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An improved self-adaptive grey wolf optimizer for the daily optimal operation of cascade pumping stations

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HIGHLIGHTS

- The parameter A of GWO is dynamically adjusted to augment its exploration.
- The Inverse Parabolic Spread Distribution is used to further improve the accuracy.
- A strategy is proposed to dynamically adjust the feasible region of variables.
- The IAGWO is proposed to solve the operation problem of cascade pumping stations.

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ABSTRACT

Cascade pumping stations play a particularly important role in a water diversion project, and even a small increase in pumping efficiency would bring considerable economic and social benefits. In order to optimize the daily operation of cascade pumping stations to minimize the total daily cost and maximize the efficiency, an improved self-adaptive Grey Wolf optimizer (IAGWO) is proposed. The parameter A of IAGWO is dynamically adjusted to reduce the percentage of wolves moving out of the feasible area (AGWO), and the Inverse Parabolic Spread Distribution, which can maintain the diversity and bring wolves back into the feasible region, is used to further improve the accuracy. The proposed IAGWO and AGWO algorithms are tested using 23 benchmark functions, and the results show that the exploration of the proposed IAGWO and AGWO algorithms is augmented and their exploitation is competitive compared with other algorithms examined in this study. Moreover, a strategy is proposed to dynamically adjust the feasible region of variables in order to reduce unnecessary search for the optimization model of daily cost. The proposed IAGWO and AGWO algorithms are applied to a cascade pumping station system consisting of six pumping stations. Compared to the present scheme, the optimized schemes by IAGWO, AGWO, GWO and PSO can save 0.80268%, 0.80189%, 0.77369% and 0.45331% of daily operating expenses, respectively, which show that the proposed algorithms can obtain more efficient and economic solutions for the daily operation of cascade pumping stations.

1. Introduction

Cascade pumping stations play a particularly important role in a water diversion project, as they are responsible for pumping water to the reservoirs or demand nodes in order to obtain a better spatial distribution of water resources and further meet the demands of water supply, irrigation and so on. However, given that these pumping systems consume a large amount of electric energy, even a small increase in pumping efficiency would bring considerable economic benefits. Thus, there is a need to optimize the daily operation of cascade pumping stations in order to minimize the total daily cost and maximize the efficiency while satisfying various constraints, such as mass balance constraints, hydraulic balance constraints, and inequality constraints.

The daily optimal operation of cascade pumping stations is by its very nature a complex non-convex, nonlinear and high dimensional optimization problem. A large number of meta-heuristic optimization methods have been proposed in their basic structure to solve such difficult optimization problems, such as genetic algorithm (GA) [1–4], simulated annealing (SA) [5–8], particle swarm optimization (PSO) [9–11], ant colony optimization (ACO) [12,13], fuzzy optimization [14,15] and gravitational search algorithm (GSA) [16,17]. However, each algorithm has its own merits and demerits, and none of them can be claimed as absolutely better than others [18]. Thus, there is an incentive to use more advanced

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meta-heuristic algorithm for the daily optimal operation of cascade pumping stations.

Grey wolf optimizer (GWO) is a recently developed powerful evolutionary algorithm which mimics the social leadership and hunting behaviour of grey wolves in their natural habitat [19]. The reduced number of search parameters is an important advantage of GWO algorithm which is reflected in various science and engineering applications, such as optimal dispatch problem [20,21], feature subset selection [22], time series forecasting [23], strategic bidding problem in energy market [24], optimal power flow [25] and engineering design problems [26]. Undoubtedly, GWO has the potential to solve the problem of the daily optimal operation of cascade pumping stations. However, the objective function is not defined in the infeasible region for cascade pumping stations, because the solutions of the infeasible region imply that the water level is higher than the highest water level of the channel or lower than the lowest water level of the forebay, or the pump runs in the vibration zone and so on, which may cause serious accidents in the actual operation, such as channel overtopping, cavitation erosion, and pump vibration. Therefore, the solutions of the infeasible region will increase needless objective function evaluations, complicate the problem and affect the performance of the algorithm.

In general, the problem can be solved by (1) preventing the position to reach the infeasible searching space or (2) an explicit mechanism to repair the infeasible solution. Miettinen et al. [27] analysed five penalty function-based constraint handling techniques to be used with GAs in global optimization. Huang et al. [28] proposed a simple hybrid "damping" boundary condition that combined the characteristics of existing "absorbing" and "reflecting" boundaries, and thus much robust and consistent optimization performance could be obtained for PSO regardless of the dimensionality and location of the global optimum solution. Helwig et al. [29] showed that bound handling technique had a significant impact on the performance of the PSO algorithm, and thus proposed the Bounded Mirror method. Padhye et al. [30] proposed Inverse Parabolic Spread Distribution and Inverse Parabolic Confined Distribution for PSO, and found that the former was the most robust method and would never fail. Cheng et al. [31] observed the position diversity time-changing curves of PSOs with different topologies and boundary constraints handling techniques, and then analysed their impacts on the exploration and exploitation ability of the algorithm. Clerc [32] modified the PSO algorithm that could prevent particles to leave the search space. Meyer et al. [33] discussed how the ideas underlying stochastic ranking could be used in ACO and evaluated two different ways to achieve this integration. Although many attempts have been made to improve the performance of GWO algorithm, such as dynamically updating the parameters of GWO nonlinearly [18,24,34] or though reinforcement learning and neural networks [35], modifying the updating mechanisms with Lévy flight or astrophysics strategy [26,36], hybridizing GWO with other algorithms [37,38] and proposing the binary version [39], there are few studies on the bound handling techniques of GWO algorithm for daily optimal operation of cascade pumping stations.

In the study, an improved self-adaptive grey wolf optimizer (IAGWO) is proposed based on the two bound handling strategies. Specifically, based on the first strategy, a simple augmentation is proposed for the exploration of the GWO algorithm (AGWO). The exploration is augmented by decreasing the solutions in the infeasible space in order to increase the valid search and convergence precision. Then, the Inverse Parabolic Spread Distribution [30] is applied to GWO to obtain more robust performance. The proposed IAGWO and AGWO algorithms are tested with 23 benchmark test functions (CEC2005 [40]), and the results show that they hit the best minima more than PSO, GSA, Fast Evolutionary Programming (FEP) [19], Cuckoo Search Algorithm (CS), GWO and improved EGWO [18], AGWO [18] algorithms. The proposed IAGWO and AGWO algorithm are applied to a cascade pumping station consisting of 6 pumping stations, and the results show that they are more feasible and efficient than GWO and PSO algorithms.

The rest of this paper is organized as follows: Section 2 describes the daily optimal operation model of cascade pumping stations; in Section 3, the proposed IAGWO algorithm is presented and verified; in Section 4, the IAGWO algorithm is applied to the daily optimal operation of cascade pumping stations; and conclusions are drawn in Section 5.

2. The daily optimal operation model of cascade pumping stations

For large-scale long-term water diversion projects, only a short time is needed for the coordination of hydraulic elements such as the flow rate and water level of the cascade pumping stations during the start-up or adjustment phase, and the energy consumption is very small, which has a small impact on the economic benefit of the project. Thus, energy is consumed mainly in the steady state operation of cascade pumping stations, which can be affected primarily by the flow rate and head of each pumping station. In this study, the pumping stations are mainly considered and other buildings are generalized. The main objective of the daily optimal operation of cascade pumping stations is to minimize the total daily cost and to maximize their efficiency while satisfying various equality and inequality constraints. Such a problem can be solved by a decomposition–coordination model. The large system can be divided into three subsystems, each of which is first optimized, and then the global optimization of the large system can be achieved according to its overall objective and the relationship among the three subsystems. Fig. 1 shows the three-layer structure of the decomposition–coordination model of a large system. The first layer is the optimization model of single-stage pumping station, by which the optimal flow rate distribution of each single-stage pumping station can be obtained. The second layer is the optimization model of cascade pumping stations, by which the optimal head distribution of cascade pumping stations can be obtained, and the hydraulic losses of channels are calculated by the hydrodynamic model. The third layer is the optimization model of daily cost, by which the optimal daily cost can be obtained.

2.1. The optimization model of the single-stage pumping station

2.1.1. Objective function

The objective of the first layer model is to distribute the flow rate among pumps in a single-stage pumping station to maximize its total efficiency while satisfying various equality and inequality constraints. The objective function for n pump units is expressed as follows:

$$\eta_{pumpstation\_\max_j} = \frac{Q_{total_k} H_j}{\sum_{i=1}^{n} \frac{Q_i H_j}{\eta_i}} \tag{2.1}$$

where $\eta_{\text{pumpstation\_max}_j}$ is the total efficiency of the j^{th} pumping station under the condition of Q_{total_k} and H_j; Q_{total_k} is the total flow rate in the k^{th} period, which is fixed by the third layer model; H_j is the head of the j^{th} pumping station, which is fixed by the second layer model; n is the total number of pumps in the j^{th} pumping station; Q_i and η_i are the flow rate and efficiency of the i^{th} single pump in the j^{th} pumping station, respectively.

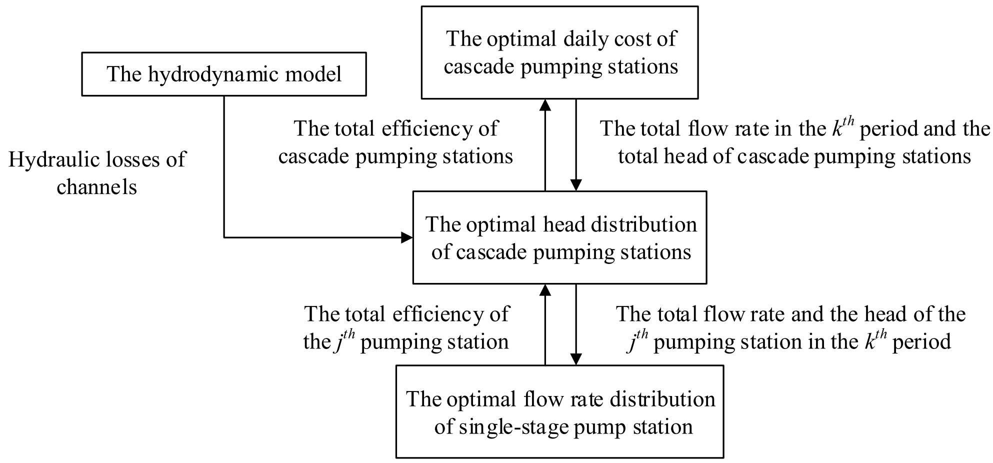

Fig. 1. The three-layer structure of the decomposition–coordination model of the large system.

2.1.2. Constraints

(1) The mass balance constraint

$$Q_{\text{total}_k} = \sum_{i=1}^{n} Q_i \tag{2.2}$$

(2) Flow rate limits of a single pump

$$Q_{i,\min} \le Q_i \le Q_{i,\max} \tag{2.3}$$

where $Q_{i,\min}$ and $Q_{i,\max}$ are the minimum and maximum flow rate of the i^{th} single pump, respectively.

2.2. The optimization model of cascade pumping stations

2.2.1. Objective function

The objective of the second layer model is to distribute the heads among cascade pumping stations to maximize the total efficiency while satisfying various equality and inequality constraints. The objective function for m pumping stations is expressed as follows:

$$\eta_{cascade\_pumpstation\_\max_k} = \frac{Q_{total_k} H_{total}}{\sum_{j=1}^{m} \frac{Q_{total_k} H_j}{\eta_{pumpstation\_\max_j}}} \tag{2.4}$$

where $\eta_{cascade\_pumpstation\_\max_k}$ is the total efficiency of cascade pumping stations in the k^{th} period under the condition of Q_{total_k} and H_{total}; H_{total} is the total head of cascade pumping stations; m is the total number of pumping stations; and $\eta_{\text{pumpstation\_}\max_j}$ is the maximum total efficiency of the j^{th} pumping station, respectively.

2.2.2. Constraints

(1) Real hydraulic balance constraint

$$H_{\text{total}} = Z_{out_m} - Z_{in_1} \tag{2.5}$$

$$\sum_{j=1}^{m} H_j = H_{\text{total}} + \sum_{j=1}^{m-1} h_{j,j+1} \tag{2.6}$$

where Z_{out_m} is the water level of the outlet pond of the m^{th} pumping station; Z_{in_1} is the water level of the forebay of the first pumping station; and $h_{j,j+1}$ is the hydraulic loss of the channel between j^{th} and $j+1^{\text{th}}$ pumping stations.

(2) Head limits of the single pumping station

$$H_{j_{\min}} \le H_j \le H_{j_{\max}} \tag{2.7}$$

where $H_{j_{\min}}$ and $H_{j_{\max}}$ are the minimum and maximum head of the j^{th} pumping station, respectively.

(3) Water level limits of the forebay of the pumping station

$$Z_{in_{j,\min}} \le Z_{in_j} \le Z_{in_{j,\max}} \tag{2.8}$$

where $Z_{in_{j,\min}}$ and $Z_{in_{j,\max}}$ are the minimum and maximum water level of the forebay of the j^{th} pumping station, respectively.

