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EEET 4075 Mechatronic System Design 2 - Project Report

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I. INTRODUCTION

MULTIRATE Extended Kalman Filters and absolute versus relative (dead reckoning) localisation.

II. RELATED WORKS

Several different ways to combine the estimations of different sensors, difficulties with multi-rate systems. Variants or modifications on EKF and UKF, compare and contrast, computational cost, relative accuracy, limitations. maybe compare to particle filters as well? Need for an initial estimate of position and heading. Beacon based navigation.

Using Alexander's Method for fusion of absolute and dead reckoning system due to update rate of LiDaR being much slower than that of odometry and IMU.

III. METHODOLOGY

A. SYSTEM MODEL

$$\boldsymbol{x}_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} + Tv\cos\left(\theta_{k-1}\right) \\ y_{k-1} + Tv\sin\left(\theta_{k-1}\right) \\ \theta_{k-1} + T\omega \end{bmatrix} \tag{1}$$

$$\boldsymbol{F}_k = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x_{k-1}}} = \begin{bmatrix} 1 & 0 & -Tv\sin\left(\theta_{k-1}\right) \\ 0 & 1 & Tv\cos\left(\theta_{k-1}\right) \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

B. MEASUREMENT

$$\delta s_k = \frac{s_{r,k} - s_{r,k-1} + s_{l,k} - s_{l,k-1}}{2} \tag{3}$$

$$\delta \theta_k = \frac{s_{r,k} - s_{r,k-1} + s_{l,k} - s_{l,k-1}}{b} \tag{4}$$

where b is the wheelbase in m,

$$b = 0.287 \tag{5}$$

$$\boldsymbol{h}_k = \begin{bmatrix} \delta s_k \\ \delta\theta_k \\ \omega_k \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_k - x_{k-1}\right)^2 + \left(y_k - y_{k-1}\right)^2} \\ \theta_k - \theta_{k-1} \\ \frac{\theta_k - \theta_{k-1}}{T} \end{bmatrix} \tag{6}$$

$$\boldsymbol{H}_k = \begin{bmatrix} \frac{x_k - x_{k-1}}{\sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}} & 0 & 0 \\ \frac{y_k - y_{k-1}}{\sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}} & 0 & 0 \\ 0 & 1 & \frac{1}{T} \end{bmatrix}^\top \tag{7}$$

C. LIDAR

To estimate the orientation of the turtlebot, the angle to the closest beacon is used as a correction.

To estimate the position of the turtlebot, the intersection of two circles is calculated knowing their center coordinates and radii.

$$\begin{aligned} \left(x - x_1\right)^2 + \left(y - y_1\right)^2 &= r_1^2 \\ \left(x - x_2\right)^2 + \left(y - y_2\right)^2 &= r_2^2 \end{aligned} \tag{8}$$

Rearranging, and with the assumption that $x_1 = x_2$ in both cases we get

$$y = -\frac{r_1^2 - r_2^2 - y_1^2 + y_2^2}{2\left(y_1 - y_2\right)} \tag{9}$$

$$\begin{aligned} x &= \pm\frac{\sqrt{\beta_1\beta_2}}{2\left(y_1 - y_2\right)} \\ \beta_1 &= \left(r_1 + r_2\right)^2 - \left(y_1 - y_2\right)^2 \\ \beta_2 &= \left(y_1 - y_2\right)^2 - \left(r_1 - r_2\right)^2 \end{aligned} \tag{10}$$

As we know the fence around which the bot cannot escape, one of the possible positions can be eliminated, leaving BLAH if using the left beacons, and BLAH if using the right beacons.

$$x = \frac{\sqrt{\beta_1\beta_2}}{2\left(y_1 - y_2\right)} \tag{11}$$

$$x = 3.5 - \frac{\sqrt{\beta_1\beta_2}}{2\left(y_1 - y_2\right)} \tag{12}$$

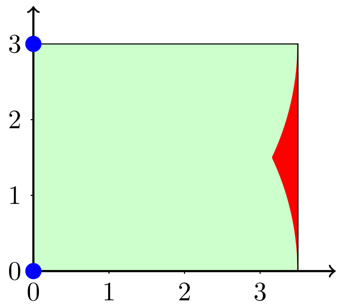

FIGURE 1: Left beacons showing measurable locations in green.

