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Financial Risk Management&Regulation (M5320)
Fall 2016
https://courseworks.columbia.edu

Homework 5

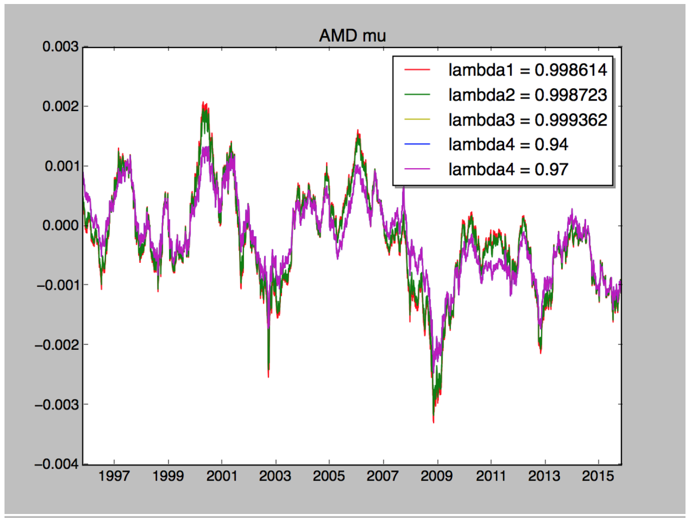
Due: 1:00 pm Thursday 13 Oct 2016

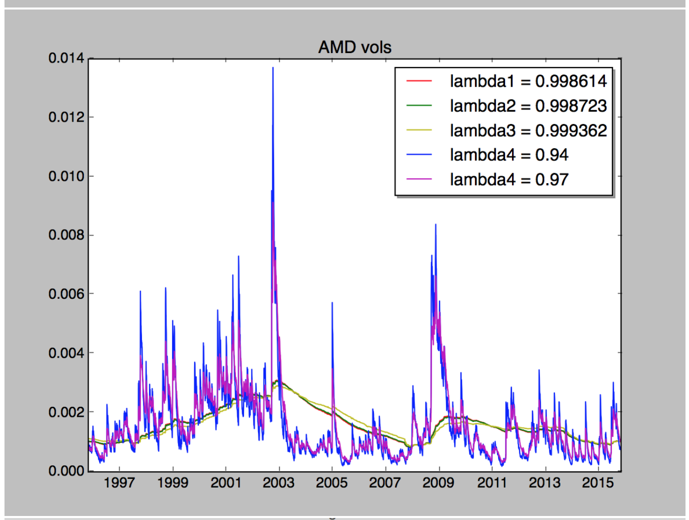
1. **Equivalence**
   In last week's homework, you computed the mean and variance of a data set for different window lengths as well as for exponential weighting with two different values for λ. What λs should be used for exponential weighting to get results most similar to using windows of 2, 5, and 10 years, respectively?

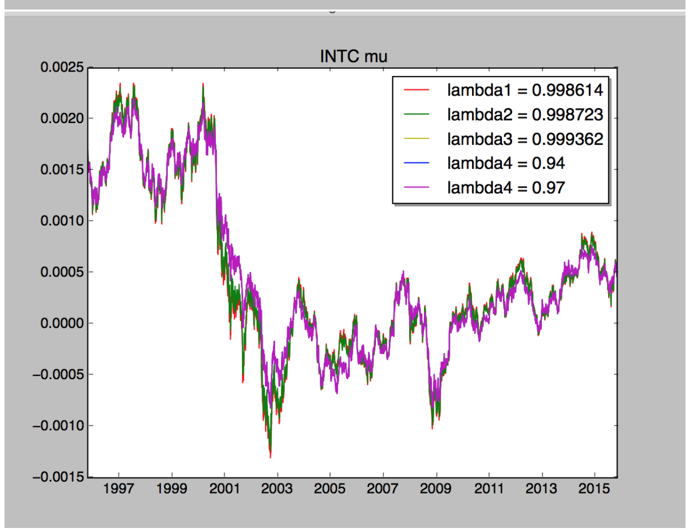
   Solution:
   We guesstimated that to make exponentially weighted averages comparable to win- dowed averages, we want the exponentially weighted ones to drop off to a certain level at the end of the window. If we want a weight of X% at the end of a window of size N, we need to solve $\lambda^N = X$. Experimenting with different values of X and comparing the exponentially weighted results to the windowed results indicate that X = 20% looks about right. Solving λN = .2 for $n = 252 \times 2$, $n = 252 \times 5$ and $n = 252 \times 10$ yields a lambda of 0.998614116 for a 2 year window, 0.9987234838 for a 5 year window, and 0.9993615381 for a 10 year window.
   It appears that λ needs to be over 0.998 to get usable results. Solving $0.9987^N = 0.5$ gives that this value of lambda has a half life of about 2 years.

