= \frac{1}{N} \sum a_i = \frac{S_N - S_1}{N}$.
- Taking more samples in a time period does not improve confidence!
- Confidence only increases by taking a longer window.

Volatility

Variance estimation is easier:

- BM, all paths have the same quadratic variation.
- QV accrues as $\sigma^2 T$.
- Confidence is a function of the number of points – more points, more confidence.

Estimates of variance are much better — sample variance works well, but there are issues.

Volatility estimation issues

Estimating volatility

- Still need substantial amounts of data.
- How much history to use?
  - Too much history – variance is not representative of current market (too old).
  - Too little history – variance is not representative of current market (too noisy).

Need to answer the questions:

- How long before a change in the markets is incorporated into estimates?
- How quickly should VaR adapt to the markets?

Estimating Volatility

Volatility estimation is based on the sample variance:

- N daily samples
- S_i, $i = N$ is today, $i = N - 1$ is yesterday, etc.
- Arithmetic returns: $a_i = S_{i+1} - S_i$
- Log returns: $l_i = \log(S_{i+1}/S_i)$
- Sample arithmetic return mean: $\bar{a} = \frac{S_N - S_1}{N}$
- Sample SD: $\sqrt{\frac{1}{N}\sum a_i^2 - (\bar{a})^2}$

Weighted variance

Consider weighting history so that recent history counts more:

- Adapts faster to current markets.
- Similar to using less data – makes estimates noisier.

Weighted variance:

- Weights: w_i, with $w = \sum w_i$.
- Probabilities $p_i = w_i/w$.
- Arithmetic returns: $a_i = (S_{i+1} - S_i)$.
- Sample weighted arithmetic return mean: $\bar{a} = \sum p_i a_i$.
- Sample SD: $\sqrt{\sum p_i a_i^2 - (\bar{a})^2}$.

Exponential weighting

One commonly used weighting scheme is exponential weighting:

- Daily updates are easier, which was important before computers.
- Used for maintaining vol estimates for bad quote detection in ticker plants.

Exponential weighting:

- On day N, weight $a_N = S_N - S_{N-1}$ by 1, a_{N-1} by λ, a_{N-2} by λ^2, ...
- Total weight $w = \sum_{i=0}^{\infty} \lambda^i = 1/(1-\lambda)$.
- Mean m_N estimated on day N: $m_N = (1-\lambda)\sum_{i=0}^{\infty} \lambda^i a_{N-i}$.
- Mean recurrence: $m_N = (1-\lambda)a_N + \lambda m_{N-1}$.
- 2nd moment recurrence: $r_N = (1-\lambda)a_N^2 + \lambda r_{N-1}$.
- Variance: $v_N = r_N - m_N^2$.
- SD: $\sigma_N = \sqrt{v_N}$.

Correlation

Correlation is worse.

Consider 600 stocks, 1 year of daily data.

Appears easy - dot product of returns. But:

- 151,200 individual price observations.
- 179,700 individual correlations to solve for.

Not enough data – correlations are underspecified!

Even with 200 stocks:

- 50,400 individual price observations.
- 19,900 individual correlations to solve for.

2.5 data points for each correlation – how accurate can that be?

Correlation tweaks

Fixes for correlations:

- Longer data series.
- Smoothing first:
  - Correlations of averages.
  - Filtering.
  - SSA.

Complications

Complications

- What exactly should we compute the vol of?
  - Prices?
  - Changes in prices?
  - Log of price changes?
  - Something else?
- For that mater, is vol constant? No...
- Why exactly are we computing vol anyway?

Still haven't specified the model.

Models

Easiest assumption:

- Nonnegative things follow GBM.
- Everything else follows ABM.
- Fit each model to historical data – can derive based on statistics and calculations.
- Correlation is between BM drivers.

Once model is assumed and fit to data, we can proceed by:

- Estimating based on factors and assumptions (e.g. – parametric VaR).
- Simulating changes on model and actual pricers (e.g. – Monte Carlo VaR).

Fitting

How exactly do we fit GBM S to history? We need to relate the parameters of the model to statistics of the historical data.

$$dS = \mu S dt + \sigma S dW$$

$$S_T = S_0 e^{(\mu - \frac{\sigma^2}{2})T + \sigma W_T}$$

$$\log(S_{t_2}/S_{t_1}) = \left(\mu - \frac{\sigma^2}{2}\right)(t_2 - t_1) + \sigma(W_{t_2} - W_{t_1})$$

With sample mean and variance of daily log returns being $\bar{\mu}$ and $\bar{\sigma}^2$:

$$\bar{\mu} \approx E[\log(S_{t_2}/S_{t_1})] = \left(\mu - \frac{\sigma^2}{2}\right)(t_2 - t_1)$$

$$\bar{\sigma}^2 \approx \operatorname{var}[\log(S_{t_2}/S_{t_1})] = \sigma^2(t_2 - t_1)$$

