EKF - SLAM : PREDICTION

Remember the landmark coordinates don't have the subscript "t"! (x_{Lj}, y_{Lj}). When I first created this document I used x_{it}, y_{it} → NO!! → (x_{Lj}, y_{Lj})

Now:

Coordinates of different landmarks the robot discovers while it's travelling around the environment

$$\vec{X}_t = g\Big(\underbrace{x_{t-1}, y_{t-1}, \theta_{t-1}, x_{ot-1}, y_{ot-1}, x_{1t-1}, y_{1t-1}, \ldots x_{it-1}, y_{it-1} \ldots}_{\text{state}\,:\,\vec{X}_{t-1}}, \underset{\substack{u_t\\ \text{control}}}{l_t, r_t}\Big) + \varepsilon_{R,t}\,;\ P(\varepsilon_{R,t}) = N(0, R_t),\ R_t = V_t \cdot \Sigma_{control,t} \cdot V_t^T$$

$$\Sigma_{control,t} = \begin{bmatrix} \sigma_{l_t}^2 & 0 \\ 0 & \sigma_{r_t}^2 \end{bmatrix},\quad \sigma_{l_t}^2 = (p_1 l_t)^2 + (p_2(l_t - r_t))^2,\quad \sigma_{r_t}^2 = (p_1 r_t)^2 + (p_2(l_t - r_t))^2$$

$$\vec{X}_t = \begin{bmatrix} X_t \\ Y_t \\ \theta_t \\ X_{L1} \\ Y_{L1} \\ \vdots \\ X_{Lj} \\ Y_{Lj} \\ \vdots \\ X_{LN} \\ Y_{LN} \end{bmatrix} = \overbrace{\begin{bmatrix} X_{t-1} \\ Y_{t-1} \\ \theta_{t-1} \\ X_{L1} \\ Y_{L1} \\ \vdots \\ X_{Lj} \\ Y_{Lj} \\ \vdots \\ X_{LN} \\ Y_{LN} \end{bmatrix}}^{\vec{X}_{t-1}} + \begin{bmatrix} (Rad_t + \frac{W}{2}) \cdot (\cdots) \\ (Rad_t + \frac{W}{2}) \cdot (\cdots) \\ \alpha_t \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

old equations we know, $\alpha_t = \frac{r_t - l_t}{W}$, $Rad_t = \frac{l_t}{\alpha_t}$

The landmarks' coordinates are copied directly from state $\vec{X}_{t-1}$ to $\vec{X}_t$

$$G_t = \frac{\partial g(\cdot)}{\partial state} = \frac{\partial g(\cdot)}{\partial \vec{X}_{t-1}} = \quad , G_t \in \mathbb{R}^{(3+2N)\times(3+2N)}$$

