MATH GR 5320
Financial Risk Management and Regulation

Lecture 8: Counterparty Risk

Department of Mathematics
Columbia University

Harvey J. Stein

Head, Quantitative Risk Analytics
Bloomberg LP

Fall 2016

Compilation: October 27, 2016 at 15:40

If errors are found, please return them to hjstein@columbia.edu.

Outline

Review

Review

Credit risk:

- Explicitly model defaults for pricing risky instruments.
- Two kinds of models:
  - Structural models – model assets of firm:
    - Merton
    - Black-Cox
  - Reduced form – model statistics of time to default.
- Alternative view – information about default.
- Alternative view – everything is a reduced form model.

Summary

Reduced form:

- Model the properties of default the time τ.
- Assume a deterministic hazard rate $\lambda(t)$.
- Survival time $s(t) = \text{Prob}(\tau > t) = e^{-\int_0^t \lambda(u)du}$.
- Default time pdf $p(t) = -ds/dt$.

Properties

- Default time is unpredictable.
- Deterministic forward spreads.

Usage:

- Piecewise constant λ used for pricing CDS.
- Used for CVA when pricing is independent of default time.
- Can't be used for market risk (no spread volatility).

Extend by making hazard rate stochastic.

Review

Merton:

- Assets of firm follow GBM: $dV = \mu V dt + \sigma V dW$.
- Default at time T if $V_T < B$.

Properties:

- Equity is call option on assets with strike B and maturity T.
- Debt is ZCB paying B at T minus a put on the assets with strike B.
- Default time is predictable – announcing sequence given by τ_n being the first $t \geq T - 1/n$ for which $V_t \leq B$.
- Can be rephrased as a reduced form model:
  - $\tau = T 1_{V_T < B} + \infty 1_{V_T \geq B}$
  - $s_t(u) = 1 - \phi(-d_2) 1_{u=T}$
  - $p_t(u) = \Phi(-d_2)\delta(u - T)$
  - $\lambda_t(u) = \frac{\Phi(-d_2)\delta(u-T)}{1-\Phi(-d_2)}$

Review

Black-Cox:

- Assets of firm follow GBM: $dV = \mu V dt + \sigma V dW$.
- Default if $V_t \leq B$ or if $V_T \leq K$.
- Formulas:
  - $a = \log(B/V_0)$.
  - $b = \sigma/2 - r/\sigma$.
  - PDF(τ): $-\frac{a}{T^{3/2}}\phi\left(\frac{a+bt}{\sqrt{T}}\right)$.
  - $CDF(\tau)$: $\frac{e^{-ab-|ab|}}{2|a|}(Erfc[\frac{|a|-|b|x}{\sqrt{2x}}] + e^{2|ab|}Erfc[\frac{|a|+|b|x}{\sqrt{2x}}] - 2)$

Properties:

- Spreads still always start out at 0.
- Default is predictable:
  - Let τ_n^1 be the hitting time for the barrier $B + 1/n$.
  - Let τ_n^2 be as in Merton.
  - Let $\tau_n = \min(\tau_n^1, \tau_n^2)$.
- Better match to real world.
- Still does not calibrate well.
- Formula derivations complex – involve reflection principle, usage of martingales, etc.

Summary

Joint default modeling:

- Needed for CDOs, portfolio losses, etc.
- Options:
  - Stochastic correlated hazard rates.
  - Joint default stopping time.
  - Correlated BMs in Black-Cox.
  - Copula method.

In general:

- Complicated.
- Issues in switching between risk neutral and real world.

Overview

Counterparty risk

Pre crisis:

- Swaps between banks often traded as if they were risk free (no credit risk).

Crisis:

- Bank failures.
- Global recession.
- Libor rates far from the "risk free" rate.

Post crisis:

- Credit risk is factored into *all* derivative trades.

