

Theory of Storage, Inventory and Volatility in the LME Base Metals

Hélyette Geman

Director, Commodity Finance Centre
Birkbeck, University of London & ESCP
Europe

William O. Smith¹

Doctoral Programme
Birkbeck, University of London

Abstract

The theory of storage, as related to commodities, makes two predictions involving the quantity of the commodity held in inventory. When inventory is low (i.e. , a situation of scarcity), spot prices will exceed futures prices, and spot price volatility will exceed futures price volatility. Conversely, during periods of no scarcity, both spot prices and spot price volatility will remain relatively subdued. We test these relationships for the six base metals traded on the London Metal Exchange (aluminium, copper, lead, nickel, tin and zinc), and find strong validation for the theory. Moreover, and in contrast to widespread claims that Chinese inventory data are opaque, we find that including Chinese inventories strengthens the relationship further. We also introduce the concepts of excess volatility, inventory-implied spot price and inventory-implied spot volatility and illustrate some applications.

JEL Categories: B 26, C22, G13, G31, N50, Q31

Keywords:

Working curve, storage, base metals, inventory, volatility, convenience yield, forward curve

1. Introduction

The aim of this paper is to examine the six base metals traded on the LME (aluminium, copper, lead, nickel, tin and zinc), and examine the relationship between price, volatility and the quantity held in inventory, for both the spot and futures markets. A relationship, believed to exist for many storable commodities, is predicted by the *Theory of Storage*. In Section 1.1, we review briefly the base metals and futures trading. In Section 1.2, we review the theory of storage and its literature across a number of commodities. In Section 2, we review our data and develop a gauge for inventory which permits comparison between commodities and over long time periods. In Section 3, we examine the relationship between price and inventory, and in Section 4 the relationship between volatility and inventory. In Section 5, we consider some applications, and Section 6 concludes.

1.1 Base Metals and Futures Trading

Firstly, we review the base metals, futures markets (particularly the London Metal Exchange) and the existing literature on the theory of storage.

1.1.1 The Base Metals

Unlike the precious metals such as gold and silver, which are often purchased for investment rather than commercial use, the base metals are all notable for their industrial uses, principally in automobiles (aluminium, nickel), packaging (aluminum, tin), building and infrastructure construction (aluminium, copper, nickel, zinc), electronic and electrical components (copper, lead, tin) and many other applications.

Prices of the base metals vary according to their rarity and extraction costs, ranging from around \$25,000 per tonne (nickel, tin), through \$10,000 per tonne (copper) down to around \$2,500 per tonne (aluminum, lead, zinc), observed in mid-2011. They are typically traded on the LME in the form of bars, rods or ingots, with the exact contract specifications being tailored to the typical requirements of industrial users, and at high purities in excess of 99.8%.

Unlike many commodities, the base metals show negligible seasonal variation in their supply and only minor seasonal variation in demand (related to slight variations in construction activity across the northern hemisphere year), simplifying

their analysis. They are easily storable at relatively low cost (typically $< 5\%$ of their value p.a.), and unlike agricultural commodities, suffer negligible degradation over time, again simplifying their analysis.

1.1.2 Futures Markets

Commodity markets typically have greatest liquidity in futures markets rather than spot markets, which allows participants to 'lock in' a price in advance, for example a farmer may wish to fix a price for his harvest long before harvest time, or a construction company may wish to fix the price of copper they will use some months hence. On any given trading date ' t ', a number of futures contracts are traded, one for each maturity date T_1 to T_N . Typically maturities range from 1 month to several years into the future. The purchase of a futures contract obliges the owner to pay on the maturity date T_i , $i \in \{1, \dots, N\}$, the market price $F(t, T_i)$ to the seller, and in turn (s)he will receive one contract's worth of commodities². Typically futures are traded on an exchange, and margin payments will be payable between the trade date t and the maturity date T_i to minimize the counterparty risk born by each side. In addition, for some commodities, spot markets exist with immediate delivery required. Where this is not the case, it is typical to consider the price of the futures contract which is soonest to expire (the so-called 'front month' contract) as a proxy for a spot price.

1.1.3 The London Metal Exchange

London has been the world hub of metal trading for centuries, in an area near to the former Royal Exchange. Ad-hoc metal trading was replaced with a formal exchange with the founding of the London Metal Exchange in 1877. The LME has remained the centre of world metal trading ever since. Despite competition from COMEX in the US, and the Shanghai Futures Exchange (SHFE) in China, it remains for now the most liquid venue for trading of base metals. In particular, we examine in this study its contracts for Aluminum, Copper, Lead, Nickel, Tin and Zinc.

² Typically there may be some small lag comprising several business days between maturity of the contract and the delivery date, but this is irrelevant in the present context.

The LME's trading structure is somewhat unique, resulting from its long history. Several times a day, so called 'ring' trading sessions occur, in an open-outcry format, with traders located physically in a seated circle or ring, with only a single metal traded per brief and intense 5 minute session. Electronic trading is also available during an extended business day, and telephone trading is available 24 hours per day, with all trades reported and settled through the LME (LME 2011).

Unlike most commodity exchanges where futures contracts are typically deliverable in fixed months, with only occasional 'expiry' of contracts, the LME trades constant maturity contracts. On each trading day, contracts for delivery in 2 days ('spot'), 3 months, 15 months and 27 months are traded. The 3-month contract is the most heavily traded, and was originally introduced because it took that long for tin from South-East Asia, or copper from Chile, to arrive by ship to London (Bloomberg 2011).

The LME maintains a worldwide network of over 600 warehouses. Although counterparties of a futures or spot trade are free to arrange bilaterally the delivery of metal from seller to buyer, they can also deliver to or take delivery from an LME warehouse. The warehouses are carefully chosen worldwide to be at sources of demand rather than supply, ensuring that the buyer has immediate access to the metal he has purchased (LME 2011b). However, to date, China does not allow warehouses in its territory to become LME-registered, and metal for Chinese delivery is typically shipped from Singapore or South Korea. Inventory figures across all warehouses are published daily.

1.2 Commodity Inventories and the Theory of Storage

Commodities can be categorized as storable or non-storable. Non-storable commodities include those where storage methods exist but are prohibitively expensive (in particular, the case of electricity) and where the commodity is the provision of a service (as in the shipping industry). The vast majority of commodities are storable. They are stored for several reasons:

- As a buffer against uneven or seasonal supply, as in the case of agricultural commodities, which have been stored in silos as early as 10,000 years ago

- As a buffer against uneven demand, as in the case of most energy commodities, which are typically used more in winter for heating, and midsummer for cooling
- As a buffer against any other supply or logistical disruption, which would otherwise necessitate the expensive pause of an industrial process
- In recent years, for investment purposes within physically-backed ETFs
- For arbitrage reasons, if any, as described later.

The theory of storage applies to any commodity that can be physically stored and makes two main predictions, both related to the quantity of the commodity held in inventory (also known as stocks, a term we avoid due to its confusion with equity markets).

1.2.1 Prediction 1, concerning the relationship between spot and futures *prices*.

When there is a situation of scarcity (low inventory), spot prices will rise as purchasers bid whatever is necessary to secure supply. The effect will be less pronounced in longer term futures, since market participants know that higher price will, in the long term, stimulate increased supply and allow for a rebuilding of inventory. The effect, with *spot price > futures price*, can be extreme, and is known as 'backwardation'. An example of backwardation in the crude oil futures market is shown in Figure 1, taken from the time of high oil demand and rapid price rises in 2007. Oil contracts that mature (expire) in 40 or more months are priced around \$76, whereas those expiring within 1 month (so called 'nearby') futures are priced as high as \$89.

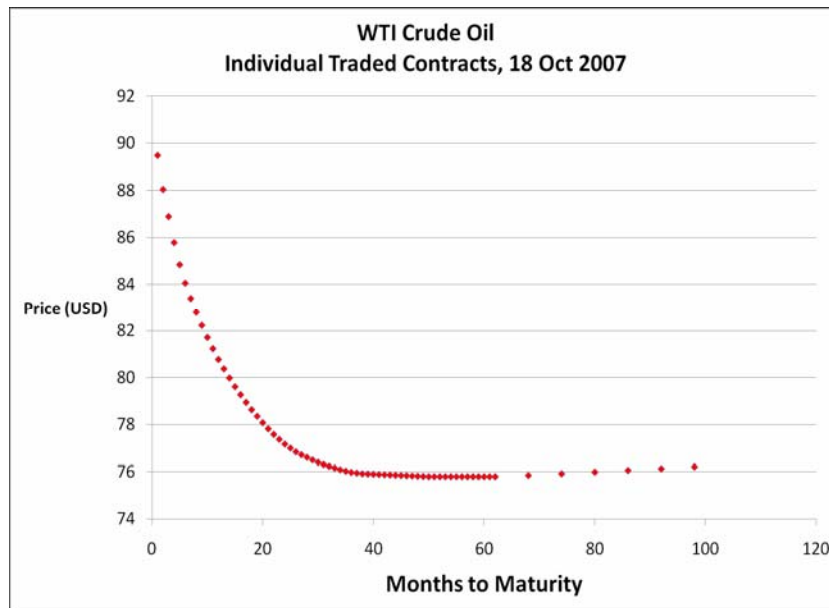


Figure 1 – extreme backwardation in the crude oil market, 2007.

