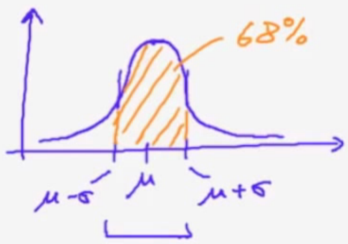
$$X \sim \mathcal{N}(x; \mu, \sigma^2)$$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$\frac{(x-\mu)^2}{\sigma^2} = 1$$

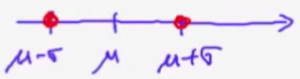
$$\Leftrightarrow \quad x = \mu \pm \sigma$$

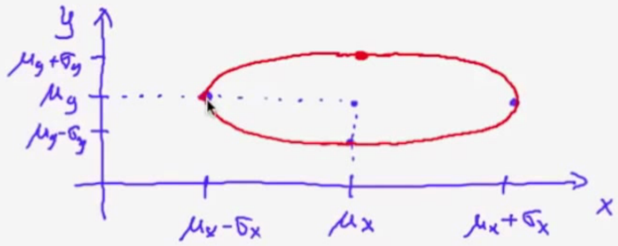


PDF

$\pm 1\sigma$: 68%
$\pm 2\sigma$: 95,5%
$\pm 3\sigma$: 99,7%

$$\underbrace{\frac{(x-\mu)^2}{\sigma^2}}_{1D} = 1 \quad \rightsquigarrow \quad \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} = 1$$





$$\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2}$$

$$[x-\mu_x, \; y-\mu_y] \; \underbrace{\overset{2\times 2}{\boxed{\begin{bmatrix} 1/\sigma_x^2 & 0 \\ 0 & 1/\sigma_y^2 \end{bmatrix}}} \begin{bmatrix} x-\mu_x \\ y-\mu_y \end{bmatrix}}$$

$$[x-\mu_x, \; y-\mu_y] \cdot \begin{bmatrix} \frac{x-\mu_x}{\sigma_x^2} \\ \frac{y-\mu_y}{\sigma_y^2} \end{bmatrix}$$

$$\frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2}$$