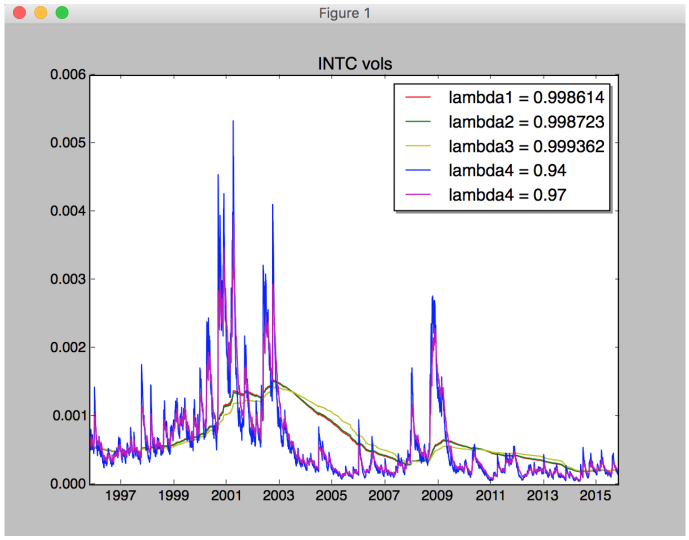
2. **Historical estimates with equivalent lambda**
   Repeat last week's exponential weighting parameter estimation using the above computed lambdas equivalent to 2 year, 5 year and 10 year windows. Compare the windowed versions to the corresponding equivalent exponentially weighted versions.

   Solution:


AMD mu
0.003
0.002
0.001
0.000
−0.001
−0.002
−0.003
−0.004
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
lambda1 = 0.998614
lambda2 = 0.998723
lambda3 = 0.999362
lambda4 = 0.94
lambda4 = 0.97


AMD vols
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
lambda1 = 0.998614
lambda2 = 0.998723
lambda3 = 0.999362
lambda4 = 0.94
lambda4 = 0.97


INTC mu
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
lambda1 = 0.998614
lambda2 = 0.998723
lambda3 = 0.999362
lambda4 = 0.94
lambda4 = 0.97



3. **Formula VaR and ES from historical estimates**
Using the estimates for the drift and volatility from last week using the various window sizes and exponential weights, tabulate and graph the VaR(S, T, p) and the ES(S, T, p') for S being A and I,using $p = 0.99, p' = 0.975$,and $T = 5days$(i.e. oneweek),foreach day over the last 20 years, or as far back as the above estimates were done. Tabulate and graph.

Unlike the parameter estimates, the computed VaRs and ESs will not be comparable between dates if you compute them on 1 share of stock. They can be normalized by either computing them assuming a $10,000 position each day, or converting them to relative losses. Do this exercise with the former normalization.

Solution:
Add VaR and ES formulas to last week?s python programs that calculated the drift and variance and plot.

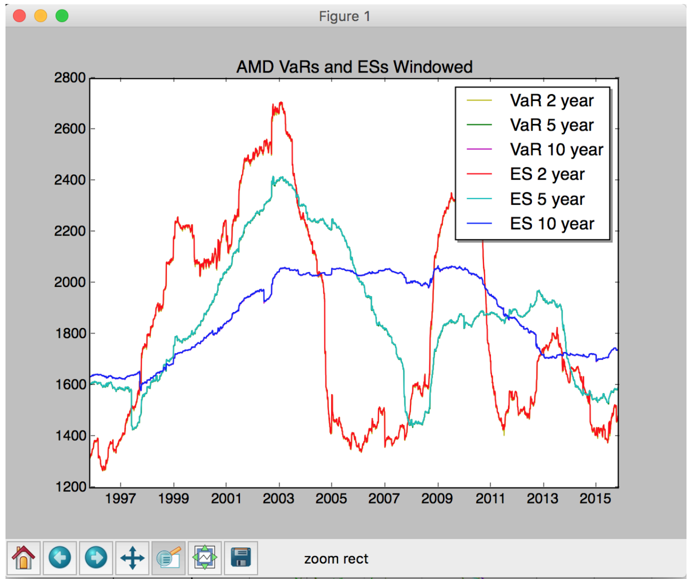


Figure 1
AMD VaRs and ESs Windowed
VaR 2 year
VaR 5 year
VaR 10 year
ES 2 year
ES 5 year
ES 10 year
2800
2600
2400
2200
2000
1800
1600
1400
1200
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015
zoom rect

