State Reduction and Assignment

Overview

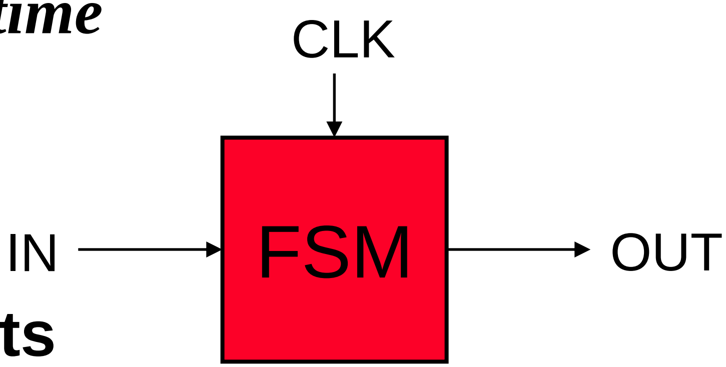
- **Important to minimize the size of digital circuitry**
- **Analysis of state machines leads to a state table (or diagram)**
- **In many cases reducing the number of states reduces the number of gates and flops**
    - **This is not true 100% of the time**
- **In this course we attempt state reduction by examining the state table**
- **Other, more advanced approaches, possible**
- **Reducing the number of states generally reduces complexity.**

Finite State Machines

- Example: Edge Detector

Bit are received one at a time (one per cycle),

such as: 000111010 → ***time***



Design a circuit that asserts

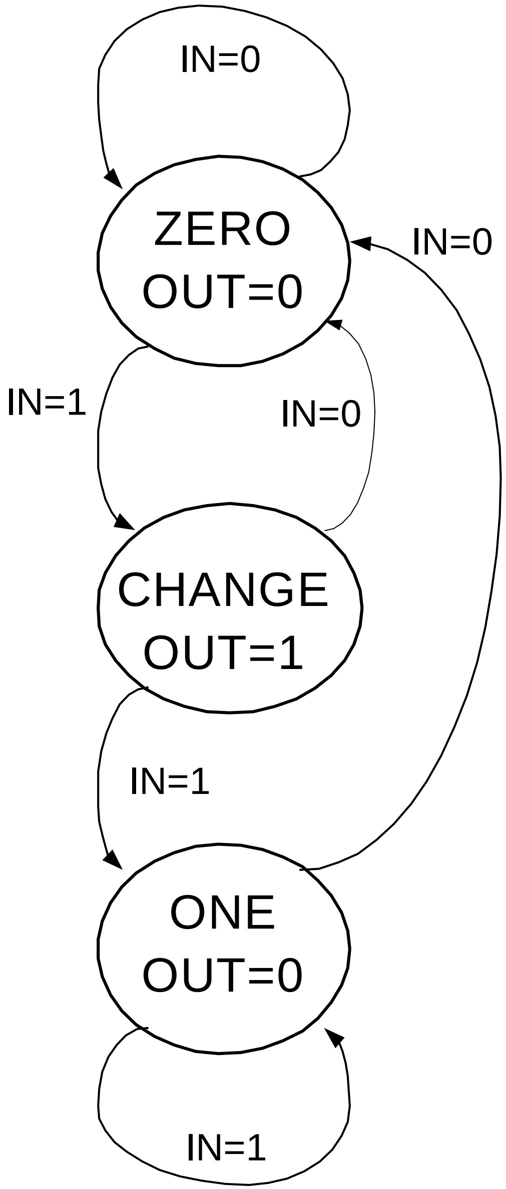
its output for one cycle when

the input bit stream changes

from 0 to 1.

Try two different solutions.

