More Karnaugh Maps and Don’t Cares

Overview

- Karnaugh maps with four inputs
  - Same basic rules as three input K-maps
- Understanding prime implicants
  - Related to minterms
- Covering all implicants
- Using Don’t Cares to simplify functions
  - Don’t care outputs are undefined
- Summarizing Karnaugh maps

Karnaugh Maps for Four Input Functions

- Represent functions of 4 inputs with 16 minterms
- Use same rules developed for 3-input functions
- Note bracketed sections shown in example.

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

wx \ yz	00	01	11	10
00	$w'x'y'z'$	$w'x'y'z$	$w'x'yz$	$w'x'yz'$
01	$w'xy'z'$	$w'xy'z$	$w'xyz$	$w'xyz'$
11	$wxy'z'$	$wxy'z$	$wxyz$	$wxyz'$
10	$wx'y'z'$	$wx'y'z$	$wx'yz$	$wx'yz'$

(b)

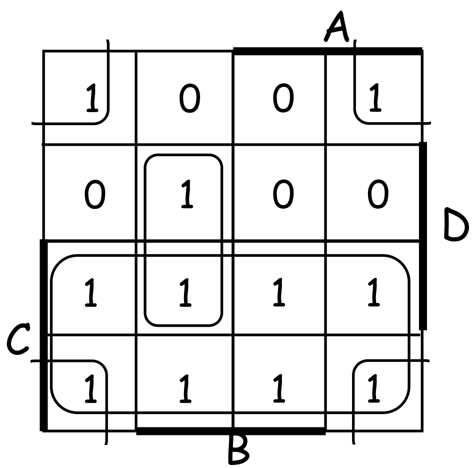
Fig. 3-8 Four-variable Map

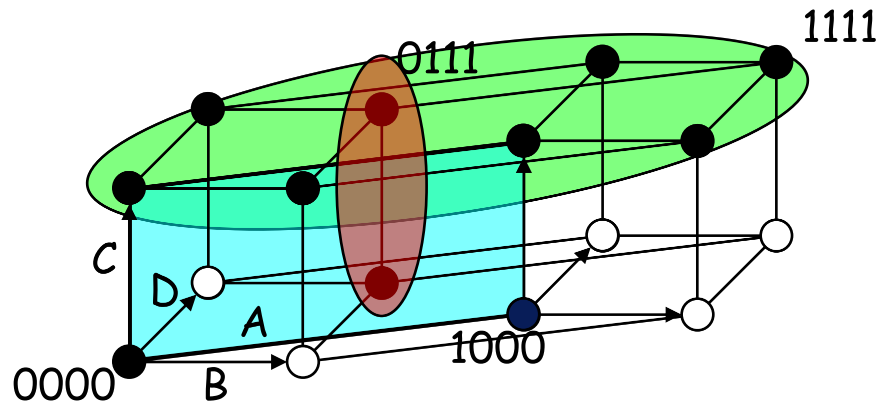
Karnaugh map: 4-variable example

- $F(A,B,C,D) = \Sigma m(0,2,3,5,6,7,8,10,11,14,15)$

  F =

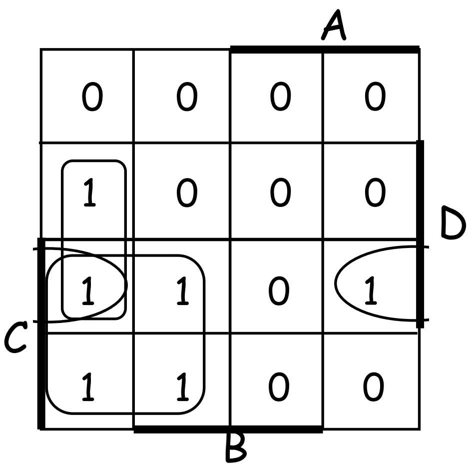
  C + A'BD + B'D'