We have:

PFE	Potential Future Exposure
EE	Expected Exposure
CVA	Credit Valuation Adjustment
DVA	Debt Valuation Adjustment
FVA	Funding Valuation Adjustment
XVA	All other valuation adjustment

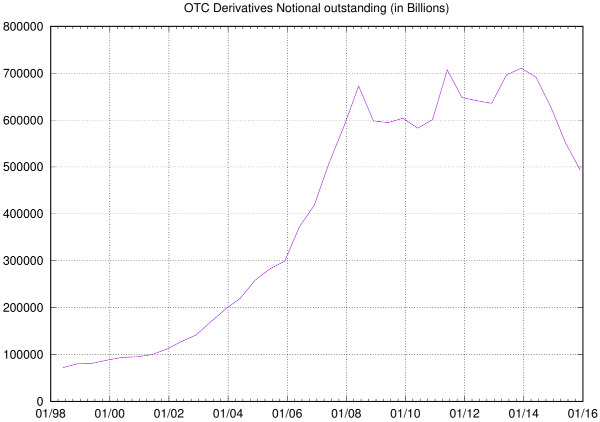
Size

Despite the financial crisis, the OTC derivatives markets continue to be a big part of the market, and interest rate derivatives are the largest part of the OTC derivatives market [Com16]:

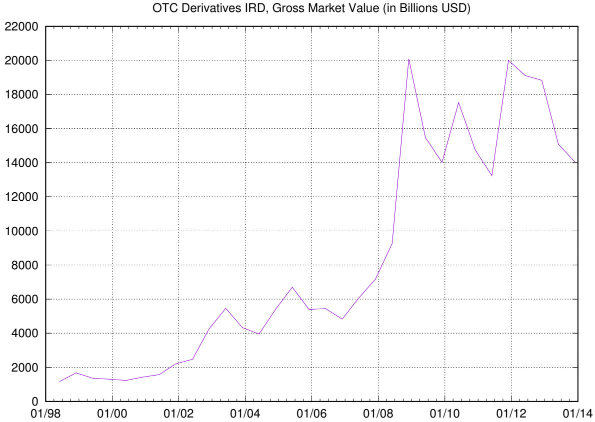
- OTC derivatives notional outstanding
  - H1 2008 — \$673 trillion
  - H2 2013 — \$710 trillion
  - H2 2016 — \$493 trillion
  - 80% in interest rate derivatives
- OTC derivatives gross market value
  - H1 2008 — \$20 trillion
  - H2 2008 — \$35 trillion — up 75%
  - H2 2016 — \$14 trillion
- IRD gross market value
  - H1 2008 — \$9 trillion
  - H2 2008 — \$20 trillion — doubled in 6 months.
  - H2 2016 — \$10 trillion

With the continued market turmoil and an unfavorable regulatory climate, the derivatives market has shrunken but is still large.

Notional outstanding



IRD market value



Demand

Counterparty credit risk management is demanded by:

- Accounting standards: – credit risk must be taken into account:
  - Financial Accounting Standards Advisory Council [Fin07]
  - Financial Accounting Standards Advisory Council [Fin09]
  - International Accounting Standards Board [Int01]
- Regulators.
- Risk managers.

The Dodd-Frank Act pushes OTC derivatives into clearing houses to mitigate counterparty risk, but:

- Risk doesn't go away — clearing houses will be bearing and collateralizing for counterparty risk.
- Corporations may still remain off of clearing houses.
- Collateralization costs must be computed and are similar to credit valuation adjustments.

Definition

Counterparty risk

Counterparty risk — the exposure to loss due to a specific counterparty failing to meet contractual obligations, i.e. defaulting.

Typically restricts to credit risk from counterparties on derivatives contracts, but doing so is myopic. If a counterparty defaults, all of their contracts are affected:

- OTC derivatives
- bond issues
- stock issues
- debts, loans, ...

Compartmentalization of risks should be based on the type of risk, not the type of security.

So, we will analyze counterparty risk from a general credit risk perspective.

References: Canabarro and Duffie [CD03] and Pykhtin and Zhu [PZ07]

Bond credit risk

Example — Investor is long a bond:

- Counterparty is issuer.
- Issuer defaults — investor loses bond cash flows, but gets recovery on the bond.
- Recovery is a percentage of the *principal* of the bond.
- Default causes loss of interest payments, and early (but partial) return of principal.
- Could be an improvement if bond is trading at a sufficient discount.

Example — Investor is short a bond:

- No credit risk.

Swap credit risk

Example — Investor enters into a swap.

- Counterparty is the entity with which the swap was transacted.
- Investor is simultaneously long one leg of the swap, and short the other leg.
- Counterparty defaults — investor loses swap cash flows, but gets recovery on the swap.
- Recovery is a percentage of the *market value* of the swap.
- If swap value is positive then there's a loss.
- If swap value is negative, then there is *no loss*.

Credit exposure on a swap is nonlinear – like a call option!

Risk mitigation

Risk modifications — Netting

Netting agreements:

- Optional part of the ISDA master agreement.
- In the event of default, recovery is on the net market value of all contracts covered by the agreement.