State Transition Diagram Solution A



	IN	PS	NS	OUT
ZERO	0	00	00	0
	1	00	01	0
CHANGE	0	01	00	1
	1	01	11	1
ONE	0	11	00	0
	1	11	11	0

Solution A, circuit derivation

	IN	PS	NS	OUT
ZERO	0	00	00	0
	1	00	01	0
CHANGE	0	01	00	1
	1	01	11	1
ONE	0	11	00	0
	1	11	11	0

PS

IN \ PS	00	01	11	10
0	0	0	0	-
1	0	1	1	-

$NS_1 = IN\ PS_0$

PS

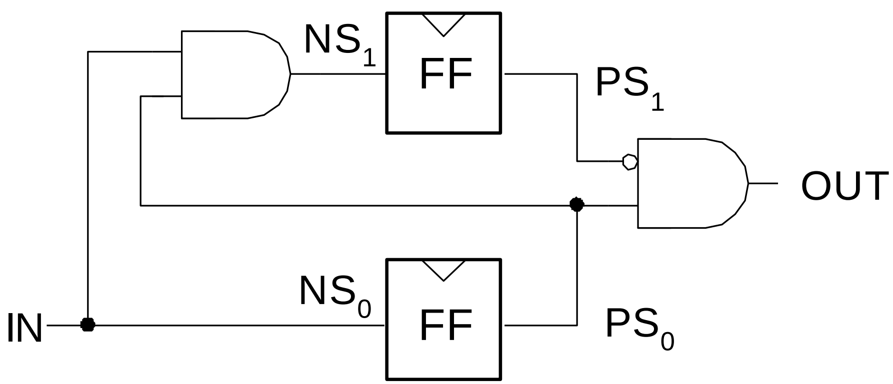
IN \ PS	00	01	11	10
0	0	0	0	-
1	1	1	1	-

$NS_0 = IN$

PS

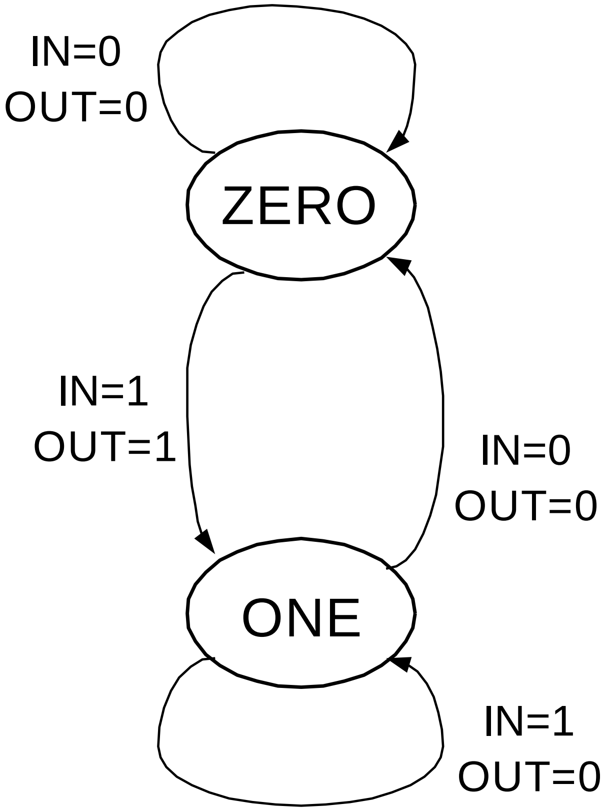
IN \ PS	00	01	11	10
0	0	1	0	-
1	0	1	0	-

$OUT = \overline{PS_1}\ PS_0$



Solution B

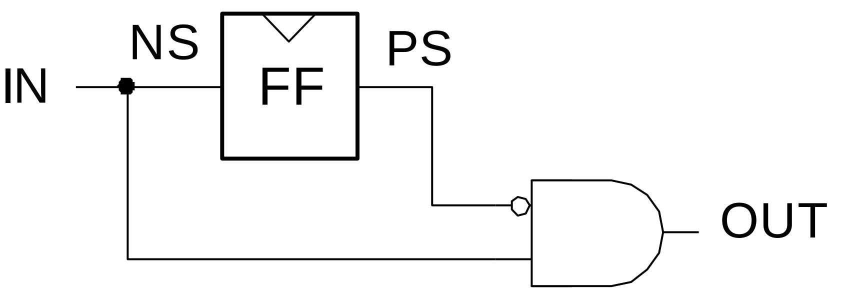
Output depends non only on PS but also on input, IN



