IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Date of current version Monday 20, 2020.

Digital Object Identifier PLACEHOLDER

EEET 4075 Mechatronic System Design 2 - Project Report

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This work was supported in part by the University of South Australia

I. INTRODUCTION

MULTIRATE Extended Kalman Filters and absolute versus relative (dead reckoning) localisation.

II. RELATED WORKS

Several different ways to combine the estimations of different sensors, difficulties with multi-rate systems. Variants or modifications on EKF and UKF, compare and contrast, computational cost, relative accuracy, limitations. maybe compare to particle filters as well? Need for an initial estimate of position and heading. Beacon based navigation.

III. METHODOLOGY

A. SYSTEM MODEL

$$\boldsymbol{x}_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ \phi_{1,\,k} \\ r_{1,\,k} \\ \phi_{2,\,k} \\ r_{2,\,k} \\ \phi_{3,\,k} \\ r_{3,\,k} \\ \phi_{4,\,k} \\ r_{4,\,k} \end{bmatrix} = \begin{bmatrix} x_{k-1} + Tv\cos\left(\theta_{k-1}\right) \\ y_{k-1} + Tv\sin\left(\theta_{k-1}\right) \\ \theta_{k-1} + T\omega \\ \operatorname{atan2}\left(\left(y_1 - y_k\right),\, \left(x_1 - x_k\right)\right) - \theta_k \\ \sqrt{\left(y_1 - y_k\right)^2 + \left(x_1 - x_k\right)^2} \\ \operatorname{atan2}\left(\left(y_2 - y_k\right)\left(x_2 - x_k\right)\right) - \theta_k \\ \sqrt{\left(y_2 - y_k\right)^2 + \left(x_2 - x_k\right)^2} \\ \operatorname{atan2}\left(\left(y_3 - y_k\right),\, \left(x_3 - x_k\right)\right) - \theta_k \\ \sqrt{\left(y_3 - y_k\right)^2 + \left(x_3 - x_k\right)^2} \\ \operatorname{atan2}\left(\left(y_4 - y_k\right),\, \left(x_4 - x_k\right)\right) - \theta_k \\ \sqrt{\left(y_4 - y_k\right)^2 + \left(x_4 - x_k\right)^2} \end{bmatrix} \tag{1}$$

$$\boldsymbol{F}_k = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x_{k-1}}} = \begin{bmatrix} 1 & 0 & -Tv\sin\left(\theta_{k-1}\right) & 0 & 0 & \dots \\ 0 & 1 & Tv\cos\left(\theta_{k-1}\right) & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & 0 & \dots \\ \frac{\partial \phi_{1,\,k-1}}{\partial x_{k-1}} & \frac{\partial \phi_{1,\,k-1}}{\partial y_{k-1}} & \frac{\partial \phi_{1,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial r_{1,\,k-1}}{\partial x_{k-1}} & \frac{\partial r_{1,\,k-1}}{\partial y_{k-1}} & \frac{\partial r_{1,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial \phi_{2,\,k-1}}{\partial x_{k-1}} & \frac{\partial \phi_{2,\,k-1}}{\partial y_{k-1}} & \frac{\partial \phi_{2,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial r_{2,\,k-1}}{\partial x_{k-1}} & \frac{\partial r_{2,\,k-1}}{\partial y_{k-1}} & \frac{\partial r_{2,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial \phi_{3,\,k-1}}{\partial x_{k-1}} & \frac{\partial \phi_{3,\,k-1}}{\partial y_{k-1}} & \frac{\partial \phi_{3,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial r_{3,\,k-1}}{\partial x_{k-1}} & \frac{\partial r_{4,\,k-1}}{\partial y_{k-1}} & \frac{\partial r_{3,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial \phi_{4,\,k-1}}{\partial x_{k-1}} & \frac{\partial \phi_{4,\,k-1}}{\partial y_{k-1}} & \frac{\partial \phi_{4,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \\ \frac{\partial r_{4,\,k-1}}{\partial x_{k-1}} & \frac{\partial r_{5,\,k-1}}{\partial y_{k-1}} & \frac{\partial r_{4,\,k-1}}{\partial \theta_{k-1}} & 0 & 0 & \dots \end{bmatrix} \tag{2}$$