$$= \begin{array}{c|ccc|ccccccccccc|}
 & \partial X_{t-1} & \partial Y_{t-1} & \partial\theta_{t-1} & \partial X_{ot-1} & \partial Y_{ot-1} & \partial X_{1t-1} & \partial Y_{1t-1} & \cdots & \partial X_{it-1} & \partial Y_{it-1} & \cdots & \partial X_{(N-1)t-1} & \partial Y_{(N-1)t-1} \\
\partial X_t & \frac{\partial X_t}{\partial X_{t-1}} & \frac{\partial X_t}{\partial Y_{t-1}} & \frac{\partial X_t}{\partial \theta_{t-1}} & \frac{\partial X_t}{\partial X_{ot-1}} & \frac{\partial Y_t}{\partial Y_{ot-1}} & \frac{\partial X_t}{\partial X_{1t-1}} & \frac{\partial X_t}{\partial Y_{1t-1}} & \cdots & \frac{\partial X_t}{\partial X_{it-1}} & \frac{\partial X_t}{\partial Y_{it-1}} & \cdots & \frac{\partial X_t}{\partial X_{(N-1)t-1}} & \frac{\partial X_t}{\partial Y_{(N-1)t-1}} \\
\partial Y_t & \frac{\partial Y_t}{\partial X_{t-1}} & \frac{\partial Y_t}{\partial Y_{t-1}} & \frac{\partial Y_t}{\partial \theta_{t-1}} & \frac{\partial Y_t}{\partial X_{ot-1}} & \frac{\partial Y_t}{\partial Y_{ot-1}} & \frac{\partial Y_t}{\partial X_{1t-1}} & \frac{\partial Y_t}{\partial Y_{1t-1}} & \cdots & \frac{\partial Y_t}{\partial X_{it-1}} & \frac{\partial Y_t}{\partial Y_{it-1}} & \cdots & \frac{\partial Y_t}{\partial X_{(N-1)t-1}} & \frac{\partial Y_t}{\partial Y_{(N-1)t-1}} \\
\partial \theta_t & \frac{\partial \theta_t}{\partial X_{t-1}} & \frac{\partial \theta_t}{\partial Y_{t-1}} & \frac{\partial \theta_t}{\partial \theta_{t-1}} & \frac{\partial \theta_t}{\partial X_{ot-1}} & \frac{\partial \theta_t}{\partial Y_{ot-1}} & \frac{\partial \theta_t}{\partial X_{1t-1}} & \frac{\partial \theta_t}{\partial Y_{1t-1}} & \cdots & \frac{\partial \theta_t}{\partial X_{it-1}} & \frac{\partial \theta_t}{\partial Y_{it-1}} & \cdots & \frac{\partial \theta_t}{\partial X_{(N-1)t-1}} & \frac{\partial \theta_t}{\partial Y_{(N-1)t-1}} \\
\hline
\partial X_{ot} & \frac{\partial X_{ot}}{\partial X_{t-1}} & \frac{\partial X_{ot}}{\partial Y_{t-1}} & \frac{\partial X_{ot}}{\partial \theta_{t-1}} & \frac{\partial X_{L0}}{\partial X_{L1}} & \frac{\partial X_{L0}}{\partial Y_{L1}} & \frac{\partial X_{L0}}{\partial X_{L2}} & \frac{\partial X_{L0}}{\partial Y_{L2}} & \cdots & \frac{\partial X_{L0}}{\partial X_{Lj}} & \frac{\partial X_{L0}}{\partial Y_{Lj}} & \cdots & \frac{\partial X_{L0}}{\partial X_{LN}} & \frac{\partial X_{L0}}{\partial Y_{LN}} \\
\partial Y_{ot} & \frac{\partial Y_{ot}}{\partial X_{t-1}} & \frac{\partial Y_{ot}}{\partial Y_{t-1}} & \frac{\partial Y_{ot}}{\partial \theta_{t-1}} & \frac{\partial Y_{ot}}{\partial X_{L1}} & \frac{\partial Y_{ot}}{\partial Y_{L1}} & \frac{\partial Y_{ot}}{\partial X_{L2}} & \frac{\partial Y_{ot}}{\partial Y_{L2}} & \cdots & \frac{\partial Y_{ot}}{\partial X_{Lj}} & \frac{\partial Y_{ot}}{\partial Y_{Lj}} & \cdots & \frac{\partial Y_{ot}}{\partial X_{LN}} & \frac{\partial Y_{ot}}{\partial Y_{LN}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\partial X_{it} & \frac{\partial X_{it}}{\partial X_{t-1}} & \frac{\partial X_{it}}{\partial Y_{t-1}} & \frac{\partial X_{it}}{\partial \theta_{t-1}} & \frac{\partial Y_{it}}{\partial X_{ot-1}} & \frac{\partial X_{it}}{\partial Y_{ot-1}} & \frac{\partial X_{it}}{\partial X_{1t-1}} & \frac{\partial X_{it}}{\partial Y_{1t-1}} & \cdots & \frac{\partial X_{it}}{\partial X_{it-1}} & \frac{\partial X_{it}}{\partial Y_{it-1}} & \cdots & \frac{\partial X_{it}}{\partial X_{(N-1)t-1}} & \frac{\partial X_{it}}{\partial Y_{(N-1)t-1}} \\
\partial Y_{it} & \frac{\partial Y_{it}}{\partial X_{t-1}} & \frac{\partial Y_{it}}{\partial Y_{t-1}} & \frac{\partial Y_{it}}{\partial \theta_{t-1}} & \frac{\partial Y_{it}}{\partial X_{ot-1}} & \frac{\partial Y_{it}}{\partial Y_{ot-1}} & \frac{\partial Y_{it}}{\partial X_{1t-1}} & \frac{\partial Y_{it}}{\partial Y_{1t-1}} & \cdots & \frac{\partial Y_{it}}{\partial X_{it-1}} & \frac{\partial Y_{it}}{\partial Y_{it-1}} & \cdots & \frac{\partial Y_{it}}{\partial X_{(N-1)t-1}} & \frac{\partial Y_{it}}{\partial Y_{(N-1)t-1}} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\partial X_{(N-1)t} & \frac{\partial X_{(N-1)t}}{\partial X_{t-1}} & \frac{\partial X_{(N-1)t}}{\partial Y_{t-1}} & \frac{\partial X_{(N-1)t}}{\partial \theta_{t-1}} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial X_{(N-1)t}}{\partial X_{(N-1)t-1}} & \frac{\partial X_{(N-1)t}}{\partial Y_{(N-1)t-1}} \\
\partial Y_{(N-1)t} & \frac{\partial Y_{(N-1)t}}{\partial X_{t-1}} & \frac{\partial Y_{(N-1)t}}{\partial Y_{t-1}} & \frac{\partial Y_{(N-1)t}}{\partial \theta_{t-1}} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \frac{\partial Y_{(N-1)t}}{\partial X_{(N-1)t-1}} & \frac{\partial Y_{(N-1)t}}{\partial Y_{(N-1)t-1}}
\end{array}$$

