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Financial Risk Management&Regulation (M5320)

Fall 2016

https://courseworks.columbia.edu

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Homework 9

Due: 1:00 pm Thursday 10 Nov 2016

1. **Merton model again**

   Assume a Merton model for default with a constant risk free rate of r and an initial firm value of \$10,000,000. The firm issued a zero coupon bond with face value B that matures at time T:
   $dV = \mu V dt + \sigma V dW$ $\mu = 0.1$ $\sigma = 0.2$ $r = .05$ $V_0 = 10,000,000$ $T = 5$
   What is the survival probability function for time T as a function of B ? Give its formula and graph it as a function of B.
   What is the current value of the equity and of the issued zero coupon bond? Give its formula and graph it as a function of B.
   In the previous problem, the 5 year survival probability is 0.873716. What value of B gives the same 5 year survival probability?

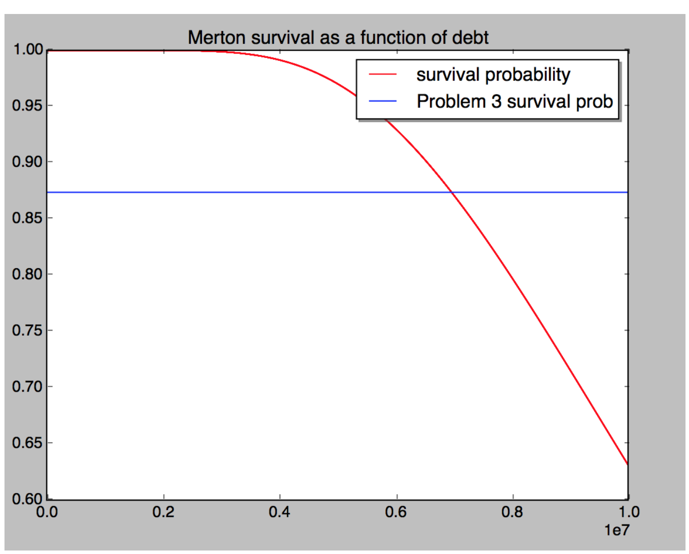
Solution: 1. The survival probabilty function for time T as a function of B is the probability that $V_T > B$, which is given by:

$$s = 1 - \Phi(-d_2)\ d_2 = \frac{log(V_0/B)+(r-\sigma^2/2)T}{\sigma\sqrt{T}}$$

With the above parameters we have:

$$s(B) = 1 - \Phi(-d_2(B))$$
$$d_2(B) = \frac{log(10000000/B)+0.15}{0.2\sqrt{5}}$$

The graph is:

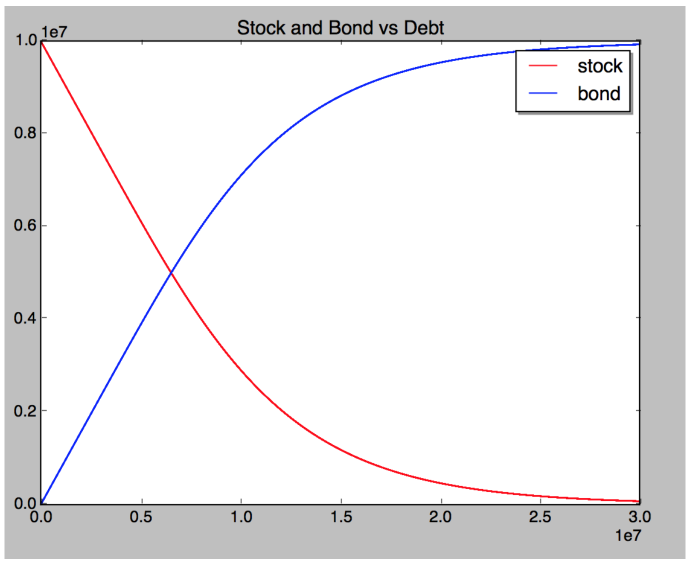


Solution: 2. The equity value is the value of a call on the assets with strike B. The debt is the value of receiving a risk free B at time T minus a put on the assets with strike B:

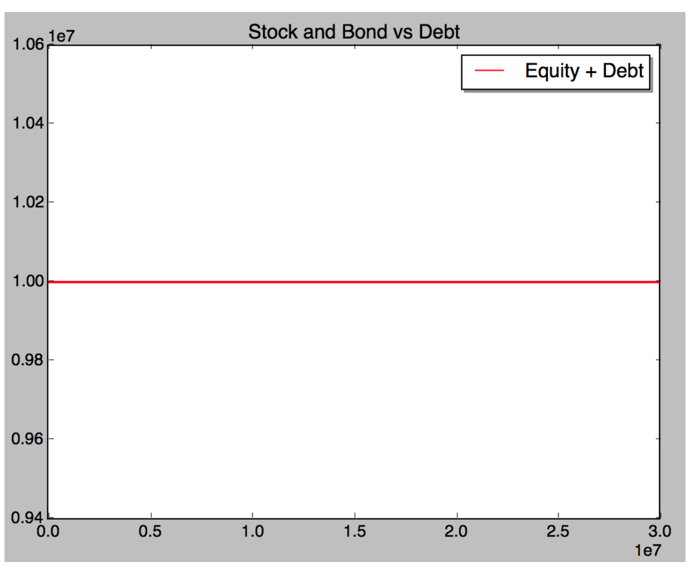
$$S(B) = \Phi(d_1(B))V_0 - \Phi(d_2(B))Be^{-rT}$$

$$D(B) = (V_0 - S(B))$$

Graph of equity and debt values as a function of B:



Plot confirming that equity + debt = firm value:



2. **CVA**
   (a) What is "CVA"?
   (b) Give the general formula for computing CVA and give the assumptions under which it holds.
   (c) What is wrong way risk?
   (d) Give an example of wrong way risk.

Solution:
(a) CVA is the credit valuation adjustment applied to an OTC derivative contract (or a set of contracts under a netting agreement). It is the market value of the embedded default risk.
(b) Let τ is the time at which the counterparty defaults, V(t) be the value of the set of contracts under the netting agreement at time t, and R be the recovery rate.
If there is no collateralization, then the loss in the event of default is: $(1 - R)max(V(\tau), 0)$
Let Q be the equivalent martingale measure with respect to numeraire N, p(t) be the default time PDF under Q, t_i be discrete increasing times, and $\bar{p}(t_i)$ be the probability of defaulting between t and t , $\bar{t} \in [t_{i-1}, t_i]$, and let $S(t) = N(0)E^Q[max(V(t), 0)/N(t)]$ be the time 0 value of the payoff max(V(t), 0). If R is a known constant, and $1_{\tau<t}$and max(V (t), 0)/N (t) are independent with respect to Q, then the CVA is:

$$(1 - R) \int_0^T S(t)p(t)dt \approx (1 - R) \sum S(\bar{t}_i)\bar{p}(t_i)$$

(c)Wrong way risk is when $1_{\tau<t}$ and max(V(t), 0)/N(t) are not independent, and V and default are positively correlated.
(d)An example of wrong way risk is entering into an oil swap with an oil company where the oil company pays a fixed amount and you pay the market price of oil. The contract value goes up when oil prices drop, which is exactly when the oil company is most likely to default. Another example of wrong way risk is a currency swap with a foreign government where the foreign counterparty pays the foreign currency and receives the local currency. When the value of the local currency goes up relative to the foreign currency, the value of the swap goes up, and the foreign government is weakest and most likely to default.

3. **CVA calcs**
   Consider a discrete world in which at time zero, the stock S and the bond B each cost \$100. At the future time T, the stock is either worth \$150 with probability 0.9 and or \$50 with probability 0.1. In both cases, the bond still costs \$100.
   You enter into a contract with a counterparty. At time T, you will pay him the cost of the stock, and he will pay you the cost of the bond.
   (a) Assuming no arbitrage, if the chance of the counterparty defaulting is zero, what is the price of this contract?
   (b) Suppose now that there is a chance that your counterparty defaults and only pays 40% of the value of the contract (assuming he owes you anything at all). According to your analysis of the cost of buying default protection, you have computed that there?s a 20% risk neutral chance of default, and that the default is independent of the stock price.What is the value of this risky contract and what is the value of the CVA?