Conversely, when supplies are ample, spot prices can become depressed with respect to futures prices. However, this effect, with *spot price < futures price*, termed 'contango', is usually less pronounced. At a certain point, the possibility of so-called 'cash and carry arbitrage' emerges, whereby a risk-free profit can be obtained by buying the commodity in the spot market, simultaneously selling a futures contract at a higher price, and storing ('carrying') the commodity until the delivery date of the futures contract. This possibility limits the degree of contango for storable commodities. This effect is asymmetrical – we cannot move a quantity of commodity from the future to the present, therefore there is no economic limit on the strength of backwardation imposed by storage. However, given sufficiently high spot prices, some consumers will cancel or postpone their demand, or possibly substitute their demand to another commodity. This weaker economic argument provides some limit to the strength of backwardation.

1.2.2 Prediction 2, concerning the relationship between spot and futures volatilities.

In conditions of scarcity, not only will spot prices be elevated, but they will also experience elevated volatility. This is because in a tight market, any news about short term supply, demand or inventory will have a large impact on the spot

market. However, there is little corresponding rise in the volatility of long term futures contracts, whose prices mainly respond to longer-term news.

In conditions of abundance, this effect will disappear, and there will be no pronounced difference between the volatility of spot and futures prices.

We note that in general, the so called 'Samuelson effect' (Samuelson 1965) states that commodity futures becomes more volatile as they approach maturity, although unlike the theory of storage, it does not mention that such conditions mainly apply during scarcity. We might expect that spot price volatility will almost always exceed futures price volatility, since long term prices mainly respond to long-term news, whereas short-term prices should respond to both short *and* long term news, as well as all kinds of "noise" induced by short term trading.

1.2.3 Development of the Theory of Storage –Inventory and Prices

We describe below the key architects of the theory of storage. In particular, early and instrumental work seems to be regularly overlooked in the literature.

Empirical observation of futures markets had long noted that near-month futures prices were often higher than long-term futures. Keynes (1930) first sought to explain the empirical data by noting that long term futures were usually sold by farmers wishing to fix a price for their harvest and therefore reduce their risk. The futures were bought by speculators, willing to take on the risk in order to realise a profit. Speculators would not enter the market, bearing risk, he argued, unless futures prices tended to rise as harvest approached, giving them a profit. Keynes' theory did not explain why the relationship he described seemed to vary from year to year, and in some years did not hold at all.

We attribute the initial development of the theory of storage to Holbrook Working. In 1927, he was a researcher at the recently established 'Food Research Institute' of Stanford University. The institute decided to focus on wheat because of its great importance as a world staple food (Johnston 1996). Little was formally known about the large fluctuations in the prices of wheat futures. Working theorized that the inventory levels of wheat, in particular the 'year-end carryover', being the inventory

still existing at the end of one 'harvest year', just prior to the arrival of the new harvest, would be instrumental in understanding the behaviour of wheat prices. Since reliable wheat inventory data, or indeed inventory data of any commodity had not been collated and aggregated up to this date, Working and the Food Research Institute began to record new data and research previous years (Working 1927). By 1933, Working had sufficient inventory data, and in two profoundly important but rarely cited papers (Working 1933, Working 1934), he lays out in detail the concepts of the theory of storage, based on his empirical research on wheat. In the first paper, he describes in detail the futures markets in wheat and calculates price spreads between nearby and distant futures. The US wheat harvest occurs mainly from June to August, with the harvesting peaking in July. During the months of June and July, before the harvest had been transported to market, shortages of wheat sometimes developed. By September, the harvest was complete and for a time there was abundance. Working plotted the July-September spread (comparing pre- and post-harvest prices), as observed in June against the year-end inventory, see Figure 2 (reproduced using Working's original data), which we will henceforth term the "Working curve". A clear pattern emerged: in years of low inventory, the prices of July futures were much higher than September futures, resulting in a negative spread. In years with no shortage, September futures were slightly more expensive, by an amount roughly equivalent to the additional cost of storing wheat for two months. This result, showing that short-term futures rise in time of scarcity, is Prediction 1. As well as this central result, Working also documented, we believe for the first time, some other features of futures markets:

- Information affecting next year's harvest (long term information) caused equal change in prices for July and September, resulting in no changes in spread. We would today term these as parallel shifts in the futures curve. Conversely, short-term information, about this year's harvest, affected short term prices (July) more than long term prices (September).
- The average weekly changes in price of July wheat (what we would now term volatility) varied more and more as harvest approached (the so-called Samuelson (1965) effect).
- In situations of scarcity, when July wheat rose in price over September, its volatility also rose greatly compared with situations of abundance (Prediction 2).

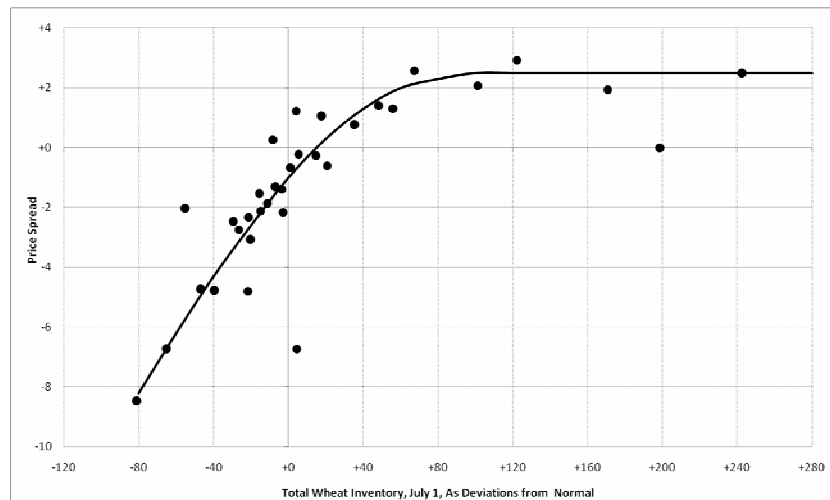


Figure 2 - Relationship between Chicago July-September Spread in June and United States Wheat Stocks on July 1 (from Working(1933))

In the second paper, Working (1934) continued developing the theory of storage. He noted that representing the spread as a percentage rather than a dollar amount, in order to facilitate comparison across long time periods, did not diminish the relationship. He also noticed that the spread would build as harvest time approached, because a situation of impending scarcity or abundance only became clear towards the end of the crop year. Finally, he devoted attention to years that deviated from the usual trend, showing that 'corners' and 'squeezes' had occurred in those years, whereby inventory was bought and withheld from the market by a single participant with the aim of distorting the market for profit. Working summarized his earlier work in two later papers (Working 1948; Working 1949), these sources are those normally cited.

Further intellectual development of the theory of storage was made by Kaldor (1939). Since Working's groundbreaking work was only published in the journal of his employer, the "Wheat Studies of the Food Research Institute", Kaldor was perhaps unaware of it, certainly he did not reference it. Kaldor noted that during backwardation, holding a physical position seems, at first glance, to be illogical, since it is clear from the futures market that prices are expected to fall. Why not simply buy later at a lower price, or buy a long-dated future rather than buy in the spot market? He introduced the term "convenience yield", i.e., the convenience or benefit derived from holding the physical commodity rather than a paper futures contract. This was measured as a percentage yield (as proposed by

Working) which the holder of the physical asset implicitly receives to offset the decline in price. Often the theory of storage is initially credited to Kaldor (Fama and French 1987; Brennan 1958 and others). We believe that much belated credit is mainly due to Working, partly because he 'got there first', and partly because Working explicitly plots graphically the relationship between spread and inventory, whereas Kaldor only discusses the relationship in general qualitative terms.

Brennan (1958) contributed further to the development of the theory of storage. He took empirical data for a number of agricultural commodities (eggs, cheese, butter, wheat and oats) over a period of years, and showed that the Working curve was observed in many markets. Whereas Working had framed the theory in terms of yearly observations, Brennan noted that it held at all times, using monthly observations.