Let ZERO=0,
ONE=1

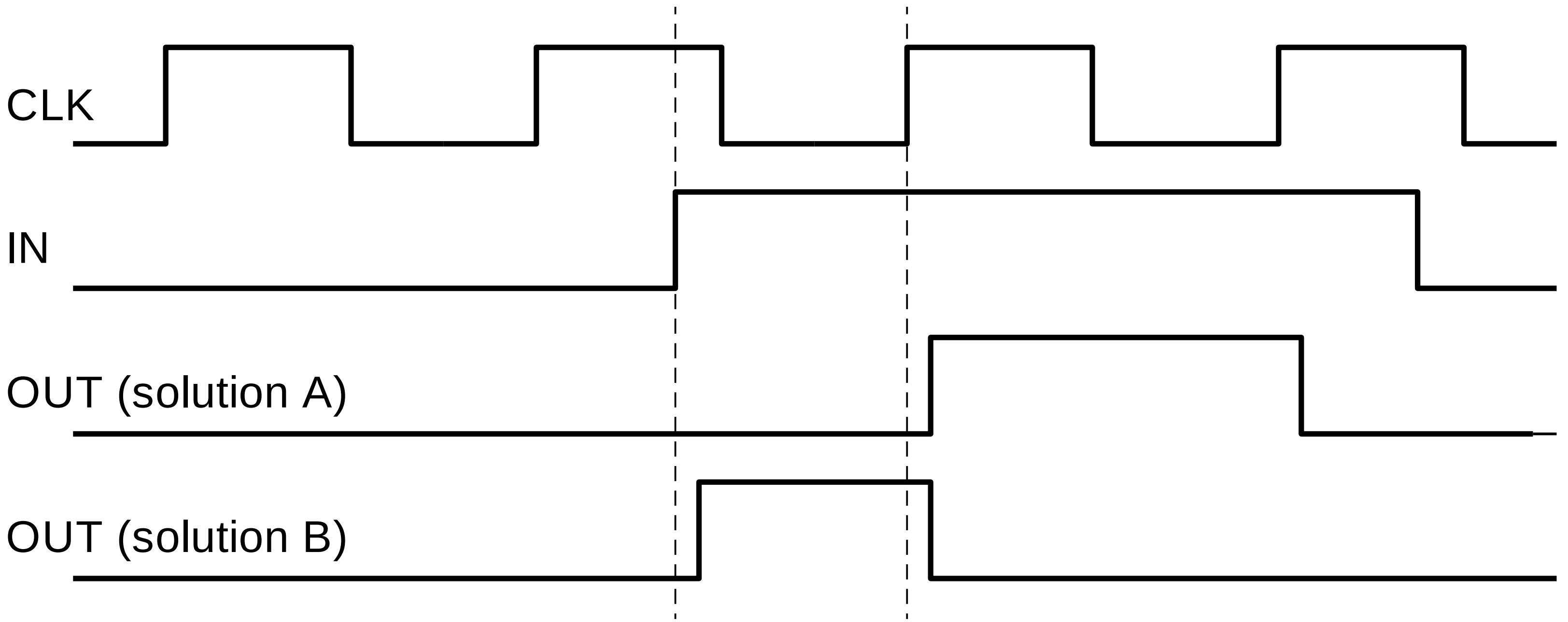
IN	PS	NS	OUT
0	0	0	0
0	1	0	0
1	0	1	1
1	1	1	0

NS = IN, OUT = IN PS’



What’s the *intuition* about this solution?

Edge detector timing diagrams



- Solution A: output follows the clock
- Solution B: output changes with input rising edge and is asynchronous wrt the clock.

