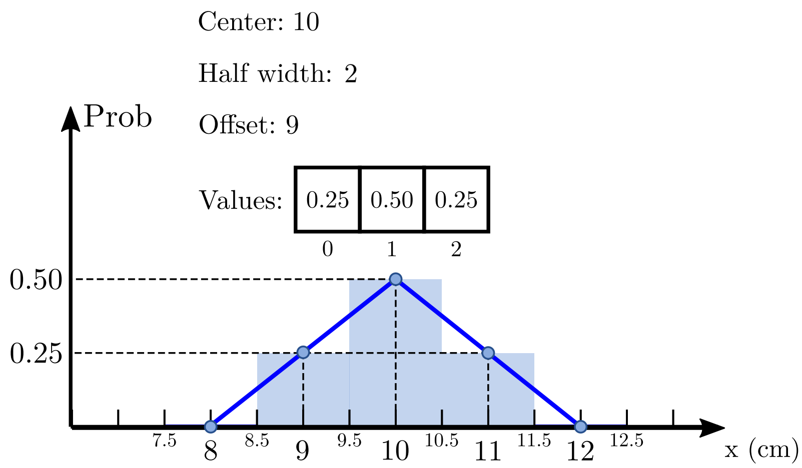
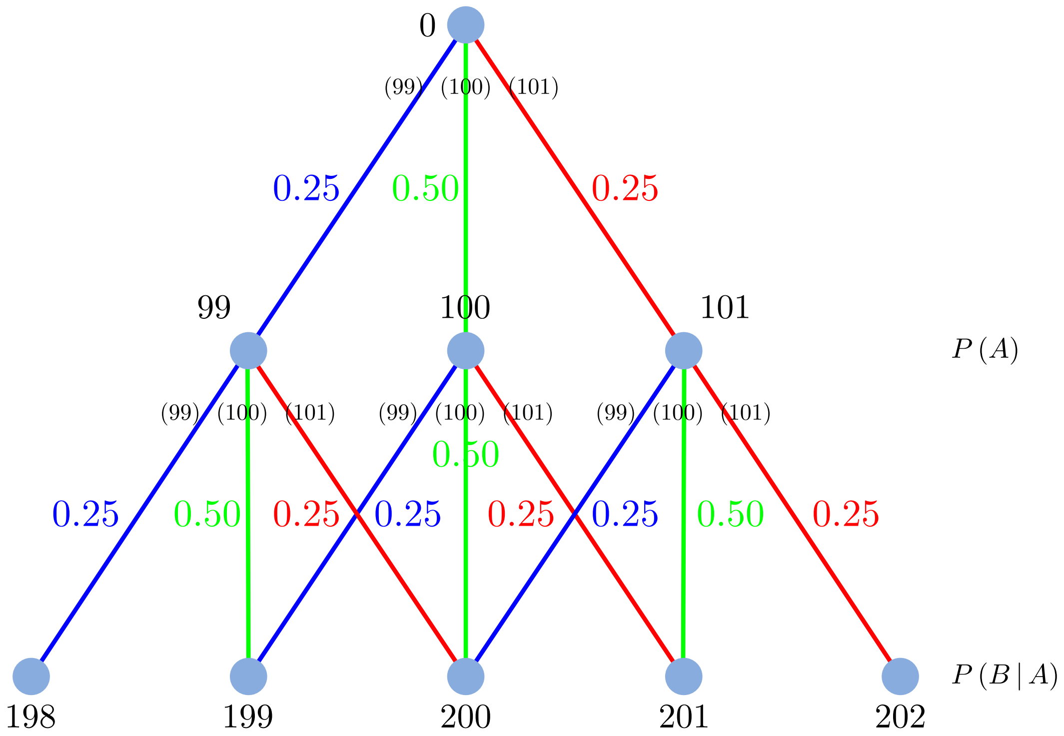
L03: FILTERING

A discrete probability distribution satisfies:

$$0 \le P(x) \le 1$$

$$\sum_{x=-\infty}^{+\infty} P(x) = 1$$





$$
\begin{aligned}
P(A,\,B) &= \;P(B,\,A) \\
P(A,\,B) &= P(A\,|\,B)\cdot P(B) \\
P(B,\,A) &= P(B\,|\,A)\cdot P(A) \\
P(A\,|\,B)\cdot P(B) &= P(B\,|\,A)\cdot P(A) \\
\frac{P(A\,|\,B)}{P(B\,|\,A)} &= \frac{P(A)}{P(B)}
\end{aligned}
$$

$$
\begin{aligned}
P(A) = \sum_{B=-\infty}^{\infty} P(A,\,B) &= \sum_{B=-\infty}^{\infty} P(A\,|\,B)\cdot P(B) \\
P(B) = \sum_{A=-\infty}^{\infty} P(B,\,A) &= \sum_{A=-\infty}^{\infty} P(B\,|\,A)\cdot P(A)
\end{aligned}
$$

$$P\,(99) = \sum_{A=-\infty}^{\infty} P\,(99, A) = P\,(99, 0) = P\,(99\,|\,0) \cdot P\,(0) = 0.25 \cdot 1.00 = 25$$

$$P\,(100) = \sum_{A=-\infty}^{\infty} P\,(100, A) = P\,(100, 0) = P\,(100\,|\,0) \cdot P\,(0) = 0.50 \cdot 1.00 = 0.50$$

$$P\,(101) = \sum_{A=-\infty}^{\infty} P\,(101, A) = P\,(101, 0) = P\,(101\,|\,0) \cdot P\,(0) = 0.25 \cdot 1.00 = 0.25$$

$$P\,(198,\,99) \;=\; P\,(198\,|\,99) \;\cdot\; P\,(99) \;=\; 0.25 \;\cdot\; 0.25 \;=\; 0.0625$$

$$P\,(199,\,99) \;=\; P\,(199\,|\,99) \;\cdot\; P\,(99) \;=\; 0.50 \;\cdot\; 0.25 \;=\; 0.1250$$

$$P\,(200,\,99) \;=\; P\,(200\,|\,99) \;\cdot\; P\,(99) \;=\; 0.25 \;\cdot\; 0.25 \;=\; 0.0625$$

$$P\,(199,\,100) \;=\; P\,(199\,|\,100) \;\cdot\; P\,(100) \;=\; 0.25 \;\cdot\; 0.50 \;=\; 0.1250$$

$$P\,(200,\,100) \;=\; P\,(200\,|\,100) \;\cdot\; P\,(100) \;=\; 0.50 \;\cdot\; 0.50 \;=\; 0.2500$$

$$P\,(201,\,100) \;=\; P\,(201\,|\,100) \;\cdot\; P\,(100) \;=\; 0.25 \;\cdot\; 0.50 \;=\; 0.1250$$

$$P\,(200,\,101) \;=\; P\,(200\,|\,101) \;\cdot\; P\,(101) \;=\; 0.25 \;\cdot\; 0.25 \;=\; 0.0625$$

$$P\,(201,\,101) \;=\; P\,(201\,|\,101) \;\cdot\; P\,(101) \;=\; 0.50 \;\cdot\; 0.25 \;=\; 0.1250$$

$$P\,(202,\,101) \;=\; P\,(202\,|\,101) \;\cdot\; P\,(101) \;=\; 0.25 \;\cdot\; 0.25 \;=\; 0.0625$$

