A Survey on the Design of Binary Pulse Compression Codes with Low Autocorrelation

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1. Introduction

Simple pulsed radar is limited in range sensitivity by the average radiation power and, in range resolution by the pulse length. The design of any radar always involves a compromise between the two constraints. Waveform design aims to seek an appropriate harmony that best suits the relevant application. The pulse compression theory has been introduced in order to get a high range resolution as well as a good detection probability.

One of the basic types of pulse compression is binary phase coding which encodes the transmitted pulse with information that is compressed (decoded) in the receiver of the radar.

The study of the peak sidelobe level (PSL) binary sequences occurs as a classical problem of signal design for digital communication and, in equivalent guise, in analytic number theory. It has also become a notorious problem of combinatorial optimization. For years mathematicians, engineers, physicists and chemists have sought a systematic way to construct long binary sequences with low PSL.

In this Chapter, we describe pulse compression technique in radar waveform design. In order to make the presentation self-contained, we start by providing a short summary of waveform design and an introduction to principle behind pulse compression by compiling the basic tools required for analyzing and comparing different radar signals.

After that, we talk about binary sequence, its desired properties and general types of methods for finding and generating such waveforms. We keep on by an overview and introducing the existing methods and search routine done.

We conclude the chapter with a brief survey of the results exhibited yet for low autocorrelation binary sequences. We mention a table of complete results presented and also introduce a histogram to unscramble them visually and predict the future.

2. Why is pulse compression needed?

One of the most important usages of radar is range finding which is done through measuring time delay, Δt; it takes a pulse to travel the two-way path between the radar and the target.

$$R = \frac{c\Delta t}{2} \qquad (1)$$

Where, c is denoted as speed light.
In general, a pulsed radar transmits and receives a train of pulses, as illustrated by Figure 1.
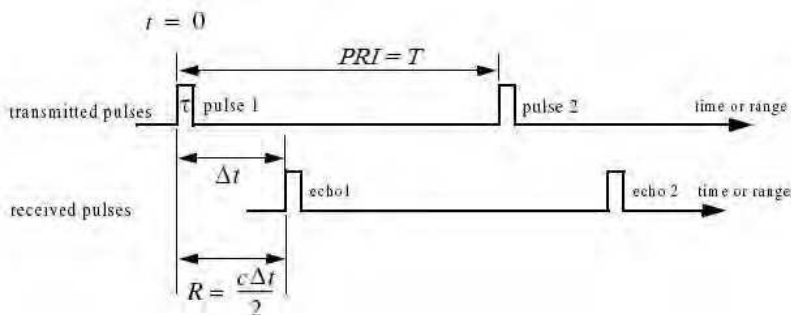

Fig. 1. Illustrating range.

By definition, Range Resolution is the ability to detect targets close proximity to each other as distinct objects only by measurement of their ranges (distances from radar) which usually expressed in terms of the minimum distance by which two targets of equal strength at the same azimuth and elevation angles must be spaced to be separately distinguishable. Resolution in the range domain ΔR corresponds to the resolution τ in the time domain, and is set by the pulsewidth according to $\Delta R = c\tau/2$ (for pulse-compression waveform, τ is the pulsewidth after pulse compression).
Without pulse compression, the instantaneous bandwidth of radar receiver, B, is equal to pulse bandwidth which is usually set to $1/\tau$; thus

$$\Delta R = \frac{c\tau}{2} = \frac{c}{2B} \qquad (2)$$

In general, radar users and designers alike seek to minimize ΔR in order to enhance the radar performance. As suggested by equation (2), in order to achieve fine range resolution one must minimize the pulse width or maximize the bandwidth.
On the other hand, as illustrated in Figure 1., during each PRI[1] the radar radiates energy only for τ seconds and listens for target returns for the rest of PRI.
Let P_{av} , P_t and E_p stand for average transmitted power, peak transmitted power and pulse energy, respectively. So,

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[1] Pulse Repetition Interval

$$P_{av} = Pt.\left(\frac{\tau}{T}\right) \tag{3}$$

$$E_p = P_t.\tau = P_{av}.T \tag{4}$$

With regard to equations (3) and (4) and also Figure 1., above solutions will reduce the average transmitted power. Furthermore, in accordance with Radar Equation (equation (5)), maximum range and thus detection probability will decrease.

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 KT_e BFL(SNR)_{o\,min}}\right)^{\frac{1}{4}} \tag{5}$$

So, it seems that the only way to account for these problems and have good range resolution is increasing the peak transmitted power, whereas there are technical limitations for the maximum peak power, such as maximum high voltage or power from the output stage, or waveguide breakdown. So, the only approach for achieving fine range resolution while maintaining adequate average transmitted power is using pulse compression techniques (Mahafza & Alabama, 2000; Skolnik, 2001) which is the main subject of this chapter and will be expatiated.

For clarity, consider this example:

Example:
Desired resolution: R = 15 cm
Required bandwidth: B = 1 GHz
Required pulse energy: E = 1 mJ
By using equation (4), two solutions are as follows:
Brute force approach:
Raw pulse duration: τ = 1 ns
Required peak transmitted power: P = **1 MW**!
Pulse compression approach:
Pulse duration: τ = 0.1 ms
Required peak transmitted power: P = 10 W

It is good to mention that, required range resolution for a given radar is dependent on its performance (detection, recognition, identification, etc.). For example, see table R26 in (Barton & Leonov, 1998) about resolution required for target interpretation tasks.