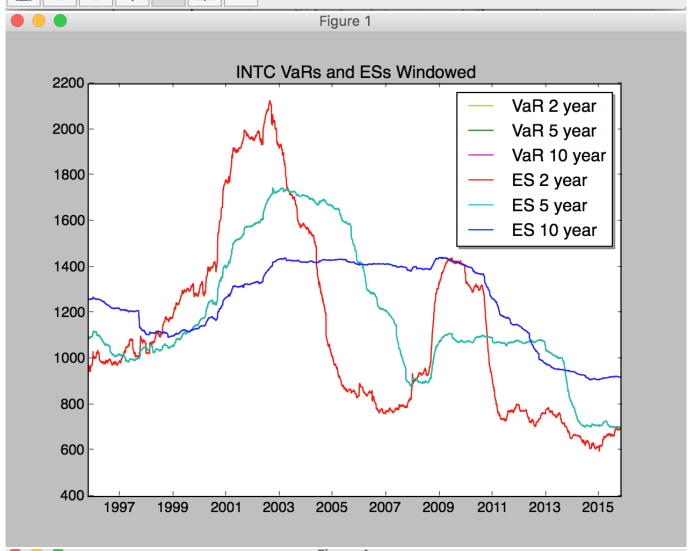


Figure 1
INTC VaRs and ESs Windowed
VaR 2 year
VaR 5 year
VaR 10 year
ES 2 year
ES 5 year
ES 10 year
2200
2000
1800
1600
1400
1200
1000
800
600
400
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015