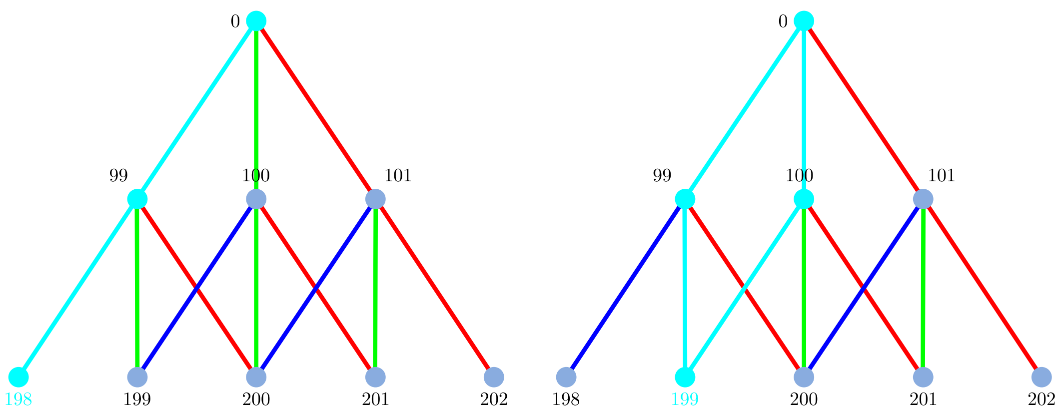
$$P\,(198) \;=\; P\,(198,\,99) \;=\; 0.0625$$

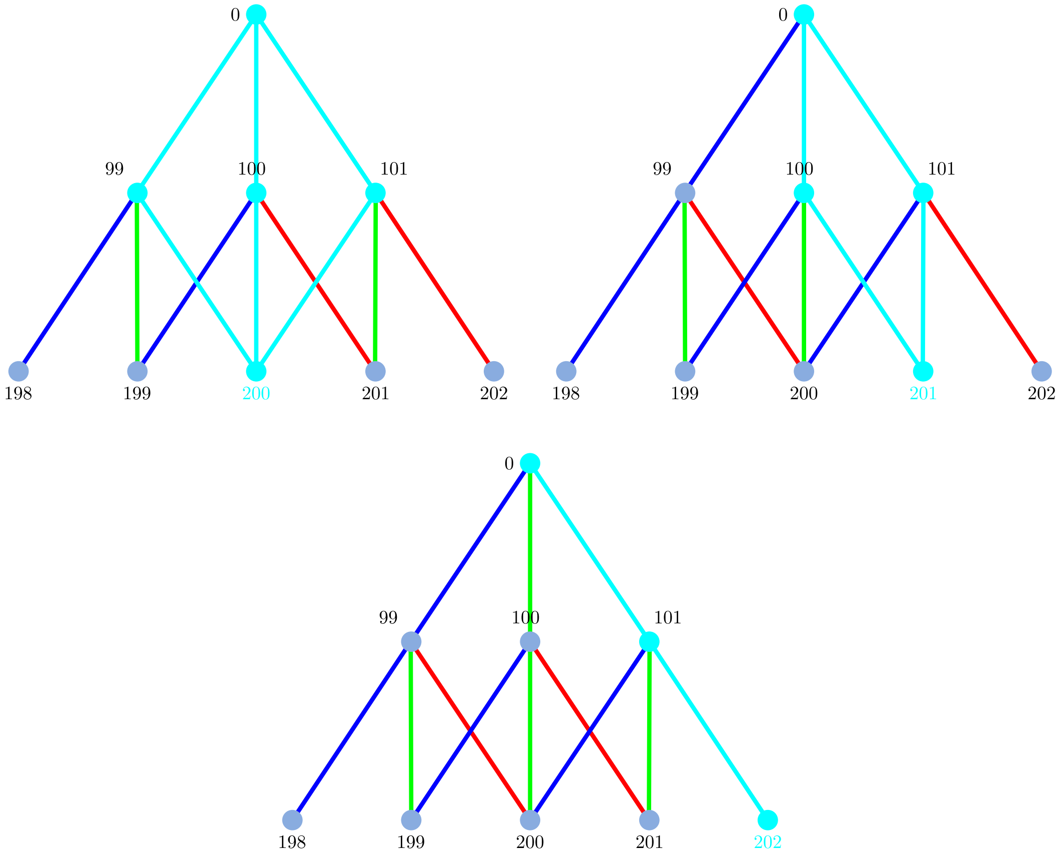
$$P\,(199) \;=\; P\,(199,\,99) \;+\; P\,(199,\,100) \;=\; 0.1250 \;+\; 0.1250 \;=\; 0.2500$$

$$P\,(200) \;=\; P\,(200,\,99) \;+\; P\,(200,\,100) \;+\; P\,(200,\,101) \;=\; 0.0625 \;+\; 0.2500 \;+\; 0.0625 \;=\; 0.3750$$

$$P\,(201) \;=\; P\,(201,\,100) \;+\; P\,(201,\,101) \;=\; 0.1250 \;+\; 0.1250 \;=\; 0.2500$$

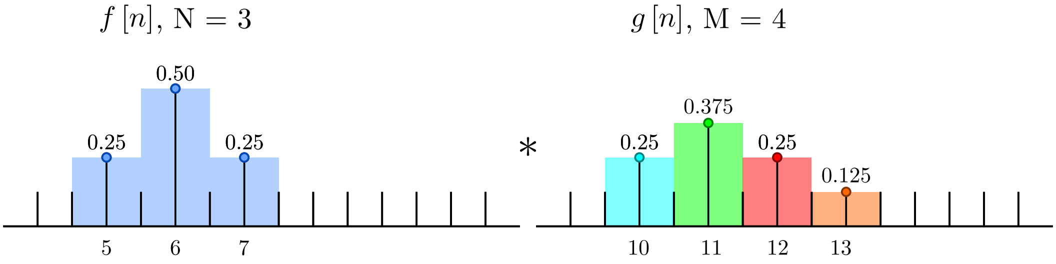
$$P\,(202) \;=\; P\,(202,\,101) \;=\; 0.0625$$





