More Logic Functions: NAND, NOR, XOR

Overview

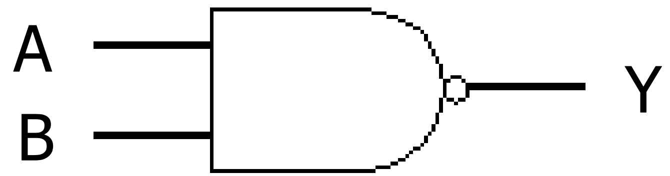
- More 2-input logic gates (NAND, NOR, XOR)
- Extensions to 3-input gates
- Converting between sum-of-products and NANDs
  - SOP to NANDs
  - NANDs to SOP
- Converting between sum-of-products and NORs
  - SOP to NORs
  - NORs to SOP
- Positive and negative logic
  - We use primarily positive logic in this course.

Logic functions of N variables

- Each truth table represents one possible function (e.g. AND, OR)
- If there are N inputs, there are 2^{2^N}
- For example, is N is 2 then there are 16 possible truth tables.
- So far, we have defined 2 of these functions
  - 14 more are possible.
- Why consider new functions?
  - Cheaper hardware, more flexibility.

x	y	G
0	0	0
0	1	0
1	0	0
1	1	1

The NAND Gate



- This is a NAND gate. It is a combination of an AND gate followed by an inverter. Its truth table shows this…
- NAND gates have several interesting properties…
  - NAND(a,a)=(aa)’ = a’ = NOT(a)
  - NAND’(a,b)=(ab)’’ = ab = AND(a,b)
  - NAND(a’,b’)=(a’b’)’ = a+b = OR(a,b)

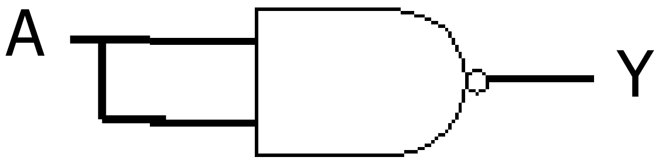
A	B	Y
0	**0**	**1**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

The NAND Gate

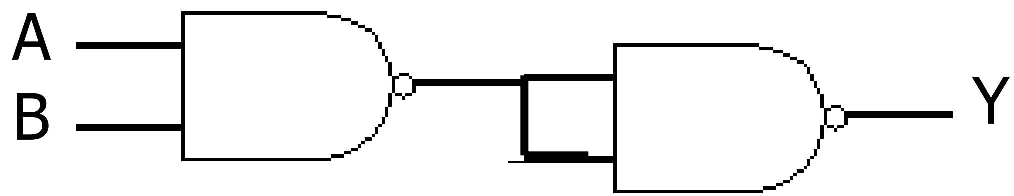
- **These three properties show that a NAND gate with both of its inputs driven by the same signal is equivalent to a NOT gate**
- **A NAND gate whose output is complemented is equivalent to an AND gate, and a NAND gate with complemented inputs acts as an OR gate.**
- **Therefore, we can use a NAND gate to implement all three of the *elementary operators* (AND,OR,NOT).**
- **Therefore,** ANY switching function can be constructed using only NAND gates**. Such a gate is said to be *primitive* or *functionally complete*.**

NAND Gates into Other Gates

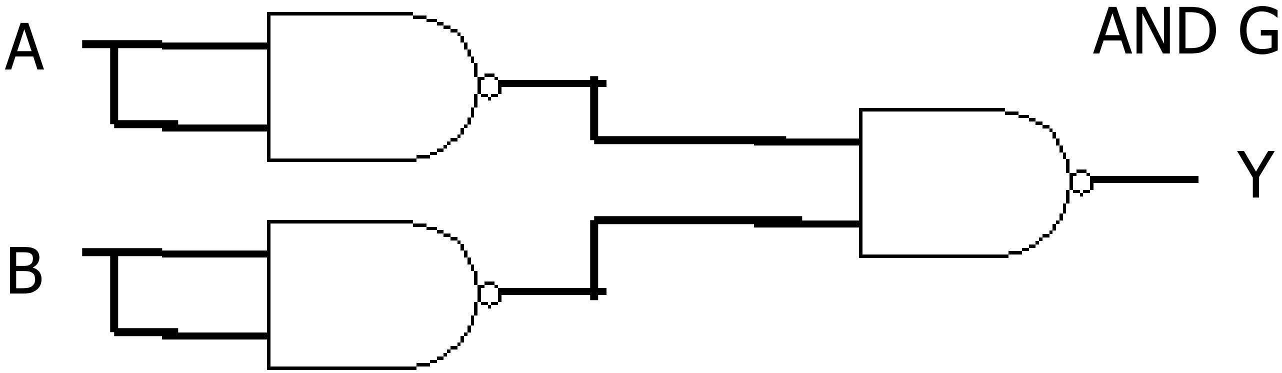
(what are these circuits?)



