P13

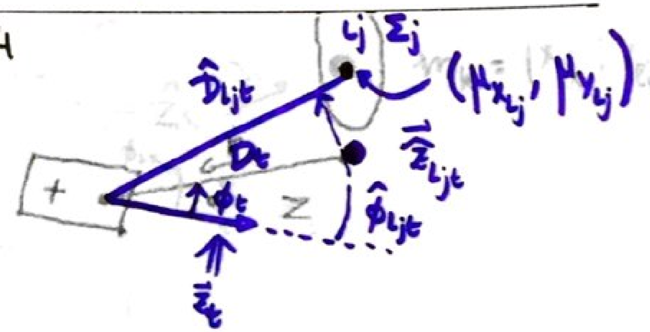
$$h(\ldots) \left\langle \begin{array}{l} D_{jt} = \sqrt{(x_{L_j}-x_\ell)^2 + (y_{L_j}-y_\ell)^2} \\ \phi_{jt} = \operatorname{atan}\left(\frac{y_{L_j}-y_\ell}{x_{L_j}-x_\ell}\right) - \theta \end{array}\right.$$

$$H_{jt} = \underbrace{\begin{bmatrix} \frac{\partial D}{\partial x_{L_j}} & \frac{\partial D}{\partial y_{L_j}} \\ \frac{\partial \phi}{\partial x_{L_j}} & \frac{\partial \phi}{\partial y_{L_j}} \end{bmatrix}}_{2\times 2,\ \mathbb{R}} \quad \frac{\partial D}{\partial x_{L_j}} = \frac{x_{L_j}-x_\ell}{\sqrt{q}} \quad \frac{\partial D}{\partial y_{L_j}} = \frac{y_{L_j}-y_\ell}{\sqrt{q}} \quad \frac{\partial \phi}{\partial x_{L_j}} = -\frac{(y_{L_j}-y_\ell)}{q} \quad \frac{\partial \phi}{\partial y_{L_j}} = \frac{x_{L_j}-x_\ell}{q}$$

$$\sqrt{q} = \sqrt{(x_{L_j}-x_\ell)^2 + (y_{L_j}-y_\ell)^2}$$

In slam lecture: $2\times(3+2N)$ H

Ⓐ Compute likelihoods.



Expected measurement: $j = 1 \ldots N$, # of landmarks; $k = 1 \ldots M$, # of particles.

$$\begin{bmatrix} \hat{D}_{jt} \\ \hat{\phi}_{jt} \end{bmatrix} = \hat{\vec{z}}_{jt} = h\Big(\overbrace{(x_{it}, y_{it}, \theta_{it})}^{\text{Particle } P_{it}},\ j\Big)$$

$$H = \frac{\partial h}{\partial \text{landmark}}\Big|_{(x_{it}, y_{it}, \theta_{it}, j)} \quad (2\times 2)$$

$x_{it}, y_{it}, \theta_{it}$

= Particle i, P_{it}, $i = 1 \ldots M$ At time t

μ	...	x_k y_k	...
Σ	...	Σ_k	...

list of landmark coordinates the particle P_i has.

Covariance of the term: $(\vec{z}_t - \vec{\hat{z}}_{L_jt})$

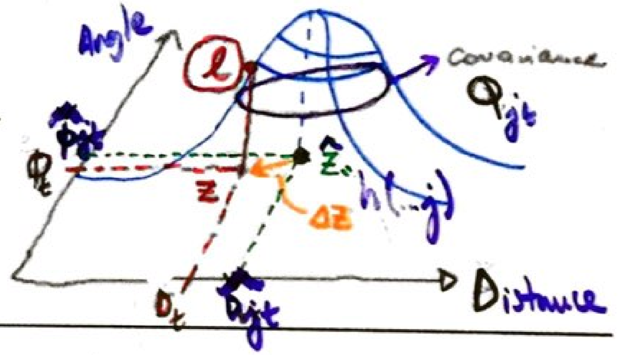
$$Q_{jt} = \underbrace{H_{jt} \underbrace{\Sigma_{i,jt}}_{\substack{\text{Variance of landmark } k \\ \text{(in } x, y)}} H_{jt}^T}_{\text{measurement variance due to the landmark variance}} + \underbrace{Q}_{\begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}}$$

aka (Q_ℓ)

In the SLAM-EKF lecture we used $H_{tk} \in \mathbb{R}^{2\times(3+2N)}$ and $\bar{\Sigma}_t \in \mathbb{R}^{(3+2N)\times(3+2N)}$

and now, in this lecture, we use $H_{tk} \in \mathbb{R}^{2\times 2}$ and $\Sigma_k \in \mathbb{R}^{2\times 2}$

variance due to actual measurement $\vec{z}_t$



$$\Delta Z_{jt} = Z_t - \hat{Z}_{jt}$$

$$\ell_{jt} = \frac{1}{2\pi\sqrt{\det(Q_{jt})}} \cdot e^{-\frac{1}{2}\Delta Z_{jt}^T Q_{jt}^{-1} \Delta Z_{jt}}$$

Likelihood that the measurement $\vec{z}_t$ is due to the observation of landmark L_j $(\mu_{x_{L_j}}, \mu_{y_{L_j}})$

~~Likelihood that the measurement Z_t is due to landmark $m_k = (x, y)$~~

$$\vec{\hat{z}}_{jt} = \begin{bmatrix} \hat{D}_{jt} \\ \hat{\phi}_{jt} \end{bmatrix} = h(\vec{P}_{it}, j)$$

$$\vec{z}_t = \begin{bmatrix} D_t \\ \phi_t \end{bmatrix}$$

$$\Sigma_K = \begin{bmatrix} \sigma^2_{x_{ek}} & \sigma_{x_{ek} y_{ek}} \\ \sigma_{x_{ek} y_{ek}} & \sigma^2_{y_{ek}} \end{bmatrix}$$

"Not"

Q_{tk} (explanation) ⇒ The uncertainty in the value $h_{tk}(P_{it}, k)$, i.e, Q_{tk}, is due to the uncertainty of the landmark "k", i.e, Σ_K, which then translates into uncertainty in a distance and bearing angle, i.e, $H_{tk}\Sigma_K H_{tk}^T$, plus the uncertainty of the actual measurement of the sensor, i.e, Q_Z