Algorithm	Problem	Scenario	Finite-time	Rate	Preference
AID[Ji et al., 2021]	BLO	Deterministic	Yes	$\mathcal{O}(K^{-1})$	-
ITD[Ji et al., 2021]	BLO	Deterministic	Yes	$\mathcal{O}(K^{-1})$	-
SOBA[Dagréou et al., 2022]	BLO	Stochastic	Yes	$\mathcal{O}(K^{-rac{1}{2}})$	-
STABLE[Chen et al., 2022]	BLO	Stochastic	Yes	$\mathcal{O}(K^{-\frac{1}{2}})$	-
stocBiO[Ji et al., 2021]	BLO	Stochastic	Yes	$\mathcal{O}(K^{-1})$	-
AmIGO[Arbel and Mairal, 2021]	BLO	Stochastic	Yes	$\mathcal{O}(K^{-1})$	-
BLMS[Li et al., 2024]	MOBL	Deterministic	No	-	No
PSP-BLEMO[Wang et al., 2024]	MOBL	Deterministic	No	-	No
OAMOML[Ye et al., 2021]	MOBL	Deterministic	No	-	No
gMOBA[Yang et al., 2024]	MOBL	Deterministic	No	-	No
FORUM[Ye et al., 2024]	MOBL	Deterministic	Yes	$\mathcal{O}(K^{-rac{1}{4}})$	No
MORBiT[Gu et al., 2023]	MOBL	Deterministic	Yes	$\mathcal{O}(\sqrt{S}K^{-\frac{2}{5}})$	No
WC-MHGD(ours)	MOBL	Deterministic	Yes	$\mathcal{O}(K^{-1})$	Yes
WC-MHGD(ours)	MOBL	Stochastic	Yes	$\mathcal{O}(K^{-1})$	Yes

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