(4) Water level limits of the outlet pond of the pumping station

$$Z_{\text{out}_{j,\min}} \le Z_{\text{out}_j} \le Z_{\text{out}_{j,\max}} \tag{2.9}$$

where $Z_{\text{out}_{j,\min}}$ and $Z_{\text{out}_{j,\max}}$ are the minimum and maximum water level of the outlet pond of the j^{th} pumping station, respectively.

2.3. The optimization model of daily cost

2.3.1. Objective function

The objective of the third layer model is to distribute the flow rate among all periods in a day to minimize the total cost while satisfying various equality and inequality constraints. The objective function for T periods of one day is described as follows:

$$F_{\min} = \sum_{k=1}^{T} \frac{\gamma Q_{\text{total}_k} H_{total}}{\eta_{cascade\_pumpstation\_\max_k}} \Delta t_k c_k \tag{2.10}$$

where $F_{\min}$ is the minimum daily cost of cascade pumping stations; T is the total periods in a day; Δt_k and c_k are the duration and electrical price in the k^{th} period; and γ is the weight of water (9.8 kN/m^3), respectively.

2.3.2. Constraints

(1) The daily total diversion balance constraint

$$W = \sum_{k=1}^{T} Q_{\text{total}_k} \Delta t_k \tag{2.11}$$

where W is the total amount of water to be diverted daily.

2.4. The hydrodynamic model

A one-dimensional hydrodynamic model is constructed using *Saint-Venant* equations [41,42], which includes complex structures such as inverted syphons and channels, and the coupling between the complex internal structures and *Saint-Venant* equations is established.

$$\begin{cases} B\dfrac{\partial Z}{\partial t} + \dfrac{\partial Q}{\partial x} = q \\ \dfrac{\partial Q}{\partial t} + \dfrac{\partial}{\partial x}\left(\dfrac{\alpha Q^2}{A}\right) + gA\dfrac{\partial Z}{\partial x} + gAS_f = 0 \end{cases} \tag{2.12}$$

where B is the cross-sectional channel width; Z is the water level; t is the time; Q is the flow rate; x is the distance along the channel; q is the lateral inflow; α is the momentum correction coefficient; A is the wetted cross-sectional area; g is the gravitational acceleration; and S_f is the friction slope, which can be calculated from the following equation:

$$S_f = \frac{n^2 Q\,|Q|}{A^2 R^{4/3}} \tag{2.13}$$

where n is the Manning's roughness coefficient, and R is the hydraulic radius of the channel section.

In the hydrodynamic model, the Preissmann four-point implicit difference scheme is adopted to discretize Eqs. (2.12) and (2.13), and then the linearized equations between the i^{th} and the $(i+1)^{\text{th}}$ cross sections can be described as follows:

$$\begin{cases} Q_{i+1}^{n+1} - Q_i^{n+1} + C_i Z_{i+1}^{n+1} + C_i Z_i^{n+1} = D_i \\ E_i Q_i^{n+1} + G_i Q_{i+1}^{n+1} + F_i Z_{i+1}^{n+1} - F_i Z_i^{n+1} = \phi_i \end{cases} \tag{2.14}$$

where Q_i^{n+1} and Z_i^{n+1} are the flow rate and water level of the i^{th} cross section at the $(n+1)^{\text{th}}$ time step, respectively; and the coefficients of C_i, D_i, E_i, G_i, F_i and ϕ_i are calculated by the model parameters and the hydraulic elements at the n^{th} time step.

Eq. (2.14) is usually solved by catch-up method. In a steady state situation, the water level of the outlet pond of the i^{th} pumping station can be calculated according to the flow rate and the water level of the forebay of the $(i+1)^{\text{th}}$ pumping station, and then the head loss between the two pump stations can be calculated.

3. Grey wolf optimizer based on the two strategies of bound handling (IAGWO)

3.1. Overview of grey wolf optimizer (GWO)

GWO is a recently developed meta-heuristic search algorithm used to solve non-convex engineering optimization problems [19]. It computationally simulates the hunting techniques and social hierarchy of grey wolves living in a pack with a strict hierarchy: on the top are the alpha wolves responsible for decision making, followed by beta and delta wolves. The rest of the pack is called omegas. The main hunting steps include encircling the prey, hunting, attacking the prey, and searching for the prey.

3.1.1. Encircling the prey

$$\vec{D} = \left| \vec{C} .* \vec{X}_p(t) - \vec{X}(t) \right| \tag{3.1}$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} .* \vec{D} \tag{3.2}$$

where $\vec{D}$ represents the distance between the position of the prey ($\vec{X}_p$) and a grey wolf ($\vec{X}$); t is the current iteration; and $\vec{A}$ and $\vec{C}$ are the coefficient vectors, which can be calculated as follows:

$$\vec{A} = 2a\vec{r}_1 - a\vec{E} \tag{3.3}$$

$$\vec{C} = 2\vec{r}_2 \tag{3.4}$$

$$a = 2 - \frac{t}{T_{\max}} \tag{3.5}$$

where the components of a are linearly decreased from 2 to 0 over the course of iterations; $T_{\max}$ is the maximum iterations; $\vec{r}_1$ and $\vec{r}_2$ are random vectors in [0,1]; and $\vec{E}$ is a vector whose elements are equal to 1. A grey wolf can update its position inside the space around the prey in any random location by using these equations.

3.1.2. Hunting

It is assumed that the alpha (best candidate solution), beta and delta wolves have better knowledge about the potential location of the prey. Therefore, the first three best solutions are obtained and oblige the other search agents (including omegas) to update their positions according to the position of the best search agent. The formulas are as follows.

$$\begin{aligned} &\vec{D}_\alpha = \left| \vec{C}_1 .* \vec{X}_\alpha - \vec{X} \right|,\ \vec{D}_\beta = \left| \vec{C}_2 .* \vec{X}_\beta - \vec{X} \right|, \\ &\vec{D}_\delta = \left| \vec{C}_3 .* \vec{X}_\delta - \vec{X} \right| \end{aligned} \tag{3.6}$$

$$\begin{aligned} &\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 .* \vec{D}_\alpha,\ \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 .* \vec{D}_\beta, \\ &\vec{X}_3 = \vec{X}_\delta - \vec{A}_3 .* \vec{D}_\delta \end{aligned} \tag{3.7}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{3.8}$$

If one dimension of the new position $\vec{X}(t+1)$ is out of the feasible search space, that search information will be abandoned. Instead, a new position will be reset as its boundary in that dimension. The equation of this strategy is as follows:

$$\begin{cases} X_{i,j}^{(t+1)} = X_{i,\max},\ \text{if}\, X_{i,j}^{(t+1)} > X_{i,\max} \\ X_{i,j}^{(t+1)} = X_{i,\min},\ \text{if}\, X_{i,j}^{(t+1)} < X_{i,\min} \\ X_{i,j}^{(t+1)} = X_{i,j}^{(t+1)},\ otherwise \end{cases} \tag{3.9}$$

where t is the number of the last iteration, and $t+1$ is the number of current iteration; $X_{i,\max}$ and $X_{i,\min}$ are the maximum and minimum boundaries of the j^{th} dimension in i^{th} agent, respectively.

3.1.3. Attacking the prey

The hunting process is terminated by attacking the prey when it stops moving. This can be done mathematically by decreasing the value of the vector a from 2 to 0 with iterations. It is found that $\left|\vec{A}\right| < 1$ can force the wolves to attack the prey. This attacking behaviour represents the exploitation or local search of the GWO algorithm [19]. However, GWO is prone to stagnation in local solutions, and it needs more operators to emphasize exploration.

3.1.4. Searching for the prey

Based on the position of α, β, and δ wolves, the grey wolves search for the prey. They diverge from each other to search for the prey and converge to attack the prey. It is observed that $\left|\vec{A}\right| > 1$ can force the wolves to search for a better prey. This divergence behaviour refers to the exploration or the global search of the GWO algorithm.

The component $\vec{C}$ provides random weights for the prey, which assists GWO to show a more random behaviour throughout optimization, and thus favours exploration and local optima avoidance.

GWO is described in Table 1.

3.2. Grey wolf optimizer based on the two bound handling strategies (IAGWO)

In this study, a new adaptive grey wolf optimizer called IACWO is proposed based on the two bound handling strategies. In IAGWO, two modifications have been made to improve the searching ability of GWO. The first strategy is used to change the updating mechanism, and the second one is used to avoid wolf sticking in the boundary that may lead to the inability to find the global optimum at the end of its search process.

Table 1
The description of GWO.

Input: dim(D), upper bounder(ub), lower bounder(lb), search agents(SA), maximum iterations ($T_{\max}$).
Result: The optimal wolf position, the optimal fitness, and convergence_curve.
(1) Initialize a population of $D * SA$ grey wolves positions randomly.
(2) $t = 0$.
(3) Initialize convergence curve.
While $t \leq T_{\max}$
For each search agent
For each dim
Keep the search contained within the feasible area by Eq. (3.9).
Calculate the fitness of each search agent.
Find α, β, and δ as the first three best solutions based on their fitness values.
End
End
Update the coefficients in Eq. (3.5).
For each search agent
For each dim
Update A, and C as in Eqs. (3.3) and (3.4).
Update current positions of wolves according to Eqs. (3.6)–(3.8).
End
End
$t = t + 1$.
Update convergence curve.
End
(4) Return the α search agent positions and its fitness.

3.2.1. The adaptive updating mechanism based on the first strategy of bound handling (AGWO)

The coefficient $\overrightarrow{A}$ is very important for the updating mechanism, the exploration and exploitation, and the searching performance. However, as a is large at the beginning of the iteration, many wolves moved out of the feasible search space and were reset to the boundary value, which may result in the sticking in the boundary. Very often, it is dangerous for the operation of cascade pumping stations at the boundary. Inspired by the strategy that can prevent the position to reach the infeasible searching space, the coefficient $\overrightarrow{A}$ is modified to change the updating mechanism. The modified coefficient $\overrightarrow{A}$ can prevent wolves from leaving the feasible area, thus resulting in an increase in the valid search and the convergence precision of the optimal solutions.

The modified coefficient $\overrightarrow{A}$ is described by

$$R_{\text{up}} = \begin{cases} 1, & \dfrac{X_p - lb}{D} \geq 1 \\ \dfrac{X_p - lb}{D}, & \dfrac{X_p - lb}{D} < 1 \end{cases} \tag{3.10}$$

$$R_{\text{low}} = \begin{cases} -1, & \dfrac{X_p - ub}{D} \leq -1 \\ \dfrac{X_p - ub}{D}, & \dfrac{X_p - ub}{D} > -1 \end{cases} \tag{3.11}$$

$$A = a * [r_1 * (R_{\text{up}} - R_{\text{low}}) + R_{\text{low}}] \tag{3.12}$$

where D represents the distance between the position of the prey (X_p) and a grey wolf (X); lb is the lower bound; and ub is the upper bound.

Fig. 2 shows how a search agent updates its position, and the new positions are almost inside the feasible region in a 2D search space. Considering random weights ($\overrightarrow{C}$), the position of the prey ($\overrightarrow{C} .* \overrightarrow{X}_p$) is a random position in the rectangular *qpou*. According to $\overrightarrow{X}_p$, $\overrightarrow{D}$ and $\overrightarrow{a}$, the new position of a grey wolf $\overrightarrow{X}(t+1)$ is one of the positions in the rectangular *abcd* by the GWO, but it is one of the positions in the rectangular *ebgf* by the AGWO.

The transition between exploration and exploitation is generated by the value of A. As the value of A decreases in GWO, a half of the iterations are devoted to exploration ($|A| \geq 1$) and the other half are devoted to exploitation ($|A| < 1$). The modified coefficient A also can adaptively balance the exploration and exploitation and increase the valid searching. As A decreases, $\frac{R_{up}-R_{low}-1}{R_{up}-R_{low}}$ and $\frac{1}{R_{up}-R_{low}}$ of iterations are devoted to exploration ($|A| \geq 1$) and exploitation ($|A| < 1$), respectively.

3.2.2. Application of the inverse parabolic spread distribution

The Inverse Parabolic Spread Distribution [30], which is one of the most robust bound handling approaches in PSO, is applied to handle inequality constraints in GWO in this study. In general, more than one bounds may be violated. Fig. 3 shows an example where two bounds are violated.

In such a case, there are two intersections between the bounds and the line joining X_{orig} (the original wolf position) and X_c (wolf position after update). The intersection point closest to X_{orig} is selected to be X_1.

$$|X_1| = \max[d_1, d_2, \ldots, d_i, \ldots, d_n] \tag{3.13}$$

where $|X_1|$ is the distance between X_1 and X_c; n is the number of bounds violated; and d_i is the distance between X_c and one of the intersections.

There is the same number of intersections between the bounds on the opposite side and the line joining X_{orig} and X_c. Similarly, the intersection point closest to X_{orig} is selected to be X_2.

$$|X_2| = \min[l_1, l_2, \ldots, l_i, \ldots, l_n] \tag{3.14}$$

where $|X_2|$ is the distance between X_2 and X_c; and l_i is the distance between X_c and one of the intersections.