FSM Comparison

Solution A

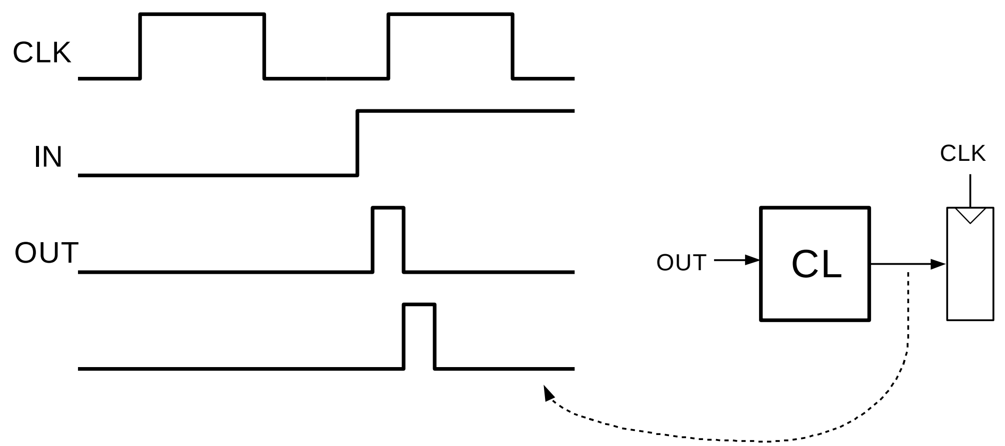
Moore Machine

- output function only of PS
- maybe <u>more</u> state
- synchronous outputs
  - no glitching
  - one cycle “delay”
  - full cycle of stable output

Solution B

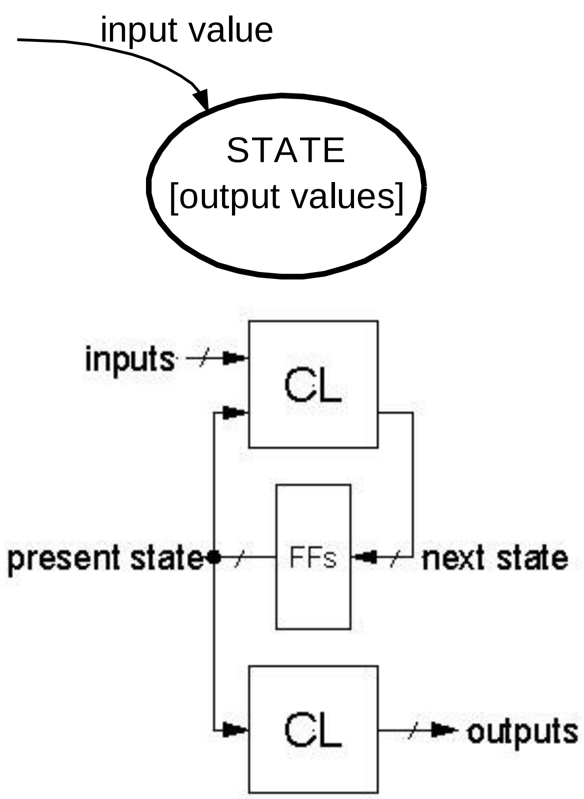
Mealy Machine

- output function of both PS & input
- maybe fewer states
- asynchronous outputs
  - if input glitches, so does output
  - output immediately available
  - output may not be stable long enough to be useful:

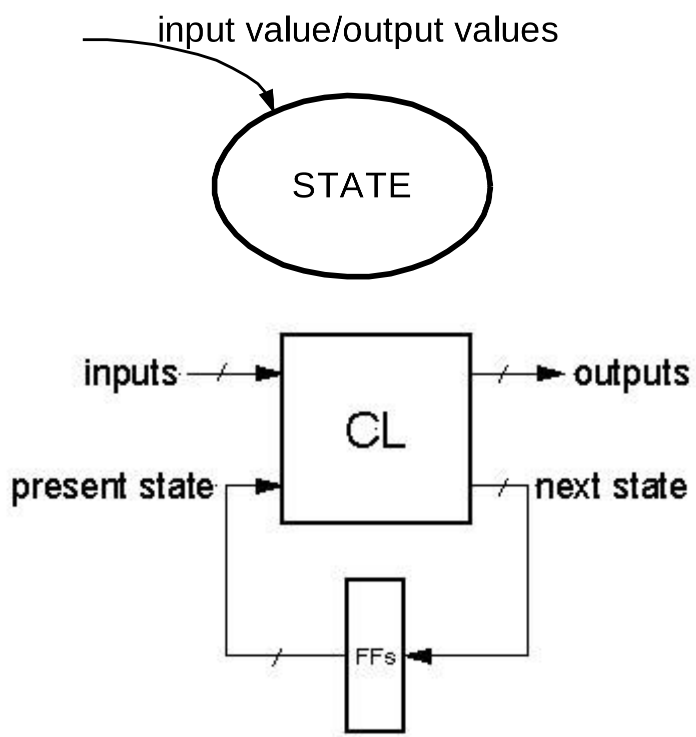


FSM Recap

Moore Machine



Mealy Machine



Both machine types allow one-hot *implementations.*

FSM Optimization

- **<u>State Reduction:</u>**

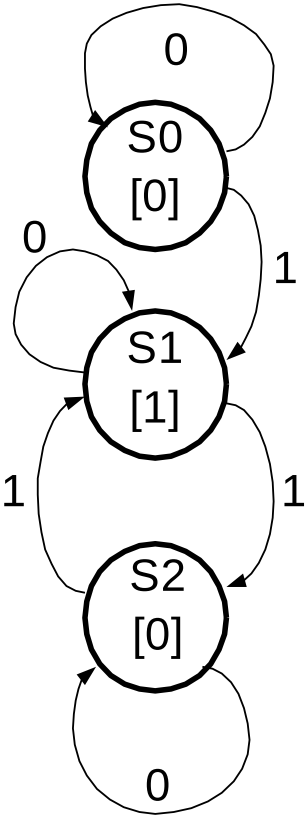
  Motivation:

  lower cost

  - **fewer flip-flops in one-hot implementations**
  - **possibly fewer flip-flops in encoded implementations**
  - **more don't cares in next state logic**
  - **fewer gates in next state logic**

  Simpler to design with extra states then reduce later.

- **Example: Odd parity checker**



Moore machine

State Reduction

- “Row Matching” is based on the state-transition table:
- **If two states**
  - **have the same output *and* both transition to the same next state**
  - ***or* both transition to each other**
  - ***or* both self-loop**
  - **then they are equivalent.**
- **Combine the equivalent states into a new renamed state.**
- **Repeat until no more states are combined**

State Transition Table

PS	NS x=0	NS x=1	output
S0	S0	S1	0
S1	S1	S2	1
S2	S2	S1	0

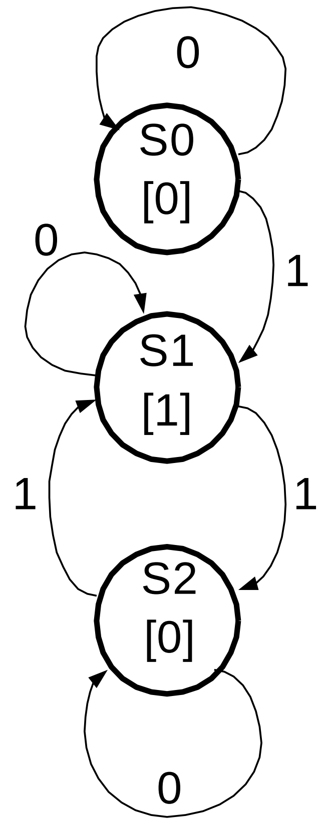
FSM Optimization

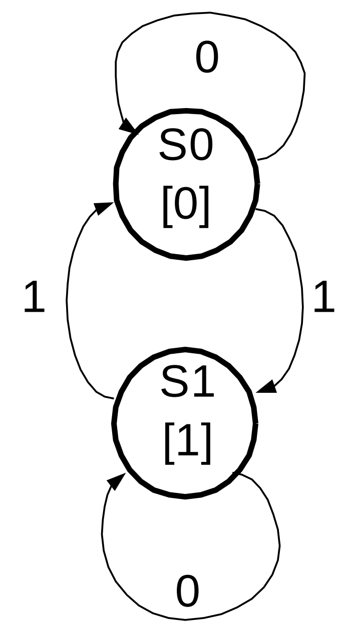
- **Merge state S2 into S0**
- **Eliminate S2**
- **New state machine shows same I/O behavior**

