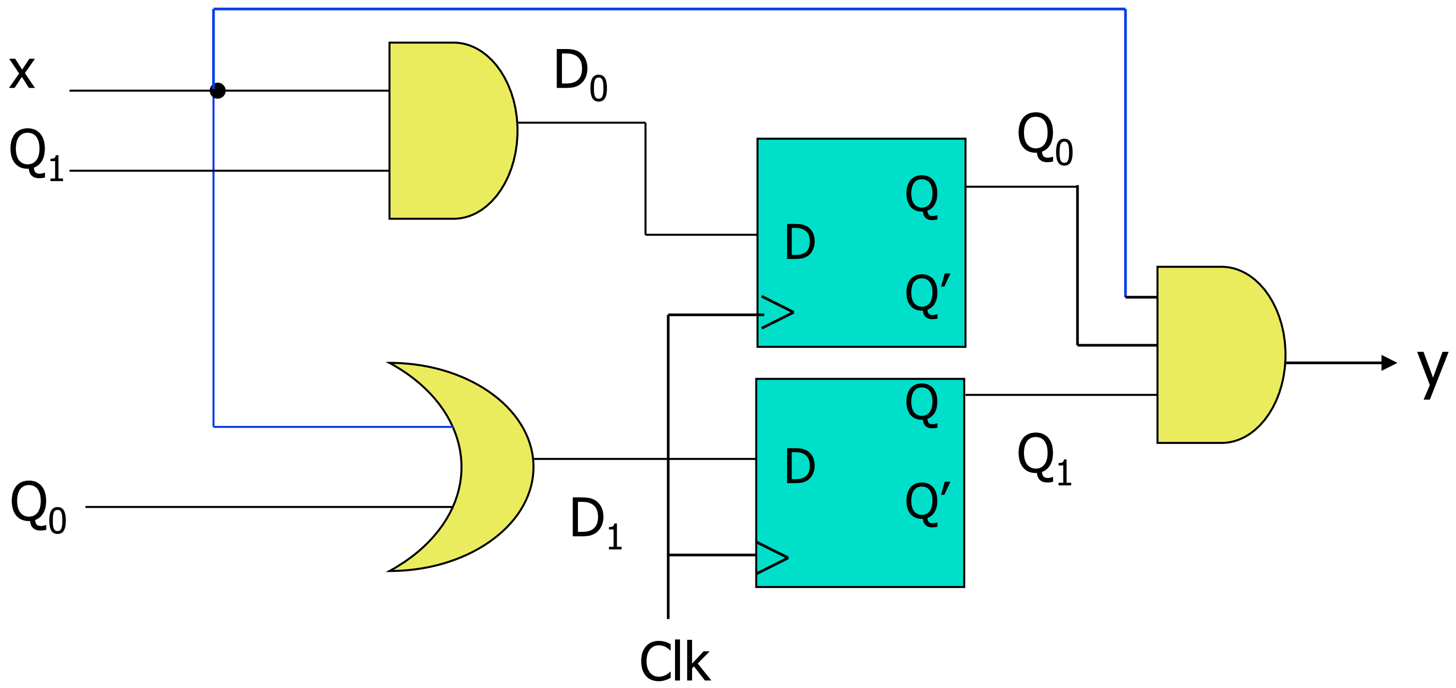
Analyzing Sequential Circuits

Overview

- Understanding flip flop state:
  - Stored values inside flip flops
- Clocked sequential circuits:
  - Contain flip flops
- Representations of state:
  - State equations
  - State table
  - State diagram
- Finite state machines
  - Mealy machine
  - Moore machine