To re-update the new vector position X' in the region between X_1 and X_2, the Inverse Parabolic Spread Distribution is used to maintain diversity and bring the wolves back into the feasible region:

$$\text{let,} \int_{|X_1|}^{|X_2|} \frac{a}{(x - |X_1|)^2 + \alpha^2 |X_1|^2} dx = p_1 \tag{3.15}$$

where α is a user defined parameter. According to Fig. 3, the proposed probability density function has a peak at location X_1. The peak is heightened if α is lowered ($\alpha = 1.2$ in this study). The calculation for the distribution constant a is done by equating the cumulative probability equal to one. The limits are chosen from X_c till infinity.

$$a = \frac{\alpha^2 |X_1|^2}{\frac{\pi}{2} + \tan^{-1} \frac{1}{\alpha}} \tag{3.16}$$

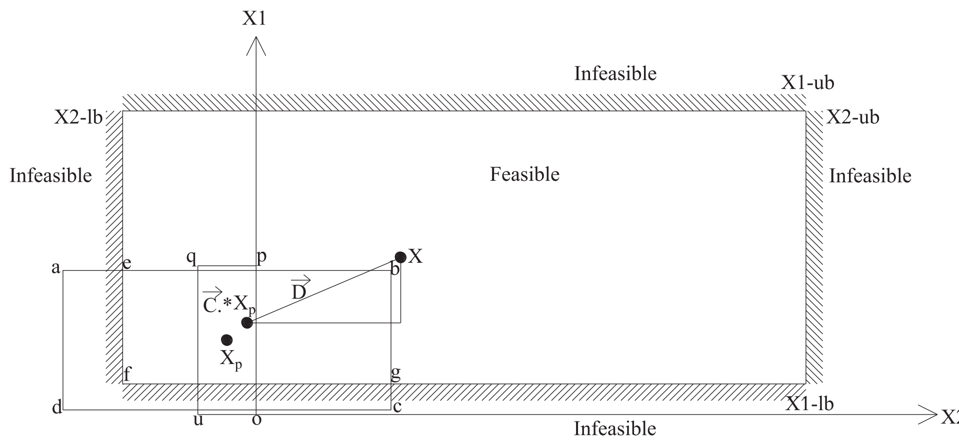

Fig. 2. A search agent updates its position in a 2D search space.

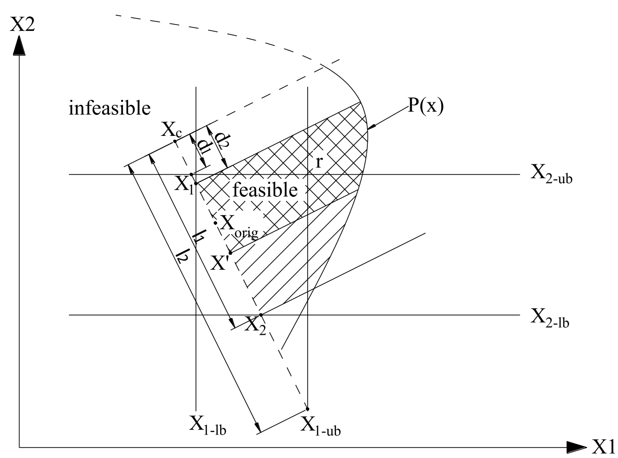

Fig. 3. Inverse Parabolic Distribution for handling bounds.

Then, the probability distribution function is reconstructed as:

$$P_1(x) = \frac{a}{p_1((x - |X_1|)^2 + \alpha^2 |X_1|^2)} \quad s.t. \, |X_1| \le x \le |X_2| \tag{3.17}$$

Let X' denote the sampled location and r be a uniformly distributed random number in [0, 1], then the distance between X_c and the new vector position X' ($|X'|$) can be found as follows:

$$r = \int_{|X_1|}^{|X'|} \frac{a}{p_1((x - |X_1|)^2 + \alpha^2 |X_1|^2)} dx \tag{3.18}$$

$$|X'| = |X_1| + \alpha |X_1| \tan(r \tan^{-1}(\frac{|X_2| - |X_1|}{\alpha |X_1|})) \tag{3.19}$$

Once $|X'|$ is calculated, the new vector position X' can be easily obtained.

3.3. Verification of the proposed IAGWO

3.3.1. Benchmark functions

In order to verify the performance of the proposed IAGWO, we applied the IAGWO to minimization functions in Table 2 and compared its results with that of PSO, GSA, FEP [19], CS and an improved EGWO, AGWO [18]. Benchmark functions F1 to F7 are unimodal functions; F8 to F13 are multi-modal functions; and F14 to F26 are fixed-dimensional multi-modal functions. In all cases, the number of search agents is 30, the maximum iteration is 500, and the run time is 30 on each benchmark function.

3.3.2. Verification of the IAGWO

In order to consider the effect of the first strategy, the study also shows the results of the improved GWO (AGWO). Tables 3–5 show that of the 23 benchmark functions, the proposed IAGWO algorithm, the proposed AGWO algorithm, AGWO [18], FEP [19], GSA [19], PSO [19], GWO [19], and EGWO [18] and CS algorithms [18] hit the best minimum solution of 7, 8, 7, 6, 5, 4, 3, 2 and 2 benchmark functions, respectively, indicating that the proposed IAGWO and AGWO algorithms are better than other algorithms on these multimodal benchmark functions, and competitive with other algorithms on unimodal benchmark. What is more, the proposed IAGWO algorithm hits the best minimum of 16 benchmark functions out of 23 compare to the proposed AGWO algorithm, indicating that the second bound handling strategy also improve the searching ability.

A two-sample t-test [43,44] is used to compare the proposed IAGWO and AGWO algorithms versus other algorithms examined (PSO, GSA, FEP, CS and an improved EGWO, AGWO [18]). A P value of less than 0.05 is considered statistically significant. In Tables 3–5, "1" indicates that IAGWO or AGWO is significantly better; "0" indicates no difference; while "-1" indicates that IAGWO or AGWO is significantly poorer. Tables 3–5 show that for the 161 functions examined in this study, the proposed AGWO algorithm shows better performance in 84 functions, similar performance in 44 functions, and poorer performance in 33 functions in comparison with other algorithms. Besides, Tables 3–5 also show that for the 161 functions examined in this study, the proposed IAGWO algorithm shows better performance in 85 functions, similar performance in 45 functions, poorer performance in 31 functions in comparison with

Table 2
Text functions.

Function	Dim	Range	$f_{\min}$
$f_1(x)=\sum_{i=1}^{n} x_i^2$	30	[−100,100]	0
$f_2(x)=\sum_{i=1}^{n}\lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	[−10,10]	0
$f_3(x)=\sum_{i=1}^{n}(\sum_{j-1}^{i} x_j)^2$	30	[−100,100]	0
$f_4(x)=\max_i\{\lvert x_i\rvert, 1\le i\le n\}$	30	[−100,100]	0
$f_5(x)=\sum_{i=1}^{n-1}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	30	[−30,30]	0
$f_6(x)=\sum_{i=1}^{n}([x_i+0.5])^2$	30	[−100,100]	0
$f_7(x)=\sum_{i=1}^{n} ix_i^4+random[0,1)$	30	[−1.28,1.28]	0
$f_8(x)=\sum_{i=1}^{n}-x_i\sin\left(\sqrt{\lvert x_i\rvert}\right)$	30	[−500,500]	−418.9829 × 5
$f_9(x)=\sum_{i=1}^{n}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	30	[−5.12,5.12]	0
$f_{10}(x)=-20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right)-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right)+20+e$	30	[−32,32]	0
$f_{11}(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	30	[−600,600]	0
$f_{12}(x)=\frac{\pi}{n}\left\{10\sin(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2\left[1+10\sin^2(\pi y_{i+1})\right]+(y_n-1)^2\right\}+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4}\quad u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m & x_i>a\\ 0 & -a<x_i<a\\ k(-x_i-a)^m & x_i<-a\end{cases}$	30	[−50,50]	0
$f_{13}(x)=0.1\left\{\sin^2(3\pi x_1)+\sum_{i=1}^{n}(x_i-1)^2\left[1+\sin^2(3\pi x_1+1)\right]+(x_n-1)^2\left[1+\sin^2(2\pi x_n)\right]\right\}+\sum_{i=1}^{n}u(x_i,5,100,4)$	30	[−50,50]	0
$f_{14}=(\frac{1}{500}+\sum_{i=1}^{25}\frac{1}{i+\sum_{j=1}^{2}(x_i-a_{ij})^6})^{-1}$	2	[−65,65]	1
$f_{15}=\sum_{i=1}^{11}[a_i-\frac{x_i(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}]^2$	4	[−5,5]	0.00030
$f_{16}=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_4-4x_2^2+4x_2^4$	2	[−5,5]	−1.0316
$f_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	2	[−5,5]	0.398
$f_{18}(x)=\left[1+(x_1+x_2+1)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]\times\left[30+(2x_1-3x_2)^2\times(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\right]$	2	[−2,2]	3
$f_{19}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}\left(x_j-p_{ij}\right)^2\right)$	3	[1,3]	−3.86
$f_{20}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}\left(x_j-p_{ij}\right)^2\right)$	6	[0,1]	−3.32
$f_{21}(x)=-\sum_{i=1}^{5}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.1532
$f_{22}(x)=-\sum_{i=1}^{7}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.4028
$f_{23}(x)=-\sum_{i=1}^{10}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.5363

other algorithms. Their convergence is also compared with that of GWO and PSO algorithms on 2 unimodal benchmark functions, 2 multimodal benchmark functions, and 2 fixed-dimensional multimodal benchmark functions, as shown in Fig. 4. All these results indicate the superior performance of the proposed IAGWO and AGWO algorithms. This is because AGWO can extensively explore the search area, and IAGWO can give promising search area rather than sticking in the boundary.

In order to further verify whether AGWO can extensively explore the search area, the F17 function with a dimension of 2 is used to illustrate the distribution of positions of wolfs at 9 stages, where the number of search agents is 40, and the maximum iteration is 100, and the results are shown in Figs. 5 and 6, respectively. The comparison between Figs. 5 and 6 shows that the percentage of wolves moving out of the feasible area using AGWO is much lower than that using GWO, and a part of wolves who move out of the feasible area in Fig. 5 are used to explore in Fig. 6. The percentages of wolves moving out of the feasible area on each benchmark function are shown in Tables 6–8. Clearly, the percentage of wolves moving out of the feasible area is decreased by 19.6078–99.4077%, and the percentage is high on most benchmark functions (64.9153–99.4077%) except that on the F16 function (19.6078%).

4. The IAGWO algorithm for the daily optimal operation of cascade pumping stations

In this section, the procedure of AGWO and IAGWO for solving the daily optimal operation problem of cascade pumping stations is described in details, and the case results are analysed.

4.1. The procedure of IAGWO for the daily optimal operation of cascade pumping stations

Due to the small number of variables in the third layer and their high repetition, the third layer is optimized using dynamic programming (DP); while the other two layers are optimized using the proposed IAGWO and AGWO. The main procedure of the proposed IAGWO and AGWO include initialization, constraint handling and updating.

4.1.1. Structure of individuals in IAGWO and AGWO

For the first layer model, the control variables are the flow rates of each pump in a single-stage pumping station. The array of the control variable vector is described as follows:

$$Q=[Q_1,Q_2,\ldots,Q_i,\ldots,Q_n] \tag{4.1}$$

Table 3
The results of unimodal benchmark functions.