Further evidence to support the Working curve has been found over the years in a range of commodities, such as heating oil, copper and lumber (Pindyck 1994), soybeans (Geman and Nguyen 2005) and crude oil and natural gas (Geman and Ohana 2009). Convenience yield is usually said not to exist in the case of electricity, due to its non-storability. However, in the special case of the Scandinavian Nordpool electricity market, unusual because a large proportion of its electricity is generated from hydroelectric dams, water stored in the dams serves as an inventory of electricity (Botterud, Kristiansen, and Ilic 2010). Watkins and McAleer (2006) examine the LME base metals in an econometric sense, and find limited support for a 'cost of carry' which is based on convenience yield and hence related to the theory of storage.

1.2.4 Development of the Theory of Storage –Inventory and Volatility

The 2nd branch of the theory of storage, described in our *Prediction 2* was again first discussed by Working in his seminal 1933 paper. However, it took some years before further empirical work was done on the relationship between volatility and inventory.

Fama and French (1988) test five bases metals (aluminium, copper, lead, tin and zinc) traded on the London Metal Exchange (LME) from 1972 to 1983, as well as

3 precious metals, gold, platinum and silver. In the absence of formal inventory data, they use interest-adjusted spread as a proxy for inventory. In the case of the base metals, they find that spot price volatility rises as inventory decreases. Gold inventories are always high (central banks and other reserves hold inventory, although the willingness of the owners to sell is sometimes in doubt), so spreads are little-varying and therefore offer little forecast power for price volatility.

Other studies of the relationship between volatility and inventory in the case of metals include:

- Ng and Pirrong (1994), who study four base metals traded on the London Metal Exchange (LME): aluminium, copper, lead and zinc, from 1986-1992. They do not have access to inventory information, so use the adjusted spread as a proxy. They find, as predicted, a strong relationship between spread and spot price volatility.
- Brunetti and Gilbert (1995) examine 6 LME-traded base metals and find that low inventory levels (adjusted for worldwide consumption) are correlated with periods of high spot price volatility.

2. Data and the Units of Inventory

The relationship between price and inventory is usually represented in terms of convenience yield. The convenience yield, usually denoted $y(t, T)$, the benefit accruing to a holder of a physical commodity between times t and T that is not enjoyed by the owner of a futures contract, can be derived from the spot-futures relationship in Equation (1) (See Appendix 1 for details).

$$F(t, T) = S(t)e^{[r(t, T) + c(t, T) - y(t, T)](T - t)} \quad (1)$$

We therefore need a historical database of prices, both spot and futures for the 6 base metals, as well as of $r(t, T)$, the cost of financing over (t, T) in the currency in which the commodity is traded, and $c(t, T)$, storage costs per unit of inventory from time t to T . Naturally we also need a historical database recording the quantity of each metal held in inventory.

2.1 Price Database

In many commodity markets, liquidity exists mainly in the futures market while spot markets are thinly traded, if at all. In these cases, it is common to use the first nearby future price as a proxy for the spot price. Still, futures prices often suffer from technicalities around rollover dates and thin liquidity as delivery date approaches. Fortunately, structural reasons related to the nature of trading imply that metal spot and futures prices reported by the LME are reliable and can be used directly (Fama and French 1988). Since the theory of storage is mainly concerned with relatively short-term effects caused by abundant or low inventory, we study the 'short' end of the futures curve using the spot and 3 month prices published by the LME.

Our price database from the LME covers the period January 1983 to June 2011, except in the case of tin and zinc, when we start in January 1990 due to absence of inventory data or suspension of trading during the earlier period. All prices were initially quoted in British Pounds during the early years of our study period, later transferring into US\$, so when necessary we convert to US\$ using the prevailing spot £/\$ spot rates.

2.2 Inventory Database

We mainly use inventory data as reported daily by the LME, being the total metal inventory held in the large number of LME-appointed warehouses worldwide. All inventory figures are in metric tonnes. We also compiled additional inventory data:

- Aluminium and Copper also trade (or were traded) for some years on the COMEX exchange in New York, although the main world market remains the LME. COMEX publishes its own daily inventory data for its warehouses.
- The US Geographical Survey (USGS) publishes its estimates of annual, year-end US commercial stocks for all 6 base metals. However, in some cases, there is risk of double-counting inventory held in US-based LME warehouses.
- In recent years, the Shanghai Futures Exchange, SHFE, has begun trading aluminium, copper and zinc and publishes its own daily warehouse figures from 2003 onwards (2007 for zinc).
- In the case of Aluminium, the International Aluminium Institute publishes results of monthly surveys of worldwide total zinc commercial and government stocks, and explicitly excludes LME warehouses.

2.3 World Consumption Database

In order to compare inventory with increasing worldwide consumption, we use an annual series of worldwide metal consumption (this includes both primary and secondary – recycled – materials), from World Metal Statistics.

2.4 Storage Costs

Historical storage costs are unavailable for the LME warehouses. At the time of writing, 2011, costs for all six of the base metals, at all warehouses worldwide, were in the range \$0.36 - \$0.46 per tonne per day (LME 2011b), summarized in Table 1. These costs are typically set once per year, with minimal variations from year to year. The costs correspond to annual storage costs ranging from <1% to 5% p.a. of the value of the metal, i.e. low numbers compared to the situation of crude oil or natural gas. Warehouse costs are highly consistent across the globe (see the low standard deviations of cost across all relevant warehouses in Table 1). There are few warehouse operators, and in recent years they have mainly been taken over by the

major banks and commodity trading houses, a situation some claim allows unfair advantage and the possibility of a single player to ‘corner’ the market (we will examine these claims in Section 6), this is certainly at least a source of asymmetry of information. We estimate historical warehousing costs by deflating the 2011 cost by the US CPI Index.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
Average Cost, 2011 (US\$ per tonne per day)	0.408	0.367	0.362	0.453	0.422	0.375
Standard Deviation across Accepting Warehouses (US\$ per tonne per day)	0.0013	0.0006	0.0013	0.0013	0.0015	0.001

Table 1 : Average LME Warehouse Daily Storage cost, 2011 (\$US Per Tonne Per Day)

2.5 Other Data

For ‘ r ’, the cost of financing, we use the relevant constant maturity US treasury rates published by the US Federal reserve and made available by the reliable ‘FRED’ service.

2.6 Data Processing

We take monthly arithmetic averages over all trading days for inventory. For spread, we first make the computations at a daily frequency then take the monthly arithmetic average. Monthly volatility is calculated as the annualized standard deviation of daily log-returns observed during the month.

2.7 Choice of Units for Inventory

Although inventory is typically published in physical units (tonnes in the case of metals, barrels for oil, etc), if we plot the LME inventory of the 6 metals over the study period (Figure 3), we see that the inventory for Aluminium dwarfs all others.

Given the large rise in worldwide consumption of each metal over the years (Figure 4), we opt to express inventory instead using the gauge of *days of worldwide consumption*, i.e., we divide inventory by annual consumption and represent it in days. Figure 5 shows LME inventory in this representation. We can now easily compare each metal and see trends across the decades. We observe that inventory of each metal has tended to vary between circa 2 and 50-60 days of consumption, with notable peaks in 1994, 2003 and 2010. It is interesting to note that the inventory builds begin contemporaneously with a US recession, as in 1990, 2001 and 2008, but take some time to peak, showing that production responds only very slowly to reduced demand during such shocks. The shocks in demand are also visible as brief pauses in the upward consumption trend (Figure 4). We also note the metal inventories do not move in lockstep — at times we see inventory builds in some metals but not others. With the exception of zinc, inventories were universally low during the noted commodity price boom period of 2004-2008. Copper, the most important non-precious metal in terms of its share of world trade, has not seen inventories above 10 days' worth since 2004.