Flip Flop State

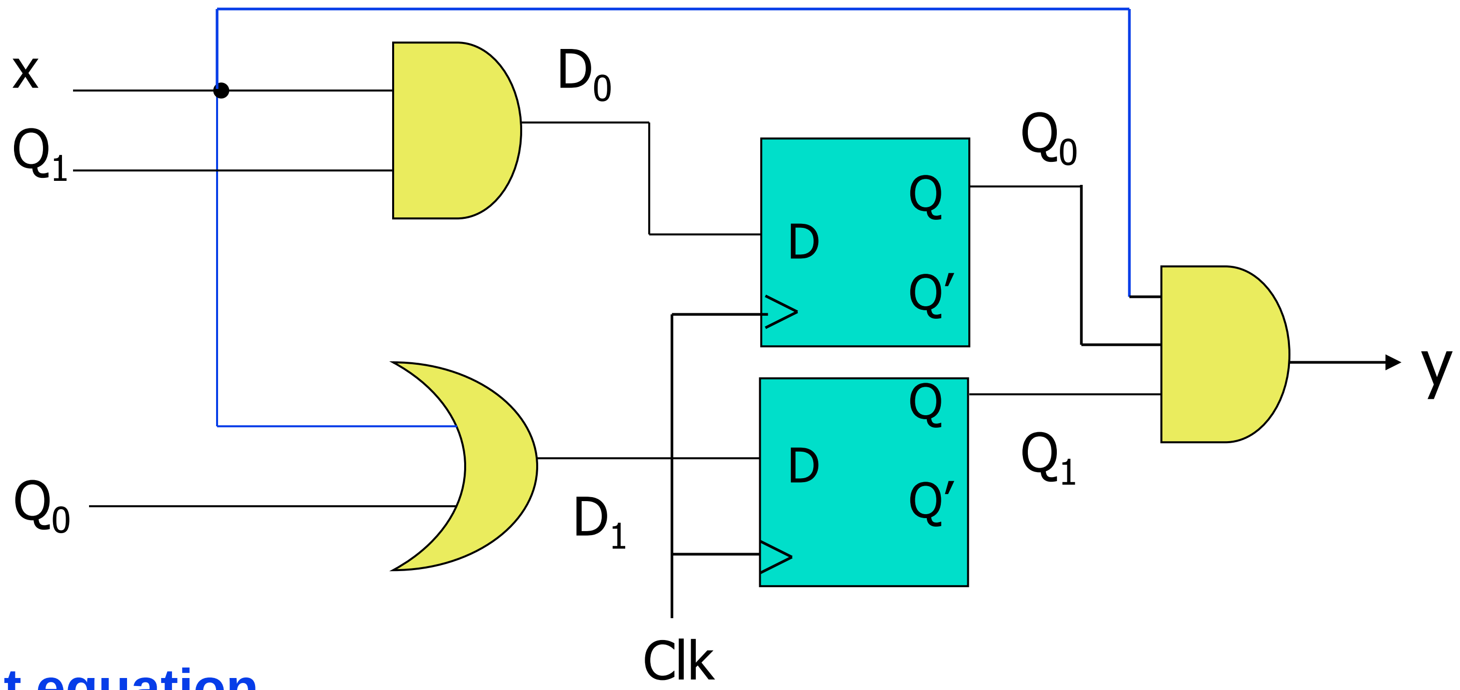
- **Behavior of clocked sequential circuit can be determined from inputs, outputs and FF state**



$$y(t) = x(t)Q_1(t)Q_0(t)$$
$$Q_0(t+1) = D_0(t) = x(t)Q_1(t)$$
$$Q_1(t+1) = D_1(t) = x(t) + Q_0(t)$$

Output and State Equations

- Next state dependent on previous state.



Output equation

State equations

$$y(t) = x(t)Q_1(t)Q_0(t)$$
$$Q_0(t+1) = D_0(t) = x(t)Q_1(t)$$
$$Q_1(t+1) = D_1(t) = x(t) + Q_0(t)$$

State Table

- Sequence of outputs, inputs, and flip flop states enumerated in state table
- Present state indicates current value of flip flops
- Next state indicates state after next rising clock edge
- Output is output value on current clock edge

State Table

Present State	Next State x=0	Next State x=1	Output x=0	Output x=1
0 0	00	10	0	0
0 1	10	10	0	0
1 0	00	11	0	0
1 1	10	11	0	1

$Q_1(t)$ $Q_0(t)$ $Q_1(t+1)$ $Q_0(t+1)$

State Table

- All possible input combinations enumerated
- All possible state combinations enumerated
- Separate columns for each output value.
- Sometimes easier to designate a symbol for each state.