F	Index			IAGWO	AGWO		AGWO[18]		EGWO		GWO		PSO		GSA		FEP		CS	
F1	Mean			3.506E−28	2.554E−28		**6.690E−44**		1.300E−30		1.270E−27		1.360E−04		2.530E−16		5.700E−04		5.780E−03	
	Std			3.850E−28	2.476E−28		**1.330E−43**		3.920E−30		3.110E−27		2.020E−04		9.670E−17		1.300E−04		2.410E−03	
	AGWO vs others	t-stat	t-test				5.55435	**−1**	5.52539	**−1**	−1.75125	**0**	−3.62566	**1**	−14.08942	**1**	−23.61188	**1**	−12.91546	**1**
	IAGWO vs others	t-stat	t-test		1.11941	**0**	4.90416	**−1**	4.88572	**−1**	−1.57998	**0**	−3.62566	**1**	−14.08942	**1**	−23.61188	**1**	−12.91546	**1**
F2	Mean			**5.318E−17**	7.328E−17		**8.250E−27**		9.270E−20		8.520E−17		0.0421		0.0557		8.100E−03		0.2080	
	Std			**4.840E−17**	4.773E−17		**9.410E−27**		1.260E−19		6.620E−17		0.0454		0.1941		7.700E−04		0.0317	
	AGWO vs others	t-stat	t-test				8.26770	**−1**	8.25721	**−1**	−0.78673	**0**	−4.99664	**1**	−1.54431	**0**	−56.64914	**1**	−35.33484	**1**
	IAGWO vs others	t-stat	t-test		−1.59230	**0**	5.91725	**−1**	5.90692	**−1**	−2.10281	**1**	−4.99664	**1**	−1.54431	**0**	−56.64914	**1**	−35.33484	**1**
F3	Mean			4.040E−06	2.319E−06		**2.980E−08**		4.680E−04		2.430E−05		70.1256		896.5347		0.0160		0.2630	
	Std			1.626E−05	3.609E−06		**1.080E−07**		2.200E−03		8.140E−05		22.1192		318.9559		0.0140		0.0297	
	AGWO vs others	t-stat	t-test				3.41455	**−1**	−1.13989	**0**	−1.45275	**0**	−17.07283	**1**	−15.13685	**1**	−6.15358	**1**	−47.68639	**1**
	IAGWO vs others	t-stat	t-test		0.55643	**0**	1.32840	**0**	−1.13565	**0**	−1.31440	**0**	−17.07283	**1**	−15.13685	**1**	−6.15292	**1**	−47.68607	**1**
F4	Mean			**4.427E−07**	4.954E−07		**1.250E−10**		0.1427		7.690E−07		1.0865		7.3549		0.3000		1.430E−05	
	Std			**4.351E−07**	4.152E−07		**5.700E−10**		0.6166		6.510E−07		0.3170		1.7415		0.5000		4.830E−06	
	AGWO vs others	t-stat	t-test				6.42349	**−1**	−1.24602	**0**	−1.90789	**0**	−18.45475	**1**	−22.74377	**1**	−3.23109	**1**	−15.33471	**1**
	IAGWO vs others	t-stat	t-test		−0.47193	**0**	5.47853	**−1**	−1.24603	**0**	−2.24398	**1**	−18.45475	**1**	−22.74377	**1**	−3.23109	**1**	−15.38774	**1**
F5	Mean			27.1257	26.6467		26.9654		27.7802		27.1786		96.7183		67.5431		5.0600		**8.000E−03**	
	Std			0.5733	0.4662		0.6973		0.9073		0.814		60.1156		62.2253		5.8700		**5.400E−02**	
	AGWO vs others	t-stat	t-test				−2.04640	**1**	−5.98426	**1**	−3.05380	**1**	−6.27684	**1**	−3.53920	**1**	19.74153	**−1**	305.66882	**−1**
	IAGWO vs others	t-stat	t-test		3.49141	**−1**	0.95648	**0**	−3.28383	**1**	−0.28593	**0**	−6.23383	**1**	−3.49769	**1**	20.14734	**−1**	253.59497	**−1**
F6	Mean			**0.5322**	0.5517		1.4381		3.1384		0.7076		1.020E−04		2.500E−16		**0**		6.170E−04	
	Std			**0.2925**	0.2732		0.3314		0.6103		0.3632		8.280E−05		1.740E−16		**0**		2.800E−05	
	AGWO vs others	t-stat	t-test				−11.11429	**1**	−20.83261	**1**	−1.84712	**0**	10.87246	**−1**	10.87447	**−1**	10.87447	**−1**	10.86231	**−1**
	IAGWO vs others	t-stat	t-test		−0.26259	**0**	−11.03638	**1**	−20.73756	**1**	−2.02543	**0**	9.79431	**−1**	9.79618	**−1**	9.79618	**−1**	9.78483	**−1**
F7	Mean			1.441E−03	**1.333E−03**		**1.420E−03**		5.730E−03		1.720E−03		0.1229		0.0894		0.1415		0.0286	
	Std			1.033E−03	**6.094E−04**		**8.210E−04**		3.100E−03		1.100E−03		0.0450		0.0434		0.3522		1.277E−03	
	AGWO vs others	t-stat	t-test				−0.45981	**0**	−7.49534	**1**	−1.65858	**0**	−14.55507	**1**	−10.93411	**1**	−2.14317	**1**	−103.58754	**1**
	IAGWO vs others	t-stat	t-test		0.48698	**0**	0.08634	**0**	−7.06828	**1**	−0.99512	**0**	−14.53957	**1**	−10.91863	**1**	−2.14150	**1**	−88.88074	**1**
BEST f_{min}	IAGWO vs others(AGWO vs others)			**0/7**	**1/7**		**5/7 (4/7)**		**0/7 (0/7)**		**0/7 (0/7)**		**0/7 (0/7)**		**0/7 (0/7)**		**1/7 (1/7)**		**1/7 (1/7)**	
sig better	IAGWO vs others(AGWO vs others)				0/7		1/7 (1/7)		3/7 (3/7)		1/7 (2/7)		6/7 (6/7)		5/7 (5/7)		5/7 (5/7)		5/7 (5/7)	
sig equiv	IAGWO vs others(AGWO vs others)				6/7		3/7 (2/7)		2/7 (2/7)		6/7 (5/7)		0/7 (0/7)		1/7 (1/7)		0/7 (0/7)		0/7 (0/7)	
sig worse	IAGWO vs others(AGWO vs others)				1/7		3/7 (4/7)		2/7 (2/7)		0/7 (0/7)		1/7 (1/7)		1/7 (1/7)		2/7 (2/7)		2/7 (2/7)	

Table 4
The results of multimodal benchmark functions.

F	Index			IAGWO	AGWO		AGWO[18]		EGWO		GWO		PSO		GSA		FEP		CS	
F8	Mean			**−3967.9635**	−3752.7392		−3633.3600		−6511.1900		−6101.5865		−4841.2900		−2821.0700		−**12554.5000**		−2128.9130	
	Std			**848.9211**	981.7270		442.5815		786.8525		782.7968		1152.8140		493.0375		**52.6000**		0.0084	
	AGWO vs others	t-stat	t-test				−0.59698	**0**	11.80685	**−1**	10.07393	**−1**	3.87140	**−1**	−4.56699	**1**	48.21202	**−1**	−8.90734	**1**
	IAGWO vs others	t-stat	t-test		−0.89302	**0**	−1.88214	**0**	11.83214	**−1**	9.95017	**−1**	3.28501	**−1**	−6.29128	**1**	54.36478	**−1**	−11.66609	**1**
F9	Mean			**0.5862**	1.4438		0.9140		145.7781		2.4436		46.7042		25.9684		**0.0460**		0.2460	
	Std			**0.9868**	1.9620		5.0060		39.4368		2.8657		11.6294		7.4701		**0.0120**		1.800E−03	
	AGWO vs others	t-stat	t-test				0.53067	**0**	−19.68476	**1**	−1.55034	**0**	−20.66649	**1**	−17.09981	**1**	3.83660	**−1**	3.28772	**−1**
	IAGWO vs others	t-stat	t-test		−2.10282	**1**	−0.34590	**0**	−19.82000	**1**	−3.30022	**1**	−21.27917	**1**	−18.14037	**1**	2.94787	**−1**	1.85668	**0**
F10	Mean			**9.219E−14**	9.788E−14		**1.930E−15**		3.400E−13		1.060E−13		0.2760		0.0621		0.0180		4.010E−10	
	Std			**1.482E−14**	1.979E−14		**2.550E−15**		1.760E−12		0.0778		0.5090		0.2363		0.0021		5.210E−09	
	AGWO vs others	t-stat	t-test				25.88826	**−1**	−0.74079	**0**	0.00000	**0**	−2.92015	**1**	−1.41523	**0**	−46.15856	**1**	−0.41438	**0**
	IAGWO vs others	t-stat	t-test		−1.23806	**0**	32.33426	**−1**	−0.75820	**0**	0.00000	**0**	−2.92015	**1**	−1.41523	**0**	−46.15856	**1**	−0.41439	**0**
F11	Mean			**3.420E−03**	3.806E−03		**1.180E−03**		0.0104		4.485E−03		9.215E−03		27.7015		0.0160		0.1852	
	Std			**7.460E−03**	0.0107		**4.500E−03**		0.0201		6.659E−03		7.724E−03		5.0403		0.0220		0.0390	
	AGWO vs others	t-stat	t-test				1.21547	**0**	−1.55840	**0**	−0.28941	**0**	−2.20300	**1**	−29.59253	**1**	−2.68268	**1**	−24.14962	**1**
	IAGWO vs others	t-stat	t-test		−0.15918	**0**	1.38462	**0**	−1.75322	**0**	−0.57355	**0**	−2.90618	**1**	−29.59298	**1**	−2.91625	**1**	−24.65345	**1**
F12	Mean			0.0531	0.0386		0.1024		3.1690		0.0534		6.917E−03		1.7996		**9.200E−06**		1.258E−02	
	Std			0.0261	9.444E−03		0.0312		2.8708		0.0207		0.0263		0.9511		**3.600E−06**		4.120E−09	
	AGWO vs others	t-stat	t-test				−10.54447	**1**	−5.87215	**1**	−3.51392	**1**	6.10000	**−1**	−9.97020	**1**	21.99001	**−1**	14.82155	**−1**
	IAGWO v others	t-stat	t-test		2.81372	**−1**	−6.53763	**1**	−5.84478	**1**	−0.06244	**0**	6.71071	**−1**	−9.88499	**1**	10.96289	**−1**	8.36475	**−1**
F13	Mean			**0.5284**	0.6018		1.1287		2.4960		6.545E−01		6.675E−03		8.8991		**1.600E−04**		4.850E−01	
	Std			**0.1953**	0.2263		0.2193		0.4061		4.474E−02		8.907E−03		7.1262		**7.300E−05**		6.850E−08	
	AGWO vs others	t-stat	t-test				−9.00406	**1**	−21.94113	**1**	−1.23029	**0**	14.14711	**−1**	−6.26697	**1**	14.31307	**−1**	2.77779	**−1**
	IAGWO vs others	t-stat	t-test		−1.32163	**0**	−11.00873	**1**	−23.51405	**1**	−3.38872	**1**	14.37045	**−1**	−6.32321	**1**	14.56502	**−1**	1.19624	**0**
BEST $f_{\min}$	IAGWO vs others(AGWO vs others)			**0/6**	**0/6**		**2/6 (2/6)**		**0/6 (0/6)**		**0/6 (0/6)**		**0/6 (0/6)**		**0/6 (0/6)**		**4/6 (4/6)**		**0/6 (0/6)**	
sig better	IAGWO vs others(AGWO vs others)				1/6		2/6 (2/6)		3/6 (3/6)		2/6 (1/6)		3/6 (3/6)		5/6 (5/6)		2/6 (2/6)		2/6 (2/6)	
sig equiv	IAGWO vs others(AGWO vs others)				4/6		3/6 (3/6)		2/6 (2/6)		3/6 (4/6)		0/6 (0/6)		1/6 (1/6)		0/6 (0/6)		3/6 (1/6)	
sig worse	IAGWO vs others(AGWO vs others)				1/6		1/6 (1/6)		1/6 (1/6)		1/6 (1/6)		3/6 (3/6)		0/6 (0/6)		4/6 (4/6)		1/6 (3/6)	

Table 5
The results of fixed-dimension multimodal benchmark functions.

