Pushdown Automata

- A *pushdown automata (PDA)* is like an NFA except it has a *stack* or *pushdown store* that can be used to record an unbounded amount of information.

- The input is read left to right as in an NFA.

- In each step, the machine is in some state. It can read an input symbol and it can read and pop the top of the stack or it can transition without reading any input or without reading any stack symbol.

Pushown Automata

- Based on this symbol, the input symbol it is currently reading, and its current state, it replaces the top symbol on the stack with a sequence of symbols, which may consist of ϵ, or one symbol, goes to the next input symbol unless ϵ is used for the input, and enters a new state (which might be the same state), according to the transition relations of the machine.

- It can also make a move without pushing anything on the stack.

- The PDA accepts a string if after reading the string the machine is in an accept state.

Example: The even-length palindromes

- Start by putting $\$$ on the stack.
- In state q_0 we're on the first half of the string.
- In state q_1 we've on the second half of the string.
- Pop $\$$ and go to accept state.

Formal definition of a PDA

A PDA is a 6-tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$ where

$$
\begin{aligned}
Q &= \text{finite set of states;} \\
\Sigma &= \text{alphabet of input symbols;} \\
\Gamma &= \text{alphabet of stack symbols;} \\
q_0 &= \text{initial state;} \\
F &= \text{accept states} \\
\delta &= \text{transition function}
\end{aligned}
$$

Formal definition of a PDA

- $\delta(q, a, X)$ contains a set of pairs of the form (p_i, Y) where $q \in Q,\ a \in \Sigma \cup \{\epsilon\}$ and $X \in \Gamma \cup \{\epsilon\}$; $p \in Q$ is the new state; Y is the new top of the stack, after popping X.

- Note that ϵ appears as an option in three places, with the following meainings:

  1. Not reading an input symbol
  2. Not reading (popping) a stack symbol
  3. Not pushing a stack symbol on the stack.

Example: Even length palindromes

$P = (\{q_0, q_1, q_2\}, \{1, 0\}, \{1, 0, \$\}, \delta, q_0, q_3)$, where δ is described below:

$$\begin{aligned}
\delta(q_0, \epsilon, \epsilon) &= \{(q_1, \$)\} \text{ put bottom of stack symbol on stack} \\
\delta(q_1, 1, \epsilon) &= \{(q_1, 1)\} \text{ push symbols on stack /transition} \\
\delta(q_1, 0, \epsilon) &= \{(q_1, 0)\} \\
\delta(q_1, \epsilon, \epsilon) &= \{(q_2, \epsilon)\} \\
\delta(q_2, 0, 0) &= \{(q_2, \epsilon)\} \text{ pop symbols from stack} \\
\delta(q_2, 1, 1) &= \{(q_2, \epsilon)\} \\
\delta(q_2, \epsilon, \$) &= \{(q_3, \epsilon)\}
\end{aligned}$$

Graphical Representation of a PDA

Like an NFA, except an arc from state p to state q is labeled $a, X \to Y$ to indicate that $\delta(p, a, X)$ contains (q, Y).

$$L = \{0^n 1^n \mid n \geq 1\}$$

$$\{a^i b^j c^k \mid i, j, k \geq 0 \quad and \quad i = j \quad or \quad i = k\}$$

$L = \{x \mid x \in \{a, b\}$ **is not of the form** $ww\}$

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