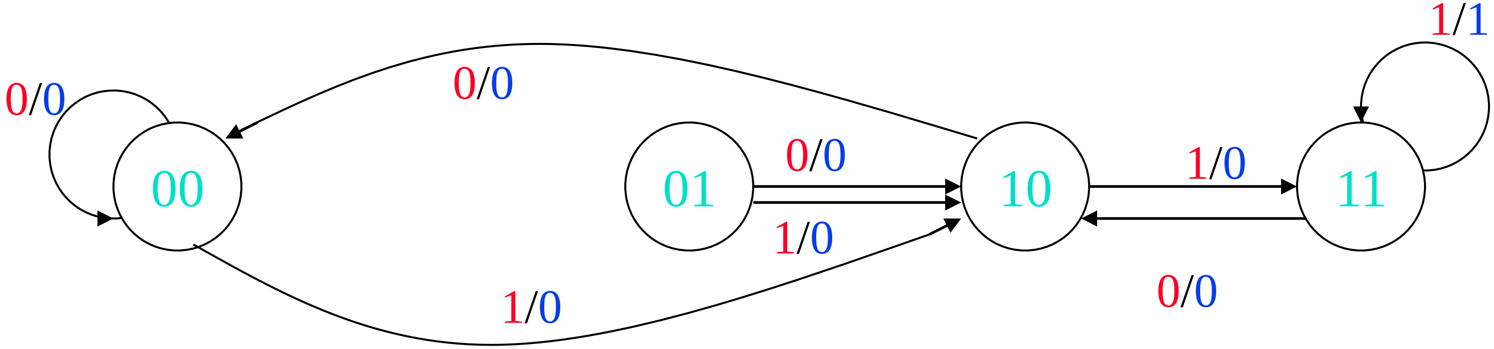
Let:
$s_0 = 00$
$s_1 = 01$
$s_2 = 10$
$s_3 = 11$

Present State	Next State x=0	Next State x=1	Output x=0	Output x=1
s_0	s_0	s_2	0	0
s_1	s_2	s_2	0	0
s_2	s_0	s_3	0	0
s_3	s_2	s_3	0	1

State Diagram

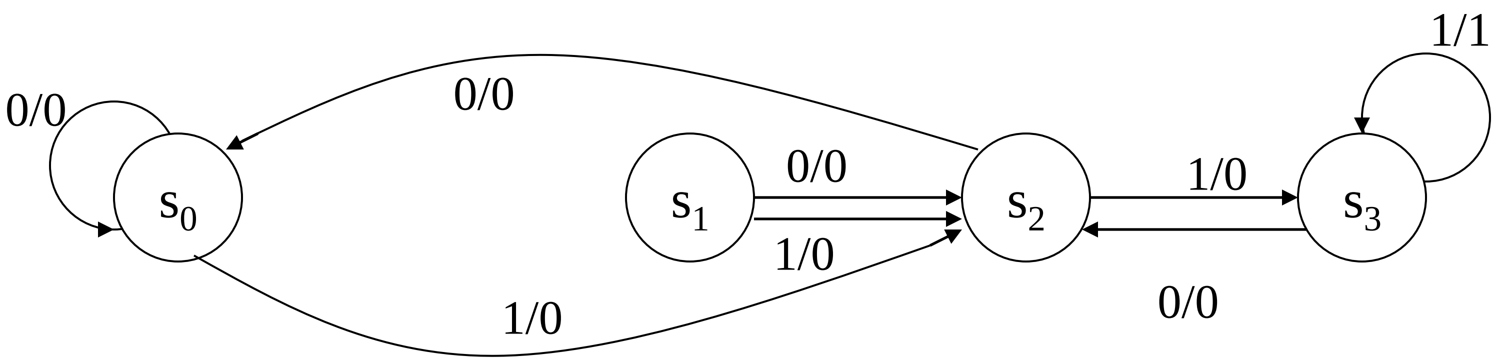
- Circles indicate current state
- Arrows point to next state
- For x/y, x is input and y is output

Present State	Next State x=0	Next State x=1	Output x=0	Output x=1
0 0	00	10	0	0
0 1	10	10	0	0
1 0	00	11	0	0
1 1	10	11	0	1