A discrete probability distribution can be calculated using a discrete convolution operation:

$$P\left[B\right] = \left(f * g\right)\left[B\right] = \sum_{A=-\infty}^{\infty} f\left[B - A\right] \cdot g\left[A\right]$$



$$\begin{aligned} P\left[B\right] = {} & f\left[B - (-\infty)\right] \cdot g\left[-\infty\right] + \ldots + f\left[B - 9\right] \cdot g\left[9\right] + \\ & + f\left[B - 10\right] \cdot g\left[10\right] + f\left[B - 11\right] \cdot g\left[11\right] + f\left[B - 12\right] \cdot g\left[12\right] + f\left[B - 13\right] \cdot g\left[13\right] + \\ & + f\left[B - 14\right] \cdot g\left[14\right] + \ldots + f\left[B - \infty\right] \cdot g\left[\infty\right] \end{aligned}$$

$g\left[\cdot\right] = 0$ for$A = (-\infty, 9] \cup [14, \infty)$
The first non-zero value of the function$g\left[\cdot\right]$ appears at$A = 10$,$g\left[10\right] = 0.25$, and the last non-zero value appears at$A = 13$,$g\left[13\right] = 0.125$

The first non-zero value of the function$f\left[\cdot\right]$ appears at:

$$B - 10 = 5 \rightarrow B = 15$$

and the last non-zero value appears at

$$B - 13 = 7 \rightarrow B = 20$$

The length of the discrete probability distribution$P\left[B\right]$ is

$$N + M - 1 = 3 + 4 - 1 = 6$$

The first non-zero value of$P\left[B\right]$ is

$$\texttt{first n where } g\left[n\right] \neq 0 + \texttt{ first m where } f\left[m\right] \neq 0$$

$$5 + 10 = 15$$

The last non-zero value of$P\left[B\right]$ is

$$\texttt{last n where } g\left[n\right] \neq 0 + \texttt{ last m where } f\left[m\right] \neq 0$$

$$7 + 13 = 20$$

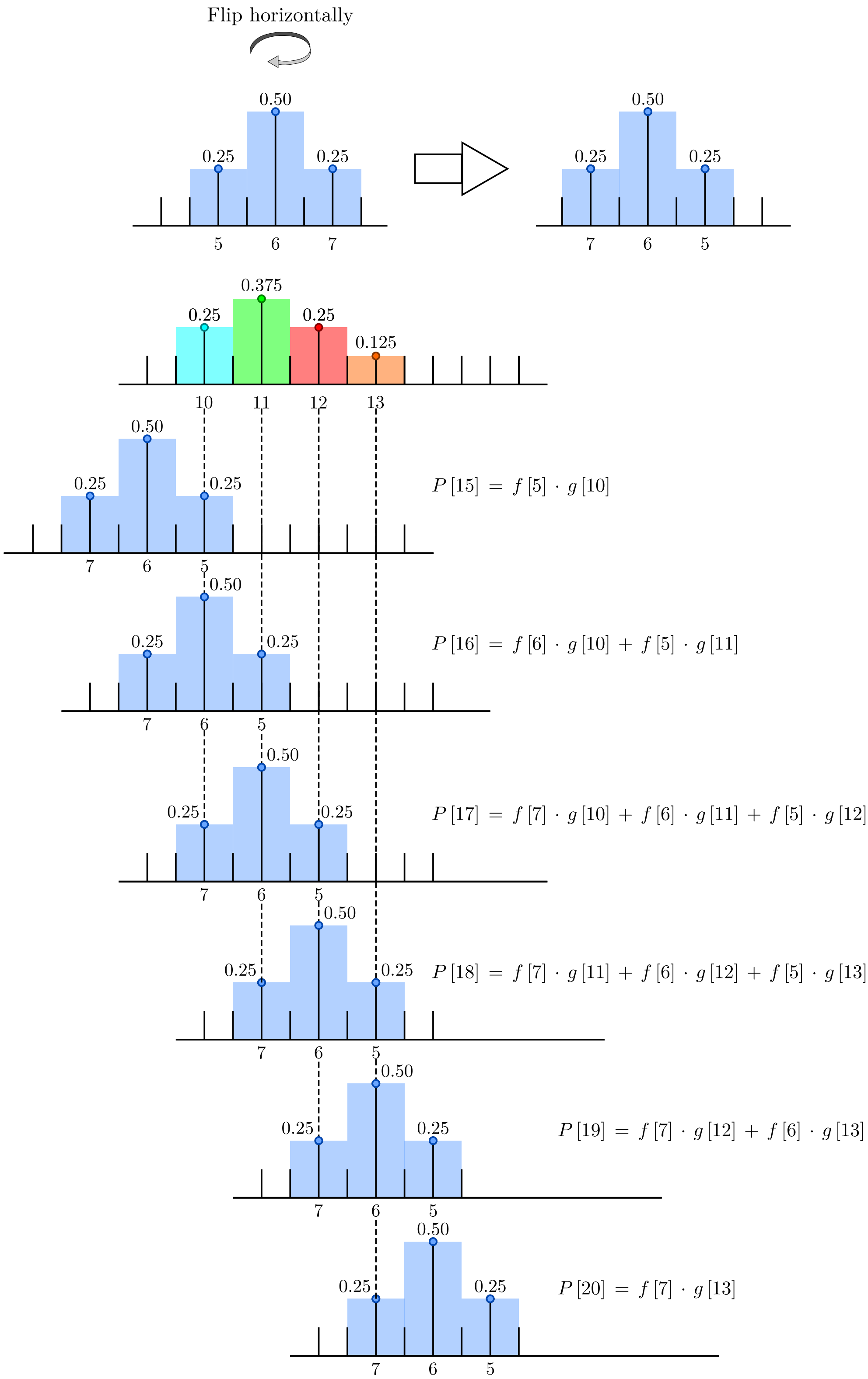
or alternatively

$$\texttt{The first non-zero value of } P\left[B\right] + length\left(P\left[B\right]\right) - 1$$

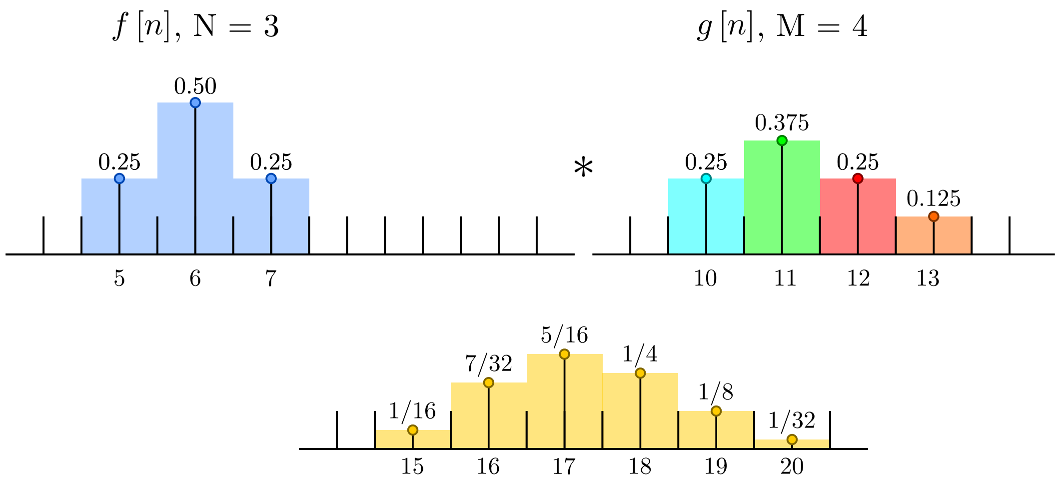
$$15 + 6 - 1 = 20$$

Graphical calculation of the convolution operation:

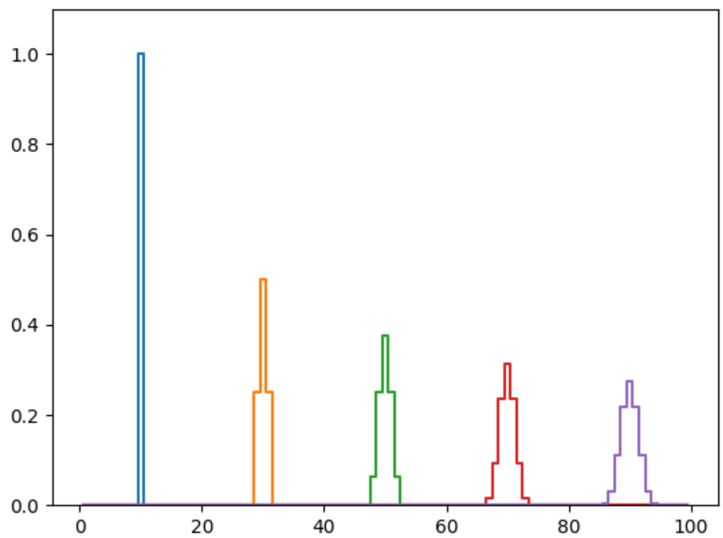
Graphical calculation of the convolution operation

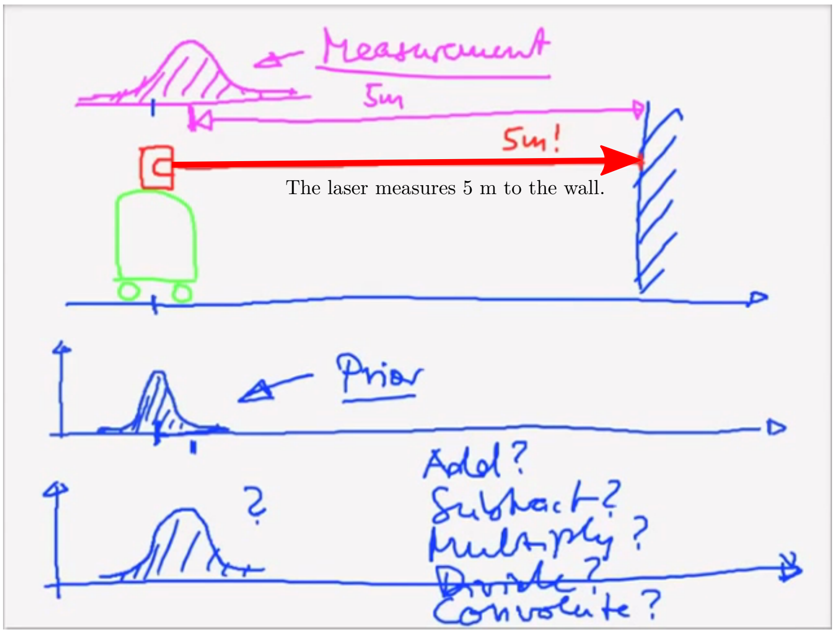


$$P\left[B\right] = \left(f * g\right)\left[B\right] = \sum_{A=-\infty}^{\infty} f\left[B - A\right] \cdot g\left[A\right]$$

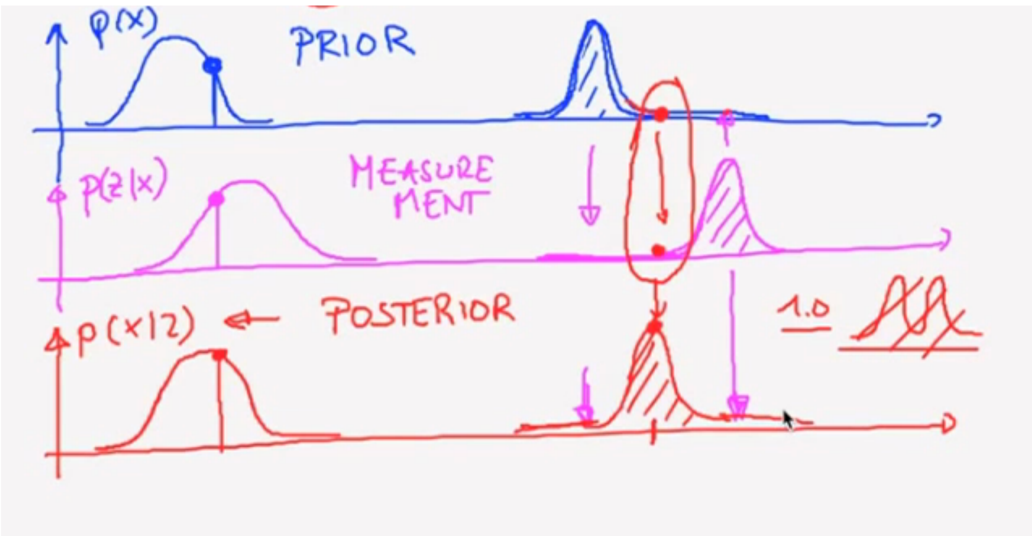


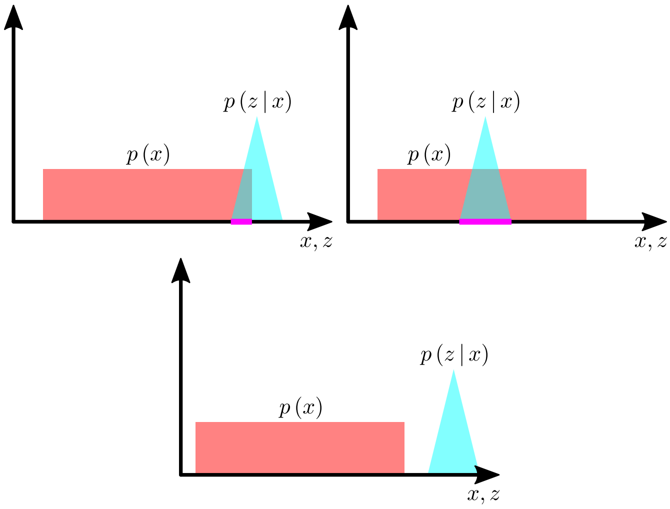

```
Pseudo-code to compute a discrete convolution:
total_length = len(f.values) + len(g.values) - 1
P = [0] * total_length
fsp = f.start_pos # Position of the first non-zero value in f.values
gsp = g.start_pos # Position of the first non-zero value in g.values
psp = tsp + gsp    # Position of the first non-zero value in P.values
for i in xrange(0, total_length): #From 0 to total_length-1
    for j in xrange(i, -1, -1):   #From i to 0
        P[i] += f.values[i -j] * g.values[j]
```