State Transition Table

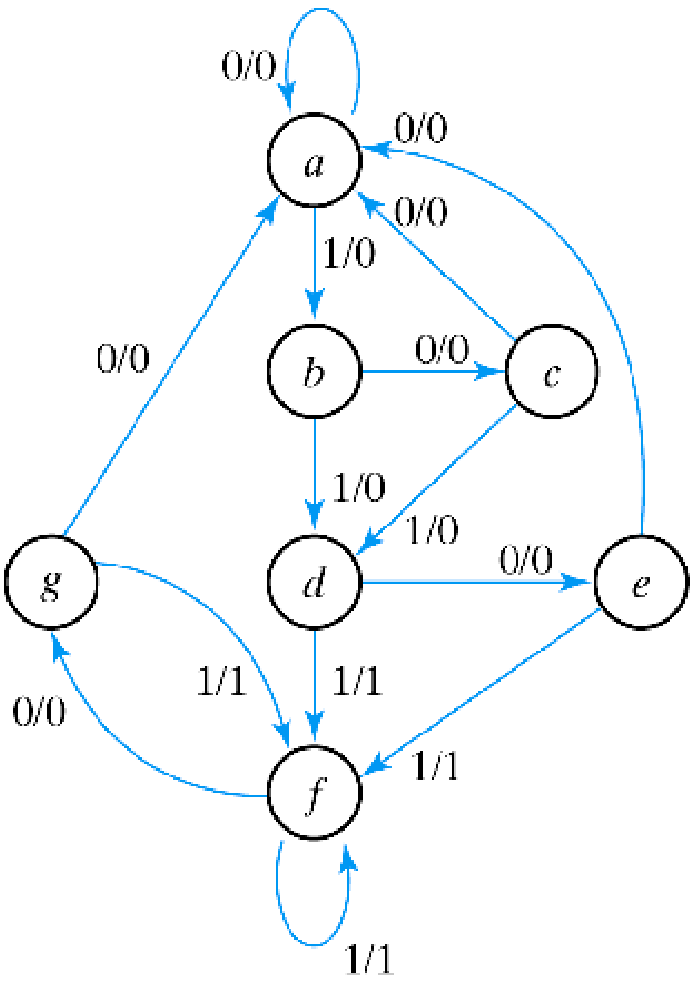
PS	NS x=0	NS x=1	output
S0	S0	S1	0
S1	S1	S0	1

- **Example: Odd parity checker.**





Row Matching Example



State Transition Table

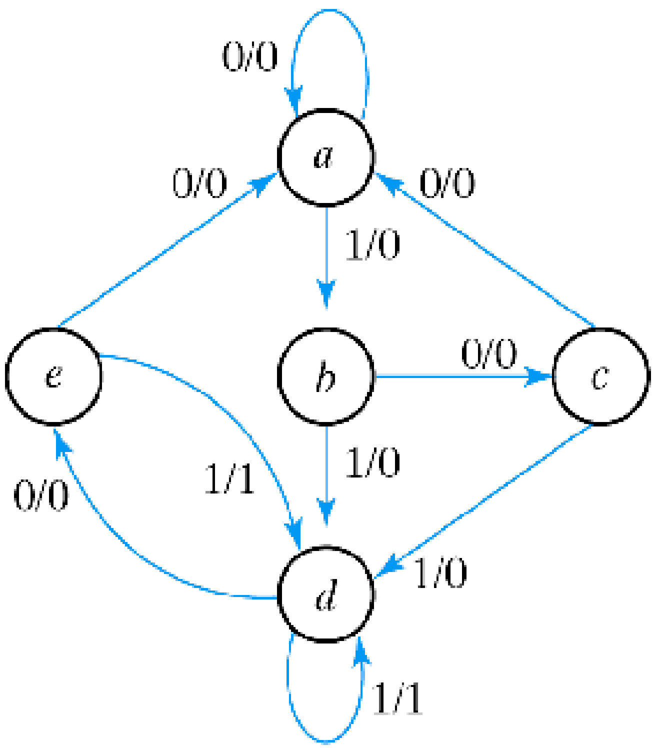
	NS		output	
PS	x=0	x=1	x=0	x=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	g	f	0	1
g	a	f	0	1

Row Matching Example

	NS		output	
PS	x=0	x=1	x=0	x=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	f	0	1
e	a	f	0	1
f	e	f	0	1