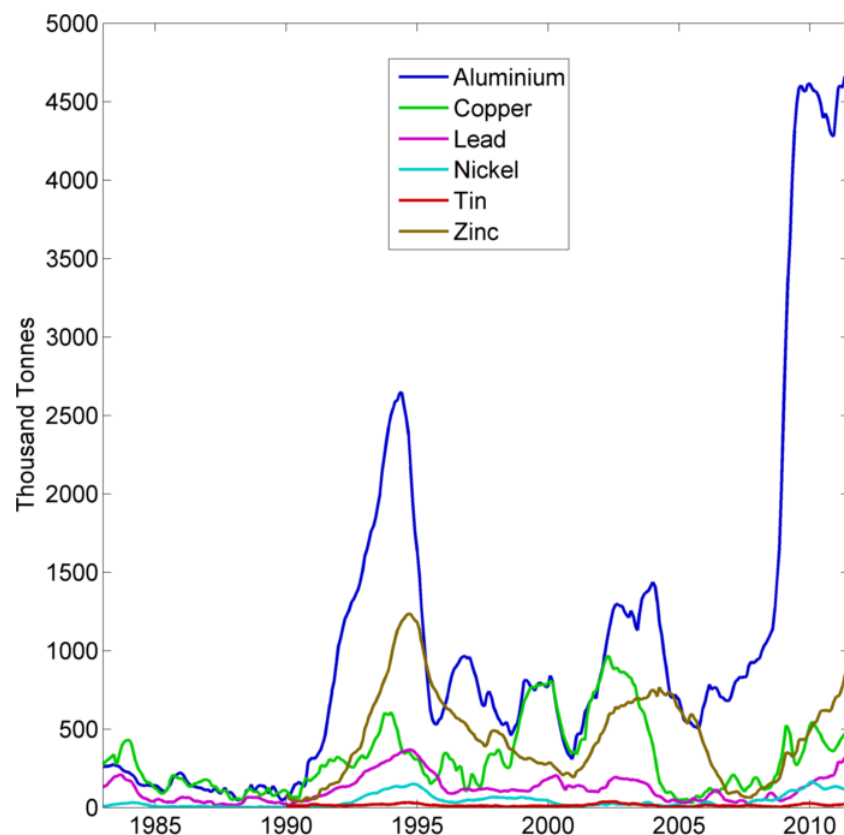


Figure 3 – LME Inventory in Thousand Tonnes for the 6 base metals over the study period

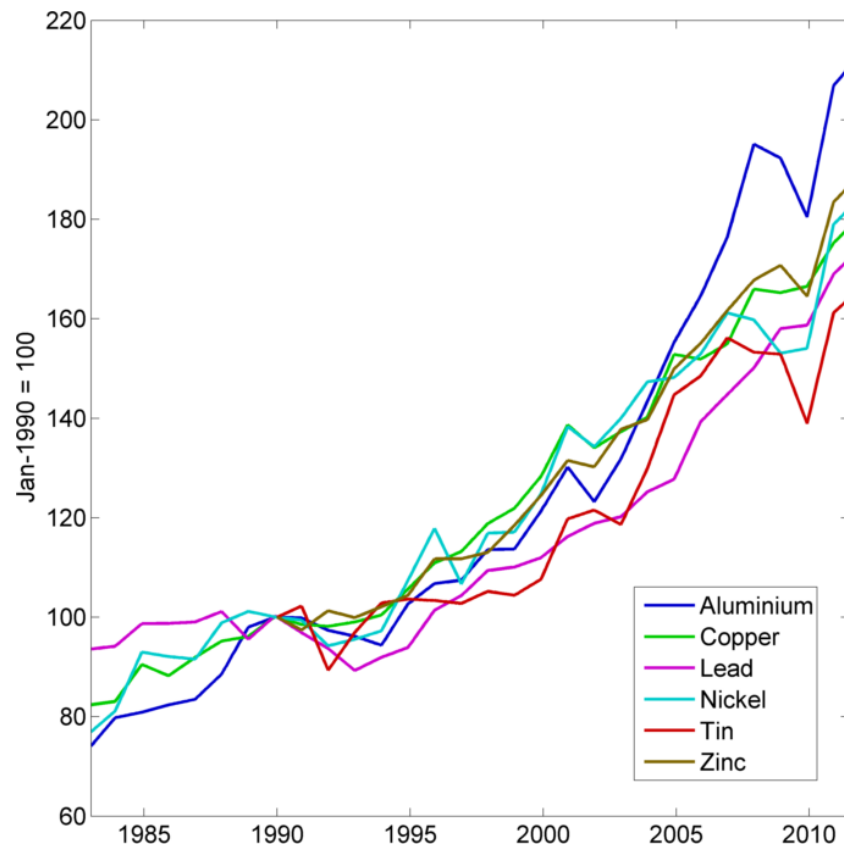


Figure 4 - Increasing worldwide consumption of each base metal over the study period (1990=100)

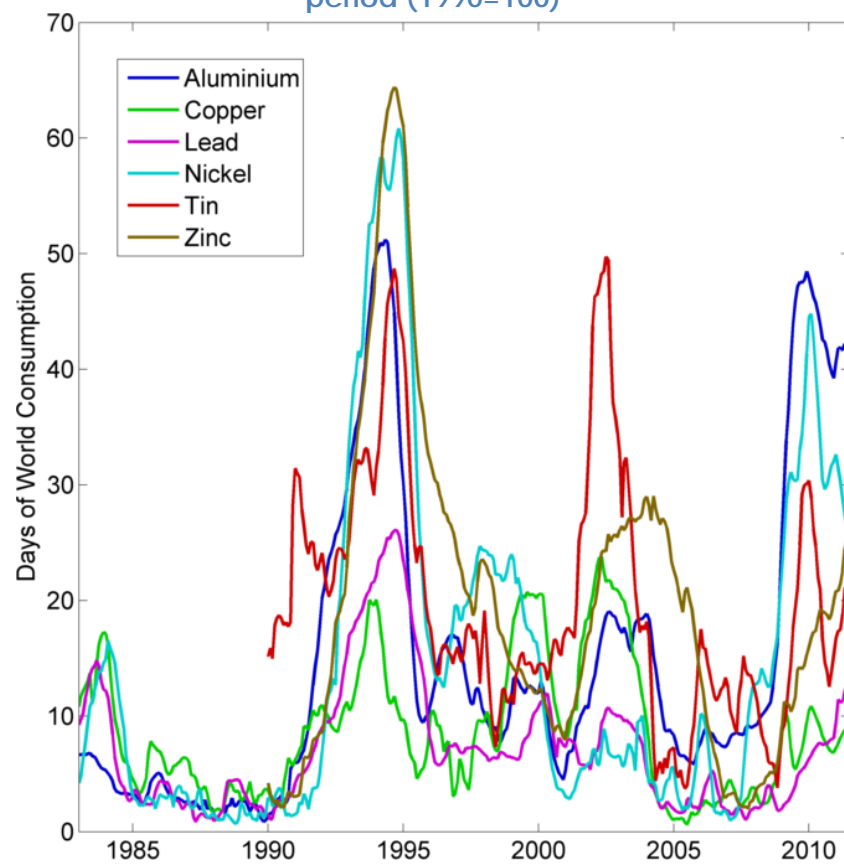


Figure 5 - Inventory measured in days of world consumption

3. Results : The Relationship between Price and Inventory

As in Working (1933) we represent the relationship between spot and futures prices in terms of a spread, rather than looking at convenience yield. More precisely, we follow Geman and Ohana (2009) and others by calculating an 'interest and storage adjusted spread'

$$\psi(t,T) = \frac{F(t,T) - S(t)e^{[r(t,T)+c(t,T)](T-t)}}{S(t)} \quad (2)$$

i.e., it is a ratio, representing the growth from the current spot price $S(t)$ to the futures price $F(t,T)$ for maturity $T > t$, adjusted for financing and storage costs. As a ratio, it can be thought of as a return relative to $S(t)$ from holding the physical commodity between t and T , and selling one future contract at date t for delivery at T .

In Figure 6, we plot, for each metal, a scattergram of monthly observations of the interest and storage adjusted spread (henceforth, 'spread') $\psi(t,T)$ against its contemporaneous inventory $i(t)$. A very clear and consistent picture emerges, relatively identical for each metal. We note the following features:

1. The vertical axis represents the extent to which the spot price is below the futures price, after funding and storage costs are removed. That is, a negative spread represents backwardation, with spot > futures.
2. The spread almost never exceeds 0, as expected; otherwise this would represent an arbitrage opportunity, obtained by purchasing the spot asset and simultaneously selling a 3 month future, paying funding and storage costs for 3 months, and delivering the metal to the futures counterparty after 3 months.
3. Whenever inventory exceeds around 30 days of worldwide consumption, we see a negligible spread, i.e., the futures curve is neither in strong backwardation nor contango.
4. When inventory falls below 10 days of worldwide consumption, an extreme spread often occurs, with spot 5% or even 10% above 3 month futures.
5. The curve is exactly as Working observed for wheat. However, he expressed inventory in terms of 'variation from the usual' whereas we measured absolute inventory in days of worldwide consumption.