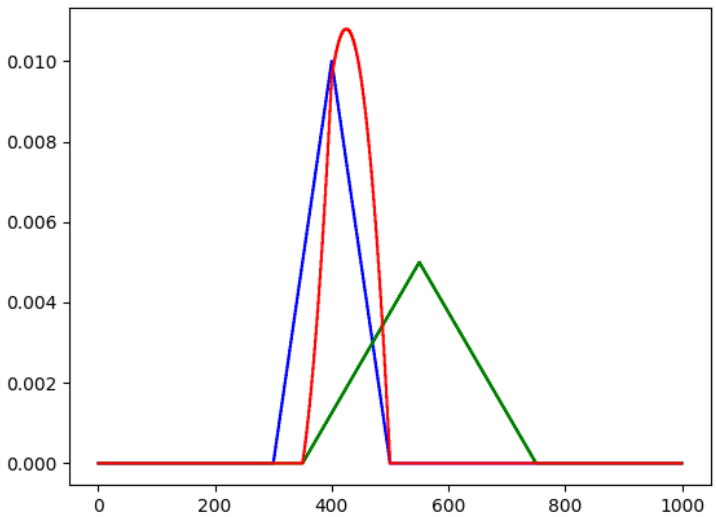




$$p\,(x\,|\,z) \;=\; \frac{p\,(z\,|\,x)\,\cdot\,p\,(x)}{p\,(z)} \;=\; \frac{p\,(z\,|\,x)\,\cdot\,p\,(x)}{\sum_{\tau=-\infty}^{+\infty} p\,(z,\,\tau)} \;=\; \frac{p\,(z\,|\,x)\,\cdot\,p\,(x)}{\sum_{\tau=-\infty}^{+\infty} p\,(z\,|\,\tau)\,\cdot\,p\,(\tau)} \;=\; \alpha\,\cdot\,p\,(z\,|\,x)\,\cdot\,p\,(x)$$







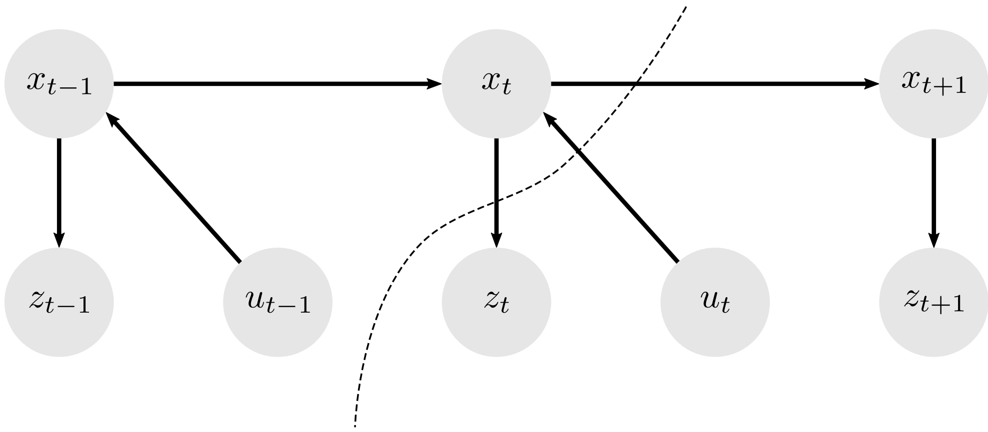
Motion - Convolution

$$p\,(x) \;=\; \sum_{y\,=\,-\infty}^{+\infty} p\,(x,\,y) \;=\; \sum_{y\,=\,-\infty}^{+\infty} p\,(x\,|\,y)\,\cdot\,p\,(y)$$

Measurement - Multiplication

$$p\,(x\,|\,z) \;=\; \alpha\,\cdot\,p\,(z\,|\,x)\,\cdot\,p\,(x)$$

THE BAYES FILTER IN 1 DIMENSION



Belief with bar,$\overline{bel}\left(x_t\right)$: after the robot has moved and before the robot takes measurements:

$$\begin{aligned} p\left(x_t \,|\, u_t,\, z_{t-1}\right) \;&=\; \int_{x_{t-1}=-\infty}^{+\infty} p\left(x_t,\, x_{t-1} \,|\, u_t,\, z_{t-1}\right)\, dx_{t-1} \;= \\ &=\; \int_{x_{t-1}=-\infty}^{+\infty} p\left(x_t \,|\, x_{t-1},\, u_t,\, z_{t-1}\right) \,\cdot\, p\left(x_{t-1} \,|\, u_t,\, z_{t-1}\right)\, dx_{t-1} \end{aligned}$$

$$\begin{aligned} p\left(x_t \,|\, u_t,\, z_{t-1}\right) \;&\longrightarrow\; p\left(x_t\right) \\ p\left(x_t \,|\, x_{t-1},\, u_t,\, z_{t-1}\right) \;&\longrightarrow\; p\left(x_t \,|\, x_{t-1},\, u_t\right) \\ p\left(x_{t-1} \,|\, u_t,\, z_{t-1}\right) \;&\longrightarrow\; p\left(x_{t-1} \,|\, z_{t-1}\right) \end{aligned}$$

$$\begin{aligned} \overline{bel}\left(x_t\right) \;&=\; p\left(x_t\right) \\ bel\left(x_{t-1}\right) \;&=\; p\left(x_{t-1} \,|\, z_{t-1}\right) \end{aligned}$$

$$\overline{bel}\left(x_t\right) \;=\; \int_{x_{t-1}=-\infty}^{+\infty} p\left(x_t \,|\, x_{t-1},\, u_t\right) \,\cdot\, bel\left(x_{t-1}\right)\, dx_{t-1}$$