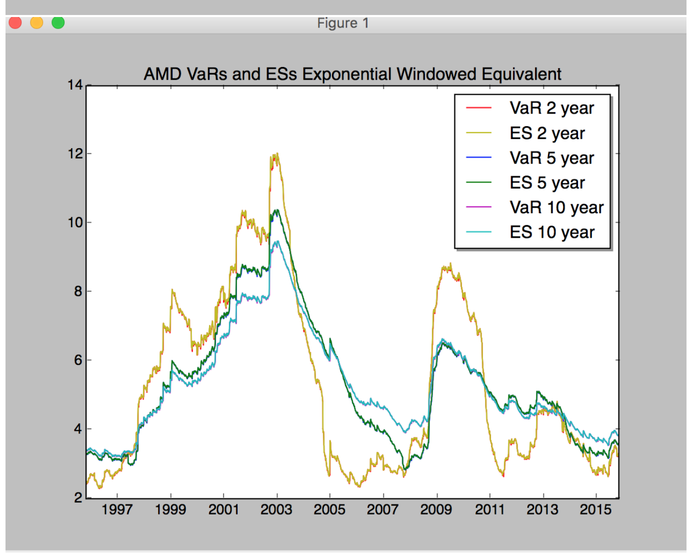
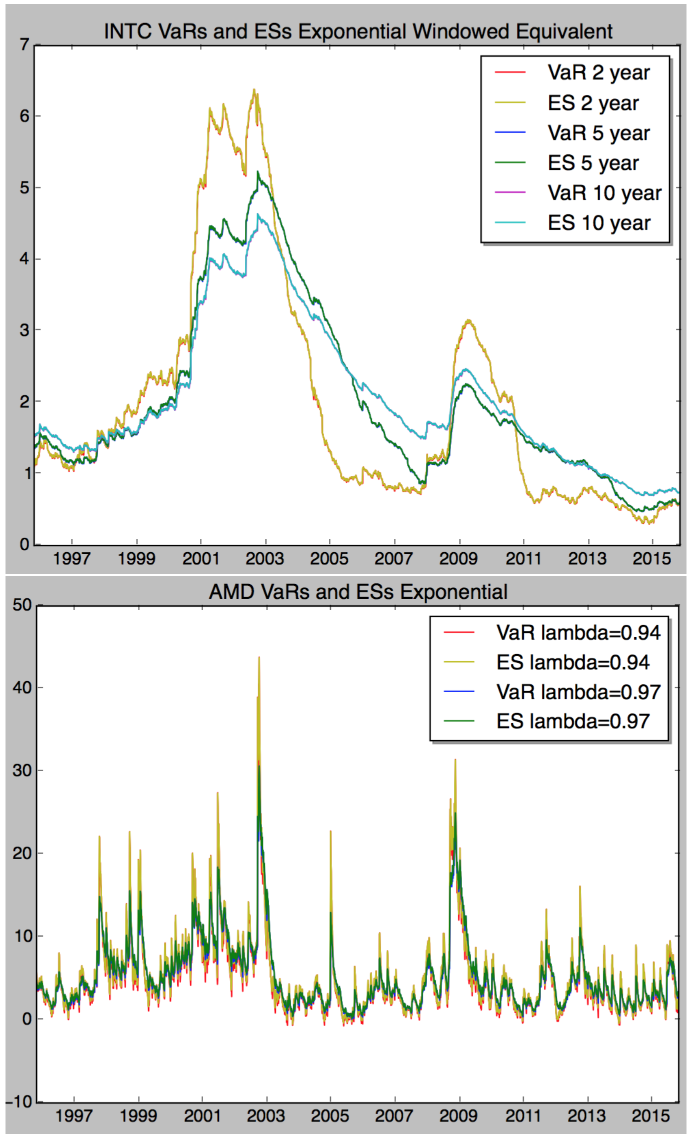
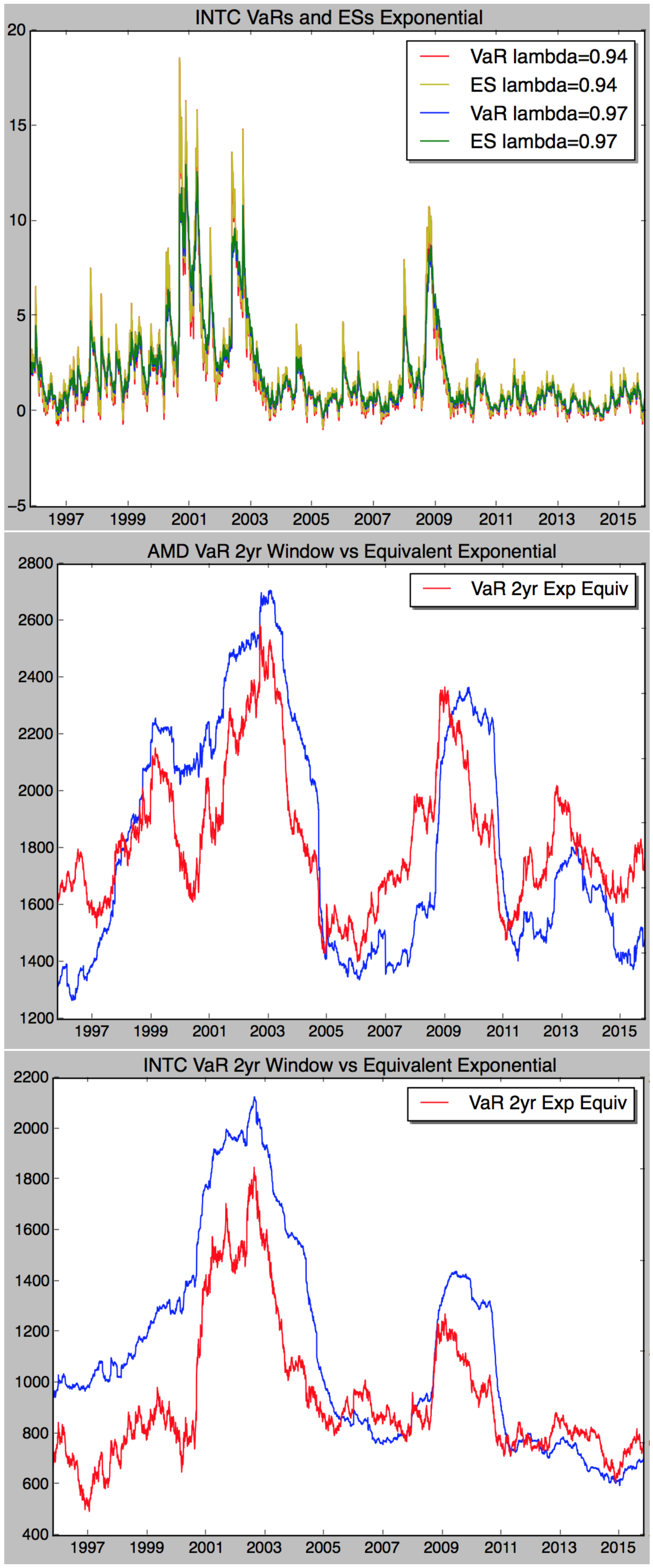
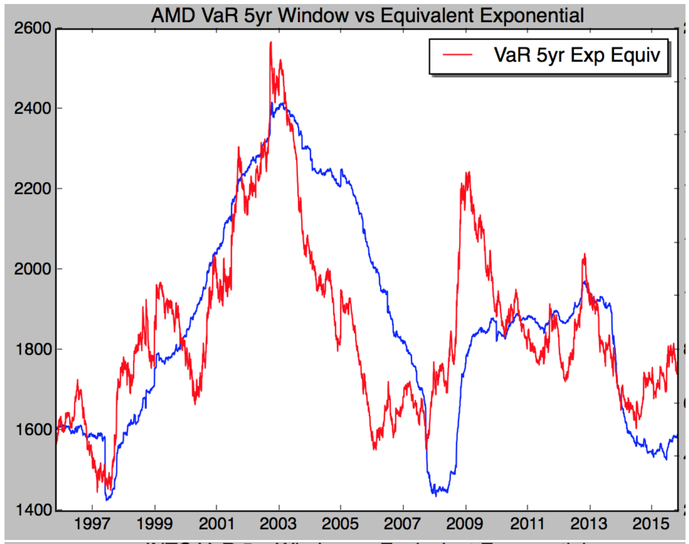