NOT Gate

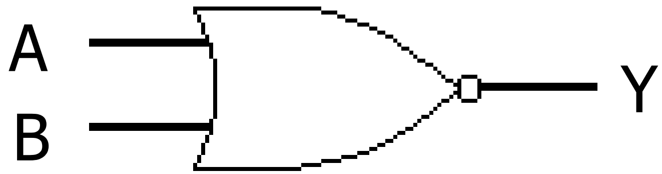


AND Gate



OR Gate

The NOR Gate



- This is a NOR gate. It is a combination of an OR gate followed by an inverter. It's truth table shows this…
- NOR gates also have several interesting properties…
  - NOR(a,a)=(a+a)' = a' = NOT(a)
  - NOR'(a,b)=(a+b)'' = a+b = OR(a,b)
  - NOR(a',b')=(a'+b')' = ab = AND(a,b)

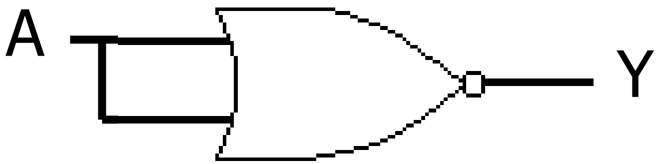
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Functionally Complete Gates

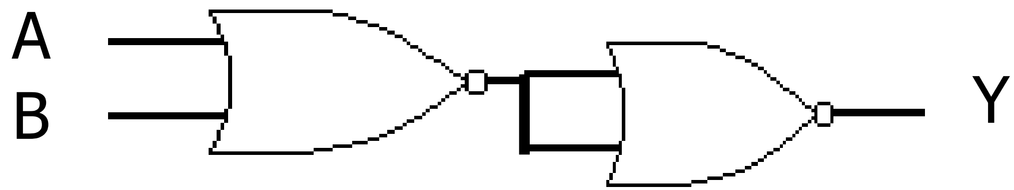
- **Just like the NAND gate, the NOR gate is functionally complete…any logic function can be implemented using just NOR gates.**
- **Both NAND and NOR gates are very valuable as any design can be realized using either one.**
- **It is easier to build an IC chip using all NAND or NOR gates than to combine AND,OR, and NOT gates.**
- **NAND/NOR gates are typically faster at switching and cheaper to produce.**

NOR Gates into Other Gates

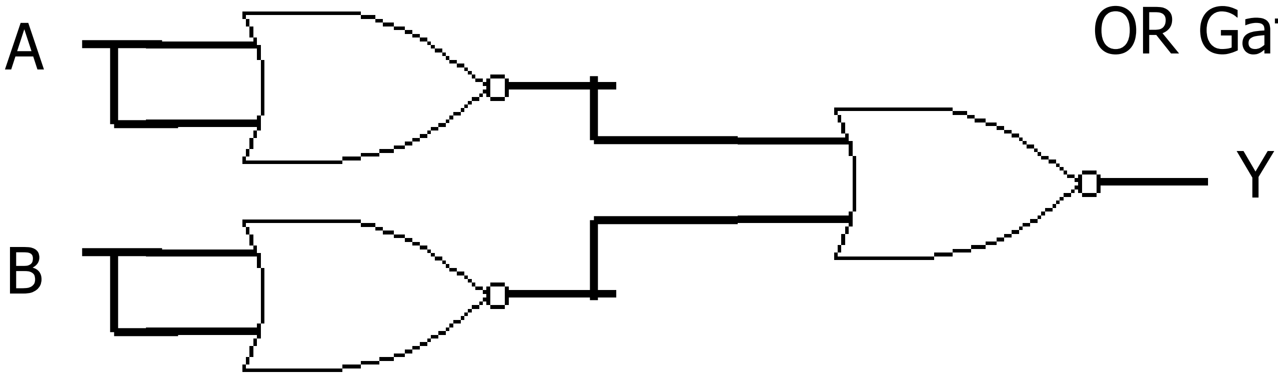
(what are these circuits?)