No netting agreement:

- Investor owns two 5 year 5% swaps to counterparty — one is pay fixed, the other is receive fixed.
- No net market exposure — the two positions cancel out.
- Substantial counterparty exposure:
    - Get recovery on the market value of the positive swap
    - Still owe full value on the market value of the negative swap

With netting agreements, exposure at any given time is on the net market value of all securities covered. Risk is reduced.

Risk modifications — Collateralization

Collateralization

- ISDA credit support annex (CSA).
- At the end of the period (day, week, etc), if net position value exceeds a threshold, collateral must be posted.
- Post deficit minus threshold, but only if amount exceeds minimum transfer amount (MTA).
- Exposure is to the threshold plus the movement of the market value over the period, plus potentially MTA.
- Leads to requirement of initial margin from clearing houses.

Collateralization example

Collateralization example.

Banks A and B enter into a swap.

- Current market value (MV) = 0.
- $500 thousand threshold.
- $100 thousand minimum transfer amount.

A's MV	A collat	B collat	Note
0	0	0	
400,000	0	0	Less than threshold
550,000	0	0	Transfer $<$ threshold
600,000	0	100,000	Paid exposure - threshold
650,000	0	100,000	Transfer $<$ threshold
690,000	0	100,000	Transfer $<$ threshold
700,000	0	200,000	Topped off to exposure - threshold
610,000	0	200,000	Transfer $<$ threshold
600,000	0	100,000	Paid back to exposure - threshold

Unintended consequences

Risk modifications change effective seniority structure of debt.

- Netting — contracts which are assets are used to pay some contracts which are debts in advance of other debts.
- Collateralization — cash and other assets of firm used as collateral are used to pay corresponding contracts in advance of other debts.
- Reduces value to bond holders and other creditors.

So, to a certain extent, using risk mitigants on derivatives debt is really transfering the risk to the other creditors.

Clearing houses

Clearing house:

- Third party that stands between transacting parties.
- Also known as Central Clearing Counterparties (CCCPs, or more commonly CCPs).

Example:

- A enters into a pay fixed swap with B.
- Swap moves to clearing house.
- A has pay fixed swap with clearing house.
- B has receive fixed swap with clearing house.

Goal is to improve market robustness.

References: Steigerwald [Ste13], Nosal and Steigerwald [NS10], Nosal [Nos11], Gregory [Gre10]. Gregory book is available online through clio.

Clearing houses

Clearing house risk:

- Trades are balanced – no market risk (until default).
- Credit is not balanced – clearing house holds default risk.

Market robustness impact:

- Transparency (clearing house knows everyone's exposures).
- Standardization.
- Clearing house can use various risk mitigation tools to smooth impact of defaults.

Clearing house risk mitigation

Clearing house risk mitigation tools:

- Collateralization.
- Netting.
- Default settlement rules.
- Clearing house capital.

Clearing house collateralization

Clearing house collateralization is called margining.

- Initial margin – Collateral to cover market moves between default and close-out.
- Variational margin – Collateral posted daily for changes in MV.

Collateral is intended to prevent losses in the event of a default.

Risk:

- Variational margin is insufficient.

Clearing house netting

Clearing house potentially nets exposure.

Without clearing house:

- A transacts with B, C, and D.
- Each netted separately.

With clearing house:

- A's trades are all on clearing house.
- Trades enacted with B, C and D get netted together for margining.

Risk:

- Default can cause large, sudden increase in margin.
- Multiple clearing houses can substantially reduce netting – e.g. separate IR clearing house and FX clearing house.

Clearing house default rules

Default of a member:

- Auction off trades to remaining members.
- Reassign trades (if possible)
- Distribute losses as unpaid collateral – Variation Margin Gains Haircut (VMHG).
- Tear-ups – Close out contracts and pay members, with members potentially taking losses if margin was insufficient.

Clearing house losses

Potential for losses:

- Initial margin plus daily variational margin attempts to make needed cash available in the event of a default.
- If that proves insufficient, coverage comes from backers of clearing house and equity of the firm itself.
- Reassignment of contracts reduces losses due to market impact. But in a real crisis, reassignment might fail.

Risk mitigation clauses

Another approach to risk mitigation that is becoming popular is to include risk mitigation clauses (A.K.A. Additional Termination Events):

- Contract can be closed out at replacement value if the counterparty's rating drops.
- Contract can be closed out at market value prior to maturity.