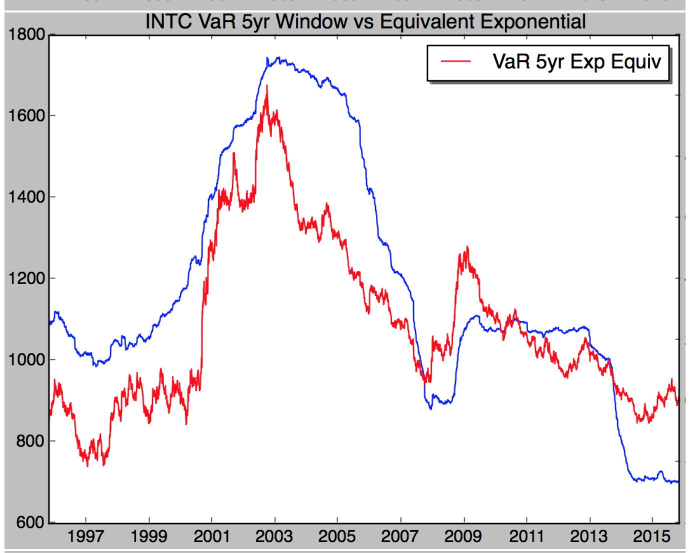
Figure 1
AMD VaRs and ESs Exponential Windowed Equivalent
VaR 2 year
ES 2 year
VaR 5 year
ES 5 year
VaR 10 year
ES 10 year
14
12
10
8
6
4
2
1997 1999 2001 2003 2005 2007 2009 2011 2013 2015

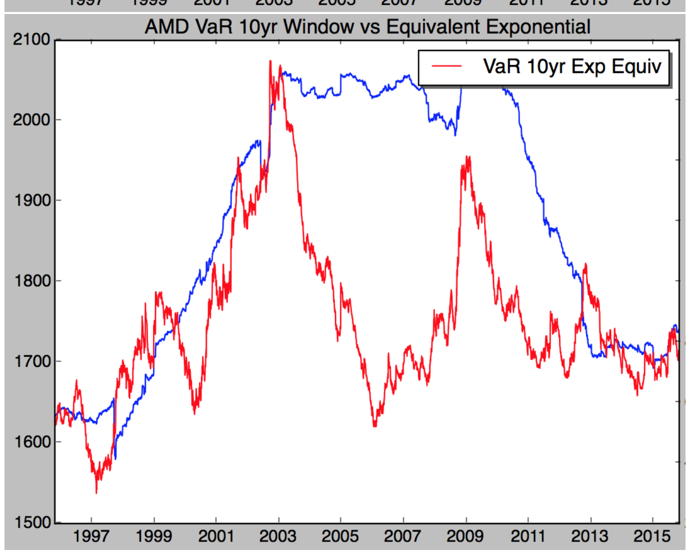

INTC VaRs and ESs Exponential Windowed Equivalent
VaR 2 year
ES 2 year
VaR 5 year
ES 5 year
VaR 10 year
ES 10 year
AMD VaRs and ESs Exponential
VaR lambda=0.94
ES lambda=0.94
VaR lambda=0.97
ES lambda=0.97