3. Pulse compression

Pulse compression allows radar to use long waveforms in order to obtain high energy and simultaneously achieve the resolution of a short pulse by internal modulation of the long pulse. This technique can increase signal bandwidth through frequency or phase coding. Although, amplitude modulation is not forbidden but usually is not used. The received echo is processed in the receiver matched filter to produce a short pulse with duration 1/B, where

B is bandwidth of compressed pulse. This technique is of interest when the radar is not able to generate enough required power. So, a concise summary for pulse compression is gathering two opposite benefits "High Range Resolution" and "high detection probability" concurrently. It can be stated that "radar pulse compression" is a substitute for "short pulse radar", although, each one has its own advantages and difficulties.

Some privileges of short-pulse radar are as follows (Skolnik, 2001):

- range resolution
- range accuracy
- clutter reduction
- interclutter visibility
- glint reduction
- multipath resolution
- multipath height-finding
- target classification
- doppler tolerance
- ECCM
- Minimum range

and some defects of short-pulse are given below:

- Interference with other frequncy bands
- Limited maximum range

Pulse compression has all advantages related to short pulse except short minimum range. Furthermore, this technique has obviated limitation in average transmitted power belonging to short pulse. In other hand, it has two disadvantages:

- Increased complexity for generating, transmitting and processing which cause more expense.
- Appearing sidelobes in compressed pulse which result in decreased range resolution.

A twin good and bad effect of pulse compression technique can be shown by Figure 2.
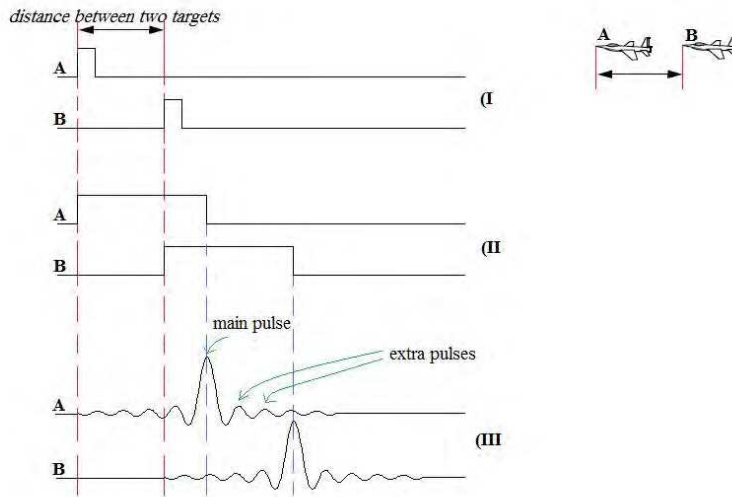
Fig. 2. Resolving targets in range (I) two resolved targets by short-pulse (II) two unresolved targets by long-pulse (III) two resolved target by using pulse compression with long-pulse

Consider two targets which can receive and reflect radar pulse. If these two reflected pulses are narrow enough, they will be separated; A-pulse and B-pulse are indicated reflected pulse from target A and B respectively (Figure 2-I). But, if these pulses are wide, they may overlap and may not be separable (Figure 2-II). If these wide pulses are passed through compression filter, two narrow pulses will be generated which can be distinguished easily (Figure 2-III). This is an efficacy of pulse compression but, one must tolerate a bad effect along with this advantage which is appearing extra pulses around the main one at the output of compression filter (Figure 2-III). This is obvious that if these side pulses have large amplitude, the radar will mistake.

Another parameter needed to introduce is pulse compression ratio which is define here:

$$\text{Pulse compression ratio} = \frac{\text{uncompressed pulse width}}{\text{compressed pulse width}}$$

And can be stated as follow:

$$pulse\ compression\ ratio = B\tau \tag{6}$$

In equation (6), B and τ are denoted as pulse bandwidth and compressed pulse width and usually $B\tau \gg 1$.

4. Different types of pulse compression technique

There are various kinds of pulse compression technique which can be categorized in two general sets. In order to be familiar to these categorizes, some common types of them are considered and since these signals have been discussed in details in many literatures, only a synoptic account of them is cited including its benefits and difficulties. For more information, the reader is referred to (Barton & Leonov, 1998; Skolnik, 2001; Farnett & Stevens, 1991; Levanon & Mozeson, 2004).

4.1 Frequency Modulation

These waveforms can entail linear (LFM) or nonlinear (NLFM) modulation of the frequency of the transmitted waveform. The summary of general characteristics of them given here (Nathanson & Cohen, 1999 ; Barton & Leonov, 1998; Farnett & Stevens, 1991):

4.1.1 Linear frequency modulation (Chirp)

Advantages:

- It is quite insensitive to doppler shifts.
- It is the easiest waveform to generate.
- There is a variety of hardware being avaiable to form and process it.

Disadvantages:

- It has range-doppler cross coupling, resulting in measurement errors unless one of the coordinates (range or doppler) is determined.
- Range sidelobes are high, compared with nonlinear FM and phase-coded waveforms. To reduce sidelobe level, weighting is usually required, resulting 1-to-2-db loss in signal to noise ratio.

4.1.2 Nonlinear frequency modulation

- It has very low range sidelobes without necessitating the use of special weighting for their supression, and hence has no signal-to-noise ratio loss as does the LFM waveform.
- It is more sensitive to doppler frequency shifts.
- It is complex and its generation techniques has been developed limitedly.