Issues:

- What exactly is "replacement value"?
- How is closing out a swap early any different from entering into the reversing swap and novation?
- By forcing closeout at a rating change, are you decreasing counterparty risk while increasing systemic risk?
- Will this really work at the next crisis — what if the market is illiquid?

Basic default pricing and CDS

Counterparty risk and CVA

CVA – Credit Valuation Adjustments.

- The price of the embedded default risk in a netted portfolio of OTC derivatives.
- Hinges on default pricing – Credit Default Swaps (CDS).

Before discussing CVA, we will need to discuss CDS pricing.

CDS contracts

A Credit Default Swap (CDS) contract consists of two legs:

- A fixed leg which pays a constant spread until default of a specified reference instrument or maturity, whichever comes first.
- A floating leg which covers the loss incurred by the reference instrument.

Details

- For CDSs, the reference instrument is a bond.
- Bond recovery is on principal.
- The recovery rate is $R = \frac{\text{Received amount}}{\text{Principal}}$.
- R is determined by bankruptcy proceedings.
- The fixed leg also pays accrued interest at default time (the portion of the next coupon that is due).

The par CDS spread is the spread that makes the prices of the two legs match.

CDS cashflows

CDS specification:

- t_i — i^{th} coupon payment date.
- C — Coupon (in decimal).
- $\alpha(t_i)$ — Coverage for the i^{th} coupon (e.g. roughly 1/2 for semiannual payments).
- $\alpha^*(t)$ — is the accrued interest factor at time t.

Typically, $\alpha^*(t) \approx \frac{t-t_{i-1}}{t_i-t_{i-1}}\alpha(t_i)$ when $t_{i-1} < t < t_i$.

Assume a notional amount of \$1.

Fixed leg payments:

- At t_i: Pay $C\alpha(t_i)$ if default hasn't occurred.
- At default time τ: Pay $C\alpha^*(\tau)$.

Floating leg payments:

- At default time τ: Pay $(1-R)$.

Pricing formula

CDS contract pricing.

- Default time τ.
- Pick an equivalent martingale measure Q with respect to a numéraire N.

Valuation of cashflows:

- Value of each coupon: $N_0 E^Q[C\alpha(t_i)1_{\tau>t_i}/N_{t_i}]$
- Value of accrued interest: $N_0 E^Q[C\alpha^*(\tau)1_{\tau\le t_n}/N_\tau]$
- Value of floating leg payment: $N_0 E^Q[(1-R)1_{\tau\le t_n}/N_\tau]$

Value of CDS contract (receiving fixed and paying float):

$$N_0 E^Q\left[\sum^n C\alpha(t_i)1_{\tau>t_i}/N_{t_i} + C\alpha^*(\tau)1_{\tau\le t_n}/N_\tau - (1-R)1_{\tau\le t_n}/N_\tau\right]$$

Simplifying

The computation of expressions evaluated at default times is complicated:

- Values of stochastic processes at different times.
- Expectations are across times.

Prefer to take expectations at a fixed time.

So, simplify using:

$$
\begin{aligned}
E[f(\tau)] &= E\left[\int f(t)\delta(t-\tau)dt\right] \\
&= \int E[f(t)\delta(t-\tau)]dt
\end{aligned}
$$

where $\delta(t)$ is the Dirac delta function, defined by:

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

Simplifying step 1

Expanding and converting to integrals:

$$
\begin{aligned}
N_0 E^Q &\left[\sum^n C\alpha(t_i)1_{\tau>t_i}/N_{t_i} + C\alpha^*(\tau)1_{\tau\le t_n}/N_\tau - (1-R)1_{\tau\le t_n}/N_\tau\right] \\
&= \sum^n N_0 E^Q[C\alpha(t_i)/N_{t_i}1_{\tau>t_i}] \\
&\quad + \int_0^{t_n} N_0 E^Q\left[C\alpha^*(t)/N_t\delta(t-\tau)\right]dt \\
&\quad - \int_0^{t_n} N_0 E^Q\left[(1-R)/N_t\delta(t-\tau)\right]dt
\end{aligned}
$$

This much holds for all arbitrage free models!

We could potentially compute this if we model the joint distributions of τ, R, and N under Q ...

