More Number Systems

Overview

- **Hexadecimal numbers**
  - **Related to binary and octal numbers**
- **Conversion between hexadecimal, octal and binary**
- **Value ranges of numbers**
- **Representing positive and negative numbers**
- **Creating the complement of a number**
  - **Make a positive number negative (and vice versa)**
- **Why binary?**

Understanding Binary Numbers

- **Binary numbers are made of <u>b</u>inary dig<u>it</u>s (bits):**
  - **0 and 1**
- **How many items does an binary number represent?**
  - $(1011)_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 = (11)_{10}$
- **What about fractions?**
  - $(110.10)_2 = 1x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 0x2^{-2}$
- **Groups of eight bits are called a *byte***
  - $(11001001)_2$
- **Groups of four bits are called a *nibble.***
  - $(1101)_2$

Understanding Hexadecimal Numbers

- **Hexadecimal numbers are made of <u>16</u> digits:**
  - **(0,1,2,3,4,5,6,7,8,9,A, B, C, D, E, F)**
- **How many items does an hex number represent?**
  - $(3A9F)_{16} = 3x16^{3} + 10x16^{2} + 9x16^{1} + 15x16^{0} = 14999_{10}$
- **What about fractions?**
  - $(2D3.5)_{16} = 2x16^{2} + 13x16^{1} + 3x16^{0} + 5x16^{-1} = 723.3125_{10}$
- **Note that *each* hexadecimal digit can be represented with four bits.**
  - $(1110)_{2} = (E)_{16}$
- **Groups of four bits are called a *nibble.***
  - $(1110)_{2}$

Putting It All Together

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

- **Binary, octal, and hexadecimal similar**
- **Easy to build circuits to operate on these representations**
- **Possible to convert between the three formats**

Converting Between Base 16 and Base 2

$$3A9F_{16} = \underbrace{0011}_{3}\ \underbrace{1010}_{A}\ \underbrace{1001}_{9}\ \underbrace{1111}_{F}{}_2$$

- **Conversion is easy!**
  - **Determine 4-bit value for each hex digit**
- **Note that there are $2^4 = 16$ different values of four bits**
- **Easier to read and write in hexadecimal.**
- **Representations are equivalent!**

Converting Between Base 16 and Base 8

$$3A9F_{16} = \underbrace{0011}_{3}\ \underbrace{1010}_{A}\ \underbrace{1001}_{9}\ \underbrace{1111}_{F}{}_2$$

$$35237_8 = \underbrace{011}_{3}\ \underbrace{101}_{5}\ \underbrace{010}_{2}\ \underbrace{011}_{3}\ \underbrace{111}_{7}{}_2$$

1. **Convert from Base 16 to Base 2**
2. **Regroup bits into groups of three starting from right**
3. **Ignore leading zeros**
4. **Each group of three bits forms an octal digit.**

How To Represent Signed Numbers

- **Plus and minus sign used for decimal numbers: 25 (or +25), -16, etc.**
- **For computers, desirable to represent everything as *bits*.**
- **Three types of signed binary number representations: signed magnitude, 1's complement, 2's complement.**
- **In each case: left-most bit indicates sign: positive (0) or negative (1).**

Consider ***signed magnitude***:

$0\underline{0001100}_2 = 12_{10}$

Sign bit

Magnitude

$1\underline{0001100}_2 = -12_{10}$

Sign bit

Magnitude

One's Complement Representation

- The one's complement of a binary number involves inverting all bits.
  - 1's comp of 00110011 is 11001100
  - 1's comp of 10101010 is 01010101
- For an n bit number N the 1's complement is $(2^n-1) - N$.
- Called diminished radix complement by Mano since 1's complement for base (radix 2).
- To find negative of 1's complement number take the 1's complement.

$0\underline{0001100}_2 = 12_{10}$

Sign bit Magnitude

$1\underline{1110011}_2 = -12_{10}$

Sign bit Magnitude

Two's Complement Representation

- **The two's complement of a binary number involves inverting all bits and adding 1.**
  - **2's comp of 00110011 is 11001101**
  - **2's comp of 10101010 is 01010110**
- **For an n bit number N the 2's complement is $(2^n-1) - N + 1$.**
- **Called radix complement by Mano since 2's complement for base (radix 2).**
- **To find negative of 2's complement number take the 2's complement.**

$00001100_2 = 12_{10}$

Sign bit Magnitude

$11110100_2 = -12_{10}$

Sign bit Magnitude

Two's Complement Shortcuts

- Algorithm 1 **– Simply complement each bit and then add 1 to the result.**
  - **Finding the 2's complement of $(01100101)_2$ and of its 2's complement…**

N = 01100101	**[N] =**	**10011011**
10011010		**01100100**
+ 1		**+ 1**
---------------		**---------------**
10011011		**01100101**

- Algorithm 2 **– Starting with the least significant bit, copy all of the bits up to and including the first 1 bit and then complementing the remaining bits.**
  - **N = 0 1 1 0 0 1 0 1**