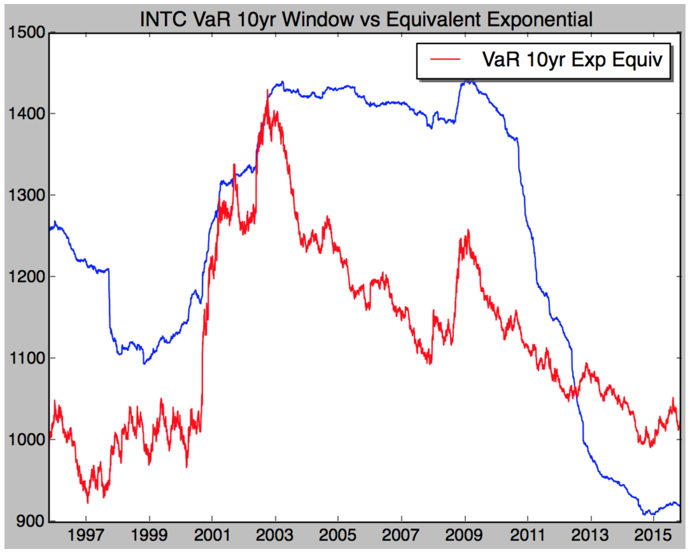


INTC VaRs and ESs Exponential
VaR lambda=0.94
ES lambda=0.94
VaR lambda=0.97
ES lambda=0.97
20
15
10
5
0
−5
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
AMD VaR 2yr Window vs Equivalent Exponential
VaR 2yr Exp Equiv
2800
2600
2400
2200
2000
1800
1600
1400
1200
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015
INTC VaR 2yr Window vs Equivalent Exponential
VaR 2yr Exp Equiv
2200
2000
1800
1600
1400
1200
1000
800
600
400
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


AMD VaR 5yr Window vs Equivalent Exponential
VaR 5yr Exp Equiv
2600
2400
2200
2000
1800
1600
1400
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


INTC VaR 5yr Window vs Equivalent Exponential
VaR 5yr Exp Equiv
1800
1600
1400
1200
1000
800
600
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


AMD VaR 10yr Window vs Equivalent Exponential
VaR 10yr Exp Equiv
2100
2000
1900
1800
1700
1600
1500
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015



4. **Derivative errors for normal CDF**

Let F(x) be a function, and let f(x) be its derivative. Define the difference derivative of F with a step size h as

$$D(F, x, h) = (F(x + h) - F(x - h))/(2h)$$

and then

$$D(F, x, h) \approx f(x)$$

We often use the above to approximate f. The error at x is then:

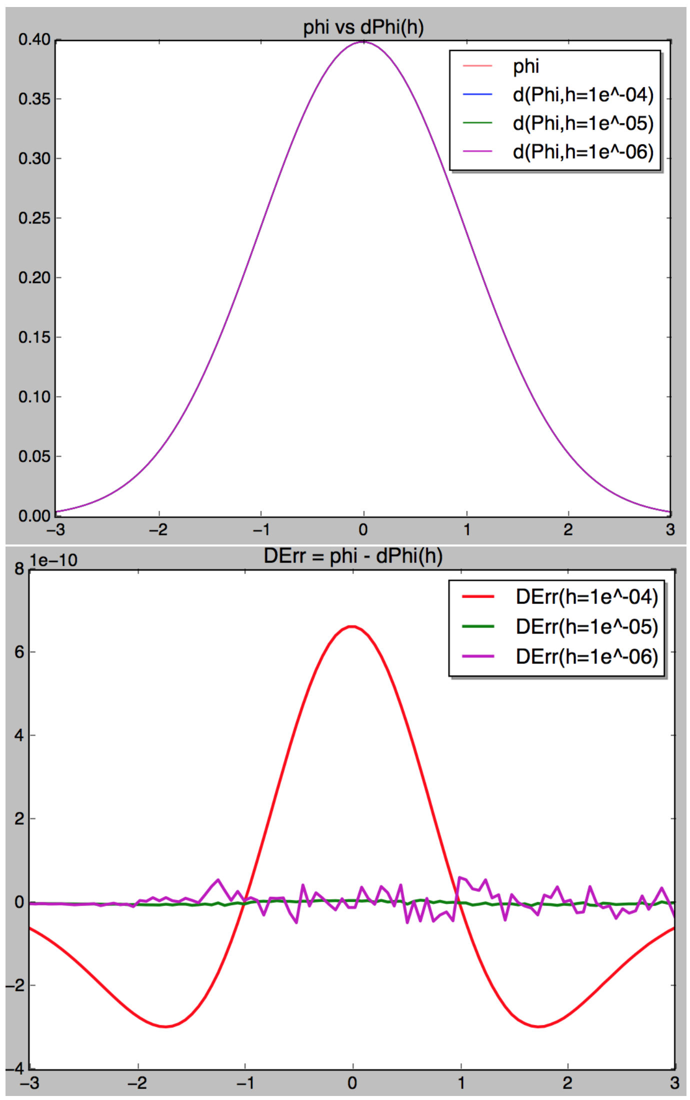
$$DErr(F, f, x, h) = f(x) - D(F, x, h)$$

