Math GR 5320: Financial Risk Management and Regulation

Assignment 9

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Due next Thursday by 1:00 pm.

For help. the preferred approach is to post questions on the Q&A tab in Piazza:

https://piazza.com/columbia/fall2016/mathg5320_001_2016_3/home

These will be quickly responded to and will be helpful to others in the class.

Otherwise, attend TA office hours, email a TA or the professor, or schedule a meeting.

1. **Merton model again**

   Assume a Merton model for default with a constant risk free rate of r and an initial firm value of \$10,000,000. The firm issued a zero coupon bond with face value B that matures at time T:

$$\begin{aligned} dV &= \mu V dt + \sigma V dW \\ \mu &= 0.1 \\ \sigma &= 0.2 \\ r &= .05 \\ V_0 &= 10,000,000 \\ T &= 5 \end{aligned}$$

   What is the survival probability function for time T as a function of B? Give its formula and graph it as a function of B.

   What is the current value of the equity and of the issued zero coupon bond? Give its formula and graph it as a function of B.

   In the previous problem, the 5 year survival probability is 0.873716. What value of B gives the same 5 year survival probability?

2. **CVA**

   (a) What is "CVA"?

   (b) Give the general formula for computing CVA and give the assumptions under which it holds.

   (c) What is wrong way risk?

   (d) Give an example of wrong way risk.

3. **CVA calcs** Consider a discrete world in which at time zero, the stock S and the bond B each cost \$100. At the future time T, the stock is either worth \$150 with probability 0.9 and or \$50 with probability 0.1. In both cases, the bond still costs \$100.

   You enter into a contract with a counterparty. At time T, you will pay him the cost of the stock, and he will pay you the cost of the bond.

   (a) Assuming no arbitrage, if the chance of the counterparty defaulting is zero, what is the price of this contract?

   (b) Suppose now that there is a chance that your counterparty defaults and only pays 40% of the value of the contract (assuming he owes you anything at all). According to your analysis of the cost of buying default protection, you have computed that

there's a 20% risk neutral chance of default, and that the default is independent of the stock price.

What is the value of this risky contract and what is the value of the CVA?

4. **Short portfolio, VaR and some ES**

   Setting as in previous VaR calculation problems.

   Repeat the VaR calculations for short portfolios. Instead of having positive positions in A, I, and P, consider having equal but negative positions in each. Use the 5 year period parameter estimates.

   Work with shorts as follows. If you have \$1 and buy a stock that costs \$1, you then have \$1 worth of stock. If you have \$1 and short the stock, you then have \$2 in cash and -\$1 worth of stock. We ignore interest rate impact, so the cash component doesn't change in value when computing VaRs.

   Compute VaR for \$10,000 invested in each on each date. Compute using formulas (assuming portfolios follow GBM).

   How do the VaRs for short portfolios compare to those for long portfolios? Which are more risky, and why?

   How do the VaRs compare for a short position in P when computing using the volatility estimates from the history of P vs pricing the portfolio by modeling A and I? To answer this, compare the portfolio VaR computed above with the Monte Carlo VaR computed by directly modeling A and I.

5. **Short portfolio, formula ES**

   Work out a formula for the ES of a short position in a portfolio following GBM. Use it to compare the 97.5% ES on a short position to the 99% VaRs computed using formulas in the previous problem (i.e. - for A, I and P across history calibrated to the last 5 years of data, equally weighted).

   Are the 97.5% ESs and 99% VaRs on short portfolios close to each other?