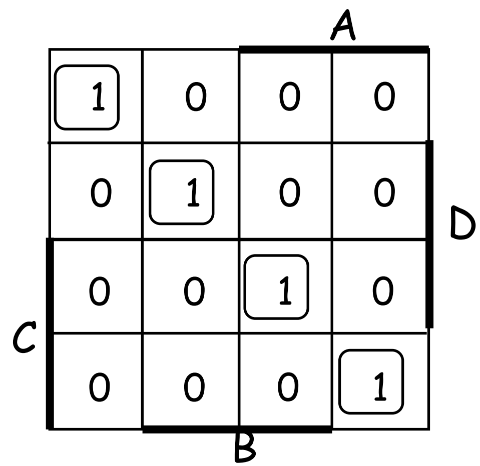


Solution set can be considered as a coordinate System!

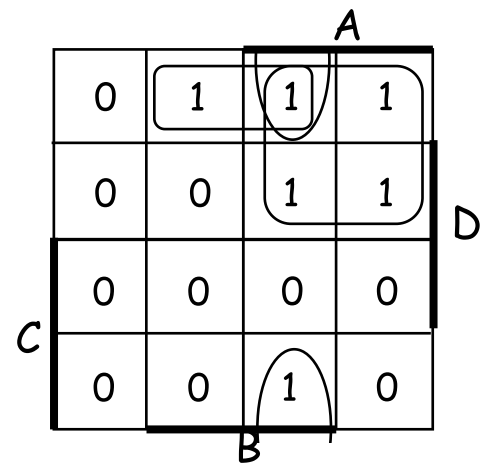
Design examples



K-map for LT



K-map for EQ



K-map for GT

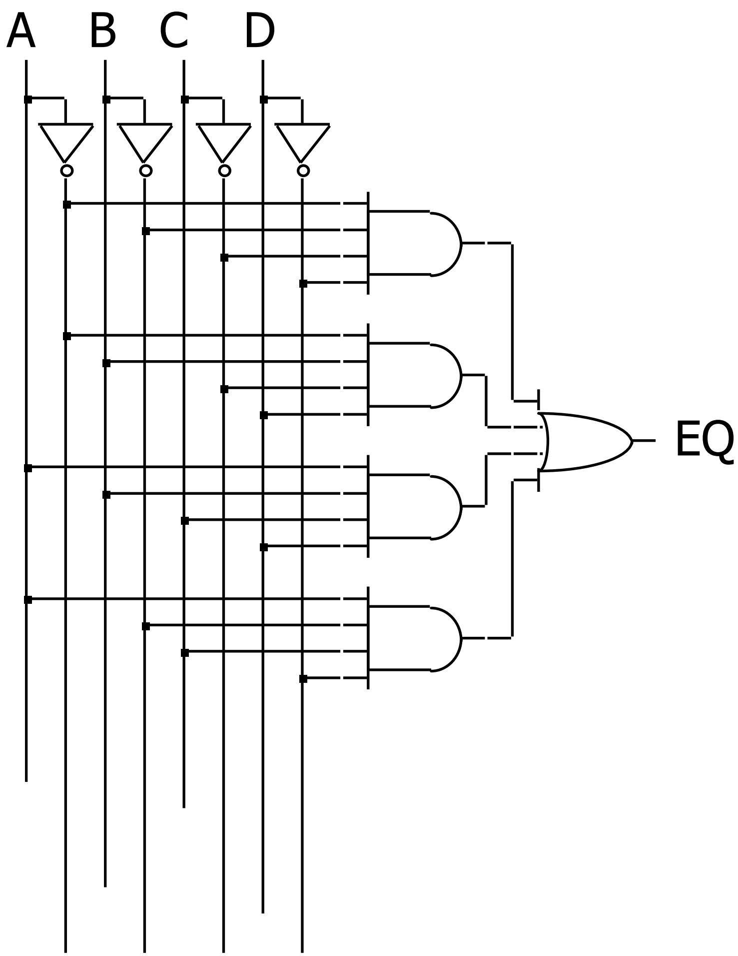
$$LT = A'\,B'\,D + A'\,C + B'\,C\,D$$

$$EQ = A'B'C'D' + A'BC'D + ABCD + AB'CD'$$

$$GT = B\,C'\,D' + A\,C' + A\,B\,D'$$

Can you draw the truth table for these examples?