4.2 Phase coding

This waveform is one in which intrapulse modulation is obtained by subdividing the pulse into subpulses of equal duration, each having a particular phase. The phase of each subpulse is set in accordance with a given code or code sequence. Common characteristics of phase coded radar are as follows (Barton & Leonov, 1998; Farnett & Stevens, 1991):

- In comparison with LFM waveforms, they have lower range sidelobes.
- They are preferred in jamming conditions, as the coding of the transmitted signal gives an additional degree of protection against ECM.
- Their resolution performance in a dense target environment or in presence of distributed clutter can be rather poor.

- The implementation of phase coded waveforms is more complex than that of the LFM waveforms.

  This modulation can be categorized in two subsets:

4.2.1 Polyphase coding

This waveform uses codes with the number of discrete phase values greater than two. Some properties of this modulation are:

- The range sidelobes of polyphase coded waveforms can be lower than those of the binary coded ones of the same length, but the performance of these waveforms deteriorates rapildly in the presence of doppler frequency shift.
- Generation and processing of polyphase-coded waveforms use technique similar to those of frequency-coded waveforms, but their range sidelobe parameters are much better than for unweighted FM waveforms.

4.2.2 Binary coding

It is the most widely used phase coded waveform which employs two phase. In this type of pulse compression method, a long pulse with duration τ is divided into N "subpulse", each with duration τ_n, where

$$\tau_n = \frac{\tau}{N} \tag{7}$$

The phase of each segment is set to 0° and 180° in accordance with the sequence of element in the phase code, as indicated in Figure 3.
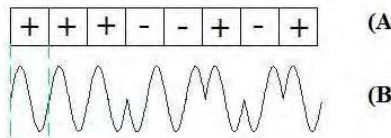

Fig. 3. Binary coded waveform.
(A) binary sequence (modulator). (B) phase coded signal.

If these pahses are selected randomly, the resulted waveform will be a noise modulated one and if they are chosen in accordance with some special patterns, the generated binary coded signal will have better function and the goal of this chapter is finding such sequences (Barton & Leonov, 1998; Farnett & Stevens, 1991; Skolnik, 2001).

By using bianry coded waveform, the effective width of matched filter output pulse is τ_n and its amplitude is N times greater than primary one. Thus, pulse compression ratio is

$$pulse\ compression\ ratio = B\tau = \frac{\tau}{\tau_n} = N \qquad (8)$$

Where, B is banwidth of modulated pulse and can be calculated by

$$B = \frac{1}{\tau_n} \qquad (9)$$

The duration of matched filter output is indeed 2τ. i.e.; in addition to main peak with width τ_n the output of this filter spreads over a 2τ period in both sides of the main lobe. These extra pulses are named time sidelobes.

Table 1. is shown a brief comparison between LFM and Biphas-coded signals (Skolnik 2001). Also, the reader can refer to a table including summary of performance of various pulse compression implementation in (Farnett & Stevens, 1991) and a good and depth overview of HRR[1] radar and comparison between several modulations mentioned in previous sections in (Cohen, 1991; Levanon & Getz, 1994).

Property	LFM	Binary phase coded pulses
Time sidelobes	Good when weighting on receive, and when a loss of about 1 dB can be tolerant	Can be equal to 1/2N, and are not easy to improve; poor doppler sidelobes
Dopller	Doppler tolerant	Requires filter bank
Ambiguity diagram	Ridge	Thumbtack (but with high sidelobes in plateau)
Pulse compression filter	Single filter can be used for transmit and receive; usually analog for high resolution	Single filter can be used for transmit and receive, but with input at opposite end; usually digital
Complexity	Less complex, especially if Strech can be used	More complex, (requires filter bank)
Application	High resolution (wide bandwidth)	Long pulses
Other	Range-doppler coupling; has been more widely used than other pulse compression	Bandwidth limited by availability of A/D converter; erroneously thought to be less susceptible to ECM spoofing

Table 1. Comparison of linear FM and Binary phase-coded pulse compression waveforms

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[1] High Range Resolution

5. Matched filter

Matched filter have a principle position in pulse compression technique, so before starting to talk about main question, its properties and characteristics are considered briefly.

The most unique characteristic of the matched filter is that it produces the maximum achievable instantaneous SNR[1] at its output when a signal plus additive white noise is present at the input. The peak instantaneous SNR at the receiver output can be achieved by matching the radar receiver transfer function to the received signal and this peak value can be calculated by

$$SNR = \frac{2E}{N_o} \tag{10}$$

Where, E and N_o are denoted as input signal energy and input noise power respectively.

Thus, we can draw the conclusion that the peak instantaneous SNR depends only on the signal energy and input noise power, and is independent of the waveform utilized by the radar.

For this peak instantaneous SNR, matched filter impulse response is:

$$h(t) = s_i^*(\tau - t) \tag{11}$$

Where, $s_i(t)$ is radar transmitted signal. Equation (11) indicates that the peak occurs at τ second after entering signal to matched filter.

Now, consider a radar system that uses signal $s_i(t)$, and assume that a matched filter receiver is utilized. The matched filter input signal can then be represented by

$$x(t) = Cs_i(t - t_1) + n_i(t) \tag{12}$$

Where C is constant, t_1 is an unknown time delay proportional to target range, and $n_i(t)$ is input white noise.