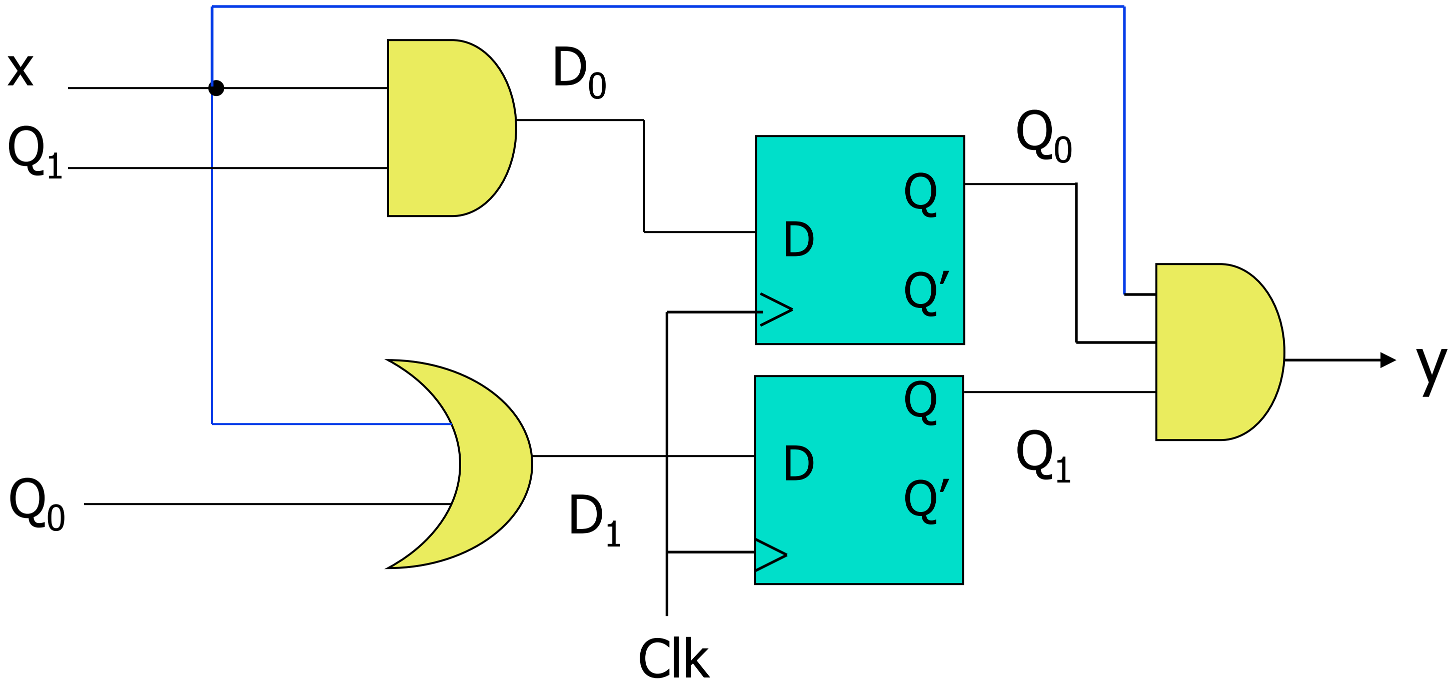
State Diagram

- Each state has two arrows leaving
  - One for x = 0 and one for x = 1
- Unlimited arrows can enter a state
- Note use of state names in this example
  - Easier to identify