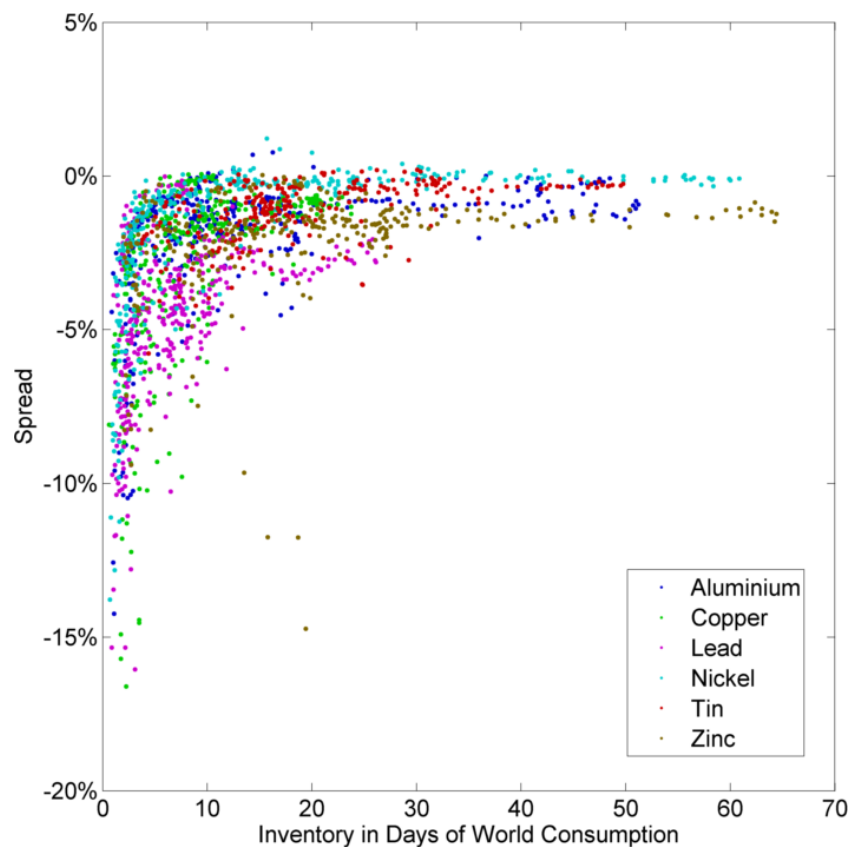


Figure 6 – Interest and Storage Adjusted Spread (spot to 3 month future) vs. Inventory

3.1 Choice of Inventory

In order to determine whether LME inventories alone produce this strong result, or whether the inclusion of additional inventory data improve the fit further, we cannot perform a simple regression due to the non-linear nature of the Working curve observed. Instead, we use the Spearman (1904) rank correlation statistic, denoted ρ_s , which measures monotonic dependence whilst being robust to non-linearities. As with the more widely used Pearson correlation, which we denote ρ_p , a value of 0 indicates no relationship, and a value of 1 indicates perfect correlation. Our results are displayed in Table 2.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
LME	0.590	0.679	0.500	0.878	0.604	0.470
LME + COMEX	0.570	0.683	–	–	–	–
LME + SHFE	0.593	0.737	–	–	–	0.579

LME +	0.578	0.731	–	–	–	0.579
COMEX +						
SHFE						
LME +	0.271	0.660	0.253	0.597	0.518	0.168
USGS						
LME+IAI	0.100	–	–	–	–	–

Table 2 – Spearman Rank Correlations between Interest- and Storage-Adjusted Spread and Inventory, using various measures of inventory

We interpret the results as follows. A strong relationship exists even when the sole LME inventories are considered (and typically the LME inventories make up the bulk of the world’s total published inventories). The addition of COMEX inventories barely changes the correlation, possibly because the COMEX inventories are small compared to those in LME warehouses. SHFE inventories, initially small, are now growing to rival those of the LME. The addition of SHFE inventories significantly improves the relationship for copper and zinc and barely changes it for Aluminium. This lends credence to the reliability of the SHFE inventory data, refuting those who claim that inventory reporting in China is highly opaque. The commercial inventory data (USGS and IAI) weaken the relationship. These cover inventory reported as ‘on-hand’ by individual companies, still in ports etc. We theorise that this inventory is available to individual market participants but *not to the industry as a whole*, and hence are not useful to the wider market, and therefore does not contribute to the market’s perception of *available* inventory.

3.2 Stability of Relationship

In order to test whether the relationship is stable over the 28-year study period (1983 to 2011), we subdivide the study period into 4 equal sub-periods, and report the Spearman correlation separately for each sub-period (Table 3). We return to the LME-only inventory figure. A value of ‘–’ indicates the relevant metal was not traded during the sub-period for sufficient time for a meaningful analysis.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
1983-1989	0.872	0.919	0.931	0.837	–	–
1990-1996	0.060	0.767	0.470	0.778	0.585	0.470

1997-2003	0.11	0.790	0.160	0.809	0.812	0.319
2004-2011	0.627	0.914	0.578	0.850	0.507	0.284

Table 3 – Spearman Rank Correlations between Interest- and Storage-Adjusted Spread and Inventory, over different sub-periods

We see that broadly, the relationship has remained solid over the entire period. There are few periods (e.g. Aluminium, 1990-1996 and 1997-2003) when the relationship is weak. Only in the case of tin and zinc is the relationship weaker in the most recent period, 2004-2011. Overall, we propose that the relationship holds as strongly now as it ever has.

We also tested whether representing inventory in days of worldwide consumption leads to a stronger relationship than simply representing inventory in tonnes. Again, we return to the LME-only inventory figure. Table 4 shows the results. Interestingly, measuring inventory in traditional tonnes gives in most cases a slightly better fit. This demonstrates that despite global consumption roughly doubling over the study period, the industry is now able to operate with the same absolute tonnage of inventory (indeed inventories, although substantially lower before 1990, have not risen greatly since 1990 in tonnes apart from the case of aluminium, see Figure 3). We hypothesize that with the rise over the years of ‘just-in-time’ stockholding techniques, and perhaps greater information flows about inventory size and location, the global metals industry is able to more efficiently allocate inventory to isolated shortages, preventing spot price spikes.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
Inventory in days of worldwide consumption	0.590	0.679	0.500	0.878	0.604	0.470
Inventory in tonnes	0.606	0.772	0.589	0.864	0.638	0.537

Table 4 – Spearman Rank Correlations between Interest- and Storage-Adjusted Spread and Inventory, using different numéraire for inventory.

4. Results : The Relationship between Volatility and Inventory

We now turn to the relationship between volatility and inventory. We calculate a monthly volatility value for the spot and the 3 month futures prices for each metal by taking the standard deviation of the daily log-returns in the given month, and multiplying by $\sqrt{252}$ to obtain an annualised volatility.

We plot in Figure 7 below the relationship between spot price volatility and inventory. Although we see that broadly, spot price volatility increases with low inventory, the picture is muddy. Clearly other factors also cause occasional spikes in volatility, evidenced by the quantity of dots for several metals with high volatility despite high inventory.

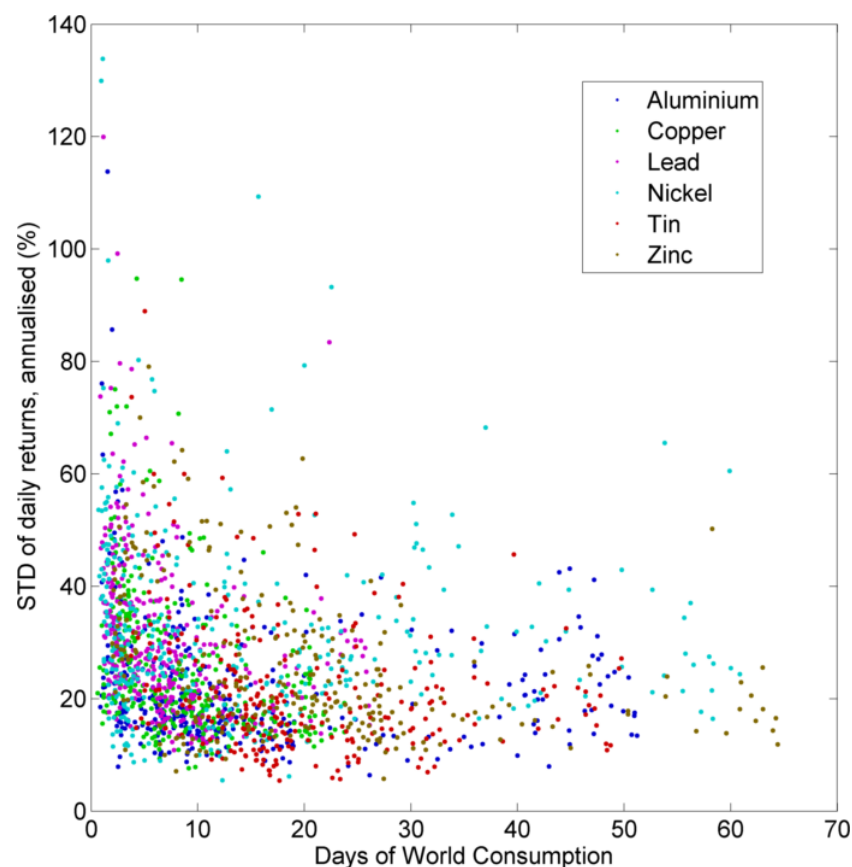


Figure 7 – Spot Volatility vs. Inventory

We now define a value we term 'excess volatility' σ_{excess} , representing the excess ratio of spot price volatility σ_{spot} over futures price volatility $\sigma_{futures}$, calculated as:

$$\sigma_{excess,t} = \frac{\sigma_{spot,t} - \sigma_{futures,t}}{\sigma_{futures,t}} \quad (3)$$

We design σ_{excess} to remove factors affecting long-term volatility, that could be expected to affect both σ_{spot} and $\sigma_{futures}$, and isolate only those short-term factors that affect only σ_{spot} .

