Math GR 5320: Financial Risk Management and Regulation

Assignment 4

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Due next Thursday by 1:00 pm.

For help. the preferred approach is to post questions on the Q&A tab in Piazza:

https://piazza.com/columbia/fall2016/mathg5320_001_2016_3/home

These will be quickly responded to and will be helpful to others in the class.

Otherwise, attend TA office hours, email a TA or the professor, or schedule a meeting.

1. **Parametric VaR, 1 stock** The mean and standard deviation of the daily log returns of a stock are S 0.03% and 1.40%, respectively. Suppose the stock follows a geometric Brownian motion with parameters μ and σ, so that

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where W_t is a standard Brownian motion.

   (a) What are good estimates for the drift and volatility, μ and σ, respectively?

   (b) What is the 1 day 99% VaR of a position of 10,000 shares in the stock if the current stock price is \$55 a share?

2. **Parametric VaR, 2 stocks**

   We have 550 shares of stock 1 and 300 shares of stock 2. Stock 1's current price is \$50 per share and stock 2's current price is \$100 per share. Stock 1 and 2 follow GBM with drift 4% and 2% and volatility 35% and 30%, respectively, each driven by Brownian motions with 20% correlation. What is the two week 98% VaR of the portfolio? Calculate it parametrically assuming that the portfolio is normally distributed.

3. **Calibration**

   For this and the remaining problems, let A be AMD stock (ticker AMD), and I be Intel stock (ticker INTC). Their historical values are in the spreadsheets `AMD-yahoo.csv` and `INTC-yahoo.csv`, respectively, which were downloaded using the script `getYahoo.sh`.

   Let $\mu(S,t,l)$ and $\sigma(S,t,l)$ be the estimated drift rate (mean) and relative volatility parameters for a GBM process S computed on date t using the last l years worth of observations (i.e. assume $dS = \mu S dt + \sigma S dW$ and that μ and σ are constant over that set of observations).

   Tabulate and graph $\mu(S,t,l)$ and $\sigma(S,t,l)$ for $S = A$ and $S = I$ for t ranging over the last 20 years, and l being 2, 5, and 10 years, using unweighted fitting. Use adjusted closing prices.

   Why are the computed parameters over time comparable even though the value invested varies?

How stable do the results look?

4. **Exponential weighting**

   Some references say to use exponential weighting with $\lambda = 0.94$ or $\lambda = 0.97$, to calibrate to the last few weeks or about a month of data. These λs correspond roughly these periods because the weight becomes $1/2$ after approximately 14 days and 22 days, respectively.

   To see the impact of exponential weighting with these weights, repeat the previous problem using exponential weighting, with $\lambda = 0.94$ and $\lambda = 0.97$.