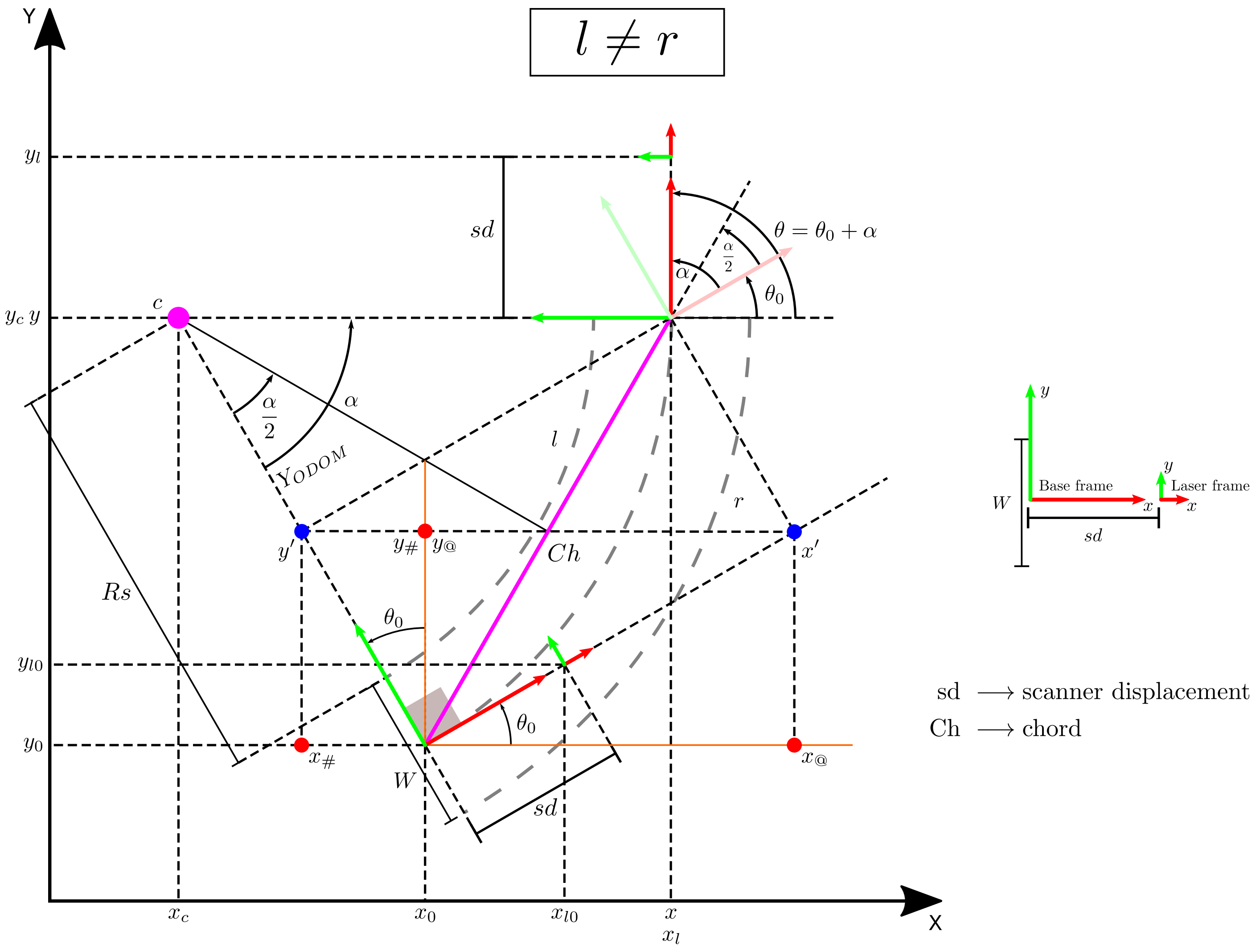
Robot's coordinates in the global reference frame



Given:

the robot's initial pose, $(x_0,\, y_0,\, \theta_0)$, and the motion commands, $(l,\, r)$.