F	Index			IAGWO	AGWO		AGWO[18]		EGWO		GWO		PSO		GSA		FEP		CS	
F14	Mean			3.2567	5.4662		2.3104		6.6337		4.4237		3.6272		5.8598		**1.2200**		1.4236	
	Std			3.3099	4.4644		2.4804		4.6570		4.2869		2.5608		3.8313		**0.5600**		0.0130	
	AGWO vs others	t-stat	t-test				3.32760	**−1**	−0.97454	**0**	0.90707	**0**	1.92426	**0**	−0.36030	**0**	5.08215	**−1**	4.87637	**−1**
	IAGWO vs others	t-stat	t-test		−2.14099	**1**	1.23207	**0**	−3.18298	**1**	−1.16037	**0**	−0.47672	**0**	−2.76876	**1**	3.26728	**−1**	2.98244	**−1**
F15	Mean			**3.178E−04**	**3.312E−04**		1.840E−03		1.030E−02		3.370E−04		5.770E−04		3.673E−03		5.000E−04		5.030E−04	
	Std			**1.325E−05**	**6.292E−05**		5.000E−03		1.320E−02		6.250E−04		2.220E−04		1.647E−03		3.200E−04		1.110E−04	
	AGWO vs others	t-stat	t-test				−1.62493	**0**	−4.06691	**1**	−0.04996	**0**	−5.73719	**1**	−10.91875	**1**	−2.78776	**1**	−7.25215	**1**
	IAGWO vs others	t-stat	t-test		−1.12037	**0**	−1.63946	**0**	−4.07241	**1**	−0.16544	**0**	−6.27650	**1**	−10.97010	**1**	−3.06364	**1**	−8.92189	**1**
F16	Mean			**−1.03163**	**−1.03163**		−1.03160		−1.03160		**−1.03163**		**−1.03163**		**−1.03163**		−1.03000		−1.03160	
	Std			**1.3716E−08**	**1.6224E−08**		4.1700E−06		5.8000E−03		**1.7644E−08**		**6.2500E−16**		**4.8800E−16**		4.9000E−07		1.4900E−08	
	AGWO vs others	t-stat	t-test				−36.71846	**1**	−0.02640	**0**	1.35339	**0**	520.06586	**−1**	520.06586	**−1**	−17886.893	**1**	−6950.940	**1**
	IAGWO vs others	t-stat	t-test		0.27063	**0**	−36.71716	**1**	−0.02640	**0**	1.70885	**0**	615.58782	**−1**	615.58782	**−1**	−17889.676	**1**	−7560.291	**1**
F17	Mean			**0.3979**	**0.3979**		0.3980		**0.3979**		**0.3979**		**0.3979**		**0.3979**		0.3980		**0.3979**	
	Std			**1.0000E−06**	**7.5519E−07**		1.3900E−05		**2.4300E−07**		**1.9237E−06**		**0.0000**		**0.0000**		1.5000E−07		**3.2400E−06**	
	AGWO vs others	t-stat	t-test				−43.31744	**1**	−13.40222	**1**	−2.01437	**0**	7.31378	**−1**	7.31378	**−1**	−783.17959	**1**	−19.38289	**1**
	IAGWO vs others	t-stat	t-test		−0.11020	**0**	−43.27940	**1**	−10.46576	**1**	−1.98379	**0**	5.38516	**−1**	5.38516	**−1**	−596.46555	**1**	−19.05797	**1**
F18	Mean			**3.0000**	**3.0000**		**3.0000**		12.9000		**3.0000**		**3.0000**		**3.0000**		3.0200		3.0014	
	Std			**1.7310E−05**	**4.9273E−05**		**3.0700E−05**		2.5052E+01		**4.1955E−05**		**1.3300E−15**		**4.1700E−15**		1.1000E−01		2.5800E−03	
	AGWO VS others	t-stat	t-test				3.50689	**−1**	−2.12807	**1**	0.75516	**0**	4.13188	**−1**	4.13188	**−1**	−0.97727	**0**	−2.84275	**1**
	IAGWO VS others	t-stat	t-test		−2.52661	**1**	2.03264	**0**	−2.12808	**1**	−1.83060	**0**	4.13855	**−1**	4.13855	**−1**	−0.97847	**0**	−2.89435	**1**
F19	Mean			−3.8627	−3.8624		−3.8590		**−3.8628**		−3.8612		**−3.8628**		**−3.8628**		−3.8600		−3.2680	
	Std			1.6035E−04	9.9511E−04		3.3000E−03		**1.1400E−05**		2.7109E−03		**2.5800E−15**		**2.2900E−15**		1.4000E−05		1.8500E−05	
	AGWO vs others	t-stat	t-test				−5.29623	**1**	2.21944	**−1**	−2.24777	**1**	2.11135	**−1**	2.11135	**−1**	−12.93172	**1**	−3216.065	**1**
	IAGWO vs others	t-stat	t-test		−1.51044	**0**	−5.98607	**1**	4.26901	**−1**	−2.95089	**1**	3.60812	**−1**	3.60812	**−1**	−89.41237	**1**	−19839.26	**1**
F20	Mean			**−3.3220**	**−3.3220**		−3.1858		−3.2572		−3.2519		−3.2663		−3.3178		−3.2700		−3.3219	
	Std			**0.0000**	**0.0000**		0.1199		0.0871		0.0738		0.0605		0.0231		0.0590		0.0072	
	AGWO vs others	t-stat	t-test				−6.11620	**1**	−4.00496	**1**	−5.11302	**1**	−4.95095	**1**	−0.97910	**0**	−4.74410	**1**	−0.09444	**0**
	IAGWO vs others	t-stat	t-test		−2.82583	**1**	−6.11665	**1**	−4.00557	**1**	−5.11374	**1**	−4.95182	**1**	−0.98140	**0**	−4.74500	**1**	−0.10182	**0**
F21	Mean			**−10.1517**	**−10.1513**		−6.7613		−5.2161		−9.9816		−6.8651		−5.9551		−5.5200		−9.7280	
	Std			**0.0009**	**0.0010**		1.7626		2.9706		0.9305		3.0196		3.7371		1.5900		0.2881	
	AGWO vs others	t-stat	t-test				−10.35732	**1**	−8.94667	**1**	−0.98248	**0**	−5.86057	**1**	−6.04676	**1**	−15.68579	**1**	−7.91263	**1**
	IAGWO vs others	t-stat	t-test		−1.40654	**0**	−10.35836	**1**	−8.94728	**1**	−0.98445	**0**	−5.86117	**1**	−6.04725	**1**	−15.68695	**1**	−7.91900	**1**
F22	Mean			**−10.4015**	**−10.4011**		−7.1128		−7.2367		−10.2243		−8.4565		−9.6845		−5.5300		−9.8730	
	Std			**0.0009**	**0.0009**		1.9801		3.5259		0.9701		3.0871		2.0141		2.1200		0.3203	
	AGWO vs others	t-stat	t-test				−8.94306	**1**	−4.83307	**1**	−0.98177	**0**	−3.39217	**1**	−1.91615	**0**	−12.37349	**1**	−8.87809	**1**
	IAGWO vs others	t-stat	t-test		−1.79570	**0**	−8.94422	**1**	−4.83373	**1**	−0.98415	**0**	−3.39291	**1**	−1.91729	**0**	−12.37457	**1**	−8.88529	**1**
F23	Mean			−10.5344	−10.5346		−8.1292		−7.6685		−10.2646		−9.9529		**−10.5364**		−6.5700		−9.7822	
	Std			0.0010	0.0007		1.0174		3.6421		1.4813		1.7828		**2.6000E−15**		3.1400		0.5002	
	AGWO vs others	t-stat	t-test				−12.73182	**1**	−4.23775	**1**	−0.98145	**0**	−1.75701	**0**	13.75158	**−1**	−6.79933	**1**	−8.10011	**1**
	IAGWO vs others	t-stat	t-test		0.70317	**0**	−12.73095	**1**	−4.23751	**1**	−0.98086	**0**	−1.75652	**0**	10.47701	**−1**	−6.79905	**1**	−8.09835	**1**
BEST $f_{\min}$	IAGWO vs others(AGWO vs others)			**7/10**	**7/10**		**1/10 (1/10)**		**2/10 (2/10)**		**3/10 (3/10)**		**4/10 (4/10)**		**5/10 (5/10)**		**1/10 (1/10)**		**1/10 (1/10)**	
sig better	IAGWO vs others(AGWO vs others)				3/10		7/10 (7/10)		8/10 (7/10)		2/10 (2/10)		4/10 (4/10)		3/10 (3/10)		8/10 (8/10)		8/10 (8/10)	
sig equiv	IAGWO vs others(AGWO vs others)				7/10		3/10 (1/10)		1/10 (2/10)		8/10 (8/10)		2/10 (2/10)		5/10 (5/10)		1/10 (1/10)		1/10 (1/10)	
sig worse	IAGWO vs others(AGWO vs others)				0/10		0/10 (2/10)		1/10 (1/10)		0/10 (0/10)		4/10 (4/10)		2/10 (2/10)		1/10 (1/10)		1/10 (1/10)	

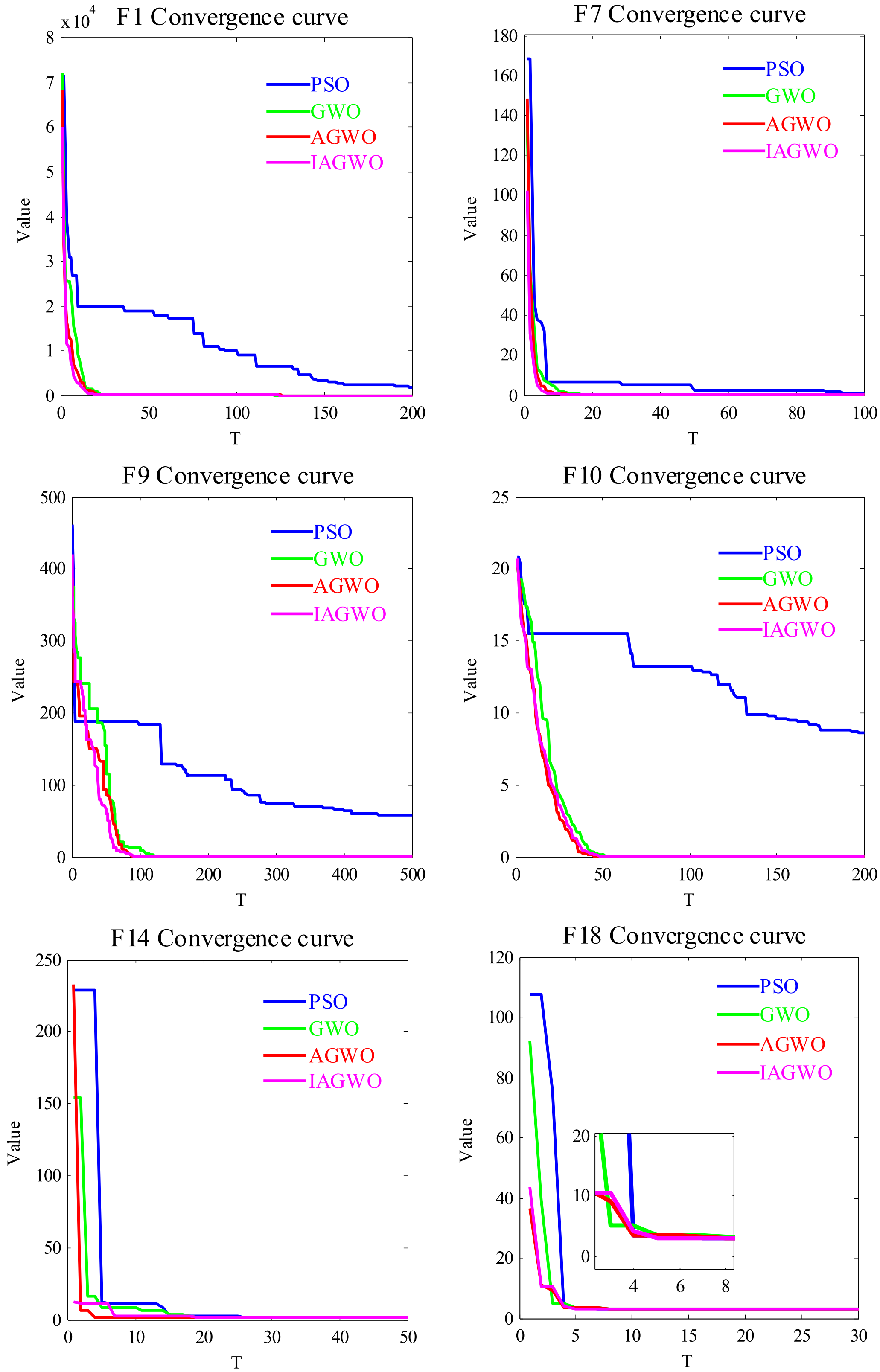

Fig. 4. The convergence curves of F1, F7, F9, F10, F14 and F18.

For the second layer model, the control variables are the heads of each single-stage pumping station. The array of the control variable vector is described as follows:

$$H = [Z_{\mathrm{in}_2}, \ldots, Z_{\mathrm{in}_j}, \ldots, Z_{\mathrm{in}_m}, Z_{\mathrm{out}_1}, Z_{\mathrm{out}_2}, \ldots, Z_{\mathrm{out}_j}, \ldots, Z_{\mathrm{out}_{m-1}}] \tag{4.2}$$

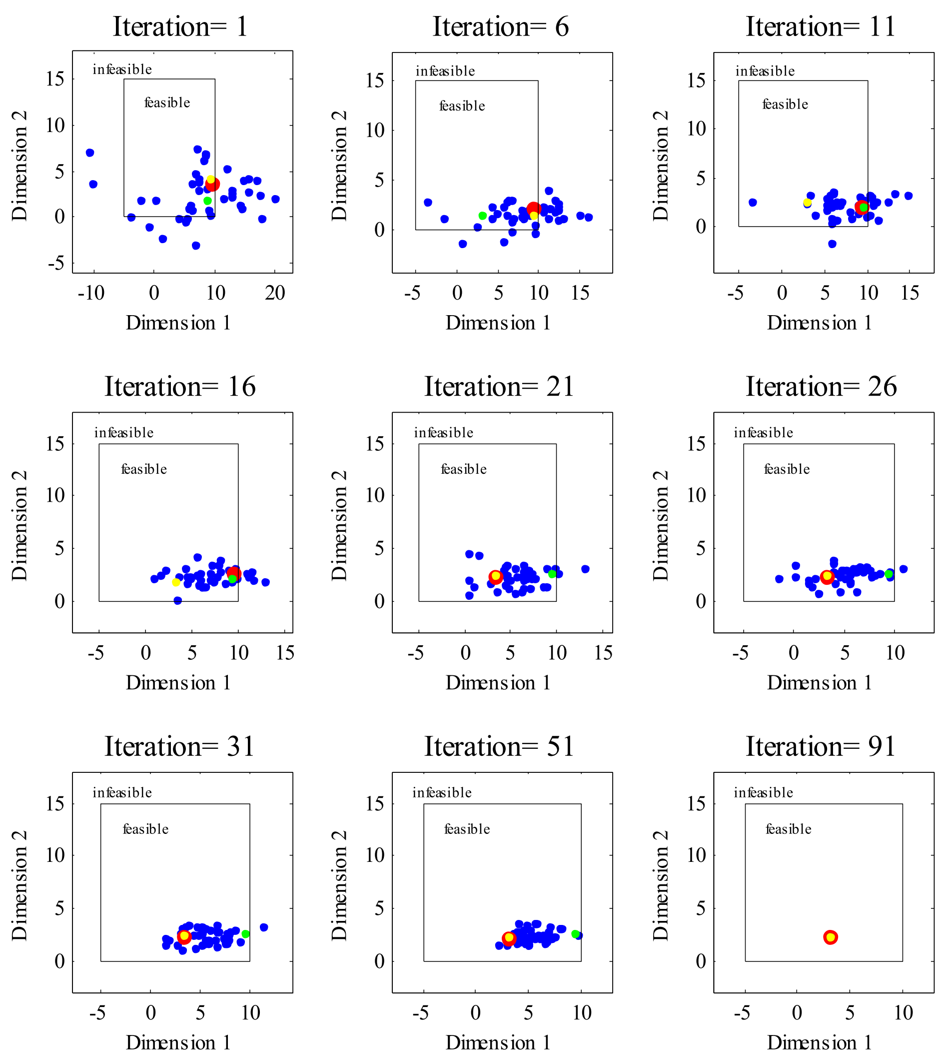

Fig. 5. The positions of wolfs observed at 9 stages of GWO for F17, where dimension = 2, and agents = 40.