3

2·N

3 2N

$$\frac{\cdot}{\partial x_{Lj}} \quad \frac{\cdot}{\partial y_{Lj}},\quad j = 1 \ldots N$$

The old G matrix $\in \mathbb{R}^{3\times3}$, we call it G_3

$$\begin{array}{c}
\begin{array}{cc} 3 & 2\cdot N \end{array} \\
\left[\begin{array}{ccc|cccccccccc}
1 & 0 & *_1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & *_2 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\hline
0 & 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1
\end{array}\right]
\end{array}
\begin{array}{l} 3 \\ 2\cdot N \end{array}$$

Identity matrix

$$R_t = V_t \cdot \Sigma_{control,t} \cdot V_t^T$$

$$V_t = \frac{\partial g(.)}{\partial Control} = \frac{\partial g(.)}{\partial U_t} = \begin{bmatrix} \frac{\partial x_t}{\partial l_t} & \frac{\partial x_t}{\partial r_t} \\ \frac{\partial y_t}{\partial l_t} & \frac{\partial y_t}{\partial r_t} \\ \frac{\partial \theta_t}{\partial l_t} & \frac{\partial \theta_t}{\partial r_t} \\ \frac{\partial x_{0t}}{\partial \theta_t} & \frac{\partial x_{0t}}{\partial r_t} \\ \frac{\partial y_{0t}}{\partial l_t} & \frac{\partial y_{0t}}{\partial r_t} \\ \vdots & \vdots \\ \frac{\partial x_{it}}{\partial l_t} & \frac{\partial x_{it}}{\partial r_t} \\ \frac{\partial y_{it}}{\partial l_t} & \frac{\partial y_{it}}{\partial r_t} \\ \vdots & \\ \frac{\partial x_{(N-1)t}}{\partial l_t} & \frac{\partial x_{(N-1)t}}{\partial r_t} \\ \frac{\partial y_{(N-1)t}}{\partial l_t} & \frac{\partial y_{(N-1)t}}{\partial r_t} \end{bmatrix} = \overbrace{\begin{bmatrix} \#_1 & \#_3 \\ \#_2 & \#_4 \\ -\frac{1}{W} & \frac{1}{W} \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}^{2} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} 3 \rightarrow \text{old } V_t \text{ matrix} \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\\a\\a\\a\\a\end{matrix}}\right\} 2N \end{matrix}$$