The matched filter output y(t) can be expressed by the convolution integral between the filter's impulse response and x(t),

$$y(t) = \int_{-\infty}^{+\infty} x(u)h(t - u)du \tag{13}$$

And by using equation (11), the matched filter output signal can be written

$$y(t) = \int_{-\infty}^{+\infty} x(u)s_i^*(\tau - t + u)du = \overline{R}_{xs_i}(t - \tau) \tag{14}$$

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[1] Signal to Noise Ratio

Where $\overline{R}_{xs_i}(t-\tau)$ is cross-correlation between x(t) and s_i(t-τ).

Therefore, the matched filter output can be computed from the cross-correlation between the radar received signal and a delayed replica of the transmitted waveform. If the input signal is the same as the transmitted signal, the output of the matched filter would be the autocorrelation function of the received (or transmitted) signal.

In practice, replicas of the transmitted waveforms are normally computed and stored in memory for use by the radar signal processor when needed (Mahafza & Alabama, 2000; Skolnik, 2001).

In pulse compression technique, initially a long pulse is generated and modulated in transmitter and in receiver, a matched filter is used to compressed signal. The matched filter output is compressed by factor equal to Bτ which is proportional to bandwidth and pulsewidth. i.e.; by utilizing long pulse and wide band modulation, it is possible to gain high compression ratio. Therefore, in using pulse compression, it is good to apply a modulation which can maximize compression ratio while having low sidelobe in compressed signal.

Sidelobe suppression technique can be used on the compressed pulse spectrum in order to reduce the side lobe levels. Usually, the cost associated with such an approach is a loss in the main lobe resolution, and a reduction in the peak value (i.e., loss in the SNR). For more information, the reader can see (Mahafza & Alabama, 2000; Baden & Cohen, 1990; Ackroyd & Ghani, 1973; Rihaczek & Golden, 1971).

6. Autocorrelation function

Since, this function have a critical position in this matter, before continuing to talk about the problem, it is good to review its definition and some important characteristics which researchers are used.

Consider a real binary sequence of length N,

$$\{a_k\}_{k=0}^{N-1}\quad,\quad a_k=\pm1 \tag{15}$$

Some definitions related to these codes are as follow (Levanon & Mozeson, 2004; Golay, 1977):

1. Its ACF[1],

$$R(n)=\sum_{i=0}^{N-|n|-1}a_ia_{i+|n|}\quad,\quad n=0,\pm1,\cdots,\pm(N-1) \tag{16}$$

2. ML[2] which is defined as the absolute maximum value of ACF,

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[1] AutoCorrelation Function
[2] MainLobe

$$ML = \max_{n}(R(n)) \tag{17}$$

3. Sidelobes which are described as maximum values of ACF except absolute one.
4. PSL which is denoted as maximum of sidelobes,

$$PSL = \max_{n \neq 0}(|R(n)|) \tag{18}$$

5. E which means energy of a sequence,

$$E = 2\sum_{n=1}^{N-1} R^2(n) \tag{19}$$

6. ISL[1] which is another parameter for measuring sidelobe levels,

$$ISL = 10\log_{10}\left(\frac{E}{N^2}\right) \tag{20}$$

7. MF[2] which is first defined by Golay,

$$MF = \frac{N^2}{E} \tag{21}$$

8. ACF is an even function,

$$R(n) = R(-n) \tag{22}$$

9. ACF is a finite length sequence,

$$R(n) = 0 \,, |n| \geq N \tag{23}$$

10. The absolute maximum value of ACF is at the origin,

$$R(n) \leq R(0) \,, \forall n \tag{24}$$

11. ACF's value for all N-bit sequences at the origion is equal and independent on codes' elements,

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[1] Integrated Sidelobe Level
[2] Merit Factor

$$R(0) = N \qquad (25)$$

12. ACF has a limitation on its values,

$$|R(n)| \le N - |n| \, , \, \forall n \qquad (26)$$

And

$$1 \le PSL \le (N-1) \qquad (27)$$

Another concept which is introduced is Allomorphic forms and psl-preserving operations. Each binary sequence can be stated in 4 forms in terms of autocorrelation function. If $R_a(n)$ is referred to ACF of sequence a_k, these three codes have ACF equal to $R_a(n)$:

- Inverse-amplitude or complement:

$$\{b_k\} \quad , \quad b_k = -a_k \quad , \quad 0 \le k \le (N-1) \qquad (28)$$

- Inverse-time or reverse:

$$\{c_k\} \quad , \quad c_k = a_{N-k-1} \quad , \quad 0 \le k \le (N-1) \qquad (29)$$

- Complement of reverse:

$$\{d_k\} \quad , \quad d_k = -a_{N-k-1} \quad , \quad 0 \le k \le (N-1) \qquad (30)$$

Then,

$$R_a(n) = R_b(n) = R_c(n) = R_d(n) \, , \, \forall n \qquad (31)$$

As PSL is related to the absolute value of ACF, there are some other forms that can have the PSL equal to a_k:

$$\{e_k\} \quad , \quad e_k = (-1)^k a_k \quad , \quad 0 \le k \le (N-1) \qquad (32)$$

$$\{f_k\} \quad , \quad f_k = (-1)^{k+1} a_k \quad , \quad 0 \le k \le (N-1) \qquad (33)$$

$$\{g_k\} \quad , \quad g_k = (-1)^k a_{N-k-1} \quad , \quad 0 \le k \le (N-1) \qquad (34)$$

$$\{h_k\} \quad , \quad h_k = (-1)^{k+1} a_{N-k-1} \quad , \quad 0 \le k \le (N-1) \qquad (35)$$