4.1.2. Initialization in IAGWO and AGWO

Individuals are initialled randomly while satisfying various constraints, which are randomly generated between the minimum and maximum values. For instance, Q_i is randomly generated between $Q_{i\max}$ and $Q_{i_{min}}$; and Z_{in_j} is randomly generated between $Z_{in_{j\_max}}$ and $Z_{in_{j\_min}}$. Generally, these newly generated individuals do not satisfy all the constraints and thus need to be modified by the constraint handling method, which will be described in the next section.

4.1.3. Constraint handling in IAGWO and AGWO

The daily optimal operation problem of cascade pumping stations has a number of equality and inequality constraints, and handling these constraints is critical to effectively solve the problem. The flowchart of constraint handling is shown in Fig. 7.

In the proposed AGWO, the inequality constraints can be easily handled, and those values out of the boundaries can be simply set to be equal to the boundaries, which is same as in the GWO and other algorithms. The Inverse Parabolic Spread Distribution is used to handle the boundaries in the proposed IAGWO. However, it is more difficult to handle equality constraints, including the mass balance constraints (Eq. (2.2)) in the first layer model and the hydraulic balance constraints (Eq. (2.6)) in the second layer model. In this study, the equality constraints are handled by the strategy proposed by Tian et al. [45] for all algorithms. First, the difference DEV_Q between the two sides of the equality constraints (2.2) is calculated.

$$DEV_Q = Q_{\text{total}_k} - \sum_{i=1}^{n} Q_i \tag{4.3}$$

Then, the difference is divided into n parts, and the flow rate will be adjusted.

$$MDEV_Q = DEV_Q/n \tag{4.4}$$

$$Q_i = Q_i + MDEV_Q \tag{4.5}$$

The next step is to determine whether the new Q_i violates the boundary conditions. If so, the inequality and equality constraints are handled again until $DEV_Q < \delta_Q$ and inequality constraints are fully satisfied, where δ_Q is far less than Q_i.

The strategy [45] used to handle the hydraulic balance constraints is similar to that for the mass balance constraints, which is described as follows.

Step 1: Calculating the difference DEV_H.

$$DEV_H = H_{\text{total}} - (Z_{out_m} - Z_{in_1} + \sum_{j=1}^{m-1} h_{j,j+1}) \tag{4.6}$$

Step 2: Dividing the difference into $2(m-1)$ parts.

$$MDEV_H = \frac{DEV_H}{2(m-1)} \tag{4.7}$$

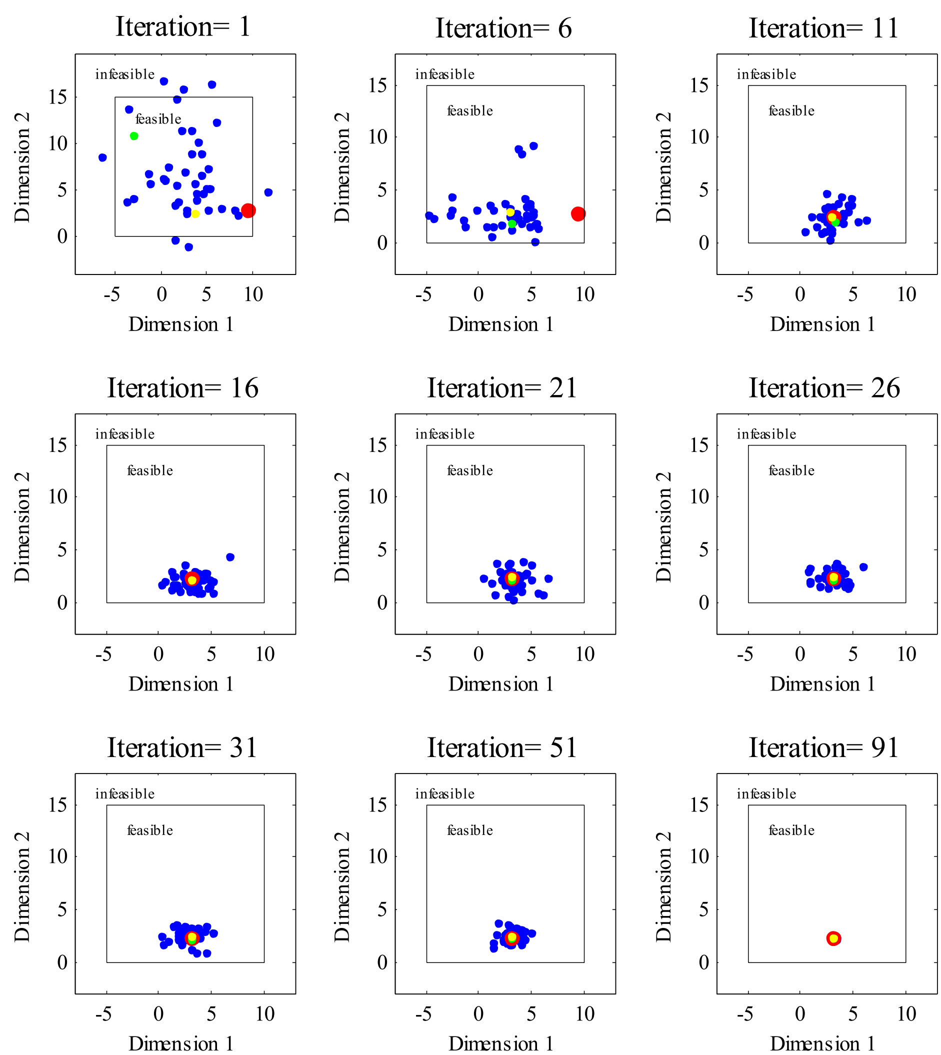

Fig. 6. The positions of wolfs observed at 9 stages of AGWO for F17, where dimension = 2, and agents = 40.

Step 3: Adjusting the water levels of the forebay and outlet pond of the j^{th} pumping station.

$$\begin{cases} Z_{out_j} = Z_{out_j} + MDEV_H \\ Z_{in_j} = Z_{in_j} - MDEV_H \end{cases} \tag{4.8}$$

Step 4: Determining whether the new Z_{in_j}, Z_{out_j} violate the boundary conditions. If so, the inequality and equality constraints are handled again until $DEV_H < \delta_H$ and inequality constraints are fully satisfied, where δ_H is far less than the water level range.

Table 6
The percentage of wolves moving out of the feasible area for unimodal benchmark functions.

F		GWO			AGWO		
		Average	Minimum	Maximum	Average	Minimum	Maximum
Unimodal benchmark functions	F1	0.0548%	0.0418%	0.0772%	0.0137%	0.0090%	0.0177%
	F2	0.0979%	0.0693%	0.1425%	0.0155%	0.0093%	0.0200%
	F3	0.2869%	0.1473%	0.5640%	0.0219%	0.0163%	0.0302%
	F4	0.1007%	0.0588%	0.1632%	0.0172%	0.0120%	0.0265%
	F5	0.0656%	0.0485%	0.0922%	0.0154%	0.0105%	0.0197%
	F6	0.0570%	0.0425%	0.0872%	0.0137%	0.0082%	0.0180%
	F7	0.0647%	0.0465%	0.0867%	0.0156%	0.0123%	0.0210%

Table 7
The percentage of wolves moving out of the feasible area for multimodal benchmark functions.

F		GWO			AGWO		
		Average	Minimum	Maximum	Average	Minimum	Maximum
Multimodal benchmark functions	F8	5.4900%	3.6882%	8.0493%	0.1104%	0.0532%	0.1763%
	F9	0.1379%	0.0793%	0.2232%	0.0188%	0.0153%	0.0248%
	F10	0.0635%	0.0425%	0.0935%	0.0139%	0.0097%	0.0180%
	F11	0.0540%	0.0358%	0.0737%	0.0140%	0.0092%	0.0180%
	F12	0.0612%	0.0485%	0.0757%	0.0141%	0.0103%	0.0193%
	F13	0.0581%	0.0460%	0.0733%	0.0145%	0.0102%	0.0185%

Table 8
The percentage of wolves moving out of the feasible area for fixed-dimension multimodal benchmark functions.

F		GWO			AGWO		
		Average	Minimum	Maximum	Average	Minimum	Maximum
Fixed-dimension multimodal benchmark functions	F14	0.1475%	0.0675%	0.2400%	0.0518%	0.0200%	0.0875%
	F15	6.4934%	0.0438%	25.0638%	0.0385%	0.0075%	0.0888%
	F16	0.0085%	0.0000%	0.0250%	0.0068%	0.0000%	0.0225%
	F17	3.9641%	0.1900%	15.8725%	0.1542%	0.0525%	0.7550%
	F18	0.1899%	0.1475%	0.2450%	0.0618%	0.0275%	0.0875%
	F19	6.4812%	6.0200%	7.2200%	0.4099%	0.3467%	0.4600%
	F20	3.5696%	0.8958%	9.3867%	0.2250%	0.1717%	0.3417%
	F21	0.7996%	0.4063%	4.0388%	0.2018%	0.1225%	0.3038%
	F22	0.5208%	0.3738%	1.1963%	0.1664%	0.1388%	0.2313%
	F23	0.5270%	0.3825%	0.9788%	0.1627%	0.1238%	0.2513%

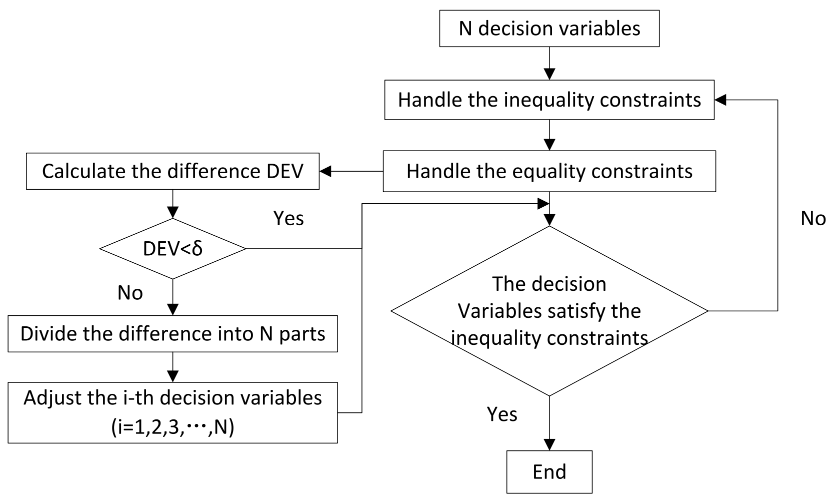

Fig. 7. The flowchart of constraint handling.

4.1.4. Updating in IAGWO and AGWO

The new updating rule is applied to the updating of the daily optimal operation of cascade pumping stations using AGWO and IAGWO.

4.1.5. The optimization of the third layer by dynamic programming (DP)

DP is used for optimization of the third layer, in which the variable is the total flow rate. The discrete step size of the total flow rate in the feasible region needs to be determined, and it is also critical to solve equality constraints. The daily total diversion balance constraint is time-related, thus making it a spatiotemporal coupling problem. As the total amount of water to be diverted daily is determined, the feasible region of the flow rates at one period varies due to different flow rates of other periods. Therefore, the strategy that the feasible region of the flow rates at different periods is dynamically adjusted in order to reduce unnecessary search is proposed.

Step 1: The variable range of the first period is discretized according to the step size.

Step 2: According to the discrete variable value of the first period and the maximum variable values of the other periods, the minimum variable value of the second period is determined (Eq. (4.9)), and then the variable range of the second period is determined. Next, according to the step size, the variable range of the second period is discretized.