$$V_t \in \mathbb{R}^{(3+2N)\times 2}$$

$$R_t = \underset{(3+2N)\times 2}{\overset{V_t}{\begin{bmatrix} \#_1 & \#_3 \\ \#_2 & \#_4 \\ -\frac{1}{W} & \frac{1}{W} \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}} \cdot \underset{2\times 2}{\overset{\Sigma_{control,t}}{\begin{bmatrix} \sigma_{l_t}^2 & 0 \\ 0 & \sigma_{r_t}^2 \end{bmatrix}}} \cdot \underset{2\times(3+2N)}{\overset{V_t^T}{\begin{bmatrix} \#_1 & \#_2 & -\frac{1}{W} & 0\,0\cdots 0\,0 \cdots 0\,0 \\ \#_3 & \#_4 & \frac{1}{W} & 0\,0\cdots 0\,0 \cdots 0\,0 \end{bmatrix}}} = \begin{array}{c|c|c} & 3 & 2N \\ \hline & \begin{matrix} R_3 \\ \text{"old } R_t \text{ matrix"} \end{matrix} & \begin{matrix} 0\,0 \cdots 0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \end{matrix} & 3 \\ \hline & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0\,0 \cdots 0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \\ \vdots \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \\ \vdots \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \\ 0\,0 \cdots 0\,0 \cdots 0\,0 \end{matrix} & 2N \end{array}$$

$$R_t \in \mathbb{R}^{(3+2N)\times(3+2N)}$$

REVIEW

N is the number of landmarks found $i = 0 \cdots N-1$ at time $t-1$

$$g(x_{t-1}, y_{t-1}, \theta_{t-1}, l_t, r_t) \longrightarrow g(x_{t-1}, y_{t-1}, \theta_{t-1}, x_{0t-1}, y_{0t-1} \cdots x_{it-1}, y_{it-1} \cdots l_t, r_t)$$

$$\underset{3\times 3}{G_t} = \begin{bmatrix} 1 & 0 & *_1 \\ 0 & 1 & *_2 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \underset{(3+2N)\times(3+2N)}{G_t} = \begin{array}{c|c|c} & 3 & 2N \\ \hline 3 & \begin{bmatrix} \text{old } G_t \end{bmatrix} & 0 \\ \hline 2N & 0 & \begin{matrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{matrix} \end{array}$$

$$R_t = V_t \, \Sigma_{control} \, V_t^T, \; 3\times 3$$

$$\begin{bmatrix} \Delta & \Delta & \Delta \end{bmatrix} \longrightarrow \underset{(3+2N)\times(3+2N)}{R_t} = \begin{array}{c|c|c} & 3 & 2N \\ \hline \begin{bmatrix} \text{old } R_t \end{bmatrix} & 0 & 3 \\ \hline 0 & 0 & 2N \end{array}$$

$$\underset{\tilde{\mathbb{X}}_t}{\underset{\text{or}}{\bar{\mu}_t}} = g\left(\underset{\hat{\mathbb{X}}_{t-1}}{\underset{\text{or}}{\mu_{t-1}}}, u_t\right)$$