NOT Gate

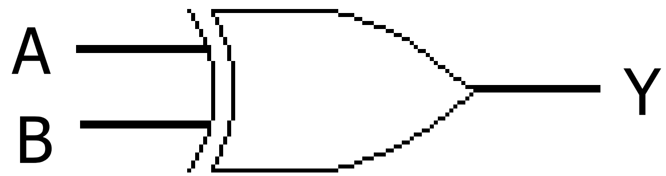


OR Gate



AND Gate

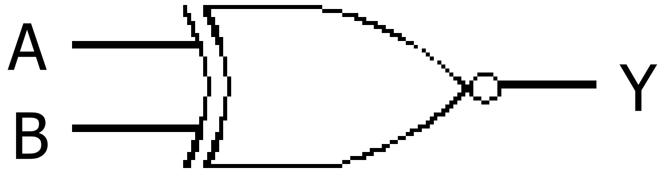
The XOR Gate (Exclusive-OR)



- **This is a XOR gate.**
- **XOR gates assert their output when exactly one of the inputs is asserted, hence the name.**
- **The switching algebra symbol for this operation is ⊕, i.e. $1 \oplus 1 = 0$ and $1 \oplus 0 = 1$.**

A	B	Y
0	**0**	**0**
0	**1**	**1**
1	**0**	**1**
1	**1**	**0**

The XNOR Gate

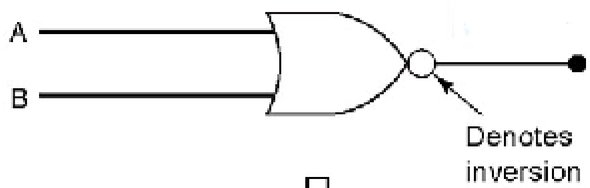


- This is a XNOR gate.
- This functions as an exclusive-NOR gate, or simply the complement of the XOR gate.
- The switching algebra symbol for this operation is ⊙, i.e. 1 ⊙ 1 = 1 and 1 ⊙ 0 = 0.

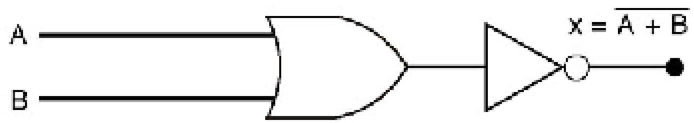
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

NOR Gate Equivalence

- **NOR Symbol, Equivalent Circuit, Truth Table**



(a)



(b)

		OR	NOR
A	B	$A + B$	$\overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

(c)

DeMorgan's Theorem

- **A key theorem in simplifying Boolean algebra expression is DeMorgan's Theorem. It states:**

  $(a + b)' = a'b'$ $(ab)' = a' + b'$

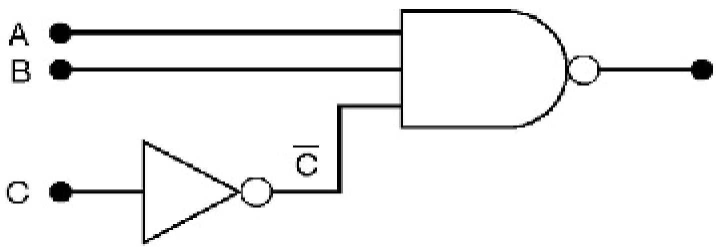
- **Complement the expression**

  $a(b + z(x + a'))$ and simplify.

$$
\begin{aligned}
(a(b+z(x + a')))' &= a' + (b + z(x + a'))' \\
&= a' + b'(z(x + a'))' \\
&= a' + b'(z' + (x + a')') \\
&= a' + b'(z' + x'a'') \\
&= a' + b'(z' + x'a)
\end{aligned}
$$