Flip Flop Input Equations

- **Boolean expressions which indicate the input to the flip flops.**

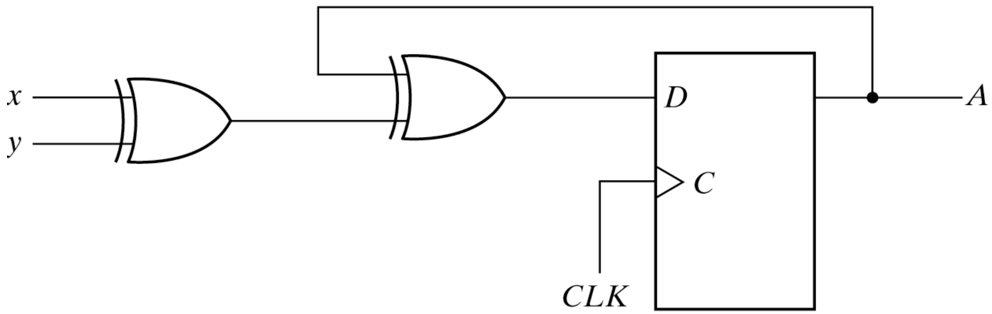


$D_{Q0} = xQ_1$
$D_{Q1} = x + Q_0$

Format implies type of flop used

Analysis with D Flip-Flops

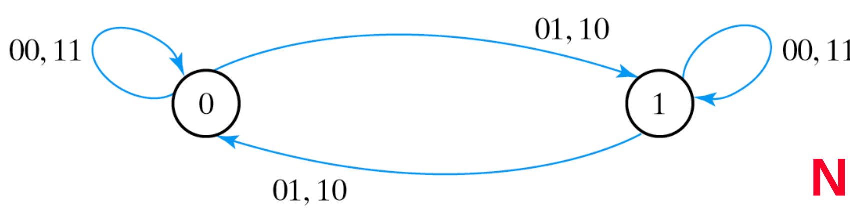
- Identify **flip flop input equations**
- Identify **output equation**