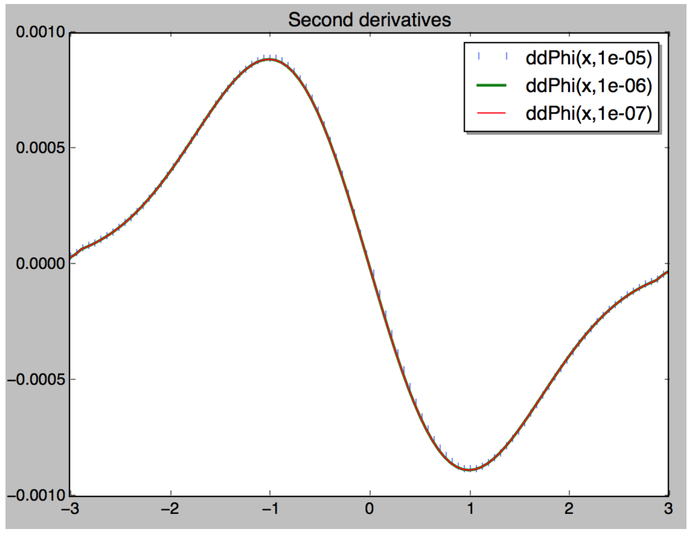
The graph of DErr(F, f, x, h) or even of D(F, x, h) can be used to choose an optimal hfor computing the derivative and to illustrate the accuracy of D(F, x, h). Let $\Phi(x)$ be the normal distribution, and let $\Phi(x)$ be the CDF of the standard normal distribution. Then

$$\phi(x) = d\Phi(x)/dx$$

In lecture 5, a graph was displayed to illustrate the error DErr($\Phi(x)$,$\phi(x)$,h) for different values of h. Reproduce this graph using your own software to estimate the optimal h to use in computing the derivative of $\phi(x)$. Also graph $\phi(x)$(x) and D($\Phi(x)$,$\phi(x)$,h) on the same plot to see if the errors are visible directly on the plots.

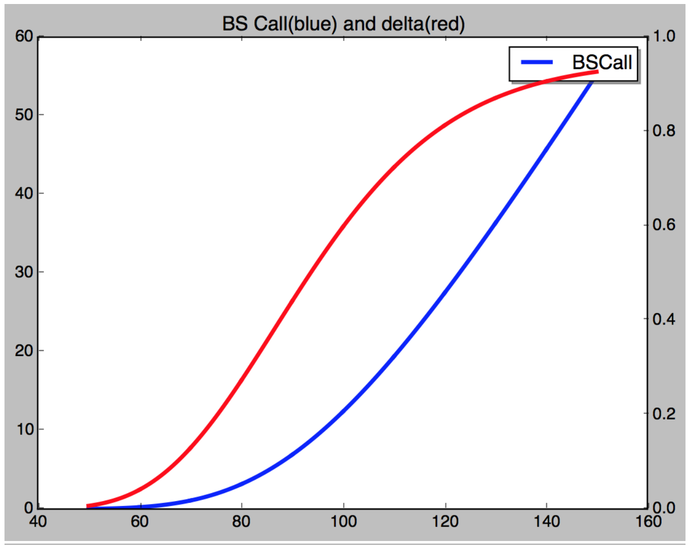
Solution:
Errors are not visible on the graph of D(($\Phi(x)$) itself. Plotting the differences indicates that the best accuracy is given using h = 1e-05. Plotting the 2nd derivative approximations shows that the first derivative clearly deteriorates at 1e-07.