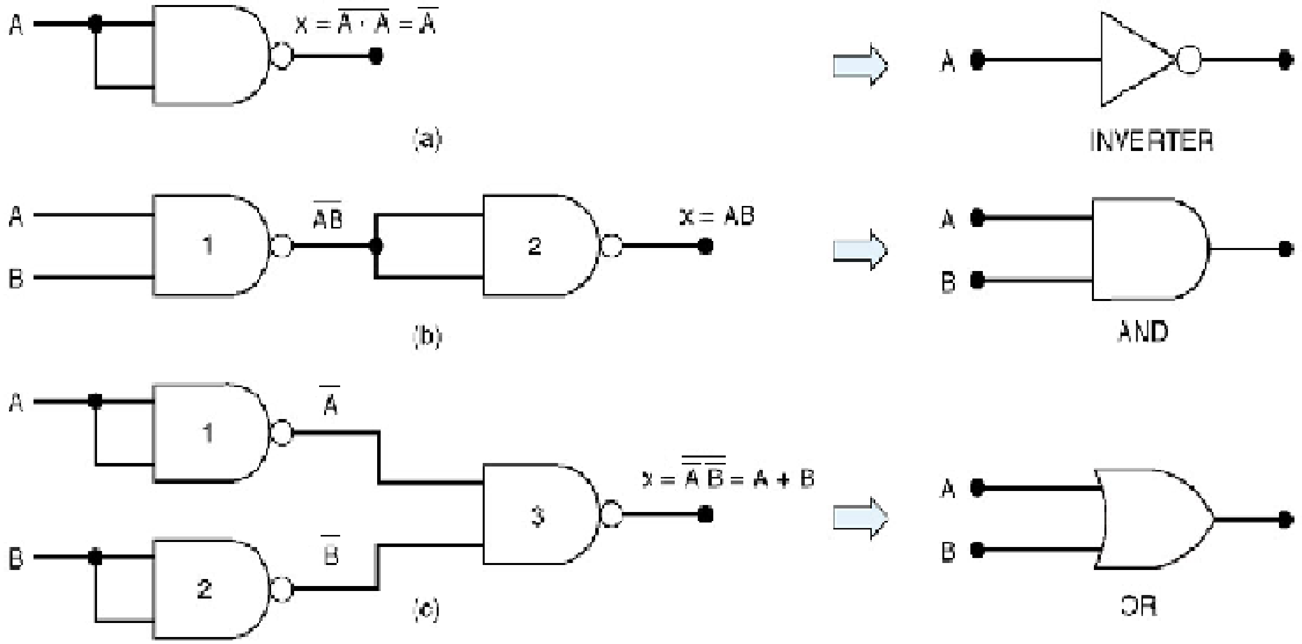
Example

- **Determine the output expression for the below circuit and simplify it using DeMorgan’s Theorem**

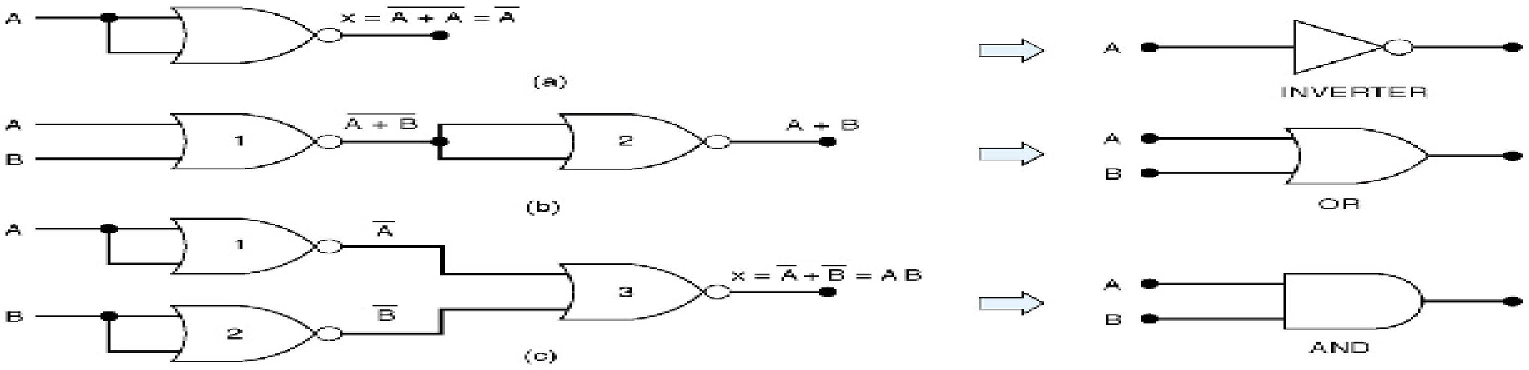


$$z = \overline{A \cdot B \cdot \overline{C}} = \overline{A} + \overline{B} + \overline{\overline{C}} = \overline{A} + \overline{B} + C$$