$$\frac{\partial \phi_{i,\,k-1}}{\partial x_{k-1}} = \frac{y_k - y_i}{\left(\frac{\left(y_k - y_i\right)^2}{\left(x_k - x_i\right)^2} + 1\right)\left(x_k - x_i\right)^2} \tag{3}$$

$$\frac{\partial \phi_{i,\,k-1}}{\partial y_{k-1}} = \frac{1}{\left(\frac{\left(y_k - y_i\right)^2}{\left(x_k - x_i\right)^2} + 1\right)\left(x_k - x_i\right)} \tag{4}$$

$$\frac{\partial \phi_{i,\,k-1}}{\partial \theta_{k-1}} = \frac{Tv\left(Tv + \cos\left(\theta_{k-1}\right)\left(x_{k-1} - x_i\right) + \sin\left(\theta_{k-1}\right)\left(y_i - y_{k-1}\right)\right)}{\left(\frac{\left(y_k - y_i\right)^2}{\left(x_k - x_i\right)^2} + 1\right)\left(x_k - x_i\right)^2} - 1 \tag{5}$$

$$\frac{\partial r_{i,\,k-1}}{\partial x_{k-1}} = \frac{x_k - x_i}{\sqrt{\left(x_k - x_i\right)^2 + \left(y_k - y_i\right)^2}} \tag{6}$$

$$\frac{\partial r_{i,\,k-1}}{\partial y_{k-1}} = \frac{y_k - y_i}{\sqrt{\left(x_k - x_i\right)^2 + \left(y_k - y_i\right)^2}} \tag{7}$$

$$\frac{\partial r_{i,\,k-1}}{\partial \theta_{k-1}} = \frac{Tv\left(\cos\left(\theta_{k-1}\right)\left(y_{k-1} - y_i\right) + \sin\left(\theta_{k-1}\right)\left(x_i - x_{k-1}\right)\right)}{\sqrt{\left(x_k - x_i\right)^2 + \left(y_k - y_i\right)^2}} \tag{8}$$

B. MEASUREMENT

$$\delta s_k = \frac{s_{r,k} - s_{r,k-1} + s_{l,k} - s_{l,k-1}}{2} \tag{9}$$

$$\delta \theta_k = \frac{s_{r,k} - s_{r,k-1} + s_{l,k} - s_{l,k-1}}{b} \tag{10}$$

where b is the wheelbase in m,

$$b = 0.287 \tag{11}$$

$$\boldsymbol{h}_k = \begin{bmatrix} \delta s_k \\ \delta \theta_k \\ \omega_k \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_k - x_{k-1}\right)^2 + \left(y_k - y_{k-1}\right)^2} \\ \theta_k - \theta_{k-1} \\ \frac{\theta_k - \theta_{k-1}}{T} \end{bmatrix} \tag{12}$$

$$\boldsymbol{H}_k = \begin{bmatrix} \frac{x_k - x_{k-1}}{\sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}} & 0 & 0 \\ \frac{y_k - y_{k-1}}{\sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}} & 0 & 0 \\ 0 & 1 & \frac{1}{T} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^\top \tag{13}$$

C. EKF

a priori estimate

$$\hat{\boldsymbol{x}}_k^- = \boldsymbol{f}_{k-1}\left(\hat{\boldsymbol{x}}_{k-1}^+, \boldsymbol{u}_{k-1}\right) \tag{14}$$

$$\boldsymbol{P}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^\top + \boldsymbol{Q}_{k-1} \tag{15}$$

Kalman gain update,

$$\boldsymbol{K}_k = \boldsymbol{P}_k^-\boldsymbol{H}_k^\top\left(\boldsymbol{H}_k\boldsymbol{P}k^-\boldsymbol{H}_k^\top + \boldsymbol{R}_k\right)^{-1} \tag{16}$$

a posteriori estimate,

$$\hat{\boldsymbol{x}}_k^+ = \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_k\left(\boldsymbol{y}_k - \boldsymbol{h}_k\right) \tag{17}$$

$$\boldsymbol{P}_k^+ = \left(\boldsymbol{I} - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{P}_k^- \tag{18}$$

IV. RESULTS AND DISCUSSION

V. CONCLUSION

VI. REFERENCES