(a) Circuit diagram

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table



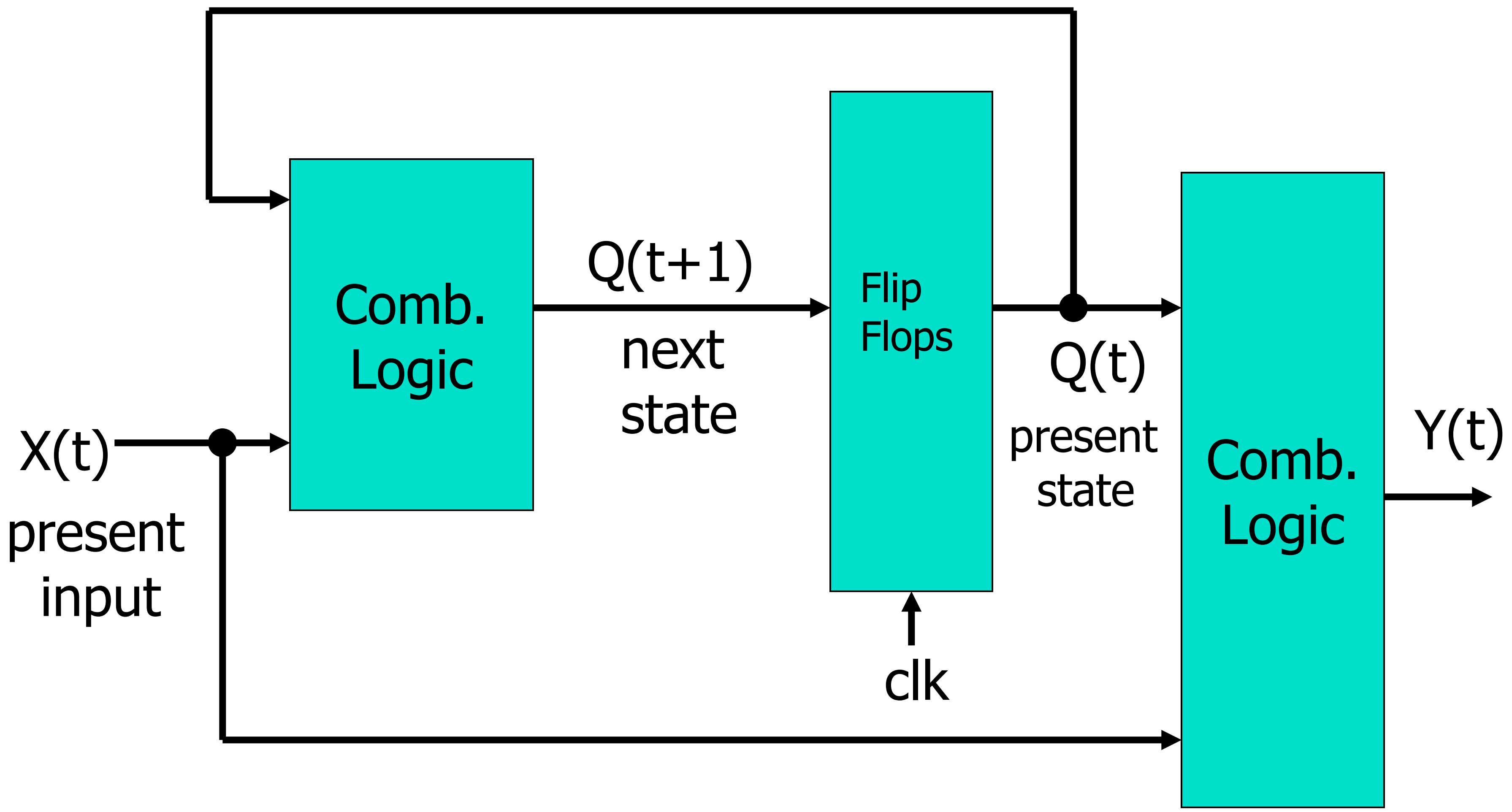
(c) State diagram

Fig. 5-17 Sequential Circuit with D Flip-Flop

Note: this example has no output

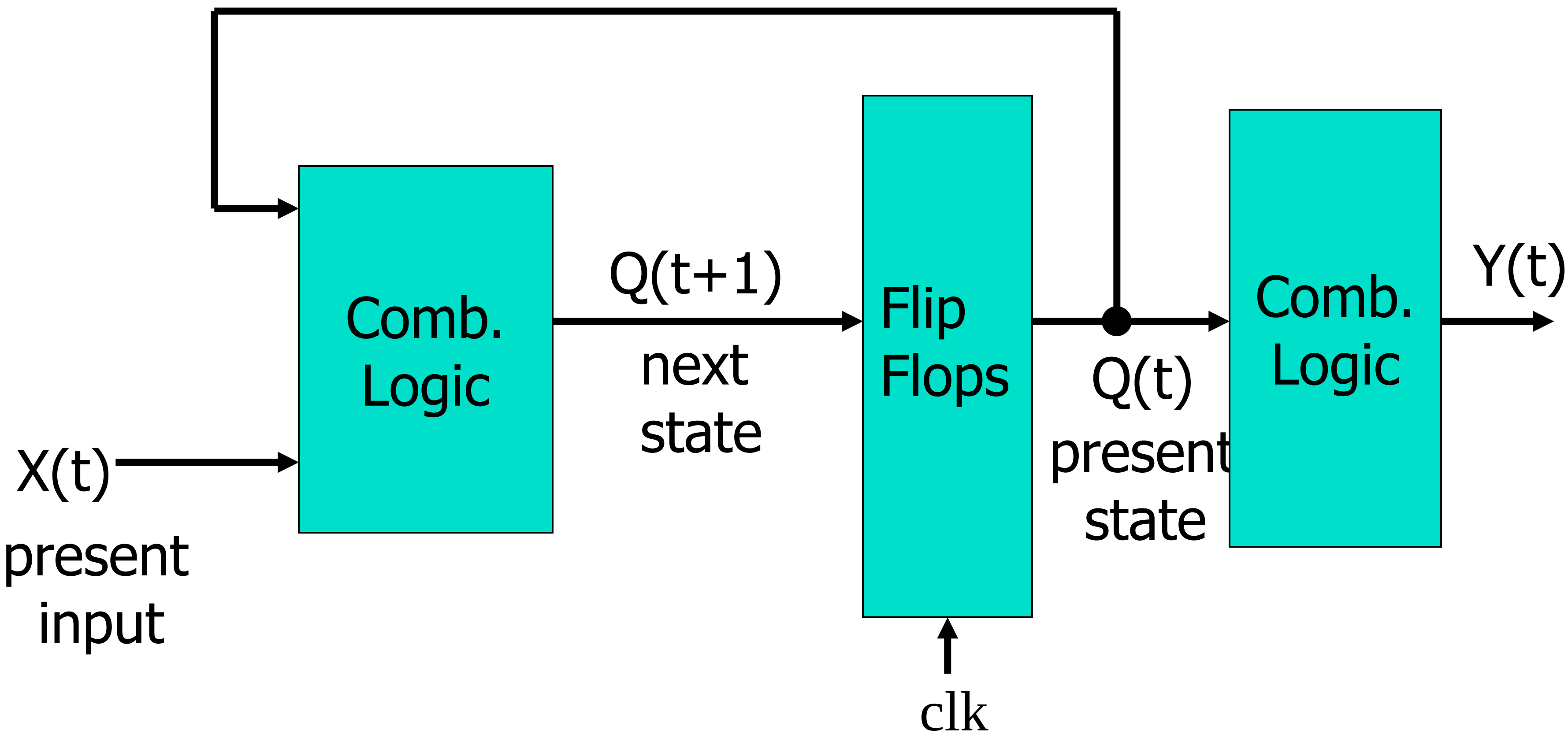
Mealy Machine

- Output based on state and present input



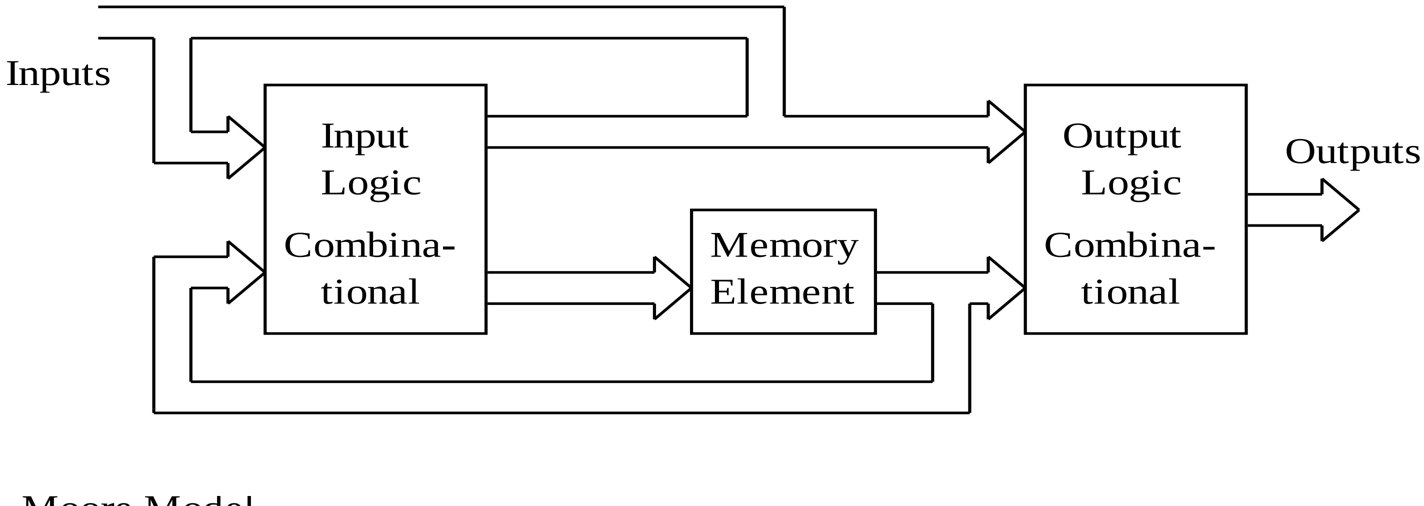
Moore Machine

- Output based on state only

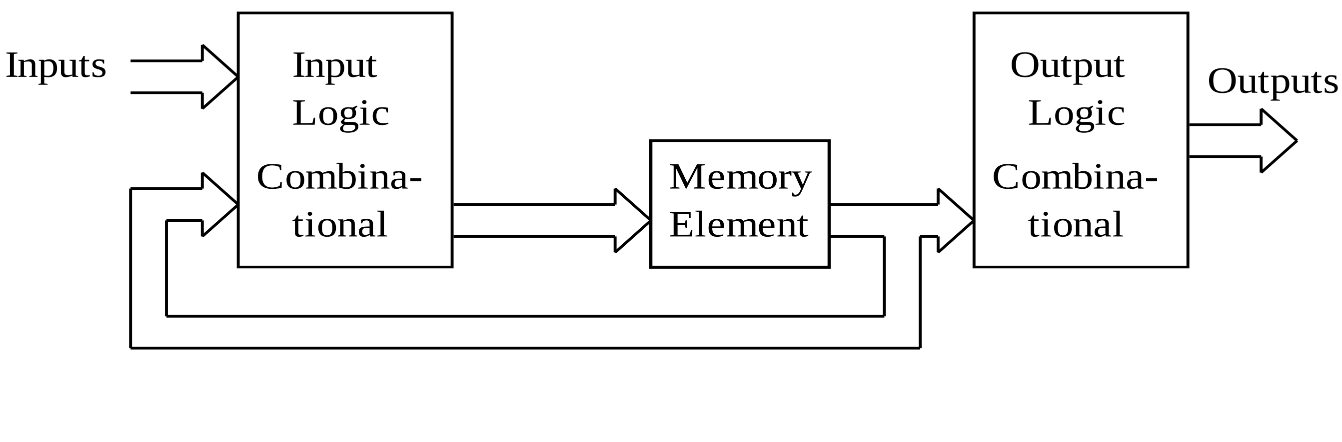


Mealy versus Moore

Mealy Model

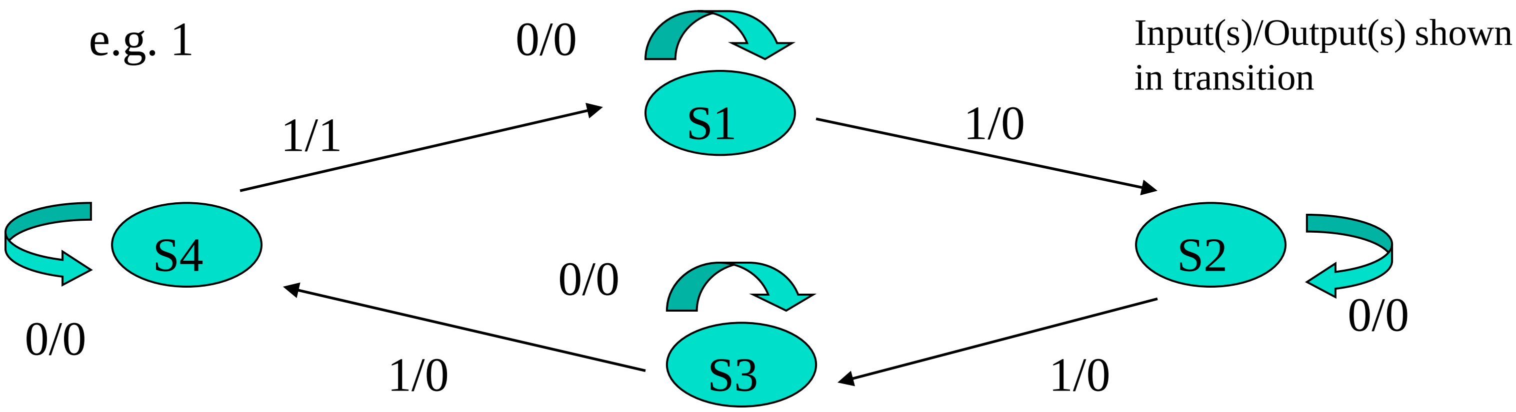


Moore Model



State Diagram with One Input & One Mealy Output

- **Mano text focuses on Mealy machines**
- **State transitions are shown as a function of inputs and current outputs.**



State Diagram with One Input & a *Moore* Output

