I'm trying to prove Lemma 2 from the previous problems with non-termination.

We are going to be working on a system with at least the following rule

$$\frac{e_1 : A \to B,\ e_2 : A}{e_1\, e_2 : B}$$

Assume a consistent system, i.e: one in which there is no x : $\perp$.

$$\text{theorem}$$
$$(f : P \to \perp) \to (\text{P is uninhabited})$$

$$\text{proof by contradiction:}$$
$$\text{assume P is inhabited. this means that there is at least one } p : P$$
$$\text{we can now construct}$$
$$\frac{f : P \to \perp,\ p : P}{f\, p : \perp}$$
$$\text{we now have an element of } \perp \text{ thus the system is inconsistent.}$$
$$\text{this is a contradiction with the assumption that we are working in a consistent system.}$$
$$\text{as we have reached a contradiction with the assumption that P is inhabiated,}$$
$$\text{p must be uninhabited } \square$$