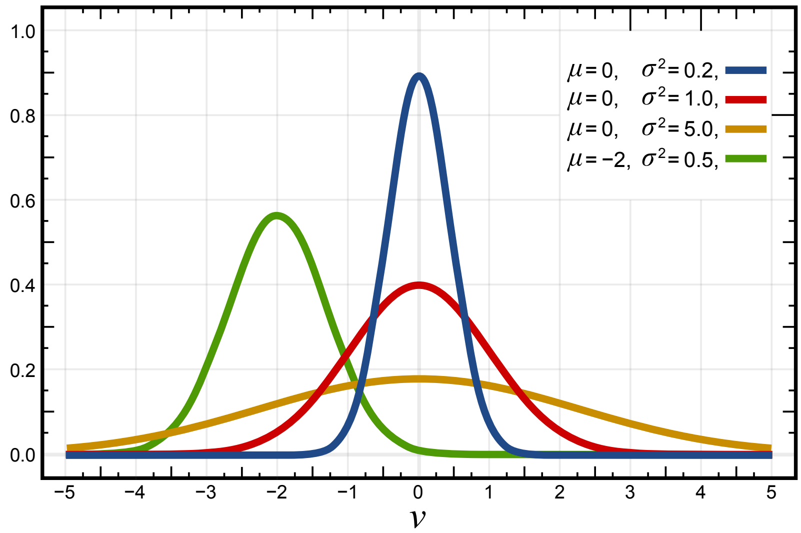
Belief without bar,$bel\left(x_t\right)$: after the robot has moved and after the robot has taken measurements:

$$bel\left(x_t\right) \;=\; p\left(x_t \,|\, z_t\right) \;=\; \frac{p\left(z_t \,|\, x_t\right) \,\cdot\, p\left(x_t\right)}{p\left(z_t\right)} \;=\; \alpha \,\cdot\, p\left(z_t \,|\, x_t\right) \,\cdot\, \overline{bel}\left(x_t\right)$$

- **INPUT:**$bel\left(x_{t-1}\right),\, u_t,\, z_t$
- **OUTPUT:**$bel\left(x_t\right)$

Normal distribution:

$$v \sim \mathcal{N}(v,\, \mu,\, \sigma^2) \;=\; \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{v\,-\,\mu}{\sigma}\right)^2}$$



$$\begin{aligned} z_t \;&=\; c_t \,\cdot\, x_t \,+\, \epsilon_{Q_t} \\ p\left(\epsilon_{Q_t}\right) \;&=\; \mathcal{N}\left(0,\, \sigma_{Q_t}^2\right) \end{aligned}$$

$$\begin{aligned} p\left(z_t \,|\, x_t\right) \;&=\; \mathcal{N}(c_t \,\cdot\, x_t,\, \sigma_{Q_t}^2) \\ \overline{bel}\left(x_t\right) \;&=\; \mathcal{N}(\overline{\mu_t},\, \overline{\sigma_t}^2) \\ bel\left(x_t\right) \;&=\; p\left(x_t \,|\, z_t\right) \;= \\ &=\; \alpha \,\cdot\, p\left(z_t \,|\, x_t\right) \,\cdot\, \overline{bel}\left(x_t\right) \;= \\ &=\; \alpha' \,\cdot\, e^{-\frac{1}{2}\left(\frac{z_t \,-\, c_t \,\cdot\, x_t}{\sigma_{Q_t}}\right)^2} \,\cdot\, e^{-\frac{1}{2}\left(\frac{x_t \,-\, \overline{\mu_t}}{\overline{\sigma_t}}\right)^2} \;= \\ &=\; \alpha' \,\cdot\, e^{-\left(\frac{1}{2}\left(\frac{z_t \,-\, c_t \,\cdot\, x_t}{\sigma_{Q_t}}\right)^2 + \frac{1}{2}\left(\frac{x_t \,-\, \overline{\mu_t}}{\overline{\sigma_t}}\right)^2\right)} \;= \\ &=\; \alpha'' \,\cdot\, e^{-\frac{1}{2}\left(\frac{x_t \,-\, \mu_t}{\sigma_t}\right)^2} \;=\; \mathcal{N}(\mu_t,\, \sigma_t^2) \end{aligned}$$

$$
\begin{aligned}
f(x) &= \frac{1}{2} \cdot A \cdot (x - B)^2 + C \\
\frac{df(x)}{dx} &= A \cdot (x - B) = 0 \rightarrow x = B \\
\frac{d^2 f(x)}{dx^2} &= A
\end{aligned}
$$

$$
\begin{aligned}
f(x_t) &= \frac{1}{2} \cdot A \cdot (x_t - B)^2 + C = \\
&= \frac{1}{2} \cdot \frac{1}{\sigma_{Q_t}^2} \cdot (z_t - c_t \cdot x_t)^2 + \frac{1}{2} \cdot \frac{1}{\overline{\sigma_t}^2} \cdot (x_t - \overline{\mu_t})^2 \\
\frac{d^2 f(x_t)}{dx_t^2} &= A = \frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2} \\
\frac{df(x_t)}{dx_t} &= A \cdot (x_t - B) = \\
&= \frac{1}{\sigma_{Q_t}^2} \cdot (z_t - c_t \cdot x_t) \cdot (-c_t) + \frac{1}{\overline{\sigma_t}^2} \cdot (x_t - \overline{\mu_t}) = \\
&= x_t \cdot \left(\frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2} \right) - \frac{z_t \cdot c_t}{\sigma_{Q_t}^2} - \frac{\overline{\mu_t}}{\overline{\sigma_t}^2} = 0
\end{aligned}
$$

$$
x_t = B = \frac{\frac{z_t \cdot c_t}{\sigma_{Q_t}^2} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2}}{\frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2}} = \frac{\frac{z_t \cdot c_t}{\sigma_{Q_t}^2} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2}}{A}
$$

$$
\alpha' \cdot e^{-\left(\frac{1}{2}\left(\frac{z_t - c_t \cdot x_t}{\sigma_{Q_t}}\right)^2 + \frac{1}{2}\left(\frac{x_t - \overline{\mu_t}}{\overline{\sigma_t}}\right)^2\right)} = \alpha' \cdot e^{-\left(\frac{1}{2} \cdot A \cdot (x_t - B)^2 + C\right)} = \alpha'' \cdot e^{-\frac{1}{2}\left(\frac{x_t - \mu_t}{\sigma_t}\right)^2}
$$

$$
\begin{aligned}
\alpha' \cdot e^{-\left(\frac{1}{2} \cdot A \cdot (x_t - B)^2 + C\right)} &= \alpha'' \cdot e^{-\frac{1}{2}\left(\frac{x_t - \mu_t}{\sigma_t}\right)^2} \\
\alpha' \cdot e^{-C} \cdot e^{-\frac{1}{2} \cdot A \cdot (x_t - B)^2} &= \alpha'' \cdot e^{-\frac{1}{2}\left(\frac{x_t - \mu_t}{\sigma_t}\right)^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\sigma_t^2} &= A \\
\sigma_t^2 &= \frac{1}{A} = \frac{1}{\frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2}} = \frac{\sigma_{Q_t}^2 \cdot \overline{\sigma_t}^2}{c_t^2 \cdot \overline{\sigma_t}^2 + \sigma_{Q_t}^2}
\end{aligned}
$$