Such that,

$$R_e(n) = R_f(n) = R_g(n) = R_h(n) = (-1)^n R_a(n) , \forall n \qquad (36)$$

7. Problem definition

Now, everything is ready in order to introduce main problem and its offered solutions. As proved before, the output of the matched filter is the autocorrelation function of the input signal (without any doppler shift frequencies and considering noise). So, a good criterion for choosing biphase codes is that their autocorrelation have sidelobes as minimum as possible. Barker sequences are one of these optimum codes whose peak sidelobe levels are equal to 1. One of these codes is shown in Figure 4.A and its autocorrelation is drawn in Figure 4.B. As indicated, level of these sidelobes is -22.3 dB below the main peak.
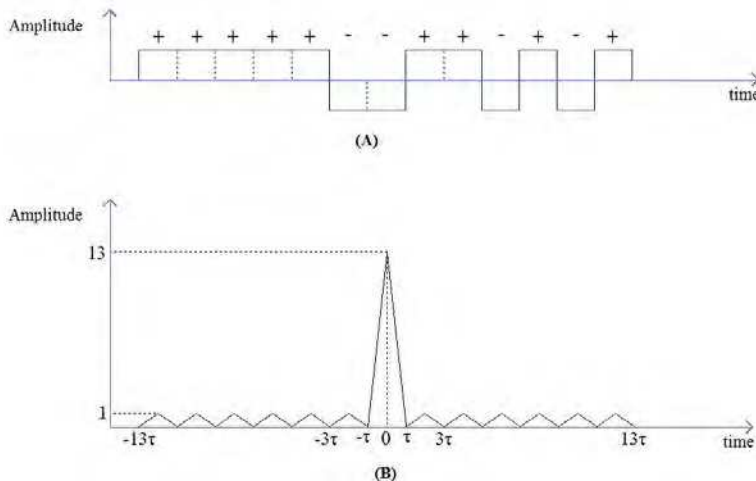

Fig. 4. Complex envelope of transmitted signal which is modulated by Barker sequence (A) Barker sequence (B) complex envelope of matched filter output

Up to now, a few numbers of these codes have been discovered as the longest found one has only 13 elements which is not appropriate for practical usage in radar. It has been shown that there is not any odd-length Barker code longer than 13. It has remained an open question for even-length Barker codes, but it is assumed that there is not any even-length one longer than 4.

Since, most practical applications require peak-to-sidelobe ratio much greater than 13, a compilation of sequences with the lowest possible sidelobes at the longer length is needed. Finding optimal sets of M phases (or codes) for different radar applications has kept radar engineers busy from the early days of radar. The number of possibilities of generating phase codes of length M is unlimited. The criteria for selecting a specific code are the resolution

properties of the resulting waveform, frequency spectrum, and the ease with which the system can be implemented. Sometimes the design is even more complicated by using different phase codes for the transmitted pulse and the reference pulse used at the receiver (possibly even with different lengths). This can improve resolution at the expense of a suboptimal signal-to-noise ratio.

The problem of finding a code that leads to a predetermined range–Doppler resolution is very complicated. A manageable problem is finding a code with a good correlation function. So, it is needed to search for codes whose autocorrelation functions have sidelobes as low as possible.

There are several parameters for measuring the sidelobe levels which are used in different conditions. The matched-filter peak sidelobe level ratio is often used to characterize the level of interference expected from point targets. For volume or surface clutter the interference level is characterized by the matched-filter integrated sidelobe level ratio. The science (or art) of designing radar signals is based on finding signals that yield a matched-filter response that matches a given application. For example, if closely separated targets are to be detected and distinguished in a low-SNR scenario, a radar signal having a matched-filter response that exhibits a narrow mainlobe (the peak) and low sidelobes is required. The mainlobe width and sidelobe level requirements are a function of the expected target separation and expected target RCS difference.

Now, the problem can be written in mathematical format:

> *It is desired to find N-bit binary sequences whose PSLs or ISLs have the minimum value among all 2^N existing codes.*

These codes are often called MPS[1] codes. Finding such these codes is classified in optimization problems and so far, no accurate and analytical solution has been found for it.

- General solutions for this optimization problem are not known (Lindner 1975).
- The search for the least autocorrelated binary sequence resembles the search of the needle in the haystack (Militzer et al.,1998).
- There is no known analytical technique to construct sequences with minimum PSL (Deng & Fan, 1999).
- Although one can identify minimum PSL sequences by conducting an exhaustive search, no general-setting solution for identifying least autocorrelated binary sequences of arbitrary bit length have been described in the literature (Ferrara, 2006).

8. A survey of the methodologies and inquiries

The search methods of finding binary sequences of desired length and PSLs are categorized in two general classes:

- Exhaustive or Global
- Partial or Local

which have their own advantages and disadvantages and are used in accordance with designer's goal.

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[1] Minimum Peak Sidelobe

8.1 Global search

Finding MPS codes involves exhaustive computer search. The only disadvantage of this method is that it takes long time, but it associates with these benefits:

1. It can reach to the absolute minimum, despite existing local ones.
2. It is able to find all optimum solutions.
3. It does not need to define any intermediate criteria and can directly search on the base of main one.

So, if all optimum solutions are required, the only key is global search. Of course, there are some limitations for this method too. Calculating of autocorrelation function for each sequence needs $N^2/2$ binary multiplications and $N^2/2$ normal summations. So, total amount of calculations needed to compute ACFs for all N-bit binary sequences are $2^N.N^2/2$ binary multiplications and the same number for normal summations. i.e.; if one bit is added to a code, the needed search time will be doubled minimally. Therefore, for large N, elapsing cpu time becomes very huge, unless special computers are used. Of course, this kind of computers has its own limitations.