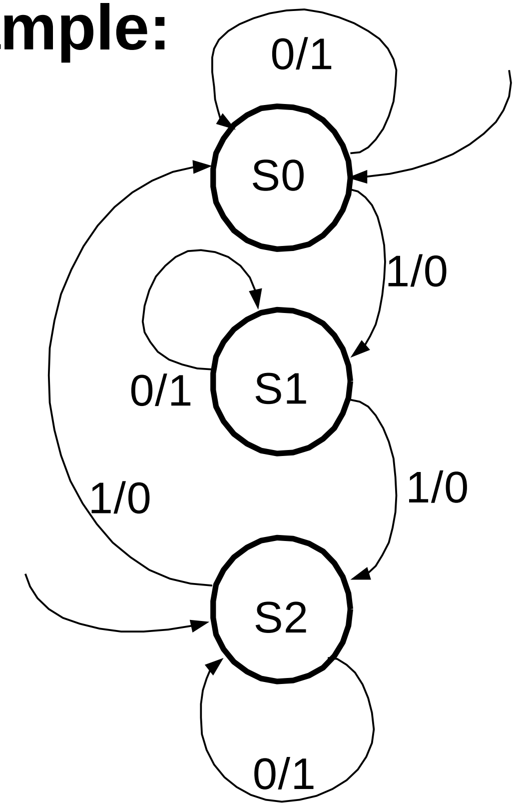
	NS		output	
PS	x=0	x=1	x=0	x=1
a	a	b	0	0
b	c	d	0	0
c	a	d	0	0
d	e	d	0	1
e	a	d	0	1

Reduced State Transition Diagram



State Reduction

- The “row matching” method is not guaranteed to result in the optimal solution in all cases, because it only looks at pairs of states.
- For example:



- **Another (more complicated) method guarantees the optimal solution:**
- “Implication table” **method:**

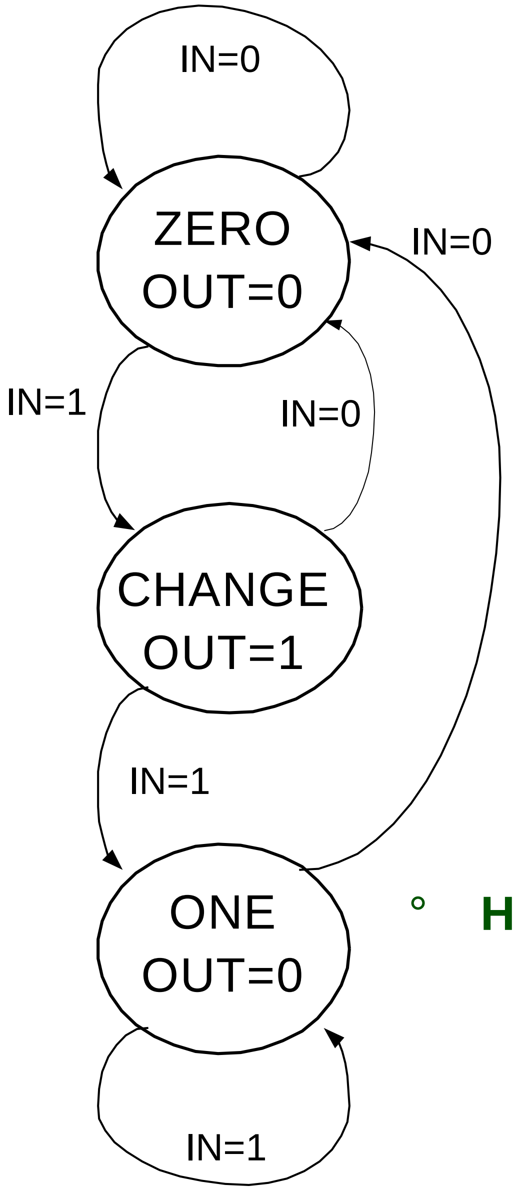
  See Mano, chapter 9.

(not responsible for chapter 9 material)

Encoding State Variables

- Option 1: Binary values
  - 000, 001, 010, 011, 100 …
- Option 2: Gray code
  - 000, 001, 011, 010, 110 …
- Option 3: One hot encoding
  - One bit for every state
  - Only one bit is a one at a given time
  - For a 5-state machine
    - 00001, 00010, 00100, 01000, 10000

State Transition Diagram Solution B



	IN	PS	NS	OUT
ZERO	0	01	01	0
	1	01	10	0
CHANGE	0	10	01	1
	1	10	00	1
ONE	0	00	01	0
	1	00	00	0

- **How does this change the combinational logic?**

Summary

- Important to create smallest possible FSMs
- This course: use visual inspection method
- Often possible to reduce logic and flip flops
- State encoding is important
  - One-hot coding is popular for flip flop intensive designs.