$\hat{\mathbb{X}}_{t-1}$ → Estimation in the step t-1

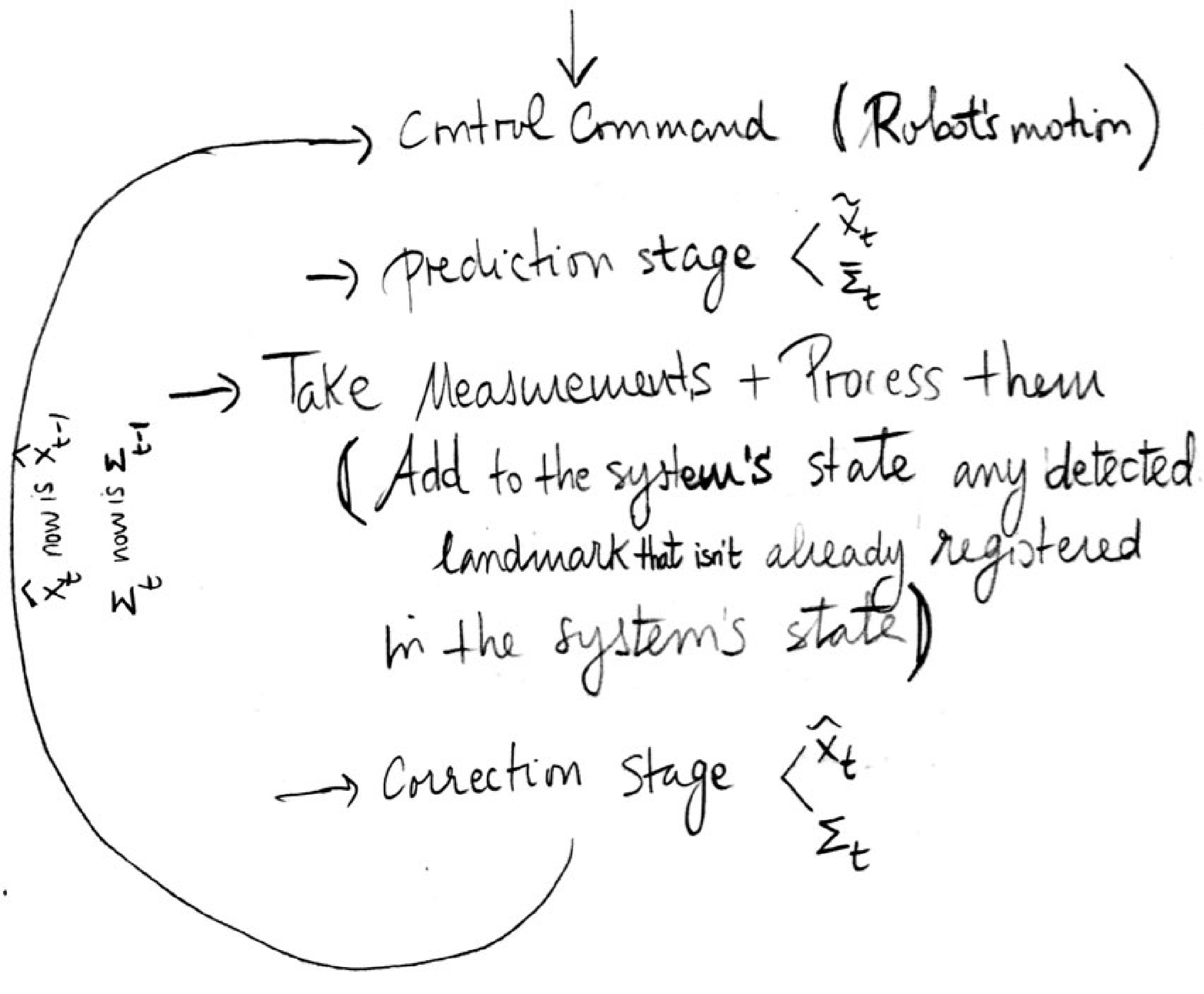
$\tilde{\mathbb{X}}_t$ → Prediction in the step t

$$\begin{array}{l} \tilde{x}_t \leftarrow \\ \tilde{y}_t \leftarrow \\ \hat{\theta}_t \leftarrow \\ \tilde{lx}_0 \leftarrow \\ \tilde{y}_{0t} \leftarrow \\ \\ \vdots \\ \tilde{x}_{it} \leftarrow \\ \tilde{y}_{it} \leftarrow \\ \vdots \\ \tilde{x}_{(N-1)t} \leftarrow \\ \tilde{y}_{(N-1)t} \leftarrow \end{array} \begin{bmatrix} \bar{\mu}_{xt} \\ \bar{\mu}_{yt} \\ \bar{\mu}_{\theta t} \\ \bar{\mu}_{lx_0} \\ \bar{\mu}_{y_0 t} \\ \\ \vdots \\ \bar{\mu}_{x_i t} \\ \bar{\mu}_{y_i t} \\ \vdots \\ \bar{\mu}_{x_{(N-1)} t} \\ \bar{\mu}_{y_{(N-1)} t} \end{bmatrix} = \begin{array}{l} \hat{x}_{t-1} \leftarrow \\ \hat{y}_{t-1} \leftarrow \\ \hat{\theta}_{t-1} \leftarrow \\ \hat{x}_{0t-1} \leftarrow \\ \hat{y}_{0t-1} \leftarrow \\ \\ \vdots \\ \hat{x}_{it-1} \leftarrow \\ \hat{y}_{it-1} \leftarrow \\ \vdots \\ \hat{x}_{(N-1)t-1} \leftarrow \\ \hat{y}_{(N-1)t-1} \leftarrow \end{array} \begin{bmatrix} \mu_{x_{t-1}} \\ \mu_{y_{t-1}} \\ \mu_{\theta t-1} \\ \mu_{x_0 t-1} \\ \mu_{y_0 t-1} \\ \\ \vdots \\ \mu_{x_i t-1} \\ \mu_{y_i t-1} \\ \vdots \\ \mu_{x_{(N-1)} t-1} \\ \mu_{y_{(N-1)} t-1} \end{bmatrix} + \begin{bmatrix} \left(Rad_t + \frac{W}{2}\right) \cdot \left(\cdots\right) \\ \left(Rad_t + \frac{W}{2}\right) \cdot \left(\cdots\right) \\ \alpha_t \\ 0 \\ 0 \\ \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