$$
\begin{aligned}
r &= (Rs + W)\,\alpha \\
l &= Rs\,\alpha \\
r - l &= W\,\alpha
\end{aligned}
$$

$$
\begin{aligned}
\alpha &= \frac{r - l}{W} \\
Rs &= \frac{l}{\alpha}
\end{aligned}
$$

$$
\begin{aligned}
x_l &= x + sd\,\cos(\theta) \\
y_l &= y + sd\,\sin(\theta)
\end{aligned}
$$

$$
\begin{aligned}
\cos(u \pm v) &= \cos u\,\cos v \mp \sin u\,\sin v \\
\sin(u \pm v) &= \sin u\,\cos v \pm \cos u\,\sin v
\end{aligned}
$$

$$
\begin{aligned}
\cos(u - v) - \cos(u + v) &= 2\,\sin(u)\,\sin(v) \\
\sin(u + v) - \sin(u - v) &= 2\,\cos(u)\,\sin(v)
\end{aligned}
$$

$$
Ch = 2\left(Rs + \frac{W}{2}\right)\,sin\left(\frac{\alpha}{2}\right)
$$

$$
\begin{aligned}
x &= x_0 + x_{@} + x_{\#} \\
&= x_0 + x' \cos(\theta_0) + y' \cos\left(\theta + 90^\circ\right) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos\left(\theta + 90^\circ\right) \\
&= x_0 + Ch \cos\left(\frac{\alpha}{2}\right) \cos(\theta_0) - Ch \sin\left(\frac{\alpha}{2}\right) \sin(\theta) \\
&= x_0 + Ch \cos\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= x_0 + 2\left(Rs + \frac{W}{2}\right) sin\left(\frac{\alpha}{2}\right) \cos\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
y &= y_0 + y_{@} + y_{\#} \\
&= y_0 + x' \sin(\theta_0) + y' \sin\left(\theta + 90^\circ\right) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \sin\left(\theta + 90^\circ\right) \\
&= y_0 + Ch \cos\left(\frac{\alpha}{2}\right) \sin(\theta_0) + Ch \sin\left(\frac{\alpha}{2}\right) \cos(\theta_0) \\
&= y_0 + Ch \sin\left(\theta_0 + \frac{\alpha}{2}\right) \\
&= y_0 + 2\left(Rs + \frac{W}{2}\right) sin\left(\frac{\alpha}{2}\right) \sin\left(\theta_0 + \frac{\alpha}{2}\right)
\end{aligned}
$$

Another way to calculate the robot's coordinates in the global reference frame:

$$\begin{aligned}
x_c &= x_0 + \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0 + 90°\right) \\
&= x_0 - \left(Rs + \frac{W}{2}\right) sin\left(\theta_0\right) \\
y_c &= y_0 + \left(Rs + \frac{W}{2}\right) \sin\left(\theta_0 + 90°\right) \\
&= y_0 + \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0\right)
\end{aligned}$$

$$\begin{aligned}
x_c &= x + \left(Rs + \frac{W}{2}\right) \cos\left(\theta + 90°\right) \\
&= x - \left(Rs + \frac{W}{2}\right) sin\left(\theta\right) \\
y_c &= y + \left(Rs + \frac{W}{2}\right) \sin\left(\theta + 90°\right) \\
&= y + \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0\right)
\end{aligned}$$

$$\begin{aligned}
x &= x_c + \left(Rs + \frac{W}{2}\right) \sin\left(\theta\right) \\
&= x_0 - \left(Rs + \frac{W}{2}\right) \sin\left(\theta_0\right) + \left(Rs + \frac{W}{2}\right) \sin\left(\theta\right) \\
&= x_0 + \left(Rs + \frac{W}{2}\right) \left(\sin\left(\theta\right) - \sin\left(\theta_0\right)\right) \\
&= x_0 + 2\left(Rs + \frac{W}{2}\right) \cos\left(\theta_0 + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)
\end{aligned}$$

$$\begin{aligned}
y &= y_c - \left(Rs + \frac{W}{2}\right) \cos\left(\theta\right) \\
&= y_0 + \left(Rs + \frac{W}{2}\right) \cos\left(\theta_0\right) - \left(Rs + \frac{W}{2}\right) \cos\left(\theta\right) \\
&= y_0 + \left(Rs + \frac{W}{2}\right) \left(\cos\left(\theta_0\right) - \cos\left(\theta\right)\right) \\
&= y_0 + 2\left(Rs + \frac{W}{2}\right) \sin\left(\theta_0 + \frac{\alpha}{2}\right) \sin\left(\frac{\alpha}{2}\right)
\end{aligned}$$