Step 3: According to the discrete variable values of the first and second periods and the maximum variable values of the other periods, the minimum variable value of the third period is determined (Eq. (4.9)), and then the variable range of the third period is determined. Next, according to the step size, the variable range of the third period is discretized. The above steps are repeated until the minimum variable value and the variable range of $T-1^{\text{th}}$ period are determined, and then the variable range of the $T-1^{\text{th}}$ period is discretized.

Step 4: The variable value of the last period is calculated by Eq. (4.10).

$$Q'_{total_{k\_\min}} = \frac{W - \sum_{i=1}^{k-1} Q_{total_i}\Delta t_i - \sum_{j=k+1}^{T} Q_{total_{j\_\max}}\Delta t_j}{\Delta t_k} \tag{4.9}$$

$$Q_{total_T} = \frac{W - \sum_{i=1}^{T-1} Q_{total_i}\Delta t_i}{\Delta t_T} \tag{4.10}$$

4.1.6. Construction of the set of external files to save computation time

In the three-layer structure, the same calculation results of the second or the first layer model may be calculated many times, depending on the combination of variables in the three or the second layer, respectively. Therefore, the set of external files is constructed to save the results calculated in the solution process in order to save computation time.

4.1.7. Flow chart of the proposed algorithm for the daily optimal operation

The flow chart for the proposed algorithm for the daily optimal operation of cascade pumping stations is shown in Fig. 8.

4.2. Case study and results

4.2.1. Case description

A case study is performed in a cascade pumping station system consisting of six pumping stations, as shown in Fig. 9. The water volume to be diverted daily is 1.71072×10^6 m^3; the water level

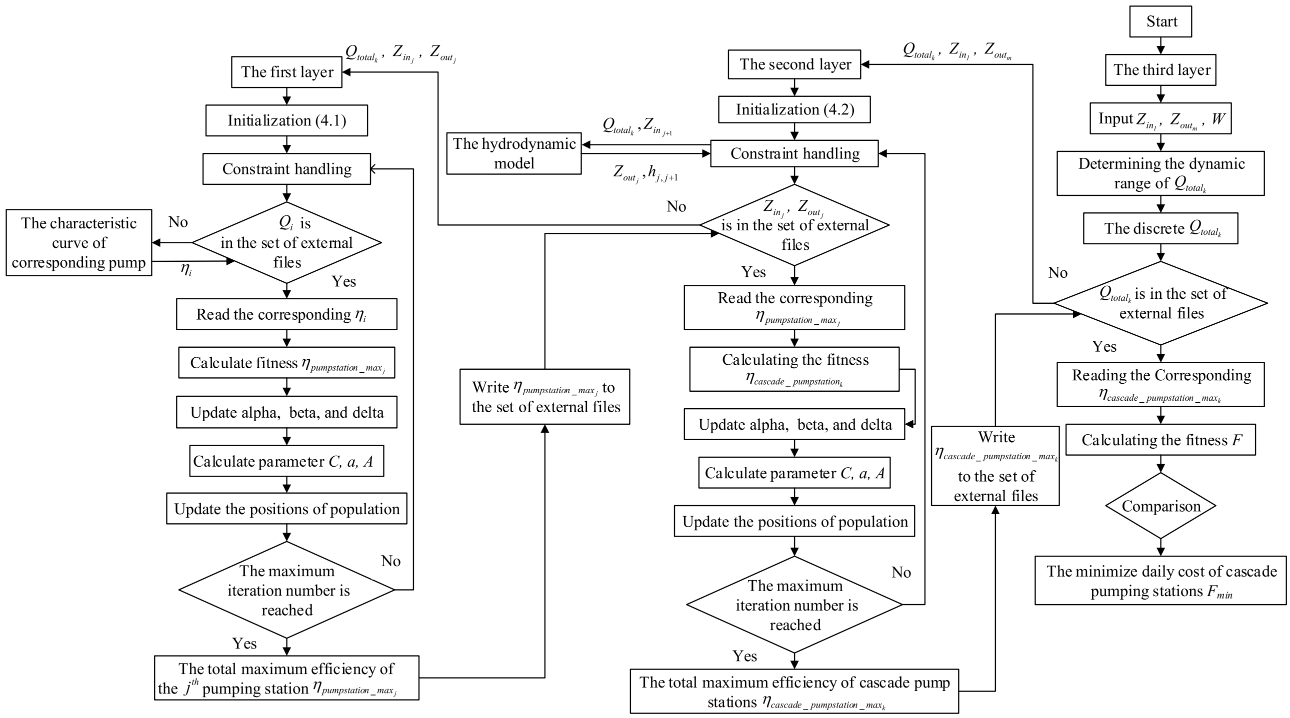

Fig. 8. The flow chart of the proposed algorithm for the daily optimal operation of cascade pumping stations.

of the forebay of the first-stage pumping station and the outlet pond of the sixth-stage pumping station are 48.6 and 58.81 m, respectively; and the cost of the present scheme is ¥94957.24/d. The two water levels are assumed to be fixed in a day. Each pumping station is equipped with four water pumps of the same type (3 workers and 1 standby) with a design flow rate of 6.67 m^3. The water levels, heads and flow rates are shown in Table 9; the electricity prices at different periods are shown in Table 10; the constraint parameters are shown in Table 11; and the information of the present scheme is shown in Table 12, respectively.

4.2.2. Simulation results

In order to verify the effectiveness of the GWO algorithm and the proposed algorithms, 100 and 50 trials are simulated for the first and second layer to obtain the maximum efficiency and minimum cost, respectively, which takes no account of distributing the flow rate among all periods according to the electricity prices at different periods. In the simulation, the population size is 30 and 20 in each trial for the first and second layer, respectively. Table 13

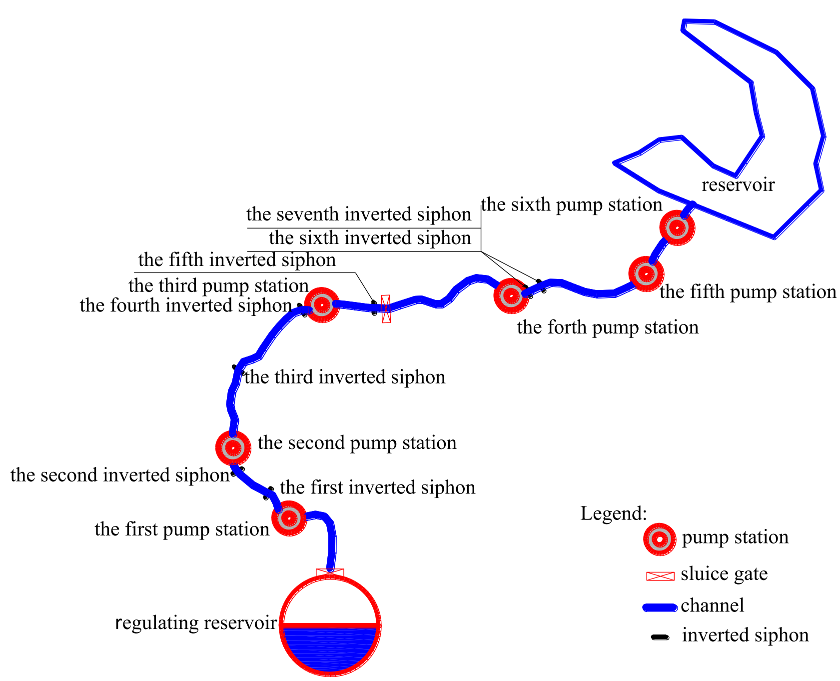

Fig. 9. The floor plan of cascade pumping stations.

Table 9
The water levels and heads.

Pumping stations		1	2	3	4	5	6
Head	max(m)	1.50	2.20	2.45	2.21	2.04	8.18
	min(m)	0.11	1.27	2.06	1.82	1.29	4.31
The water level of the forebay of pumping station	max(m)	48.38	48.60	49.42	50.53	51.30	51.92
	min(m)	49.26	49.39	49.72	50.83	51.60	52.92
The water level of the outlet pond of pumping station	max(m)	49.37	50.66	51.78	52.65	52.89	57.23
	min(m)	49.88	50.80	51.87	52.74	53.34	60.10
The flow rate of single pump	max(m^3/s)	8.4	7.7	7.1	7.4	7.7	10.4
	min(m^3/s)	5.8	5.3	5.0	5.3	5.3	5.5

Table 10
The electricity prices at different periods.

Periods	1	2	3
Time	10:00–15:00 18:00-21:00	7:00–10:00 15:00–18:00 21:00–23:00	23:00–next day 7:00
Price(¥/kw.h)	1.3222	0.8395	0.3818

Table 11
The constraint parameters of the test system.

δ_H	δ_Q	Step size of the total flow rate
10^{-4}	10^{-4}	0.1

Table 12
The information of the present scheme.

Pumping station	Present scheme (flow is 19.8 m^3/s)					
	Present head /m	Present water level/m		Hydraulic loss/m	Efficiency of single pumping station	Efficiency of cascaded pumping station
1	1.07	Forebay	48.6		39.27%	
		Outlet pond	49.67	0.52		
2	1.6	Forebay	49.14		52.21%	
		Outlet pond	50.74	1.13		
3	2.21	Forebay	49.61		63.82%	
		Outlet pond	51.82	1.11		42.4965%
4	1.97	Forebay	50.71		64.03%	
		Outlet pond	52.68	1.19		
5	1.59	Forebay	51.50		51.89%	
		Outlet pond	53.09	0.46		
6	6.18	Forebay	52.63		71.59%	
		Outlet pond	58.81			

Table 13
The optimized schemes by different methods.

Method	The total flow (m^3/s)	The water level of forebay(m)	The water level of outlet pond(m)	Efficiency of cascaded pumping station(m)	The cost of cascaded pumping station(¥/d)	Saved cost compared with present scheme	The percentage of positions moving out of the feasible area
IAGWO	19.8	48.6	58.81	42.63484%	94649.228	0.32437%	4.2800%
AGWO	19.8	48.6	58.81	42.63292%	94653.470	0.31990%	5.5200%
GWO	19.8	48.6	58.81	42.62448%	94672.218	0.30016%	95.5800%
PSO	19.8	48.6	58.81	42.60410%	94717.510	0.25310%	97.8200%

Table 14
The efficiency, head and flow rate calculated by different methods (The total flow is 19.8m^3/s).

Methods	Pumping station	Water level of forebay(m)	Water level of outlet pond(m)	Head(m)	Flow rate of each pump(m^3/s)			Efficiency of each pump			Efficiency of each pumping station
					1	2	3	1	2	3	
IAGWO	1	48.60000	49.60000	1.00000	6.78234	6.71701	6.30065	38.21747%	37.85167%	35.54553%	37.20553%
	2	48.86118	50.69924	1.83806	6.59976	6.60045	6.59978	60.86447%	60.87014%	60.86466%	60.86643%
	3	49.62572	51.78480	2.15908	6.59968	6.60112	6.59920	63.12102%	63.13343%	63.11665%	63.12370%
	4	50.76414	52.66009	1.89595	6.60491	6.59493	6.60016	62.74660%	62.66754%	62.71169%	62.70861%
	5	51.54379	53.15790	1.61411	6.59943	6.60306	6.59751	52.71582%	52.74718%	52.69815%	52.72038%
	6	52.73565	58.81000	6.07435	6.59990	6.59521	6.60489	70.34067%	70.27249%	70.39276%	70.33530%
AGWO	1	48.60000	49.60000	1.00000	6.30005	6.77103	6.72892	35.18024%	37.81483%	37.57483%	36.85660%
	2	48.86118	50.69924	1.83806	6.60206	6.59844	6.59950	60.08857%	60.05802%	60.06744%	60.07134%
	3	49.62572	51.78480	2.15908	6.59862	6.60085	6.60053	62.40741%	62.42794%	62.42512%	62.42016%
	4	50.76414	52.66009	1.89595	6.59954	6.60019	6.60026	62.21591%	62.22129%	62.22182%	62.21967%
	5	51.54379	53.15790	1.61411	6.60044	6.59759	6.60197	52.56807%	52.54215%	52.58137%	52.56386%
	6	52.73565	58.81000	6.07435	6.59589	6.60235	6.60176	70.59419%	70.66968%	70.66388%	70.64258%
GWO	1	48.60000	49.83460	1.23460	6.60000	6.60000	6.60000	45.65070%	45.65070%	45.65070%	45.65070%
	2	49.34898	50.79670	1.44772	6.79755	6.20489	6.79755	48.54436%	44.21046%	48.54436%	47.09752%
	3	49.67898	51.84370	2.16472	6.60000	6.60000	6.60000	62.58871%	62.58871%	62.58871%	62.58871%
	4	50.78898	52.69282	1.90384	6.60000	6.60000	6.60000	62.46109%	62.46109%	62.46109%	62.46109%
	5	51.55898	53.09812	1.53914	6.60000	6.60000	6.60000	50.25651%	50.25651%	50.25651%	50.25651%
	6	52.87898	58.81000	5.93102	6.60000	6.60000	6.60000	68.91912%	68.91912%	68.91912%	68.91912%
PSO	1	48.60000	49.63462	1.03462	6.7006	6.7981	6.3003	38.49004%	39.01522%	36.30677%	37.94124%
	2	49.16674	50.70322	1.53648	6.5999	6.6000	6.6000	50.16994%	50.17099%	50.17095%	50.17101%
	3	49.71551	51.78541	2.06990	6.6000	6.6000	6.6000	59.66667%	59.66667%	59.66667%	59.66667%
	4	50.81476	52.69276	1.87800	6.5997	6.5997	6.5996	61.55138%	61.55076%	61.55035%	61.55381%
	5	51.47974	53.28967	1.80993	6.6000	6.6000	6.6000	59.02842%	59.02811%	59.02847%	59.02848%
	6	52.77056	58.81000	6.03944	6.6000	6.6000	6.6000	70.31467%	70.31467%	70.31467%	70.31467%

Table 15
The daily optimized schemes by different methods.