If we now plot inventory (as above, LME inventory in days of worldwide consumption) against σ_{excess} , see Figure 8, we obtain a much clearer relationship.

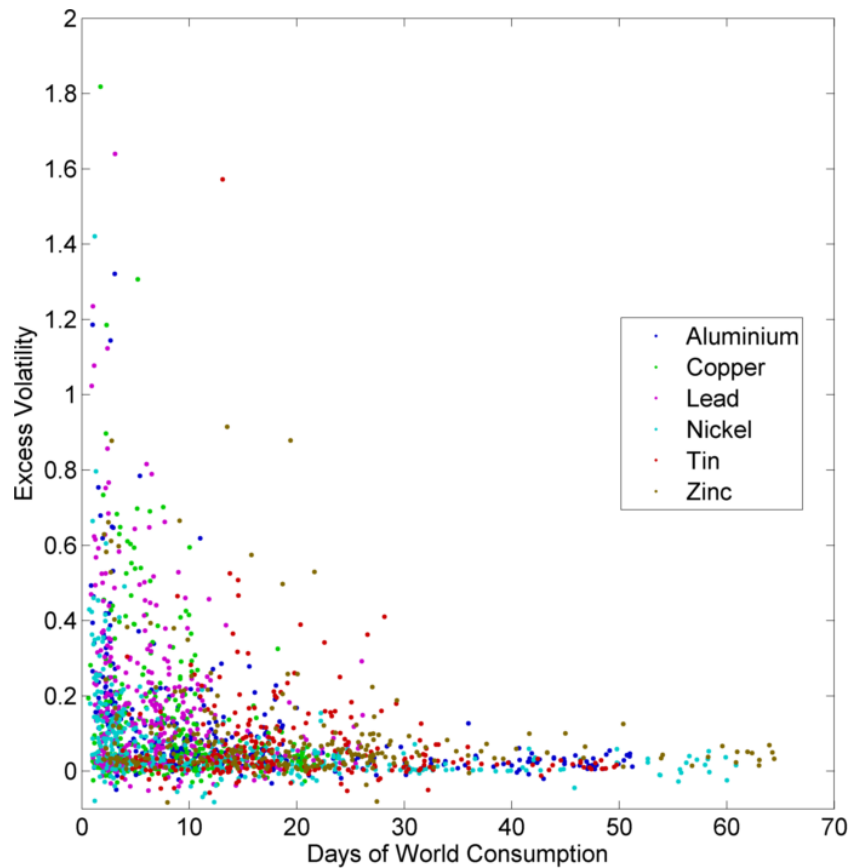


Figure 8 – Excess Volatility vs. Inventory

We interpret the graph as follows:

- σ_{spot} is almost never less than 3 month $\sigma_{futures}$ (i.e., σ_{excess} is almost never < 0).

This supports the ‘Samuelson effect’ (Samuelson 1965), noted as early as Working (1933), that volatility rises towards maturity.

- When inventory is more than 30 days of world consumption, σ_{spot} and $\sigma_{futures}$ are almost identical. We would see this as “parallel shifts” of the futures curve at such times.
- When inventory is less than 10 days, σ_{spot} may rise to 0.2x (20%) or more above $\sigma_{futures}$. However, unlike the Working curve, this “excess volatility curve” does not have a rounded corner near the origin, i.e., there are times of low inventory when σ_{spot} does not exceed $\sigma_{futures}$ despite low inventory.

To test numerically whether this “excess volatility curve” relationship holds true for all metals, we display in Table 4 the Spearman correlation between volatility and inventory, both in the case of σ_{spot} and σ_{excess} . We observe, as expected, negative correlations in all cases. Similar to the most commonly used Pearson measure of correlation, the negative correlation indicates a high value for volatility corresponds to a low value for inventory. The relationship is stronger between inventory and σ_{spot} for some metals (copper, lead, tin, zinc) and between inventory and σ_{excess} in others (aluminium, nickel). Although the correlations are pronounced, they indicate a weaker relationship between volatility and inventory than for the strong spread-inventory relationship noted earlier.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
$\rho_S(\sigma_{spot}, inventory_{days})$	-0.253	-0.466	-0.469	-0.165	-0.244	-0.406
$\rho_S(\sigma_{excess}, inventory_{days})$	-0.482	-0.205	-0.208	-0.571	-0.078	-0.166

Table 4 – Spearman Rank Correlations between volatility and inventory in days.

Finally, we again check whether inventory is best expressed in number of days or tonnes, see Table 5. For every metal, using raw tonnes weakens the relationship

between σ_{spot} and inventory, but strengthens the relationship between σ_{excess} and inventory. How do we interpret this? For spot volatility (which may capture both rises in short term volatility, and rises in volatility across the entire futures curve), the inventory in days is most important, i.e., increasing inventories have been required over time commensurate with increased consumption in order to dampen overall volatility. However, excess volatility is more susceptible to absolute inventory in tonnes, i.e., it has adapted over the years to accept the smaller inventories held per unit of consumption.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
$\rho_S(\sigma_{spot}, inventory_{tonnes})$	-0.177	-0.384	-0.376	-0.073	-0.029	-0.274
$\rho_S(\sigma_{excess}, inventory_{tonnes})$	-0.494	-0.333	-0.313	-0.593	-0.261	-0.298

Table 5 – Spearman Rank Correlations between volatility and inventory in tonnes.

5. Applications

We see several applications for these results, detailed below.

5.1 Forecasting

Trajectories of inventory, as demonstrated in Figures 3 and 5, appear to be mean-reverting, and are certainly not random walks. They display high levels of autocorrelation in their changes, i.e., inventory rises or falls continuously for many weeks before reverting. Over the short term of several weeks, inventory changes are therefore fairly predictable. Given the strong influence of inventory over both the spot-futures price spread and various measures of volatility, a model for inventory could thereby predict likely spreads and volatilities out to an horizon of perhaps 1 or 2 months. We propose that the use of stochastic differential equations with autocorrelation of returns and mean-reversion of levels would be a good starting point for modelling base metal inventories.

5.2 Investigating Market Abnormalities

The seminal paper of Black and Scholes (1973) showed that given several parameters, namely the price of an underlying asset and its volatility, the risk free interest rate, and a duration, the value of an option contingent on that asset could be expressed as a simple closed-form formula. It was soon observed that given an option price already quoted in the market, and the other parameters excluding volatility, an *implied volatility* could be calculated, namely the (unique) volatility that would give that option price had the Black-Scholes option pricing formula been used. This *implied volatility* can in turn be used to price more complex options, or it can be used to confirm that the various options on an underlying are neither under- or over-priced.

We propose a similar approach; we derive *inventory implied spot price* and *inventory implied spot volatility* for a commodity. We can compare these with observed values of spot price and spot price volatility. If the implied and market values are in agreement, we can conclude that the market is functioning “normally” with respect to inventory, i.e., the inventory in warehouses is considered “useful” to

the market in the normal way. If inventory has been “cornered” by a market participant, or another market abnormality implies that inventory is not “available” to the wider market, we would expect to see elevated values for spot price and spot price volatility.

5.3 Inventory Implied Spot Price

We fit a functional form between spread and inventory, as plotted in Figure 6. We choose for the functional form:

$$\psi(t, T) = Ae^{Bi(t)} + C \quad (4)$$

where we expect $A < 0$, $B < 0$, $C \approx 0$ with $C \leq 0$ reflecting no cash-and-carry arbitrage opportunities, and $T = t + 3$ months as in the rest of our analysis. Rather than calibrate using simple least-square methodology, which fits the curve based on the bulk of the observations and are of moderate inventory values in each case, we employ a variation of least-squares described in Appendix 2.

Others have fitted different functional forms to the relationship between inventory and spread to convert it to a linear one. Geman and Nguyen (2005) used

$$K \frac{1}{i(t)}, K < 0 \quad (5)$$

for soybeans, where scarcity is defined as inverse inventory, and obtain conclusive results.