Physical Implementation



- Step 1: Truth table
- Step 2: K-map
- Step 3: Minimized sum-of-products
- Step 4: Physical implementation with gates

1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

K-map for EQ

Karnaugh Maps

- **Four variable maps.**

AB \ CD	00	01	11	10
00	0	0	0	1
01	1	1	0	1
11	1	1	1	1
10	1	0	1	1

$$F = A'BC' + A'CD' + ABC$$
$$+ AB'C'D' + ABC' + AB'C$$

$$F = BC' + CD' + AC + AD'$$

- **Need to make sure all 1's are covered**
- **Try to minimize total product terms.**
- **Design could be implemented using NANDs and NORs**

Karnaugh maps: Don’t cares

- **In some cases, outputs are undefined**
- **We “don’t care” if the logic produces a 0 or a 1**
- **This knowledge can be used to simplify functions.**

A

CD \ AB	00	01	11	10
00	0	0	X	0
01	1	1	X	1
11	1	1	0	0
10	0	X	0	0

B

C

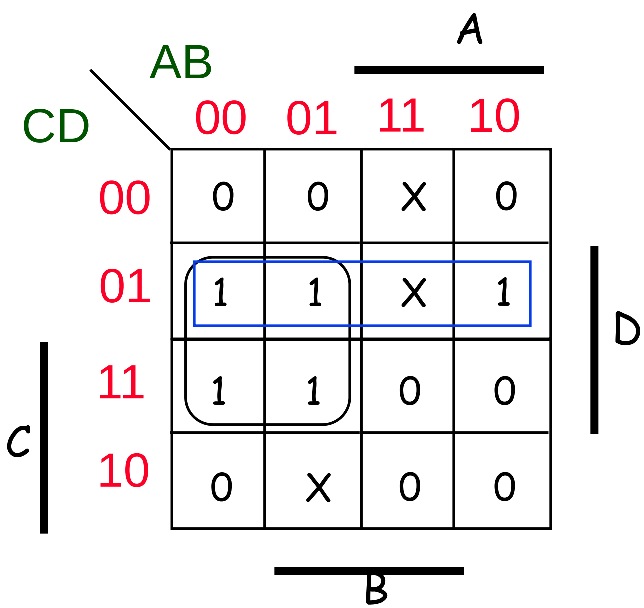
D

- Treat X’s like either 1’s or 0’s
- Very useful
- OK to leave some X’s uncovered

Karnaugh maps: Don’t cares

- $f(A,B,C,D) = \Sigma\, m(1,3,5,7,9) + d(6,12,13)$
  - without don't cares
    - f =

A'D + C'D



A	B	C	D	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	X
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	X
1	1	0	1	X
1	1	1	0	0
1	1	1	1	0

Don’t Care Conditions

- **In some situations, we don’t care about the value of a function for certain combinations of the variables.**
    - **these combinations may be impossible in certain contexts**
    - **or the value of the function may not matter in when the combinations occur**
- **In such situations we say the function is *incompletely specified* and there are multiple (completely specified) logic functions that can be used in the design.**
    - **so we can select a function that gives the simplest circuit**
- **When constructing the terms in the simplification procedure, we can choose to either cover or not cover the don’t care conditions.**

Map Simplification with Don't Cares

AB \ CD	00	01	11	10
00	0	1	0	0
01	x	x	x	1
11	1	1	1	x
10	x	0	1	1

$$F = A'C'D + B + AC$$

- **Alternative covering.**

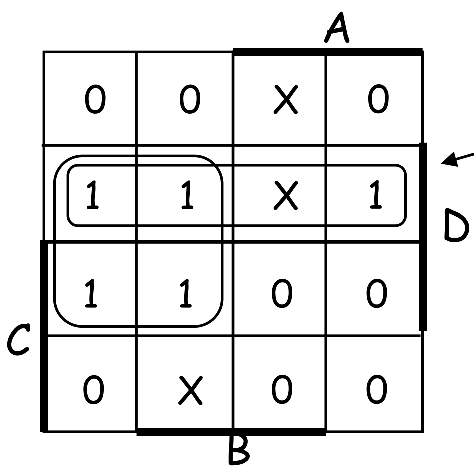
AB \ CD	00	01	11	10
00	0	1	0	0
01	x	x	x	1
11	1	1	1	x
10	x	0	1	1

$$F = A'B'C'D + ABC' + BC + AC$$

Karnaugh maps: don’t cares (cont’d)