    [N] = 1 0 0 1 1 0 1 1

Finite Number Representation

- **Machines that use 2's complement arithmetic can represent integers in the range**

  $$-2^{n-1} <= N <= 2^{n-1}-1$$

  where n is the number of bits available for representing N. Note that $2^{n-1}-1 = (011..11)_2$ and $-2^{n-1} = (100..00)_2$

- **For 2's complement more negative numbers than positive.**
- **For 1's complement two representations for zero.**
- **For an n bit number in base (radix) z there are z^n different unsigned values.**

  $$(0, 1, \ldots z^{n-1})$$

1’s Complement Addition

- Using 1’s complement numbers, adding numbers is easy.
- For example, suppose we wish to add $+(1100)_2$ and $+(0001)_2$.
- Let’s compute $(12)_{10} + (1)_{10}$.
  - $(12)_{10} = +(1100)_2 = 01100_2$ in 1’s comp.
  - $(1)_{10} = +(0001)_2 = 00001_2$ in 1’s comp.

Step 1: Add binary numbers
Step 2: Add carry to low-order bit

			0	1	1	0	0
Add	+		0	0	0	0	1
		0	0	1	1	0	1
Add carry							0
Final Result			0	1	1	0	1

1's Complement Subtraction

- **Using 1's complement numbers, subtracting numbers is also easy.**
- **For example, suppose we wish to subtract $+(0001)_2$ from $+(1100)_2$.**
- **Let's compute $(12)_{10} - (1)_{10}$.**
  - **$(12)_{10} = +(1100)_2 = 01100_2$ in 1's comp.**
  - **$(-1)_{10} = -(0001)_2 = 11110_2$ in 1's comp.**

Step 1: Take 1's complement of 2nd operand
Step 2: Add binary numbers
Step 3: Add carry to low order bit

```
            0 1 1 0 0
  -         0 0 0 0 1
  -----------------------
1's comp
            0 1 1 0 0
Add   +     1 1 1 1 0
  -----------------------
        1   0 1 0 1 0
Add carry           1
  -----------------------
Final
Result      0 1 0 1 1
```

2’s Complement Addition

- **Using 2’s complement numbers, adding numbers is easy.**
- **For example, suppose we wish to add +$(1100)_2$ and +$(0001)_2$.**
- **Let’s compute $(12)_{10}$ + $(1)_{10}$.**
  - **$(12)_{10}$ = +$(1100)_2$ = 01100_2 in 2’s comp.**
  - **$(1)_{10}$ = +$(0001)_2$ = 00001_2 in 2’s comp.**

Step 1: Add binary numbers
Step 2: Ignore carry bit

$$
\begin{array}{lrr}
 & & 0\;1\;1\;0\;0 \\
\text{Add} & + & 0\;0\;0\;0\;1 \\
\hline
\text{Final Result} & 0 & \boxed{0\;1\;1\;0\;1}
\end{array}
$$

Ignore

2's Complement Subtraction

- **Using 2's complement numbers, follow steps for subtraction**
- **For example, suppose we wish to subtract +$(0001)_2$ from +$(1100)_2$.**
- **Let's compute $(12)_{10}$ - $(1)_{10}$.**
  - **$(12)_{10}$ = +$(1100)_2$ = 01100_2 in 2's comp.**
  - **$(-1)_{10}$ = -$(0001)_2$ = 11111_2 in 2's comp.**

Step 1: Take 2's complement of 2nd operand
Step 2: Add binary numbers
Step 3: Ignore carry bit

		0 1 1 0 0
	−	0 0 0 0 1
2's comp		
		0 1 1 0 0
Add	+	1 1 1 1 1
Final Result	1	0 1 0 1 1

Ignore Carry

2's Complement Subtraction: Example #2

- **Let's compute $(13)_{10} - (5)_{10}$.**
  - $(13)_{10} = +(1101)_2 = (01101)_2$
  - $(-5)_{10} = -(0101)_2 = (11011)_2$
- **Adding these two 5-bit codes…**

```
               0 1 1 0 1
carry    +     1 1 0 1 1
         -------------
             1 0 1 0 0 0
```

- **Discarding the carry bit, the sign bit is seen to be zero, indicating a correct result. Indeed,**

  $(01000)_2 = +(1000)_2 = +(8)_{10}$.

2’s Complement Subtraction: Example #3

- **Let’s compute $(5)_{10} – (12)_{10}$.**
  - **$(-12)_{10} = -(1100)_2 = (10100)_2$**
  - **$(5)_{10} = +(0101)_2 = (00101)_2$**
- **Adding these two 5-bit codes…**

```
      0 0 1 0 1
+     1 0 1 0 0
---------------
      1 1 0 0 1
```

- **Here, there is no carry bit and the sign bit is 1. This indicates a negative result, which is what we expect. $(11001)_2 = -(7)_{10}$.**

Summary

- Binary numbers can also be represented in octal and hexadecimal
- Easy to convert between binary, octal, and hexadecimal
- Signed numbers represented in signed magnitude, 1's complement, and 2's complement
- 2's complement most important (only 1 representation for zero).
- Important to understand treatment of sign bit for 1's and 2's complement.