We displayed the results for each metal in Table 6, and the fitted curves in Figure 9. We note that the fitted curves take a similar form for each metal, although copper displays a ‘tighter’ curve (indicating tolerance of lower inventory values than for other metals) and tin displays little curvature. In the case of tin, we have never had a case of extremely low inventories, so there are no ‘low inventory’ values to fit and the fitted line does not curve sharply downwards.

Now we have a prediction for the spread $\hat{\psi}(t, T)$; given $i_{\text{days}}(t)$, we can invert equation (2), and obtain a prediction for the spot price $\hat{S}(t)$.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
Monthly samples fitted	342	342	342	342	258	258
A	-0.0160	-0.0814	-0.2357	-0.2350	-0.0276	-0.0609
B	-0.0506	-0.2081	-1.0178	-0.9225	-0.0556	-0.3324
C	-0.0097	-0.0077	-0.0343	-0.0054	-0.0007	-0.0171

Table 6 – Fitted Results Approximating the Spread as a Function of Inventory (measured in days)

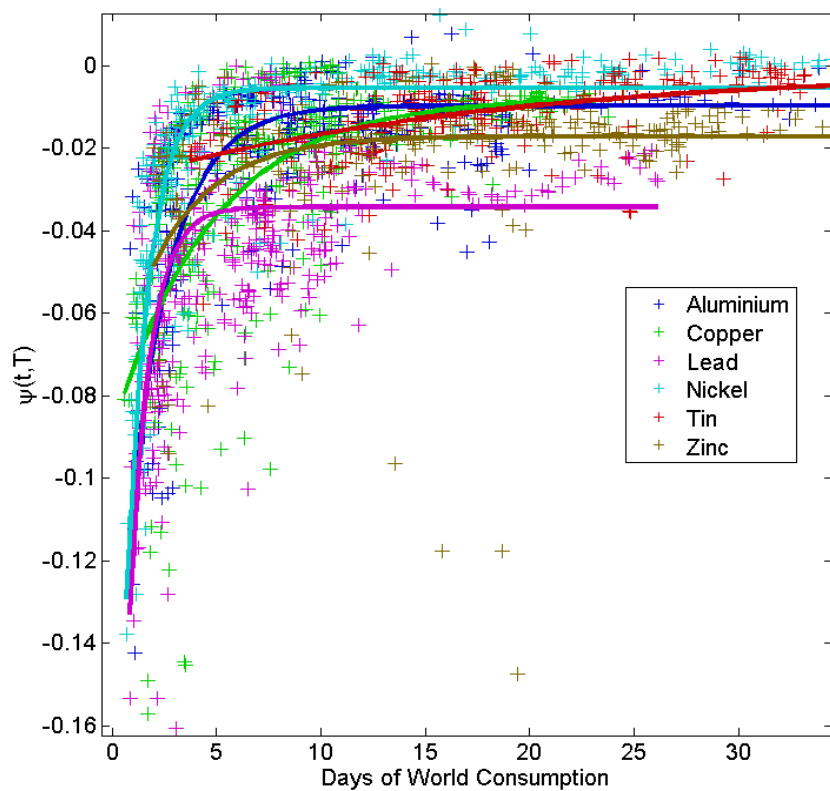


Figure 9 – Fitted Working Curves (Spread vs. Inventory)

5.4 Inventory Implied Spot Volatility

Noting from Figure 8 the similar form of the relationship between inventory and excess volatility σ_{excess} (defined in equation(3)), we repeat the above exercise, fitting a curve of the same form:

$$\sigma_{excess}(t) = Ae^{Bi(t)} + C \quad (6)$$

Where again now expect $A > 0$, $B < 0$, $C \approx 0$ but not necessarily $C > 0$ because there is no economic reason to anticipate σ_{spot} to be always higher than futures. A fitted graph is displayed in Figure 10 and the fitted values in Table 11. The fitted curves all take the same form, although again the tin 'curve' has little curvature due to the lack of low-inventory historical values for tin. We also see surprisingly little curvature for copper.

	Aluminium	Copper	Lead	Nickel	Tin	Zinc
Weekly samples to fit	342	342	342	342	258	258
A	0.6882	0.1984	3.8888	0.4979	0.0627	0.6534
B	-0.4540	-0.0636	-2.0678	-0.5839	-0.0109	-0.1087
C	0.0422	0.0047	0.1351	0.0237	0.0080	0.0189

Table 11 - Fitted Results Approximating the 'Excess Volatility' as a Function of Inventory (measured in days)

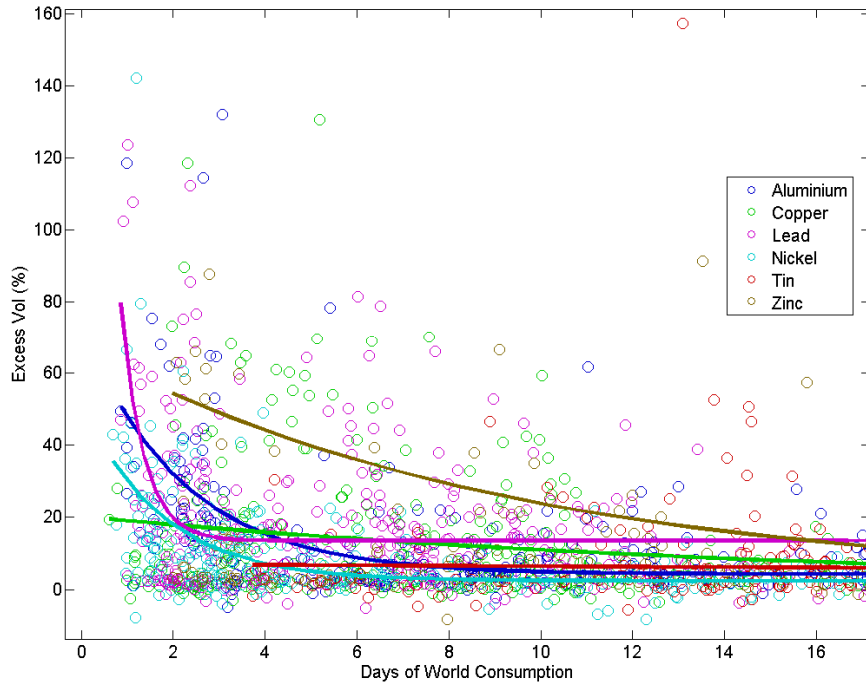


Figure 10 – Fitted Excess Volatility Curve (σ_{excess} vs. Inventory)

Similarly to the case for inventory-implied spot price, given $i_{days}(t)$, we have now a prediction for the excess volatility (denoted $\hat{\sigma}_{excess}(t)$); we can invert equation (3) and obtain a prediction for spot volatility $\hat{\sigma}_{spot}(t)$.

5.5 Using the Inventory-Implied Spot Price and Inventory-Implied Spot Volatility

We can now compare the inventory implied values for spot price and spot volatility with those observed in the market. This tells us whether the market is behaving in the ‘usual’ way with respect to the observed level of inventories. If either the empirical spot price or the empirical spot volatility exceeds that predicted, the market is acting as if the observed inventory is unavailable or only partially available. This would then tend to imply market manipulation or a corner by one inventory owner of the market. There have been many allegations of such activity in the period 2010-2011 (MSNBC.com 2011; Bloomberg 2011 etc). We plot therefore in Figure 11 and Figure 12 the ratios:

$$S_{ratio}(t) = \frac{S(t)}{\hat{S}(t)} \quad (7)$$

$$\sigma_{ratio}(t) = \frac{\sigma_{spot}(t)}{\hat{\sigma}_{spot}(t)} \quad (8)$$

for the cases of both Aluminium and Copper. In both cases, the lines hover close to 1.0 in recent years, with no upward spikes. Any discrepancy between the empirical spot values (price, volatility) and the inventory-implied values seem to have occurred prior to around 1997 and substantially disappeared since then. The graphs for the remaining four base metals (not displayed) are substantially similar. We can then conclude that there is no evidence from spot prices and volatilities that one or more major players are manipulating the base metals by keeping inventory from the market.