Solution:
(a) $100 - (150 * 0.9 + 50 * 0.1) = -\$40$
(b) value of this risky contract is $(100-150)*0.9+[(100-50)*0.8+(40-50)*0.2]*0.1 = -\$41.2$
$CVA = -40 - (-41.2) = -\$1.2$

4. **Short portfolio**

Setting as in previous VaR calculation problems.

Repeat the VaR calculations for short portfolios. Instead of having positive positions in A, I, and P, consider having equal but negative positions in each. Use the 5 year period parameter estimates.

Work with shorts as follows. If you have \$1 and buy a stock that costs \$1, you then have \$1 worth of stock. If you have \$1 and short the stock, you then have \$2 in cash and -\$1 worth of stock. We ignore interest rate impact, so the cash component doesn?t change in value when computing VaRs.

Compute VaR for \$10,000 invested in each on each date. Compute using formulas (assuming portfolios follow GBM).

How do the VaRs for short portfolios compare to those for long portfolios? Which are more risky, and why?

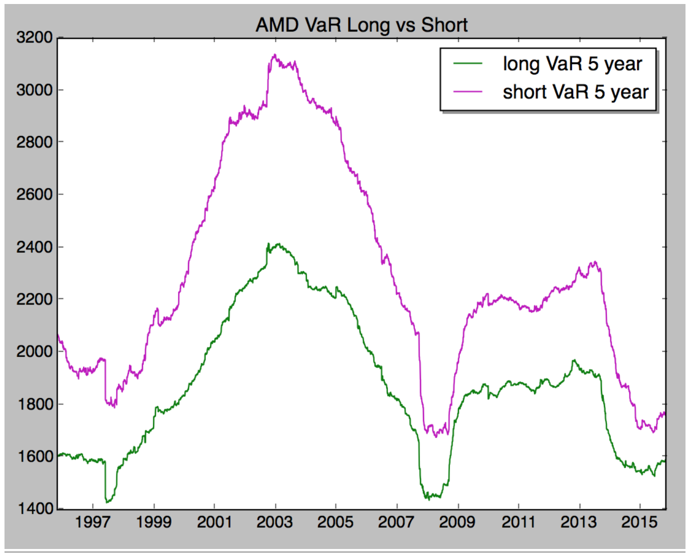
How do the VaRs compare for a short position in P when computing using the volatility estimates from the history of P vs pricing the portfolio by modeling A and I? To answer this, compare the portfolio VaR computed above with the Monte Carlo VaR computed by directly modeling A and I.

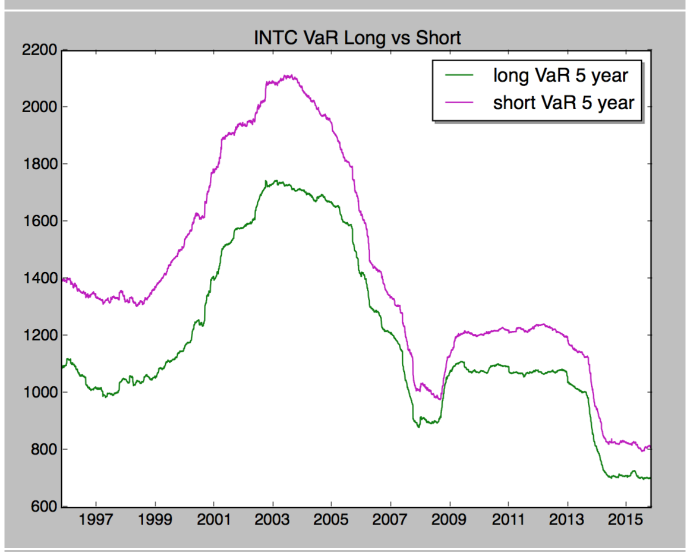
Solution:
See Code: RISK-MNGM-Homework9.ipynotebook