Simplifying step 2

...or, we can assume zero correlation between τ and N, and that R is a known constant:

$$\begin{aligned}
&\sum^{n} N_0 E^Q[C\alpha(t_i)/N_{t_i}]E^Q[1_{\tau>t_i}] \\
&\qquad + \int_0^{t_n} N_0 E^Q[C\alpha^*(t)/N_t]E^Q[\delta(t-\tau)]dt \\
&\qquad - \int_0^{t_n} N_0 E^Q[(1-R)/N_t]E^Q[\delta(t-\tau)]dt \\
&= \sum^{n} C\alpha(t_i)D(t_i)E^Q[1_{\tau>t_i}] \\
&\qquad + \int_0^{t_n} C\alpha^*(t)D(t)E^Q[\delta(t-\tau)]dt \\
&\qquad - \int_0^{t_n} (1-R)D(t)E^Q[\delta(t-\tau)]dt
\end{aligned}$$

where $D(t)$ is the current value of \$1 maturing at time t (the time zero discount factors).

Default times and default probabilities

Our final simplification step. Note that

$$E^Q[\delta(t-\tau)] = p(t)$$
$$E^Q[1_{\tau>t}] = s(t)$$

where

$p(t)$ Default probability density function.

$s(t)$ Survival probability for time t. $(s(t) = 1 - \int^t p(u)du)$

Critical: The default probability pdf p is under the pricing measure Q, not under the real world measure P.

Very simple idea in zero recovery case. If the value of receiving a risky dollar at time t is $\overline{D}(t)$, then

$$\frac{\overline{D}(t)}{D(t)} = s(t)$$

Simplifying step 3

Then

$$
\begin{aligned}
\sum^{n} C\alpha(t_i)D(t_i)E^{Q}[1_{\tau > t_i}] & \\
&+ \int_0^{t_n} C\alpha^*(t)D(t)E^{Q}[\delta(t-\tau)]dt \\
&- \int_0^{t_n} (1-R)D(t)E^{Q}[\delta(t-\tau)]dt \\
= \sum^{n} C\alpha(t_i)D(t_i)s(t) & \\
&+ \int_0^{t_n} C\alpha^*(t)D(t)p(t)dt \\
&- \int_0^{t_n} (1-R)D(t)p(t)dt
\end{aligned}
$$

Par swap rate for CDS

The par swap spread $C(t_n)$ for a CDS with payment times t_i and maturing at t_n then satisfies the following relationship:

$$\sum^{n} C(t_n)\alpha(t_i)D(t_i)s(t_i) + \int C(t_n)\alpha^*(t)D(t)p(t)dt$$
$$= \int (1-R)D(t)p(t)dt$$

Interpretation:

- The summation is the value of the fixed payments, not counting the accrued interest.
- The second term is the value of the accrued interest.
- The third term is the value of the disbursement made at default time.

Solving for the coupon

The par swap spread $C(t_n)$ satisfies:

$$\sum^{n} C(t_n)\alpha(t_i)D(t_i)s(t_i) + \int C(t_n)\alpha^*(t)D(t)p(t)dt$$
$$= \int (1-R)D(t)p(t)dt$$

Solving for $C(t_n)$, we see that the par spread is given by:

$$C(t_n) = \frac{\int (1-R)D(u)p(u)du}{\sum^{n} \alpha(t_i)D(t_i)s(t_i) + \int \alpha^*(u)D(u)p(u)du}$$

Discrete and Continuous

If we knew the CDS spread for all times t, then we could use the above relationship to convert it to a default probability curve. Similarly, we can convert probability default curves back into CDS spread curves.

However, CDS spreads are only quoted for a handful of discrete times.

Like in curve stripping, to compute implied default probabilities, we need to make an assumption about either the default probability curve or the CDS spreads.

The common assumption:

- Default probabilities are derived from piecewise constant hazard rates.

Piecewise constant hazard rates

If par CDS spreads C_i are quoted for maturities T_i, we let λ_i be discrete hazard rates, and define $\lambda(t) = \lambda_i$ for $T_{i-1} < t \leq T_i$.

Then

$$s(t) = e^{-\int_0^t \lambda(u)du}$$
$$p(t) = -ds/dt = s(t)\lambda(t)$$

For $0 < t \leq T_1$, this yields:

$$s(t) = e^{-\lambda_1 t}$$
$$p(t) = e^{-\lambda_1 t}\lambda_1$$

Using this we can solve for λ_1 given C_1, and repeat for each λ_i to construct the default probability curve.

Pricing CDS contracts

Once a default probability curve is constructed as above, it can then be used to compute the price of a CDS contract with a non-par spread.