Solving for μ and σ yields:

$$t_2 - t_1 \approx 1/252$$

$$\sigma \approx \bar{\sigma}/\sqrt{t_2 - t_1} \approx \bar{\sigma}\sqrt{252}$$

$$\mu \approx \bar{\mu}/(t_2 - t_1) + \frac{\sigma^2}{2} \approx 252\bar{\mu} + \frac{\sigma^2}{2}$$

Example 1

Consider the following weekly history of a stock:

Date	Stock
1	116.52
2	108.60
3	101.21
4	112.11
5	111.25
6	105.81
7	109.67
8	109.43
9	105.08
10	116.58
11	108.29

Fit GBM to this data.

Example 1 – unweighted

To fit GBM, we need the sample mean and variance of the log returns:

Date	Stock	Log rtn	(Log rtn)^2
11	108.29	-7.3765E-02	5.4413E-03
10	116.58	1.0386E-01	1.0786E-02
9	105.08	-4.0563E-02	1.6454E-03
8	109.43	-2.1908E-03	4.7995E-06
7	109.67	3.5831E-02	1.2838E-03
6	105.81	-5.0135E-02	2.5135E-03
5	111.25	-7.7006E-03	5.9299E-05
4	112.11	1.0228E-01	1.0462E-02
3	101.21	-7.0474E-02	4.9666E-03
2	108.60	-7.0392E-02	4.9550E-03
	Average	-0.00733	0.00423

- Log rtn $= \log(S_{t_i}/S_{t_{i-1}})$.
- $dt = 5/252$.
- $\bar{\mu} =$ avg of log rtns $=$ -0.00733.
- $\overline{\text{var}} =$ avg of sq of log rtns - sq of avg $=$ 0.00416.
- $\bar{\sigma} = \sqrt{\overline{\text{var}}} = 0.06448$.
- $\sigma = \bar{\sigma}/\sqrt{dt} = 0.4578$.
- $\mu = \bar{\mu}/dt + \sigma^2/2 =$ -0.2644.

Example 1 – weighted

GBM with geometric weight, $\lambda = .95$.

Date	Stock	Log rtn	(Log rtn)^2	weight	wgt ln rtn	wgt (ln rtn)^2
11	108.29	-7.3765E-02	5.4413E-03	1.0000E+00	-9.1916E-03	6.7802E-04
10	116.58	1.0386E-01	1.0786E-02	9.5000E-01	1.2294E-02	1.2768E-03
9	105.08	-4.0563E-02	1.6454E-03	9.0250E-01	-4.5616E-03	1.8503E-04
8	109.43	-2.1908E-03	4.7995E-06	8.5738E-01	-2.3405E-04	5.1275E-07
7	109.67	3.5831E-02	1.2838E-03	8.1451E-01	3.6366E-03	1.3030E-04
6	105.81	-5.0135E-02	2.5135E-03	7.7378E-01	-4.8339E-03	2.4235E-04
5	111.25	-7.7006E-03	5.9299E-05	7.3509E-01	-7.0535E-04	5.4317E-06
4	112.11	1.0228E-01	1.0462E-02	6.9834E-01	8.9004E-03	9.1036E-04
3	101.21	-7.0474E-02	4.9666E-03	6.6342E-01	-5.8258E-03	4.1057E-04
2	108.60	-7.0392E-02	4.9550E-03	6.3025E-01	-5.5281E-03	3.8913E-04
1	116.52					
	Sum				-0.00605	0.00423

- Log rtn $= l_i = \log(S_{t_i}/S_{t_{i-1}})$.
- Weighted log rtn $= p_i l_i$.
- $p_i = w_i/(\sum w_i)$.
- $\bar{\mu} =$ wgt avg of log rtns $= \sum p_i l_i =$ -0.00605.
- $\overline{\text{var}} =$ wgt avg of sq of log rtns - sq of wgt avg $= 0.00419$.
- $\bar{\sigma} = \sqrt{\overline{\text{var}}} = 0.06474$.
- $\sigma = \bar{\sigma}\sqrt{252} = 0.4596$.
- $\mu = 252\bar{\mu} + \sigma^2/2 =$ -0.1993.

Summary

VaR computation complications:

- Intermediate events.
- Formulas become impossible.
- Model selection and parameter estimation:
    - Mean hard to estimate.
    - Variance is easier.
    - Weighted averaging is useful.
    - Use GBM for nonnegative risk factors.
    - Use ABM for everything else.
    - Maybe factor models.

Summary

Types of VaR:

- Parametric – Formulas for approximations.
- Historical – Monte Carlo with historical perturbations. applied
- Monte Carlo – Fit models to risk factors and calculate VaR by simulation.

Still to come:

- Credit risk.
- Counterparty risk.
- Regulation.
- Case studies.
- Guest lectures.

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