Each day we start with a position of X0 = \$20, 000 - S0, where S0 is \$10,000 worth of one of the stocks or the portfolio. At the horizon time we have a position of X1 = \$20, 000 ? S1, where S1 is the value of the equity at the horizon. The loss is then X0 - X1 = S1 - S0. So, the loss for a short position is just the negative of the loss for the corresponding long position. So, by flipping signs we can easily do the calculation. For example, the VaR for the short at a percentile of p would be the negative of the VaR of the long position at a percentile 1 - p. This is easy to accomplish in the Monte Carlo as well. Flipping the formula for the ES is a little more complicated.

To compare short positions in P when using the volatility estimates from the history of P vs pricing the portfolio by modeling A and I, we compare the former using the formula for the VaR of a GBM vs Monte Carlo modeling A and I. We do not need to use Monte Carlo when directly modeling P because we know in this case that the Monte Carlo and the formulas are just different implementations of the same model.

results


AMD VaR Long vs Short
long VaR 5 year
short VaR 5 year
3200
3000
2800
2600
2400
2200
2000
1800
1600
1400
1997
1999
2001
2003
2005
2007
2009
2011
2013
2015


INTC VaR Long vs Short
long VaR 5 year
short VaR 5 year
2200
2000
1800
1600
1400
1200
1000
800
600
1997
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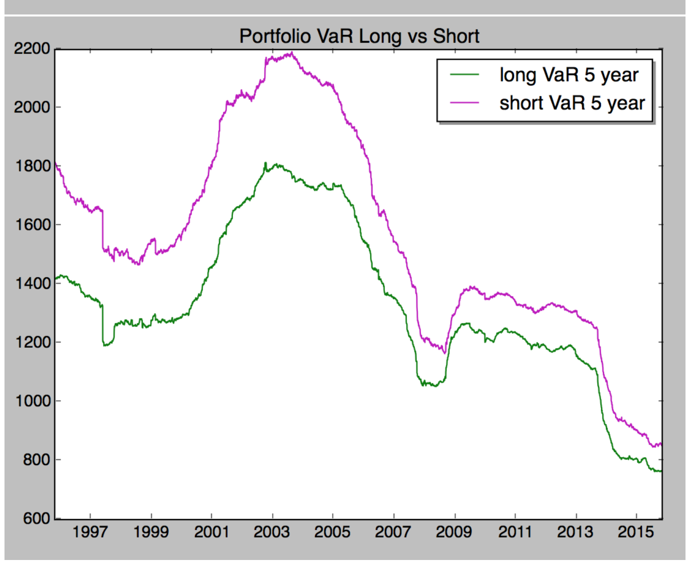
Portfolio VaR Long vs Short
long VaR 5 year
short VaR 5 year
2200
2000
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1600
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1997
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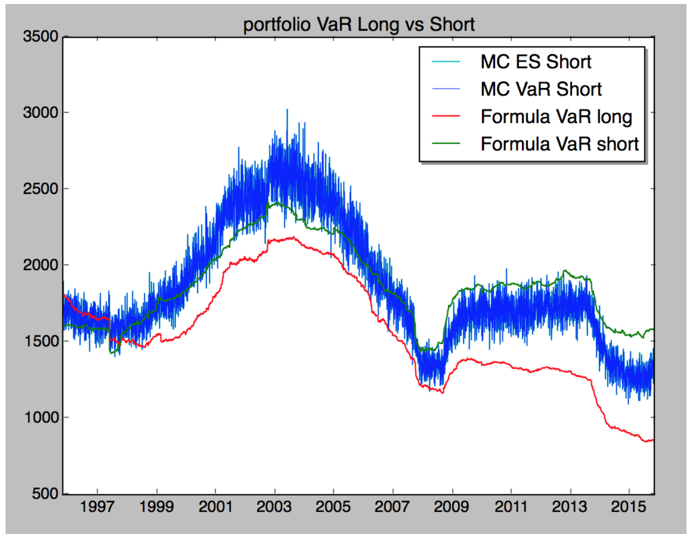


5. **Short portfolio, formula ES**
   Work out a formula for the ES of a short position in a portfolio following GBM. Use it to compare the 97.5% ES on a short position to the 99% VaRs computed using formulas in the previous problem (i.e. - for A, I and P across history calibrated to the last 5 years of data, equally weighted).
   Are the 97.5% ESs and 99% VaRs on short portfolios close to each other?