The above computations are related to *simple* full search. i.e., at first, all N-bit sequences whose number is 2^N are generated and their ACF are computed. Then, optimum codes are selected.

8.2 Local search

The only advantage of this method is that it requires relatively short time. But, it suffers from these difficulties:

1. Although it finds rather reasonable answers, it can not guarantee that it is able to reach optimum ones. Since, it involves in local minimums.
2. Even if it can find some optimal answers, it is not determined whether it has found all optimum ones or not.
3. The approximation methods usually consider a primary code, then determine next codes by using an intermediate criterion. They find new code by using previous one in this manner. The intermediate criterion must satisfy the main one. In different approach, these criteria are referred to as different names like "evaluation functions", "fitness functions", "error functions", etc. In such methods, the intermediate criteria lead to a better solution, but it can be told that up to now, no such criterion have been found so that is able to navigate the search routine to the best solution. So, definition of the intermediate criterion has important position in these methods.

However, even with the most powerful computers, enumeration algorithms are only able to globally search for the best sequences with rather small length within a reasonable amount of time. Therefore, for longer length effective optimization method should be adopted to search sequences with good rather than the best aperiodic ACF properties.

8.3 History of scientific endeavors

In 1975, Lindner searched all binary sequences up to length 40 in simple-full search method and by utilizing a fast special minicomputer constructed for extensive investigation of the

correlation functions of binary sequences. It took about 50 days. For error detection purposes, inverse-amplitude and inverse-time sequences were not excluded. He summarized his results in a table and introduced several good parameters for each code length which can be used as selection criteria (Lindner, 1975). Although, he has searched all N-bit codes till length 40, he has discarded inverse-time and inverse-amplitude sequences in stating the number of optimum codes. Later on, he published his detailed results in a restored version in 2006 which included almost all optimum found sequences. The obvious important benefits of such this table can be stated in two items:

1. Before starting to search for optimum codes, by referring to this table, one can be informed the optimum PSL of each length.
2. The number of optimum reported codes can be a good criterion to check the validity and accuracy of full search algorithms.

In none-simple exhaustive method, by considering some concepts and characteristics of autocorrelation function (usually allomorphic forms), only a portion of N-element code configuration space is searched.

In 1986, Kerdoc et al. searched sequences of length 51 and found that their minimum attainable PSL is equal to 3. As Lindner, they utilized a special-purpose digital hardware designed for the task. Also, they tried to found longest binary codes which have PSLs equal to 3, 4 and 5. They claimed that there is not any code longer than 51 with PSL equal to 3 and so far, it has remained correct (Kerdoc et al., 1986).

In 1990, Cohen et al. searched all binary sequences from bit length 41 through 48. They employed psl-preserver concept to reduce the search space. Also, they introduced an innovative and recursive algorithm to search a smaller number of codes. They used PSL as selection criterion and enumerated all MPSL codes in these lengths. They completed the efforts of Kerdoc and his co-authors and noted that there are no length 49 or 50 biphase codes with peak sidelobes of three or less. As Lindner, they have excluded inverse-time and inverse-amplitude sequences in stating the number of optimum codes, but there is a difference between these two routines. Cohen has not searched them at all (Cohen et al., 1990).

In 1996, Mertens searched all binary sequences up to length 48 again, but his criterion was minimum possible energy (E_{min}) or maximizing MF. He used psl-preserver concept to reduce search time. He compiled a table of sequences with minimum energy and suggested an asymptotic value for MF in large code length (Mertens, 1996).

In 1998, Militzer et al. introduced an evolutionary algorithm and tried to determine the most suitable values for the optimization parameters of the strategy. They used MF as a criterion and showed their highest values for some lengths of skew-symmetric sequences. They compiled a table for comparison their found MF with the highest ones which others have reported (Militzer et al., 1998).

In 1999, Deng and Fan presented another new evolutionary algorithm to generate sequences with low PSL. They obtained a list of sequences of length 49-100 which were better than the other letters in most lengths at that time (Deng & Fan, 1999).

In 2001, Coxson et al. searched binary sequences up to length 69. They exploited all psl-preserver operations and therefore, were able to reduce search space more than before. They introduced a new algorithm which is induced from Cohen's one (Cohen et al., 1990) and exhibited a new version for Lindner's and Cohen's tables. They enumerated the MPSL codes

of each length up to 48 again after excluding all allomorphic forms of a code. Also, in accordance with Kerdoc claim, they tried to provide examples of PSL=4 codes for each length between 49 and 69 but their examples are correct only up to 60 (Coxson et al., 2001).

1n 2004, Coxson et al. exhibited an efficient exhaustive algorithm which exploited all psl-preserver operations too. Also, they introduced a fast method for computing the aperiodic autocorrelation function. They established its ability by finding examples of PSL=4 codes for each length from 61 through 70. Also they searched all 64-bit sequences and found all MPSL codes and exhibited all balanced ones in a table. It is the longest power of two codes that have been fully searched (Coxson & Russo, 2004).

Next, Levanon and Mozeson provided a summary of optimal PSLs for lengths up to 69 (Levanon & Mozeson, 2004).