Method		IAGWO	AGWO	GWO	PSO
The flows of the different periods (m^3/s)	The 1st period	19.4	19.4	19.4	19.4
	The 2nd period	20.0	20.0	20.0	20.0
	The 3rd period	20.0	20.0	20.0	20.0
The water level of forebay		48.6	48.6	48.6	48.6
The water level of outlet pond		58.81	58.81	58.81	58.81
The cost of cascaded pumping station(¥/d)		94195.0412	94195.7903	94222.5648	94526.7938
Saved cost compared with present scheme		0.80268%	0.80189%	0.77369%	0.45331%
The percentage of positions moving out of the feasible area	The 1st period	9.5500%	5.7500%	95.7300%	97.8200%
	The 2nd and 3rd period	4.7500%	4.2900%	95.5100%	97.8300%

shows that the efficiency of the optimized schemes by IAGWO, AGWO, GWO and PSO for cascaded pumping station are 42.63484%, 42.63292%, 42.62448% and 42.60410%, respectively. And the costs of the optimized schemes by IAGWO, AGWO, GWO and PSO for cascaded pumping station are 94649.228 ¥/d, 94653.470 ¥/d, 94672.218 ¥/d and 94717.510 ¥/d, respectively. The optimized schemes by IAGWO, AGWO, GWO and PSO result in a cost reduction by about 0.32437%, 0.31990%, 0.30016%, and 0.25310%, respectively, compared with the present scheme. What is more, the percentage of positions moving out of the feasible area by IAGWO, AGWO, GWO and PSO are 4.2800%, 5.5200%, 95.5800% and 97.8200%, respectively. The detail information of the optimized schemes by different methods are shown in Table 14, such as efficiency, head and flow rate of each pumping station.

Considering distributing the flow rate among all periods according to the electricity prices at different periods, the flow rate of the 1st period is 19.4 m^3/s, and the 2nd, 3rd period are both 20 m^3/s, as shown in Table 15. And the costs of the optimized schemes by IAGWO, AGWO, GWO and PSO for cascaded pumping stations are 94195.0412 ¥/d, 94195.7903 ¥/d, 94222.5648 ¥/d and 94526.7938 ¥/d, respectively, which result in a cost reduction by about 0.80268%, 0.80189%, 0.77369%, and 0.45331%, respectively, compared with the present scheme. These results indicate that the flow rate during the period of low electricity price is relatively large, which can minimize the total cost of cascade pumping station. What is more, the percentage of positions moving out of the feasible area by IAGWO, AGWO, GWO and PSO are 9.5500%, 5.7500%, 95.7300% and 97.8200% in 1st period, respectively. The percentage of positions moving out of the feasible area by IAGWO, AGWO, GWO and PSO are 4.7500%, 4.2900%, 95.5100% and 97.8300% in the 2nd, 3rd period, respectively. The detail information of the optimized schemes by different methods are shown in Tables 16 and 17, such as efficiency, head and flow rate of each pumping station.

These results show that the use of IAGWO, AGWO, GWO and PSO can result in better results than the present scheme. What is more, IAGWO and AGWO can obtain better economic solutions than GWO and PSO, which indicates that IAGWO and AGWO can increase the valid search though reducing the percentage of positions moving out of the feasible area or maintaining diversity while bringing the wolves back into the feasible region. However, the magnitude of the optimized schemes is not very large, which can be attributed to the following two reasons. First, the water levels of the forebay and outlet pond of the pumping stations, as well as the heads, are maintained within a narrow range. In particular, the ranges of the water level of the outlet pond in the third and fourth pumping stations are only 9 cm. This can have a significant effect on the optimization of cascade pumping stations. Second, certain principles should be adhered to in engineering design, and the pumping stations and the water levels of channels should be coordinated. Thus, the efficiency of the optimization schemes is not so obvious.

Table 16
The efficiency, head and flow rate of first period calculated by different methods.

Methods	Pumping station	Water level of forebay(m)	Water level of outlet pond(m)	Head(m)	Flow rate of each pump(m^3/s)			Efficiency of each pump			Efficiency of each pumping station
					1	2	3	1	2	3	
IAGWO	1	48.60000	49.60973	1.00973	6.30058	6.79825	6.30117	35.49940%	38.26407%	35.50260%	36.42265%
	2	49.15702	50.66605	1.50903	6.56832	6.53234	6.29934	49.07045%	48.82122%	47.28998%	48.39562%
	3	49.62682	51.80603	2.17921	6.40389	6.59651	6.39960	61.23022%	62.98855%	61.19091%	61.80376%
	4	50.82963	52.68357	1.85394	6.47229	6.43897	6.48968	59.51621%	59.20118%	59.68061%	59.46311%
	5	51.35411	53.27031	1.91620	6.49711	6.49783	6.40507	61.87959%	61.88639%	61.00971%	61.59192%
	6	52.91986	58.81000	5.89014	9.69989	9.70011	–	68.79671%	68.79620%	–	68.79645%
AGWO	1	48.60000	49.60981	1.00981	6.79728	6.30400	6.29872	38.26098%	35.52053%	35.49076%	36.42471%
	2	49.15694	50.66570	1.50876	6.58987	6.51058	6.29955	49.21106%	48.66246%	47.28484%	48.38792%
	3	49.62674	51.80611	2.17937	6.59310	6.40036	6.40654	62.96217%	61.20303%	61.25936%	61.80869%
	4	50.83000	52.68365	1.85365	6.48609	6.45260	6.46159	59.63569%	59.31904%	59.40410%	59.45208%
	5	51.35403	53.27039	1.91636	6.45704	6.48022	6.46176	61.50479%	61.72390%	61.54938%	61.59580%
	6	52.91979	58.81000	5.89021	9.70000	9.70000	–	68.79673%	68.79673%	–	68.79673%
GWO	1	48.60000	49.61987	1.01987	6.79931	6.30035	6.30035	38.57624%	35.82761%	35.82761%	36.74523%
	2	48.81473	50.66570	1.85097	6.46667	6.46667	6.46667	59.35143%	59.35143%	59.35143%	59.35143%
	3	49.63473	51.78480	2.15007	6.60033	6.40523	6.39444	62.15492%	60.31011%	60.21023%	60.89171%
	4	50.74473	52.65180	1.90707	6.48935	6.49840	6.41224	61.58228%	61.66809%	60.85137%	61.36725%
	5	51.60000	53.13877	1.53877	6.59939	6.20122	6.59939	50.23984%	47.28965%	50.23984%	49.25757%
	6	52.92000	58.81000	5.89000	9.70000	9.70000	–	68.79600%	68.79600%	–	68.79600%
PSO	1	48.60000	49.76154	1.16154	6.52099	6.30002	6.57824	42.32061%	40.92232%	42.68964%	41.97947%
	2	49.18958	50.66570	1.47612	6.79975	6.29933	6.29998	49.45084%	46.07187%	46.07660%	47.20631%
	3	49.64260	51.78480	2.14220	6.59987	6.59998	6.20099	61.91612%	61.91727%	58.13020%	60.65126%
	4	50.69655	52.67141	1.97486	6.40363	6.49827	6.49719	62.45022%	63.32804%	63.31798%	63.03519%
	5	51.60000	53.11533	1.51533	6.59764	6.30034	6.50111	49.47541%	47.45382%	48.79164%	48.57747%
	6	52.82302	58.81000	5.98698	9.69937	9.69964	0.00000	69.17062%	69.17002%	0.00000%	69.17385%

Table 17
The efficiency, head and flow rate of second and third periods calculated by different methods.

Methods	Pumping station	Water level of forebay(m)	Water level of outlet pond(m)	Head(m)	Flow rate of each pump(m^3/s)			Efficiency of each pump			Efficiency of each pumping station
					1	2	3	1	2	3	
IAGWO	1	48.60000	49.60006	1.00006	6.79527	6.41633	6.78841	37.95473%	35.82157%	37.91563%	37.23043%
	2	48.87510	50.74304	1.86795	6.63407	6.69015	6.67509	61.44726%	61.88581%	61.76808%	61.70261%
	3	49.61825	51.79357	2.17532	6.60130	6.60247	6.79623	62.91572%	62.92571%	64.64492%	63.49621%
	4	50.73039	52.66034	1.92995	6.60547	6.69640	6.69814	63.11899%	63.77104%	63.78350%	63.55834%
	5	51.48539	53.19108	1.70570	5.61783	7.29730	7.08487	47.36751%	61.14278%	59.19653%	55.92320%
	6	52.81323	58.81000	5.99677	6.66776	6.66729	6.66496	70.63476%	70.62956%	70.60403%	70.62279%
AGWO	1	48.60000	49.59896	0.99896	6.79425	6.79812	6.40763	37.89933%	37.92143%	35.72326%	37.18107%
	2	48.83077	50.71417	1.88340	6.63018	6.68839	6.68055	61.98564%	62.43188%	62.37177%	62.26601%
	3	49.61041	51.79764	2.18723	6.79893	6.60035	6.60072	65.02684%	63.26233%	63.26550%	63.85239%
	4	50.74447	52.66120	1.91674	6.69933	6.69765	6.60302	63.49334%	63.48107%	62.78874%	63.25489%
	5	51.47689	53.18578	1.70889	7.04354	5.61576	7.34070	58.95063%	47.42553%	61.58645%	56.00866%
	6	52.78725	58.81000	6.02275	6.60423	6.69746	6.69831	70.20103%	71.18967%	71.19864%	70.86312%
GWO	1	48.60000	49.83460	1.23460	6.79994	6.60003	6.60003	46.95069%	45.65088%	45.65088%	46.08466%
	2	49.33395	50.79670	1.46275	7.39700	6.30150	6.30150	53.19288%	45.53094%	45.53094%	48.09302%
	3	49.66395	51.84370	2.17975	6.80567	6.59717	6.59717	64.85642%	63.01076%	63.01076%	63.62690%
	4	50.77395	52.70785	1.93390	6.69482	6.61251	6.69267	63.84930%	63.26265%	63.83402%	63.64906%
	5	51.54395	53.11315	1.56920	6.60655	6.79392	6.59953	51.27247%	52.74682%	51.21750%	51.74547%
	6	52.86395	58.81000	5.94605	6.66667	6.66667	6.66667	69.93297%	69.93297%	69.93297%	69.93297%
PSO	1	48.60000	49.76250	1.16250	6.79978	6.59997	6.59927	44.07366%	42.87478%	42.87030%	43.27564%
	2	48.76135	50.72904	1.96769	6.69828	6.69830	6.60252	64.63811%	64.63826%	63.99125%	64.42607%
	3	49.72000	51.78480	2.06480	6.69799	6.60328	6.69775	60.35670%	59.52714%	60.35461%	60.08250%
	4	50.81526	52.66496	1.84970	6.69759	6.69856	6.60291	61.28490%	61.29269%	60.52721%	61.03811%
	5	51.49003	53.10633	1.61630	6.66753	6.66590	6.66754	53.20138%	53.18738%	53.20151%	53.19417%
	6	52.92000	58.81000	5.89000	6.67856	6.66268	6.65976	69.34995%	69.14614%	69.10869%	69.19812%

5. Conclusions

In this study, an improved self-adaptive Grey Wolf optimizer (IAGWO) is proposed for optimization of daily operation of cascade pumping stations. The parameter *A* of IAGWO is dynamically adjusted to reduce the percentage of wolves moving out of the feasible area, and the Inverse Parabolic Spread Distribution, which can maintain the diversity and bring wolves back into the feasible region, is used to further improve the accuracy. The proposed IAGWO and AGWO algorithms are tested using 23 benchmark functions and compared with several other algorithms. The results of multimodal functions show that the exploration of the proposed IAGWO and AGWO algorithms are augmented; and the results of unimodal functions show that the proposed algorithms are competitive in exploitation compared with other algorithms. Moreover, a strategy is proposed to dynamically adjust the feasible region of variables in order to reduce unnecessary search for the optimization model of daily cost. The proposed IAGWO and AGWO algorithms are applied to a cascade pumping station system consisting of six pumping stations, and they can obtain more efficient and economic solutions for cascade pumping stations. Therefore, this study provides a good and effective approach for the daily optimal operation of cascade pumping stations. And the author will further study other current heuristic algorithms for the daily optimal operation of cascade pumping stations in the future.

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