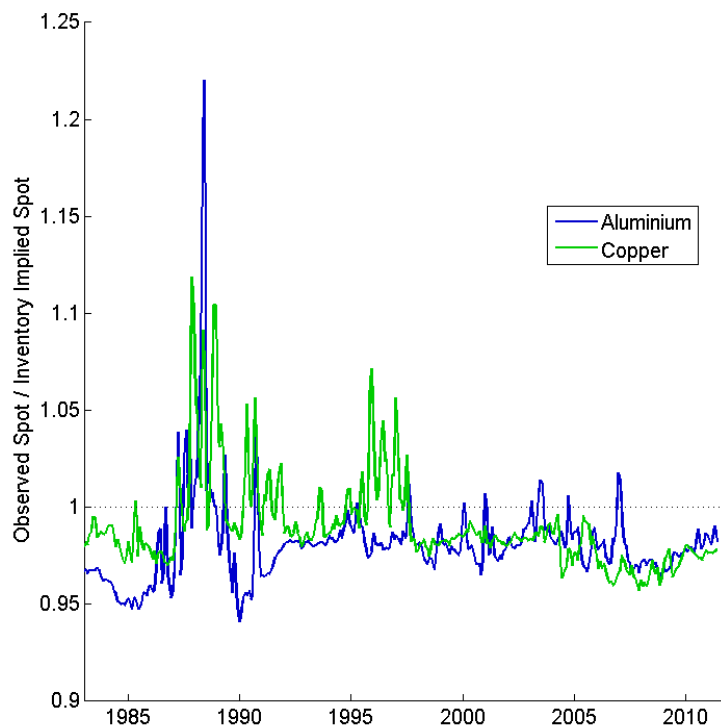


Figure 11 – Ratio of Empirical Spot Price and Inventory-Implied Spot Price over the Study Period

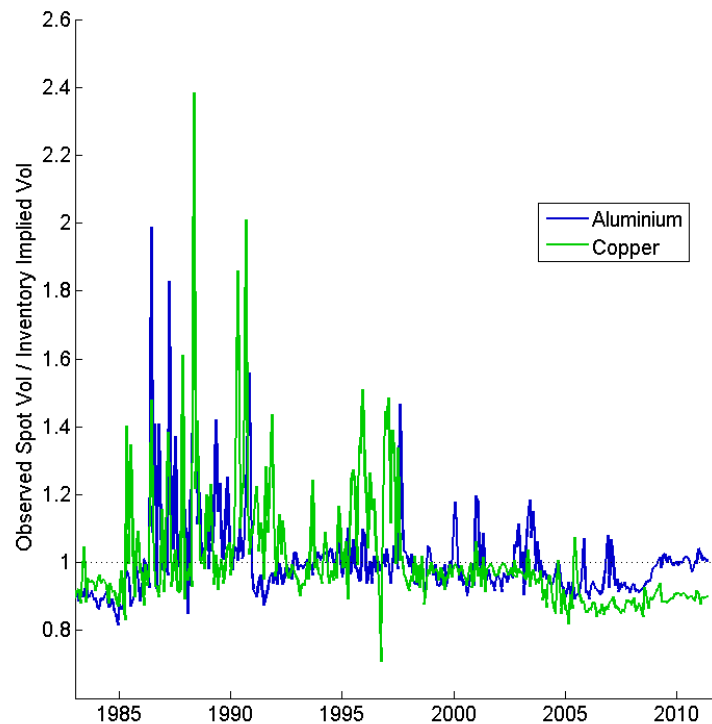


Figure 12 – Ratio of Empirical Spot Volatility and Inventory-Implied Spot Volatility over the Study Period

6. Conclusion

Working's theory of storage, and its two key predictions related to price and volatility, originally formulated for wheat, and initially validated in other agricultural markets, has been shown here to be strongly validated in the case of the six base metals traded on the LME.

We find a strong non-linear relationship between the adjusted- spread of the forward curve (based on a ratio between spot and futures prices) and inventory. In addition, we have shown that the relationship between spot volatility and inventory is strengthened further by introducing the concept of 'excess volatility'. This is analogous to the spread, in that it represents the excess of spot volatility over futures volatility.

We have shown that the inventory figures from LME warehouses alone suffice to generate the two strong relationships above. The addition of Chinese inventories

figures at the SHFE slightly strengthens the relationship further, highlighting the increasing importance of China in metal demand and trade, and refuting some suggestions that Chinese inventory data cannot be trusted. The addition of inventory figures from other exchanges and trade organisations does not improve the relationship, highlighting further that only the LME and the SHFE need be followed, for now at least.

Finally, based on our novel concepts of inventory-implied spot price and inventory-implied spot volatility, we seen no evidence that the recent allegations of major market players withholding inventory is substantiated, to the extent that LME prices are behaving as if the full inventory figures are available to the market.

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Appendix A – The Calculation of Convenience Yield and Interest-Adjusted Spread

Formally, the relationship between futures and spot prices is usually expressed as:

$$F(t, T) = S(t)e^{[r(t, T) + c(t, T) - y(t, T)](T - t)} \quad (\text{A.1})$$

where

- $F(t, T)$ is the futures price of a commodity for delivery at time T , as observed at time t
- $S(t)$ is the spot price of the commodity observed at time t
- $r(t, T)$ is the annual cost of financing the futures position from time t to T
- $c(t, T)$ is the annual cost of storage of the physical commodity from time t to T , also expressed as a rate
- $y(t, T)$ is the annual 'convenience yield' enjoyed by the holder of the stored commodity from time t to T , and is calculated to satisfy the equality, rather than observed directly.

We can understand the above relationship as follows. The convenience yield from holding a spot contract from t to T is termed the 'basis', calculated, for example by Fama and French (1987), as

$$\text{basis}(t, T) = \frac{F(t, T) - S(t)}{S(t)} \quad (\text{A.2})$$

If we taking into account the cost of financing and storing a long physical position for duration $(T - t)$ we derive a term which has been called the 'interest and storage- adjusted spread', which we term simply 'spread'

$$\psi(t, T) = \frac{F(t, T) - S(t)e^{[r(t, T) + c(t, T)](T - t)}}{S(t)} \quad (\text{A.3})$$

By expressing (A.1) and (A.3) in a discretely compounded form, we can more easily see the relationship between spread and convenience yield:

$$F(t, T) = S(t) \left(1 + [r(t, T) + c(t, T) - y(t, T)](T - t) \right) \quad (\text{A.4})$$

$$F(t, T) - S(t) = S(t) [r(t, T) + c(t, T) - y(t, T)](T - t) \quad (\text{A.5})$$

$$\frac{F(t, T) - S(t)}{S(t)} = [r(t, T) + c(t, T) - y(t, T)](T - t) \quad (\text{A.6})$$

$$y(t, T)(T - t) = [r(t, T) + c(t, T)](T - t) - \frac{F(t, T) - S(t)}{S(t)} \quad (\text{A.7})$$

$$y(t, T) = r(t, T) + c(t, T) - \frac{\frac{F(t, T) - S(t)}{S(t)}}{T - t} \quad (\text{A.8})$$

and

$$\psi(t, T) = \frac{F(t, T) - S(t) \left(1 + [r(t, T) + c(t, T)](T - t) \right)}{S(t)} \quad (\text{A.9})$$

$$= \frac{F(t, T) - S(t)}{S(t)} - \frac{S(t) [r(t, T) + c(t, T)](T - t)}{S(t)} \quad (\text{A.10})$$

$$= \frac{F(t, T) - S(t)}{S(t)} - (r(t, T) + c(t, T))(T - t) \quad (\text{A.11})$$

$$\frac{\psi(t, T)}{T - t} = \frac{\frac{F(t, T) - S(t)}{S(t)}}{T - t} - r(t, T) - c(t, T) \quad (\text{A.12})$$

$$= -y(t, T) \quad (\text{A.13})$$

From (A.13), it is clear that convenience yield is nothing more than an annualised version of the spread, but expressed with opposite sign.

Appendix B – Exponential Curve Fitting Procedure

To calibrate an exponential curve fitting the shape of the empirical Working curve (Figure 9) or excess volatility curve (Figure 10) we found that the following procedure gives a good visual fit:

- Define $j(t)$ as the log of inventory (measured in days), $j(t) = \ln(i_{days}(t))$
- Identify $n+1$ points k_m , $m \in \{0, \dots, n\}$ linearly dividing $j(t)$ into n equal 'bins', and convert back into units of inventory from log-inventory. This prioritizes the low- and moderate- inventory end of the relevant curve over the widely-spaced, high inventory values, helping to fit a close curve at the left-hand side. We chose $n = 40$:

$$k_m = e^{\min_{\tau=1}^t(j(\tau)) + \frac{m}{n} \left(\max_{\tau=1}^t(j(\tau)) - \min_{\tau=1}^t(j(\tau)) \right)} \quad (\text{B.1})$$

- For each interval k_m to k_{m+1} , identify the mean inventory figure in the 'bin':

$$\bar{i}_m = \text{mean}(i(t) | k_m \leq i(t) < k_{m+1}), m \in \{0, \dots, N-1\} \quad (\text{B.2})$$

- Finally, fit an exponential curve through the mean values in each bin \bar{i}_m , minimizing the RMS error between the fitted curve values and the mean values in each bin.

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