In 2006, Ferrara described an integer programming method for generating low autocorrelation binary codes at arbitrary bit lengths. He compared PSL values and MFs (for bit length 71 through 100) of the sequences obtained with this method to the best literature-based minimal-PSL sequences and compiled a table of best minimum-PSL binary sequences for bit lengths 71 through 100. His record of length 74 was better than the other found codes (Ferrara, 2006).

In 2008, Nunn and Coxson updated table of best minimum-PSL binary sequences from bit lengths 71 through 105. For bit lengths 71 to 82, codes with PSL 4 were found. Under the generally accepted assumption that no PSL-3 binary codes exist for lengths greater than 51, they established, with near certainty, that the optimal PSL for lengths 71-82 is 4 by searching until a single PSL-4 code is discovered for each of these lengths. PSL-5 codes were produced for all lengths from 83 to 105 (Nunn & Coxson, 2008).

8.4 The authors' efforts

The problem of finding best possible PSLs for binary sequences has triggered the authors' interest form year 2005.

The first exhibited method combined several contents and gained its efficiency from Genetic algorithm. It used some other orders of allomorphic forms which reduced search spaces more than the ordinary algorithms which only use three psl-preserver concepts. Although it was a partial search method, it does not involve in local minimum. Also, it could be implemented by a simple scheme for partitioning and parallelizing the search by the fixed upper bound on PSL. Since, it used genetic algorithm, it was possible to optimize found codes by several factors simultaneously included in fitness function. Although the presented result for this algorithm is not good (a 126-bit code with PSL 11), it seems to be able to find better codes by using the better simulation (Amin & Bastani, 2006).

The other suggested approach belonged to Global search. It utilized a branch-and-bound search strategy and PSL-preserver concept. Also, it used some rules and properties of autocorrelations which reduces the configuration space more. A fast recursive method for computing ACFs of binary sequences was presented. In addition, this algorithm could be implemented in parallel mode. All these items lead to less elapsed cpu time and faster execution. For example, it was fast enough to search all codes up to length 50 without requiring any special computer or workstation. Also, this method could be easily modified for local search (Amin & Bastani, 2007).

The recent published method was based on Local one and introduced a new innovative evolutionary algorithm inspired by the Genetic algorithm (Amin & Bastani, 2006) and

Length-Increment one (Amin & Bastani, 2007). Same as other evolutionary algorithm, at first it starts by an initial population and the better the initial codes, the faster the execution is. But, it uses some rules and lemmas which results in not depending on primary population. This proposed algorithm is fast enough to yield optimum or near optimal codes, especially in long length codes. This algorithm was used to generate optimum codes longer than 200, but in a test execution on length shorter than 100 in a matter of hours, the authors were surprised by the results which were very better than others mentioned in literatures. It was able to improve 11 records of previous best mentioned PSLs. So, the authors decided to publish these found records without trying to improve them more. At that time, the paper of Dr. Coxson (Nunn & Coxson, 2008) had been accepted but the authors could not gain access to their results. Later on, I saw mentioned paper and its results. I think it is one of our proposed method excellences that some of its fast found PSLs still have remained as minimum as accessible ones. Also, a histogram for MPSL have been represented which help to visualize the results and predict longer ones. By the histogram, it is expected, not proved, that longer codes have better mainlobe to sidelobe ratio, thus better compression ratio (Amin & Bastani, 2008).

9. Results

The summary of best found PSLs is exhibited in Table 2. These results are accurate for lengths up to 82 and for upper ones, all reported values are relative and it is the Nunn's and Coxson's opinion that many of these codes (of length 83-105) are themselves optimal for PSL.

Codes with a peak sidelobe of 2 were reported for $N \leq 28$ except ones which Barker codes have been found. The MPS codes reported for $28 < N \leq 48$ and $N = 51$ have a sidelobe level of 3, and the MPS codes of length $N = 50$ and $52 \leq N \leq 82$ have a sidelobe level of 4 and the best found PSL for all upper length to 105 is 5.

As all codes of length 1-48 and 64 have been searched exhaustively and all MPSL codes of these lengths have been found, it is possible to mention the number of best optimal existing codes. These numbers are stated after excluding all allomorphic forms of codes.

Table 2. gives a single MPS code for each length. For $M \leq 48$ the listed codes are those that have, from all those with minimum peak sidelobe, the minimum integrated sidelobe (Levanon & Mozeson, 2004).

Table 2. lists codes in Hexadecimal. Each hexadecimal digit represents four binary bits, and the convention is made that, upon base conversion, any unnecessary binary digits are removed from the left side of the sequence.

Length	PSL	Example	Number
2	1	3	1
3	1	6	1
4	1	E	1
5	1	1D	1
6	2	34	4
7	1	72	1
8	2	97	8
9	2	0D7	10

10	2	167	5
11	1	712	1
12	2	977	16
13	1	1F35	1
14	2	1483	9
15	2	182B	13
16	2	6877	10
17	2	0774B	4
18	2	190F5	2
19	2	5BB8F	1
20	2	5181B	3
21	2	16BB83	3
22	3	0E6D5F	378
23	3	38FD49	515
24	3	64AFE3	858
25	2	12540E7	1
26	3	2380AD9	242
27	3	25BBB87	388
28	2	8F1112D	2
29	3	164A80E7	283
30	3	2315240F	86
31	3	2A498C0F	251
32	3	01E5AACC	422
33	3	0CCAA587F	139
34	3	333FE1A55	51
35	3	00796AB33	111
36	3	3314A083E	161
37	3	0574276F9E	52
38	3	003C34AA66	17
39	3	13350BEF3C	30
40	3	2223DC3A5A	57
41	3	038EA520364	15
42	3	04447B874B4	4
43	3	005B2ACCE1C	12
44	3	0FECECB2AD7	15
45	3	02AF0CC6DBF6	4
46	3	03C0CF7B6556	1
47	3	069A7E851988	1
48	3	156B61E64FF3	4
49	4	012ABEC79E46F	-
50	4	025863ABC266F	-
51	3	0E3F88C89524B	-
52	4	0945AE0F3246F	-
53	4	0132AA7F8D2C6F	-