Solution:
The formula is like what we taught in lecture 2.
Here, we need to compute the

$$ES(S, T, p) = E[S_0 - S_T | S_0 - S_T > VaR(S, T, p)]$$

$$= S_0 - E[S_T | S_T < S0 - VaR(S, T, p)]$$

$$= S_0 - \frac{E[S_T 1_{S_T < S_0 - VaR(S,T,p)}]}{1-p}$$

$$= S_0 - \int_0^X S_T P(S_T) dS_T$$

where $X = S_0 - VaR(S, T, p)$

We can use BS model to derive it.

$$P = e^{-rT} E^+[max(K - S_T, 0]$$

$$= -e^{-rT} \int_K^\infty (K - S_T) P(S_T) dS_T$$

$$= e^{-rT} (\int_K^\infty S_T P(S_T) dS_T - \int_K^\infty K P(S_T) dS_T)$$

$$= \Phi(-d_2) K e^{-rT} - \Phi(-d_1) S_0$$

where $d_1 = \frac{1}{\sigma\sqrt{T}}(log(S_0/K) + (r + \sigma^2/2)T)$ and,

$$d_2 = d_1 - \sigma\sqrt{T}$$

Also, we have E[S$_T$] $= \int_0^{\infty} S_T P(S_T) dS_T$

$$= \int_0^{K} S_T P(S_T) dS_T + \int_K^{\infty} S_T P(S_T) dS_T$$

$$= \int_0^{K} S_T P(S_T) dS_T + e^{rT}\Phi(d_1)S_0$$

Plus, $S_0 e^{rT} = E[S_T]$

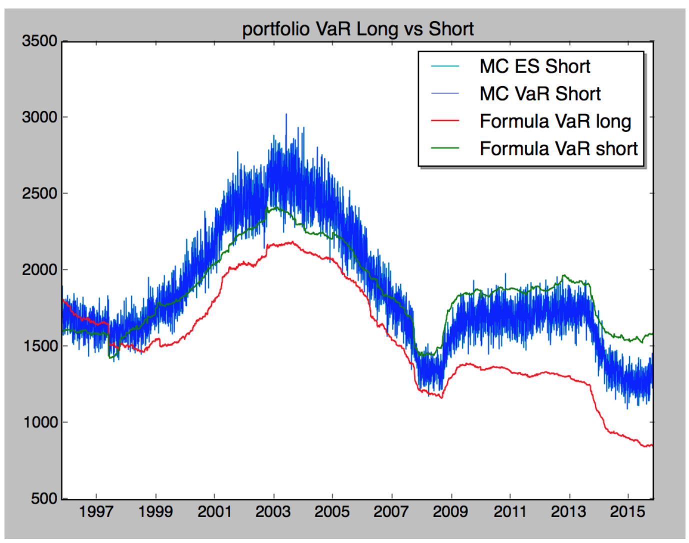
Therefore,

$\int_0^{K} S_T P(S_T) dS_T =S_0 e^{rT} - e^{rT}\Phi(d_1)S_0$

Now replace r by μ and K by X and result can be derived as:

$$ES = S_0[\frac{e^{\mu T}[1-\Phi(\Phi^{-1}(p)-\sigma)T]}{1-p} - 1]$$

Observations:



- The short positions have substantially larger downside, and hence larger VaRs.
- The VaRs and ESs are still extremely close.
- The relationship between the VaR on the short portfolio modeled directly and the VaR on the short portfolio modeled via modeling AMD and INTC remains similar to what was seen in the previous problem. Sometimes it?s lower and sometimes it?s higher.