- **f(A,B,C,D) = Σ m(1,3,5,7,9) + d(6,12,13)**
  - **f = A'D + B'C'D** **without don't cares**
  - **f =** **with don't cares**

A'D + C'D



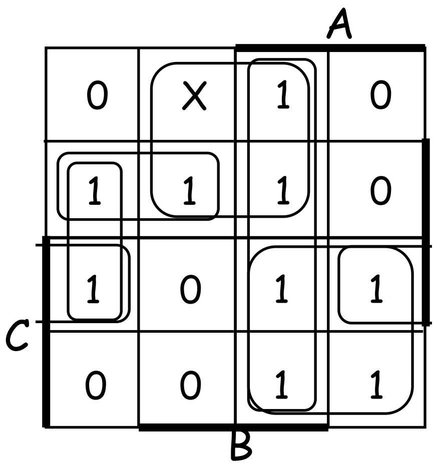
by using don't care as a "1"
a 2-cube can be formed
rather than a 1-cube to cover
this node

<u>don't cares</u> can be treated as
1s or 0s
depending on which is more
advantageous

Definition of terms for two-level simplification

- **Implicant**
  - Single product term of the ON-set (terms that create a logic 1)
- **Prime implicant**
  - Implicant that can't be combined with another to form an implicant with fewer literals.
- **Essential prime implicant**
  - Prime implicant is essential if it alone covers a minterm in the K-map
  - Remember that all squares marked with 1 must be covered
- **Objective:**
  - Grow implicant into prime implicants (minimize literals per term)
  - Cover the K-map with as few prime implicants as possible (minimize number of product terms)

Examples to illustrate terms



6 prime implicants:

A'B'D, BC', AC, A'C'D, AB, B'CD

essential

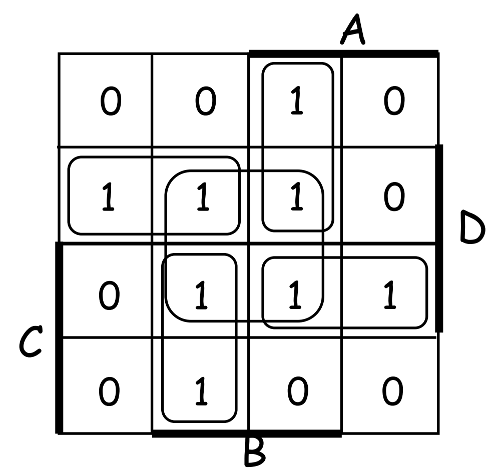
minimum cover: AC + BC' + A'B'D

5 prime implicants:

BD, ABC', ACD, A'BC, A'C'D

essential

minimum cover: 4 essential implicants

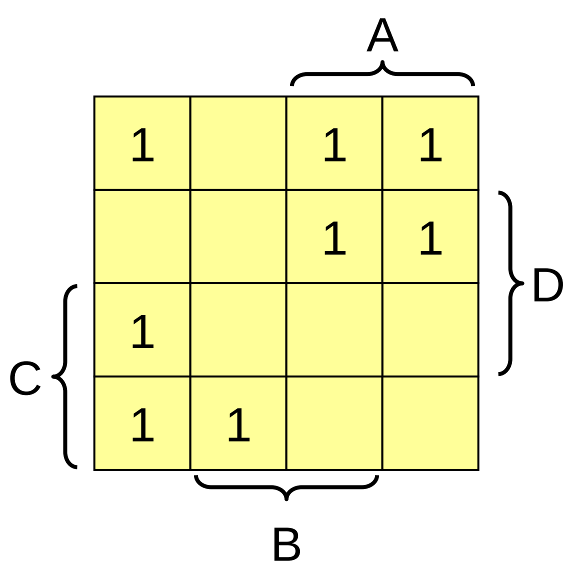


Prime Implicants

Any single 1 or group of 1s in the Karnaugh map of a function F is an implicant of F.