54	4	0266A2814B3C6F	-
55	4	04C26AA1E3246F	-
56	4	099BAACB47BC6F	-
57	4	01268A8ED623C6F	-
58	4	023CE545C9ED66F	-
59	4	049D38128A1DC6F	-
60	4	0AB8DF0C973252F	-
61	4	005B44C4C79EA350	-
62	4	002D66634CB07450	-
63	4	04CF5A2471657C6F	-
64	4	4090A2E9E63237C2	1859
65	4	002DC0B0D9BCE5450	-
66	4	0069B454739F12B42	-
67	4	20506C9AB1E909CC2	-
68	4	009E49E3662A8EA50	-
69	4	026FDB09A83A118E15	-
70	4	1A133B4E3093EDD57E	-
71	4	63383AB6B452ED93FE	-
72	4	E4CD5AF0D054433D82	-
73	4	1B66B26359C3E2BC00A	-
74	4	36DDBED681F98C70EAE	-
75	4	6399C983D03EFDB556D	-
76	4	DB69891118E2C2A1FA0	-
77	4	1961AE251DC950FDDBF4	-
78	4	328B457F0461E4ED7B73	-
79	4	76CF68F327438AC6FA80	-
80	4	CE43C8D986ED429F7D75	-
81	4	0E3C32FA1FEFD2519AB32	-
82	4	3CB25D380CE3B7765695F	-
83	5	711763AE7DBB8482D3A5A	-
84	5	CE79CCCDB6003C1E95AAA	-
85	5	19900199463E51E8B4B574	-
86	5	3603FB659181A2A52A38C7	-
87	5	7F7184F04F4E5E4D9B56AA	-
88	5	D54A9326C2C686F86F3880	-
89	5	180E09434E1BBC44ACDAC8A	-
90	5	3326D87C3A91DA8AFA84211	-
91	5	77F80E632661C3459492A55	-
92	5	CC6181859D9244A5EAA87F0	-
93	5	187B2ECB802FB4F56BCCECE5	-
94	5	319D9676CAFEADD68825F878	-
95	5	69566B2ACCC8BC3CE0DE0005	-
96	5	CF963FD09B1381657A8A098E	-
97	5	1A843DC410898B2D3AE8FC362	-

98	5	30E05C18A1525596DCCE600DF	-
99	5	72E6DB6A75E6A9E81F0846777	-
100	5	DF490FFB1F8390A54E3CD9AAE	-
101	5	DF490FFB1F8390A54E3CD9AAE	-
102	5	2945A4F11CE44FF664850D182A	-
103	5	77FAAB2C6E065AC4BE18F274CB	-
104	5	E568ED4982F9660EBA2F611184	-
105	5	1C6387FF5DA4FA325C895958DC5	-

Table 2. Best-known binary codes.

It seems that for any peak sidelobe level there is a limit of the maximal value of *N* for which a binary sequence with that sidelobe level exists. Now, it is possible to update Kerdoc's table in Table 3.

PSL	Code Length (N)	Example
1	13	1F35
2	28	DA44478
3	51	71C077376ADB4
4	82	3CB25D380CE3B7765695F
5	105	1C6387FF5DA4FA325C895958DC5

Table 3. Longest-known binary codes for PSLs from one to five.

MPSL histogram for binary sequences up to length 105 is drawn in Figure 5. This histogram shows that, the longer the code, the smaller the PSL ratio and the best record is related to length. So, it is expected, not proved, that longer codes have better mainlobe to peak sidelobe ratio, thus better compression characteristic.
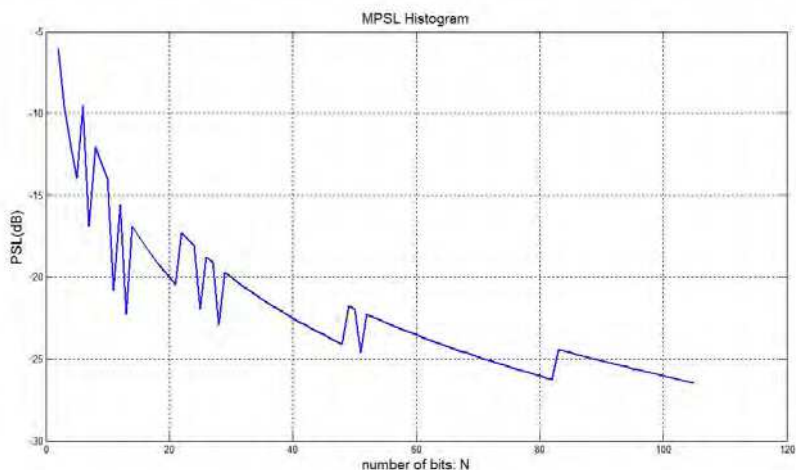

Fig. 5. MPSL Histogram in dB (absolute and relative values)

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