- Moore machine: outputs only depend on the current state
- Outputs cannot change during a clock pulse if the input variables change
- Moore Machines usually have more states.
- No direct path from inputs to outputs
- Can be more reliable

Clocked Synchronous State-machine Analysis – next class

Given the circuit diagram of a state machine:

1. **Analyze the combinational logic to determine flip-flop input (excitation) equations:** $D_i = F_i\,(Q, \text{inputs})$
   - The input to each flip-flop is based upon current state and circuit inputs.
2. **Substitute excitation equations into flip-flop characteristic equations, giving transition equations:** $Q_i(t+1) = H_i(\,D_i\,)$
3. **From the circuit, find output equations:** $Z = G\,(Q, \text{inputs})$
   - The outputs are based upon the current state and possibly the inputs.
4. **Construct a state transition/output table from the transition and output equations:**
   - Similar to truth table.
   - Present state on the left side.
   - Outputs and next state for each input value on the right side.
   - Provide meaningful names for the states in state table, if possible.
5. **Draw the state diagram which is the graphical representation of state table.**

Summary

- Flip flops contain state information
- State can be represented in several forms:
  - State equations
  - State table
  - State diagram
- Possible to convert between these forms
- Circuits with state can take on a finite set of values
  - Finite state machine
- Two types of “machines”
  - Mealy machine
  - Moore machine