The price of a receive fixed contract with spread C then follows our pricing formula:

$$\sum^{n} C\alpha(t_i)D(t_i)s(t_i) \\ + \int C\alpha^*(u)D(u)p(u)du \\ - \int (1-R)D(u)p(u)du$$

CDS on the CCP

To facilitate trading of CDS through a CCP, CDS conventions have changed.

- Quarterly maturities are traded.
- Particular spreads are traded, depending on geographic locale:
    - Standard North American Contract (SNAC) — 100 and 500 bp, with 40% recovery.
    - Standard European Contract (STEC) — 25, 100, 300, 500, 750 and 1000 bp, 40% recovery.
    - Standard Emerging Market contract (STEM) — 100 and 500 bp, with 25% recovery.

However, single name CDS just started clearing on ICE this year.

The convention is to quote a par spread for investment grade, and pay the difference between the price of the traded leg and the price of the par leg, where the prices are computed using the above model and a flat curve equal to the quoted spread. This difference is the "points up-front" payment.

For distressed credit, instead of quoting the par spread, the points up-front themselves are quoted.

Summary

Summary

Major points:

- Credit risk is factored into OTC derivative valuations.
- Risk mitigants:
  - Netting agreements.
  - Collateralization.
  - Use of clearing houses.
  - Each risk mitigant introduces other risks.
- Qualitative analysis of counterparty credit risk:
  - Long only positions (like bonds) have a very different risk profile from derivatives that can change from assets to liabilities.
  - For assets that can change (like swaps), credit exposure is nonlinear (like a call option).

Summary

Formulas, assuming zero correlation between numéraire and default, and constant known recovery rate.

CDS pricing:

$$\sum^{n} C\alpha(t_i)D(t_i)s(t_i) + \int C\alpha^*(u)D(u)p(u)du$$
$$- \int (1-R)D(u)p(u)du$$

CDS bootstrap par spread:

$$C(t_n) = \frac{\int (1-R)D(u)p(u)du}{\sum^n \alpha(t_i)D(t_i)s(t_i) + \int \alpha^*(u)D(u)p(u)du}$$
$$s(t) = e^{-\int_0^t \lambda(u)du}$$
$$p(t) = -ds/dt = s(t)\lambda(t)$$

Default probabilities are WRT pricing measure Q, not real world measure P.

References I

[CD03] Eduardo Canabarro and Darrell Duffie. "Measuring and marking counterparty risk." In: *Asset/Liability Management of Financial Institutions, ed. Leo M. Tilman, Institutional Investor Books* (2003). URL: http://www.darrellduffie.com/uploads/surveys/DuffieCanabarro2004.pdf.

[Gre10] Jon Gregory. *Counterparty credit risk: the new challenge for global financial markets*. Vol. 470. John Wiley & Sons, 2010. URL: https://clio.columbia.edu/catalog/11095981.

[Nos11] Ed Nosal. "Clearing Over-the-Counter Derivatives." In: *Economic Perspectives* 35.4 (2011). URL: https://www.chicagofed.org/publications/economic-perspectives/2011/4q-nosal.

References II

[NS10] Ed Nosal and Robert Steigerwald. "What is clearing and why is it important?" In: *Chicago Fed Letter* 278 (Sept. 2010). URL: https://www.chicagofed.org/publications/chicago-fed-letter/2010/september-278.

[PZ07] M. Pykhtin and S. Zhu. "A Guide to Modelling Counterparty Credit Risk." In: *Global Association of Risk Professionals* (37 July–Aug. 2007).

[Ste13] Robert S. Steigerwald. "Understanding Derivatives: Markets and Infrastructure." In: Federal Reserve Bank of Chicago, 2013. Chap. Chapter 2: Central Counterparty Clearing. URL: https://www.chicagofed.org/publications/understanding-derivatives/index.

[Com16] Committee on the Global Financial System. *Semiannual OTC derivatives statistics*. Tech. rep. Bank of International Settlement, May 2016. URL: http://stats.bis.org/statx/srs/table/d5.1.

References III

[Fin07] Financial Accounting Standards Advisory Council. *Statement of Financial Accounting Standards No. 157, Fair Value Measurements*. Tech. rep. Financial Accounting Standards Board, 2007.

[Fin09] Financial Accounting Standards Advisory Council. *Topic 820, Fair Value Measurements And Disclosures*. Tech. rep. Financial Accounting Standards Board, 2009.

[Int01] International Accounting Standards Board. *International Accounting Standard 39 — Financial Instruments: Recognition and Measurement*. Tech. rep. IFRS, 2001.