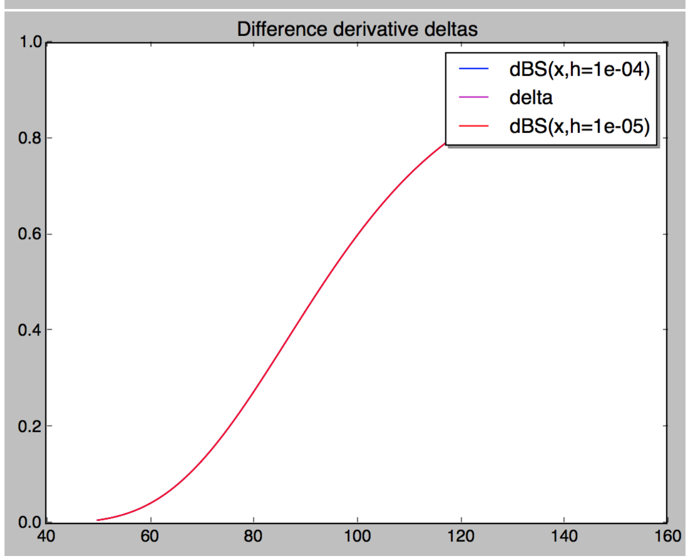


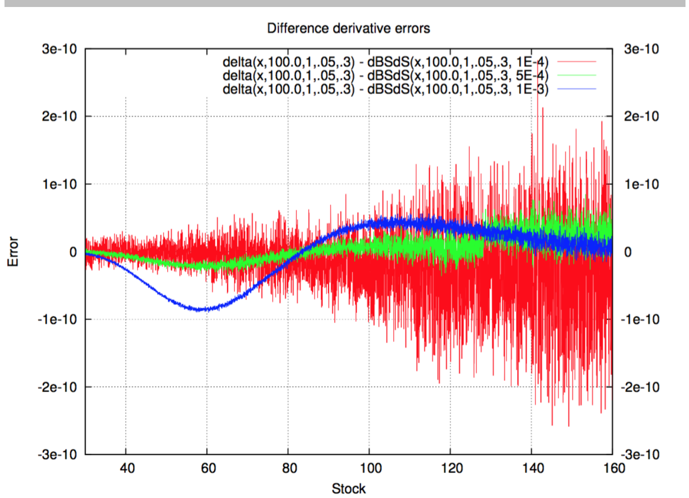


5. **Black Scholes delta error**
   Repeat the previous problem using the Black-Scholes formula for the price of a call option maturing in 1 year with a strike of 100. Use an implied volatility of 25% and a risk free rate of 0.05%. Plot the BS price and the delta itself as well to make sure your calculations make sense.

Solution:
I meant to say a strike of 100, not a price of 100, but the problem can be done by varying the initial price or varying the strike ? the results are essentially the same. Below I do the problem by varying the initial price and using a strike of 100. See plots below for the details.
In any case, the error isn't visible on the graphs of the difference derivatives, but when we compare to the actual delta, we see that the best accuracy is gotten around $h = 5 \times 10^{-4}$, although $h = 10^{-3}$does better for heavily in the money options. We also see that $h = 10^{-3}$ suffers from curvature error whereas $h = 10^{-4}$does not, but is much noisier and $h = 5 \times 10^{-4}$ is a little noisier than the larger step size and but has less curvature error.
The noise starts showing up here at a much larger step size than it did in the previous problem, indicating that the formula for the option price has less accuracy than the normal CDF calculation, which is as expected, but it is interesting to see how large the difference is.


BS Call(blue) and delta(red)
BSCall
0
10
20
30
40
50
60
40
60
80
100
120
140
160
0.0
0.2
0.4
0.6
0.8
1.0


Difference derivative deltas
dBS(x,h=1e-04)
delta
dBS(x,h=1e-05)
0.0
0.2
0.4
0.6
0.8
1.0
40
60
80
100
120
140
160


Difference derivative errors
delta(x,100.0,1,.05,.3) - dBSdS(x,100.0,1,.05,.3, 1E-4)
delta(x,100.0,1,.05,.3) - dBSdS(x,100.0,1,.05,.3, 5E-4)
delta(x,100.0,1,.05,.3) - dBSdS(x,100.0,1,.05,.3, 1E-3)
Error
Stock
3e-10
2e-10
1e-10
0
-1e-10
-2e-10
-3e-10
40
60
80
100
120
140
160