$$
\begin{aligned}
\mu_t = B = \frac{\frac{z_t \cdot c_t}{\sigma_{Q_t}^2} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2}}{A} &= \sigma_t^2 \cdot \left(\frac{z_t \cdot c_t}{\sigma_{Q_t}^2} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2} \right) \\
&= \sigma_t^2 \cdot \left(\frac{z_t \cdot c_t}{\sigma_{Q_t}^2} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2} \right) \\
&= \sigma_t^2 \cdot \left(\frac{c_t}{\sigma_{Q_t}^2} \cdot (z_t - c_t\,\overline{\mu_t} + c_t\,\overline{\mu_t}) + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2} \right) \\
&= \sigma_t^2 \cdot \left(\frac{c_t}{\sigma_{Q_t}^2} \cdot (z_t - c_t\,\overline{\mu_t}) + \frac{c_t^2}{\sigma_{Q_t}^2}\overline{\mu_t} + \frac{\overline{\mu_t}}{\overline{\sigma_t}^2} \right) \\
&= \sigma_t^2 \cdot \left(\frac{c_t}{\sigma_{Q_t}^2} \cdot (z_t - c_t\,\overline{\mu_t}) + \left(\frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2} \right) \cdot \overline{\mu_t} \right) \\
&= \sigma_t^2 \cdot \left(\frac{c_t}{\sigma_{Q_t}^2} \cdot (z_t - c_t\,\overline{\mu_t}) + \frac{1}{\sigma_t^2} \cdot \overline{\mu_t} \right) \\
&= \frac{\sigma_t^2}{\sigma_{Q_t}^2} \cdot c_t \cdot (z_t - c_t\,\overline{\mu_t}) + \overline{\mu_t}
\end{aligned}
$$

KALMAN GAIN

$$
\begin{aligned}
K_t &= \frac{\sigma_t^2}{\sigma_{Q_t}^2} \cdot c_t \\
\mu_t &= \overline{\mu_t} + K_t \cdot (z_t - c_t\,\overline{\mu_t})
\end{aligned}
$$

where:

- μ_t is the estimated state (or corrected state), aka $\hat{x}_t$.
- $\overline{\mu_t}$ is the predicted state, aka $\tilde{x}_t$.
- K_t is the Kalman gain.
- z_t is the actual measurement.
- $c_t\,\overline{\mu_t}$ is the expected measurement.
- $(z_t - c_t\,\overline{\mu_t})$ is called innovation.

$$
K_t = \frac{\sigma_t^2}{\sigma_{Q_t}^2} \cdot c_t = \frac{c_t}{\sigma_{Q_t}^2 \cdot \left(\frac{c_t^2}{\sigma_{Q_t}^2} + \frac{1}{\overline{\sigma_t}^2} \right)} = \frac{c_t \cdot \overline{\sigma_t}^2}{c_t \cdot \overline{\sigma_t}^2 + \sigma_{Q_t}^2}
$$

$$
\sigma_{Q_t}^2 \uparrow \longrightarrow K_t \downarrow
$$

$$
\begin{aligned}
K_t &= \frac{c_t \cdot \overline{\sigma_t}^2}{c_t^2 \cdot \overline{\sigma_t}^2 + \sigma_{Q_t}^2} \\
\mu_t &= \overline{\mu_t} + K_t \cdot (z_t - c_t\,\overline{\mu_t}) \\
\sigma_t^2 &= (1 - K_t \cdot c_t) \cdot \overline{\sigma_t}^2
\end{aligned}
$$

$$
\begin{aligned}
K_t = 0 &\longrightarrow \sigma_t^2 = \overline{\sigma_t}^2 \\
K_t > 0 &\longrightarrow \sigma_t^2 < \overline{\sigma_t}^2
\end{aligned}
$$

$$
x_t = a_t \cdot x_{t-1} + b_t \cdot u_t + \epsilon_{R_t}
$$

The termb_t is a coefficient that converts from control space to state space.
The termϵ_{R_t} is the system noise:

$$
\begin{aligned}
p\left(\epsilon_{R_t}\right) &= \mathcal{N}\left(0,\, \sigma_{R_t}^2\right) \\
p\left(x_t \,|\, x_{t-1},\, u_t\right) &= \mathcal{N}\left(\mu_*,\, \sigma_*^2\right)
\end{aligned}
$$

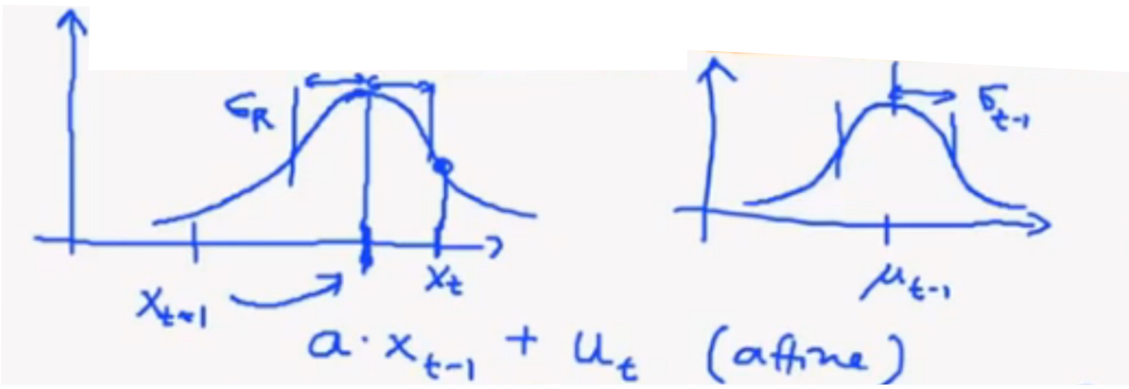
$$
\begin{aligned}
\mu_* = E\Big(x_t \,|\, x_{t-1},\, u_t\Big) &= E\Big(a_t \cdot x_{t-1} + b_t \cdot u_t + \epsilon_{R_t} \,|\, x_{t-1},\, u_t\Big) = \\
&= E\Big(a_t \cdot x_{t-1} \,|\, x_{t-1}\Big) + E\Big(b \cdot u_t \,|\, u_t\Big) + E\Big(\epsilon_{R_t}\Big) = \\
&= a_t \cdot x_{t-1} + b_t \cdot u_t
\end{aligned}
$$

$$
\begin{aligned}
\sigma_*^2 &= E\left(\left(x_t - E\left(x_t\right)\right)^2 \mid x_{t-1},\, u_t\right) = \\
&= E\left(\left(a_t \cdot x_{t-1} + b_t \cdot u_t + \epsilon_{R_t} - a_t \cdot x_{t-1} - b_t \cdot u_t\right)^2 \mid x_{t-1},\, u_t\right) \\
&= E\left(\epsilon_{R_t}^2\right) = E\left(\left(\epsilon_{R_t} - 0\right)^2\right) = \sigma_{R_t}^2
\end{aligned}
$$