A product term is called a prime implicant of F if it cannot be combined with another term to eliminate a variable.

Example:



If a function F is represented by this Karnaugh Map. Which of the following terms are implicants of F, and which ones are prime implicants of F?

(a) AC'D'
(b) BD
(c) A'B'C'D'
(d) AC'
(e) B'C'D'

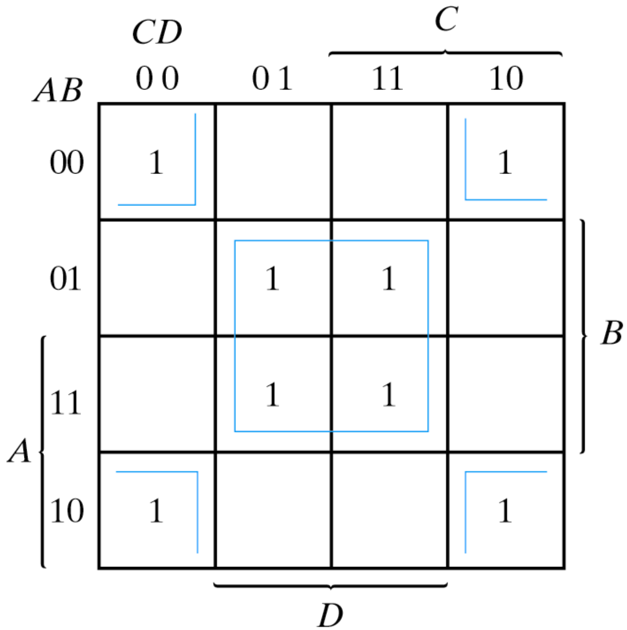
Implicants:
(a),(c),(d),(e)

Prime Implicants:
(d),(e)

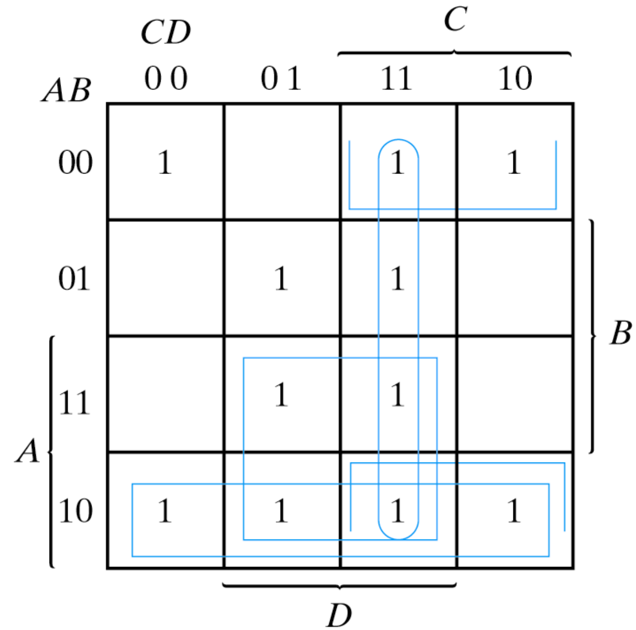
Essential Prime Implicants

A product term is an essential prime implicant if there is a minterm that is only covered by that prime implicant.

- The minimal sum-of-products form of F must include all the essential prime implicants of F.



(a) Essential prime implicants BD and B′D′



(b) Prime implicants CD, B′C AD, and AB′

Fig. 3-11 Simplification Using Prime Implicants

Summary

- K-maps of four literals considered
  - Larger examples exist
- Don’t care conditions help minimize functions
  - Output for don’t cares are undefined
- Result of minimization is minimal sum-of-products
- Result contains prime implicants
- Essential prime implicants are required in the implementation