Universality of NAND and NOR gates

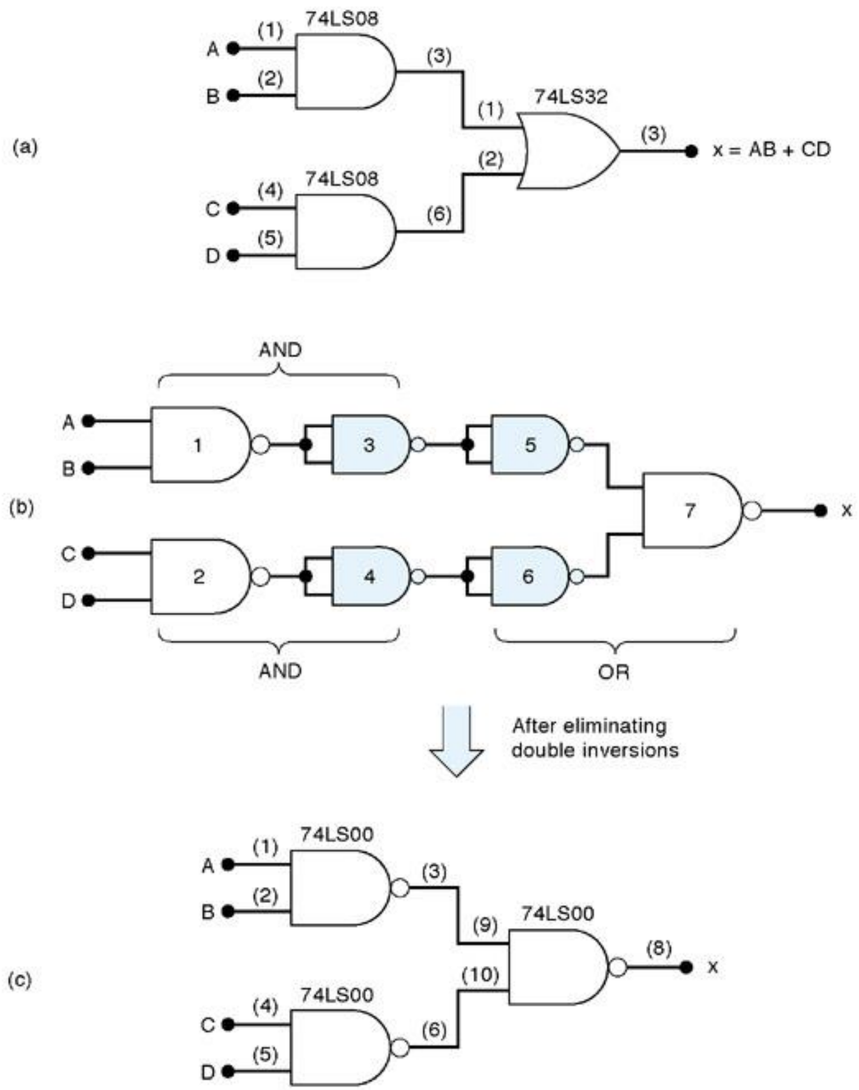


Universality of NOR gate



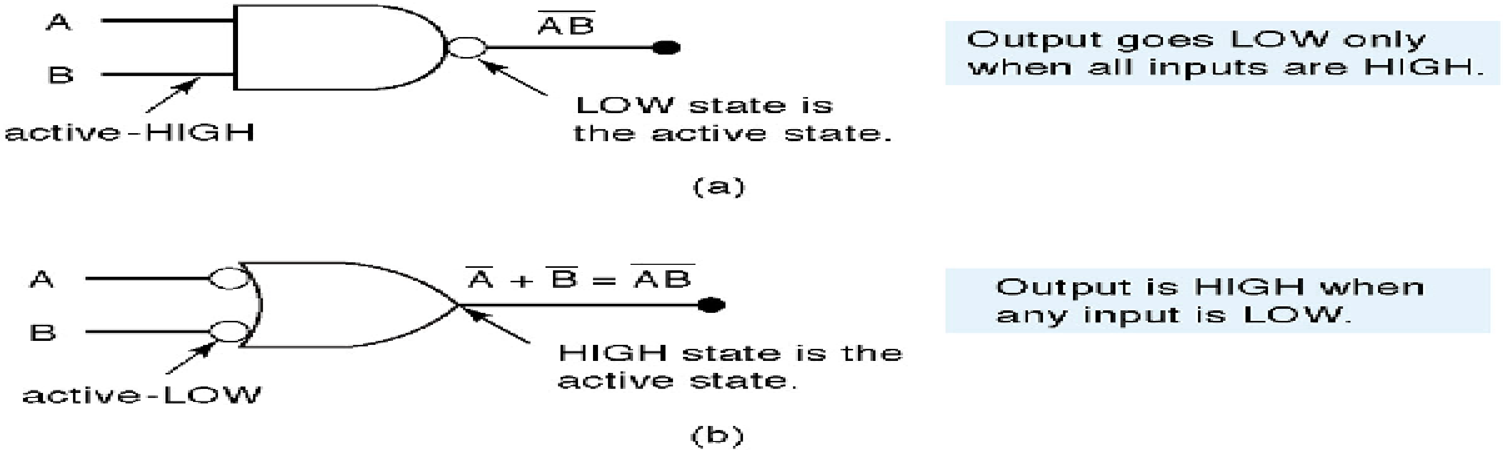
- **Equivalent representations of the AND, OR, and NOT gates**

Example



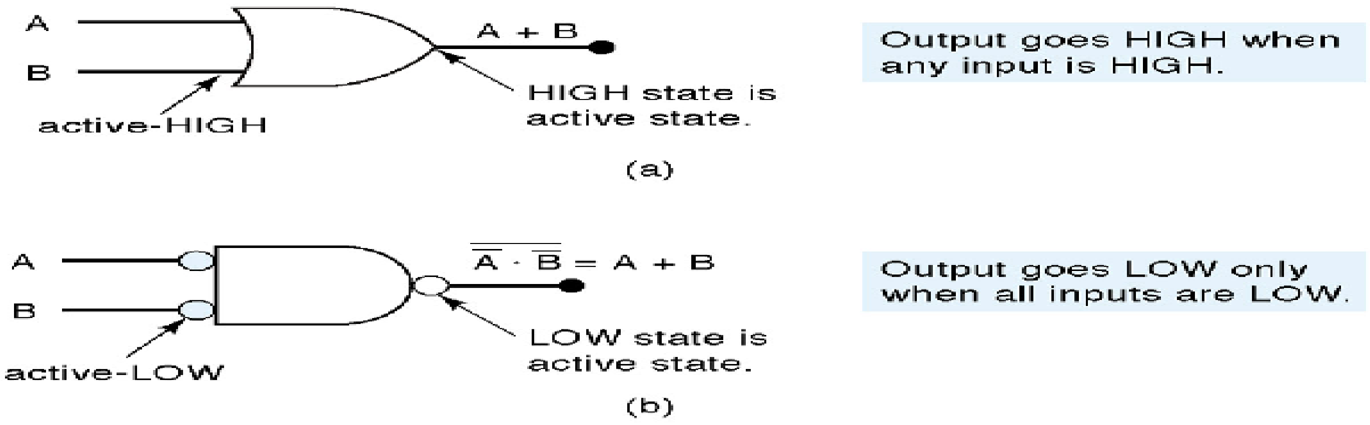
74LS08
A
B
(1)
(2)
(3)
74LS32
(1)
(2)
(3)
x = AB + CD
(a)
74LS08
C
D
(4)
(5)
(6)
AND
A
B
1
3
5
7
x
(b)
C
D
2
4
6
AND
OR
After eliminating
double inversions
74LS00
A
B
(1)
(2)
(3)
74LS00
(9)
(10)
(8)
x
(c)
74LS00
C
D
(4)
(5)
(6)

Interpretation of the two NAND gate symbols



- **Determine the output expression for circuit via DeMorgan's Theorem**

Interpretation of the two OR gate symbols



- **Determine the output expression for circuit via DeMorgan's Theorem**

Summary

- **Basic logic functions can be made from NAND, and NOR functions**
- **The behavior of digital circuits can be represented with waveforms, truth tables, or symbols**
- **Primitive gates can be combined to form larger circuits**
- **Boolean algebra defines how binary variables with NAND, NOR can be combined**
- **DeMorgan's rules are important.**
  - **Allow conversion to NAND/NOR representations**