D. EKF

a priori estimate

$$\hat{\boldsymbol{x}}_k^- = \boldsymbol{f}_{k-1}\left(\hat{\boldsymbol{x}}_{k-1}^+,\, \boldsymbol{u}_{k-1}\right) \tag{13}$$

$$\boldsymbol{P}_k^- = \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}^+\boldsymbol{F}_{k-1}^\top + \boldsymbol{Q}_{k-1} \tag{14}$$

Kalman gain update,

$$\boldsymbol{K}_{1,\,k} = \boldsymbol{P}_k^-\boldsymbol{H}_{1,\,k}^\top\left(\boldsymbol{H}_{1,\,k}\boldsymbol{P}1,\,k^-\boldsymbol{H}_{1,\,k}^\top + \boldsymbol{R}_{1,\,k}\right)^{-1} \tag{15}$$

a posteriori estimate,

$$\hat{\boldsymbol{x}}_k^+ = \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_{1,\,k}\left(\boldsymbol{y}_{1,\,k} - \boldsymbol{h}_{1,\,k}\right) \tag{16}$$

$$\boldsymbol{P}_{1,\,k}^+ = \left(\boldsymbol{I} - \boldsymbol{K}_{1,\,k}\boldsymbol{H}_{1,\,k}\right)\boldsymbol{P}_{1,\,k}^- \tag{17}$$

When the secondary measurement is received, the kalman gain, error, and state estimate are updated

$$\boldsymbol{K}_{2,\,k} = \boldsymbol{P}_{1,\,k}^-\boldsymbol{H}_{2,\,k}^\top\left(\boldsymbol{H}_{2,\,k}\boldsymbol{P}_{1,\,k}^-\boldsymbol{H}_{2,\,k}^\top + \boldsymbol{R}_{2,\,k}\right)^{-1} \tag{18}$$

$$\boldsymbol{P}_{2,\,k}^+ = \left(\boldsymbol{I} - \boldsymbol{K}_{2,\,k}\boldsymbol{H}_{2,\,k}\right)\boldsymbol{P}_{1,\,k}^- \tag{19}$$

$$\hat{\boldsymbol{x}}_k^+ = \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_{1,\,k}\left(\boldsymbol{y}_{1,\,k} - \boldsymbol{h}_{1,\,k}\right) + \boldsymbol{K}_{2,\,k}\left(\boldsymbol{y}_{2,\,k} - \boldsymbol{H}_{2,\,k}\hat{\boldsymbol{x}}_k^-\right) \tag{20}$$

If the secondary measurement is delayed by $s+N$ samples, where s is the sample which the measurement refers to, then the recalculation instead then becomes

$$\begin{aligned}\hat{\boldsymbol{x}}_k^+ =& \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_{1,\,k}\left(\boldsymbol{y}_{1,\,k} - \boldsymbol{h}_{1,\,k}\right) \\ &+ \boldsymbol{W}\boldsymbol{K}_{2,\,s}\left(\boldsymbol{y}_{2,\,k} - \boldsymbol{H}_{2,\,s}\hat{\boldsymbol{x}}_s^-\right)\end{aligned} \tag{21}$$

where

$$\boldsymbol{W} = \prod_{i=1}^{i=N}\left(\boldsymbol{I} - \boldsymbol{K}_{s+i}\boldsymbol{H}_{1,\,s+1}\right)\boldsymbol{F}_{s+i-1} \tag{22}$$

IV. RESULTS AND DISCUSSION

V. CONCLUSION

VI. REFERENCES

Trajectory Planning for Nonholonomic Mobile Robot Using Extended Kalman Filter

Integrated Indoor Navigation System for Ground Vehicles With Automatic 3-D Alignment and Position Initialization.

Incorporating delayed and infrequent measurements in Extended Kalman Filter based nonlinear state estimation.