$$
\begin{aligned}
p\left(x_t \mid x_{t-1},\, u_t\right) &= \mathcal{N}\left(\mu_*, \sigma_*^2\right) \\
&= \mathcal{N}\left(a_t \cdot x_{t-1} + b_t \cdot u_t,\, \sigma_{R_t}^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\overline{bel}\left(x_t\right) &= \int_{x_{t-1}=-\infty}^{+\infty} p\left(x_t \mid x_{t-1},\, u_t\right) \cdot bel\left(x_{t-1}\right) \cdot dx_{t-1} \\
&= \int_{x_{t-1}=-\infty}^{\infty} \mathcal{N}\left(a_t \cdot x_{t-1} + b_t \cdot u_t,\, \sigma_{R_t}^2\right) \cdot \mathcal{N}\left(\mu_{t-1},\, \sigma_{t-1}^2\right) \cdot dx_{t-1} \\
&= \gamma \cdot \int_{x_{t-1}=-\infty}^{+\infty} e^{-\frac{1}{2}\left(\frac{x_t - \left(a_t \cdot x_{t-1} + b_t \cdot u_t\right)}{\sigma_{R_t}}\right)^2} \cdot e^{-\frac{1}{2}\left(\frac{x_{t-1} - \mu_{t-1}}{\sigma_{t-1}}\right)^2} \cdot dx_{t-1} = \mathcal{N}\left(\overline{\mu_t},\, \overline{\sigma_t}^2\right)
\end{aligned}
$$

(More details about this expressions in the book“Probabilistic Robotics” (Thrun, Burgardm, Fox))



THE KALMAN FILTER IN 1 DIMENSION

1. **PREDICTION:**

$$x_t = a_t \cdot x_{t-1} + b_t \cdot u_t + \epsilon_{R_t}$$

The termϵ_{R_t} is the system noise:

$$p\left(\epsilon_{R_t}\right) = \mathcal{N}\left(0,\, \sigma_{R_t}^2\right)$$

$$\begin{aligned}
p\left(x_t \,|\, x_{t-1},\, u_t\right) &= \mathcal{N}\left(a_t \cdot x_{t-1} + b_t \cdot u_t,\, \sigma_{R_t}^2\right) \\
bel\left(x_{t-1}\right) &= p\left(x_{t-1} \,|\, z_{t-1}\right) = \frac{p\left(z_{t-1} \,|\, x_{t-1}\right) \cdot p\left(x_{t-1}\right)}{p\left(z_{t-1}\right)} = \alpha \cdot p\left(z_{t-1} \,|\, x_{t-1}\right) \cdot \overline{bel}\left(x_{t-1}\right) = \\
&= \mathcal{N}(\mu_{t-1},\, \sigma_{t-1}^2) \\
\overline{bel}\left(x_t\right) &= \int_{x_{t-1}=-\infty}^{+\infty} p\left(x_t \,|\, x_{t-1},\, u_t\right) \cdot bel\left(x_{t-1}\right) \cdot dx_{t-1} = \\
&= \int_{x_{t-1}=-\infty}^{+\infty} \mathcal{N}\left(a_t \cdot x_{t-1} + b_t \cdot u_t,\, \sigma_{R_t}^2\right) \cdot \mathcal{N}\left(\mu_{t-1},\, \sigma_{t-1}^2\right) \cdot dx_{t-1} = \mathcal{N}\left(\overline{\mu_t},\, \overline{\sigma_t}^2\right)
\end{aligned}$$

$$\begin{aligned}
\overline{\mu_t} &= a_t \cdot \mu_{t-1} + b_t \cdot u_t \\
\overline{\sigma_t}^2 &= a_t^2 \cdot \sigma_{t-1}^2 + \sigma_{R_t}^2
\end{aligned}$$

2. **CORRECTION:**

$$z_t = c_t \cdot x_t + \epsilon_{Q_t}$$

The term$\epsilon_{Q_t}$is the measurement noise:

$$p\left(\epsilon_{Q_t}\right) = \mathcal{N}\left(0,\, \sigma_{Q_t}^2\right)$$

$$\begin{aligned}
p\left(z_t \,|\, x_t\right) &= \mathcal{N}(c_t \cdot x_t,\, \sigma_{Q_t}^2) \\
\overline{bel}\left(x_t\right) &= \mathcal{N}(\overline{\mu_t},\, \overline{\sigma_t}^2) \\
bel\left(x_t\right) &= p\left(x_t \,|\, z_t\right) = \alpha \cdot p\left(z_t \,|\, x_t\right) \cdot \overline{bel}\left(x_t\right) = \\
&= \alpha \cdot \mathcal{N}(c_t \cdot x_t,\, \sigma_{Q_t}^2) \cdot \mathcal{N}(\overline{\mu_t},\, \overline{\sigma_t}^2) = \mathcal{N}(\mu_t,\, \sigma_t^2)
\end{aligned}$$

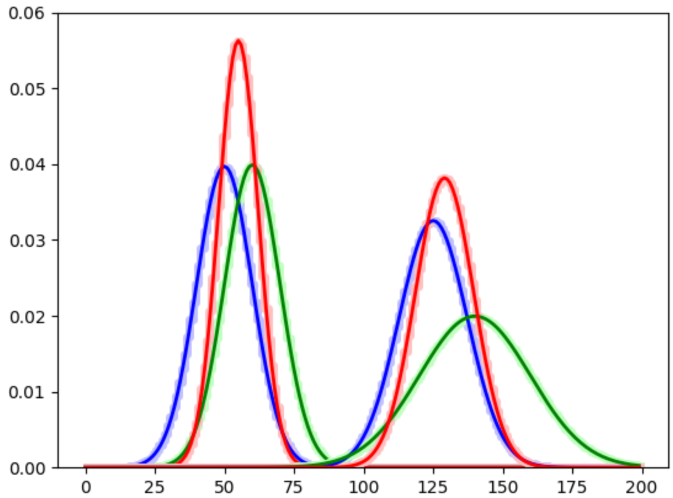
$$K_t = \frac{c_t \cdot \overline{\sigma_t}^2}{c_t^2 \cdot \overline{\sigma_t}^2 + \sigma_{Q_t}^2}$$

$$\mu_t = \overline{\mu_t} + K_t \cdot (z_t - c_t \, \overline{\mu_t})$$

$$\sigma_t^2 = (1 - K_t \cdot c_t) \cdot \overline{\sigma_t}^2$$

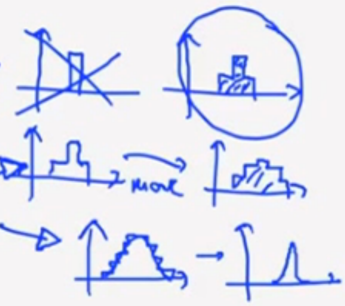
$$K_t = 0 \longrightarrow \mu_t = \overline{\mu_t}, \sigma_t^2 = \overline{\sigma_t}^2$$

$$K_t > 0 \longrightarrow \sigma_t^2 < \overline{\sigma_t}^2$$



SLAM-C : Filtering

- Assign probabilities
- Distributions
- Convolution (movement)
  Multiplication (measurement)



- Bayes filter
- Histogram filter

$N(x; \mu, \sigma^2)$

- Distributions (discrete) → Densities (continuous)
- Kalman filter