

## Homework 4–CSC 320 Fall 2015

Due in class, Wednesday November 25

1. Prove that the language

$L = \{\langle M \rangle \mid M \text{ when started on the blank tape, eventually writes a \$ somewhere on the tape}\}$   
is undecidable. Use the undecidability of  $A_{TM}$  to do this.

2. Give a reduction from

$$HALT_{TM} = \{\langle M, w \rangle \mid M \text{ is a TM which halts on } w\}$$

to

$$L = \{\langle M, j \rangle \mid M \text{ halts on all inputs with less than } j \text{ 1's}\}.$$

Prove it's a reduction.

3. Show that  $\overline{E_{TM}}$  is recognizable by giving a high level description of a nondeterministic TM which recognizes it.
4. Show that  $\{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$  is not recognizable. Use  $E_{TM}$ .
5. Say a TM  $M$  is *reversible* if for every string  $w$ ,  $M$  accepts  $w$  iff  $M$  accepts the reverse of  $w$ . (Recall that the reverse of  $w_1 w_2 \dots w_k$  is  $w_k w_{k-1} \dots w_1$ .) Prove that the language

$$T = \{\langle M \rangle \mid M \text{ is a reversible TM}\}$$

is undecidable.