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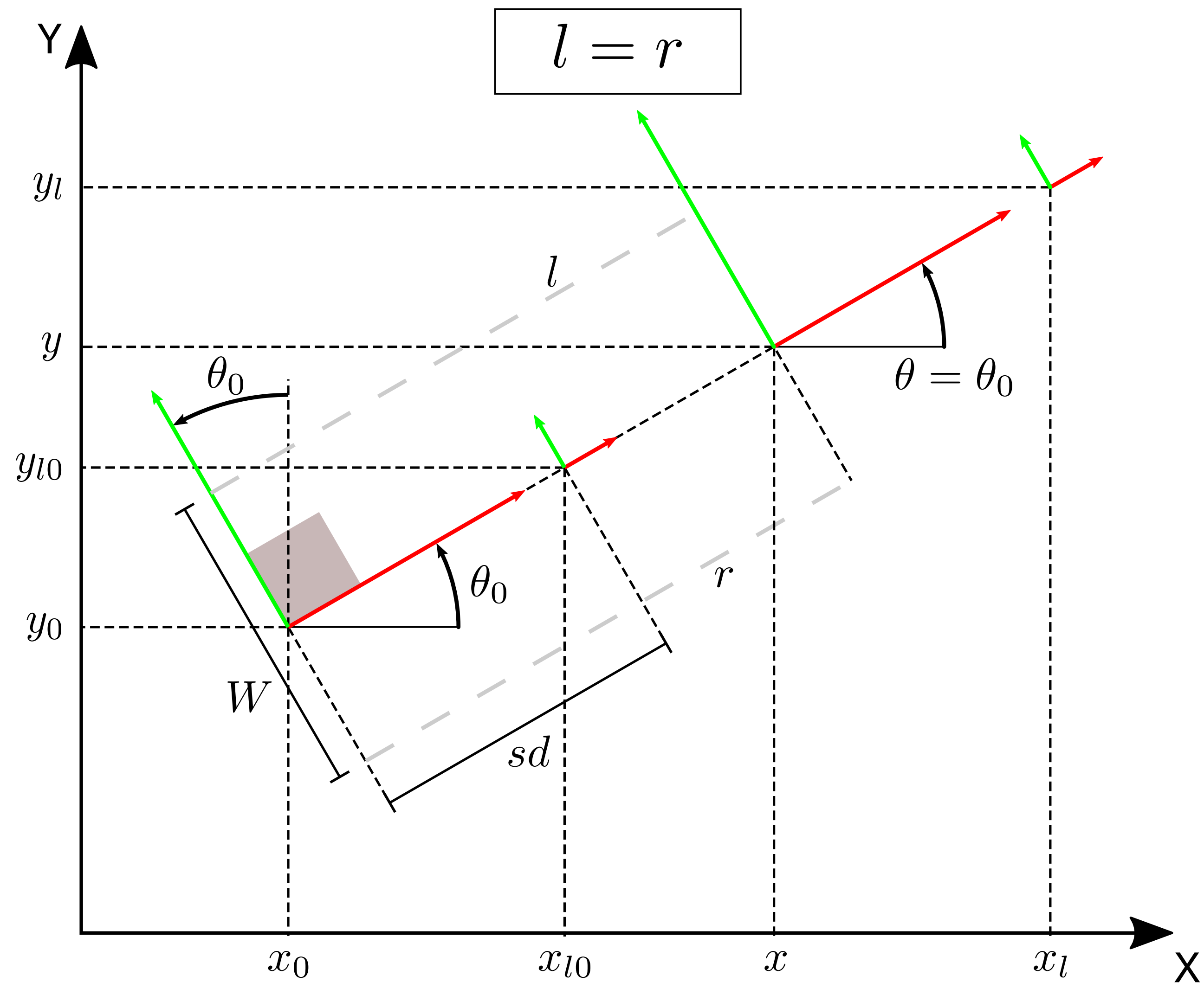

Given:

the robot's initial pose, $(x_0,\, y_0,\, \theta_0)$, and the motion commands, $(l,\, r)$.

$$
\begin{aligned}
x &= x_0 + l \cos(\theta_0) \\
y &= y_0 + l \sin(\theta_0) \\
x_l &= x + sd \cos(\theta) \\
y_l &= y + sd \sin(\theta)
\end{aligned}
$$