The landmarks' coordinates present in the estimated state at time t-1, $\overset{\mu_{t-1} \text{ or}}{\hat{\mathbb{X}}_{t-1}}$, are copied directly to the predicted state at time t $\tilde{\mathbb{X}}_t$ or $\bar{\mu}_t$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t = \underset{(3+2N)\times(3+2N)}{G_t} \cdot \underset{(3+2N)\times(3+2N)}{\Sigma_{t-1}} \cdot \underset{(3+2N)\times(3+2N)}{G_t^T} + \underbrace{\underset{(3+2N)\times 2}{V_t} \cdot \underset{2\times 2}{\Sigma_{control,t}} \cdot \underset{2\times(3+2N)}{V_t^T}}_{(3+2N)\times(3+2N)}$$

NEW DETECTED LANDMARKS ARE ADDED TO THE SYSTEM'S STATE AFTER THE PREDICTION STAGE AND BEFORE THE CORRECTION STAGE.



→ Control Command (Robot's motion)

→ Prediction stage $\langle \tilde{x}_t, \bar{\Sigma}_t$

→ Take Measurements + Process them (Add to the system's state any detected landmark that isn't already registered in the system's state)

→ Correction stage $\langle \hat{x}_t, \Sigma_t$

$\hat{x}_t$ now is $\hat{x}_{t-1}$

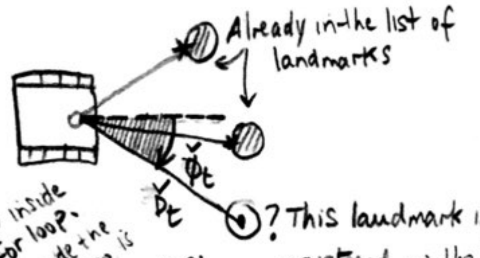
Σ_t now is Σ_{t-1}

How to add a landmark to the system state

[The addition of a landmark will happen whenever the robot observes and objett in a measurement for which it doesn't find a corresponding landmark in the current list of landmarks !!]

In previous lectures, first the algorithm detected cylinders in a scan (taken from the world by the scanner laser from the robot's pose in the world). Then the algorithm tried to match each cylinder with the closest landmark. THE COORDINATES OF EACH LANDMARK WERE GIVEN AS PART OF THE <u>INITIAL DATA</u>, i.e, THESE COORDINATES WERE KNOWN. The match between a cylinder and a landmark were based on distance. The algorithm calculated the distance between each cylinder and each landmark. Then if the minimum distance between a cylinder and a landmark were smaller than a given maximum threshold distance the match was kept, otherwise the match was rejected and the cylinder was unpaired, no match with any landmark. Now in this lecture THE LANDMARKS' COORDINATES ARE NOT KNOW, THEY ARE NOT PART OF THE INITIAL DATA ANYMORE. So, <u>now the algorithm try to match each detected cylinder in a scan with the closest landmark present in the robot's state</u>. Again, if the minimum distance between a cylinder and a landmark (present in the robot's state) is smaller than a given maximum threshold distance the match between that cylinder and that landmark is kept, otherwise the match is rejected and that cylinder is ADDED TO THE STATE AS A NEW DISCOVERED LANDMARK.

This steps are taken by the function: $[((z_{Dt}, z_{\alpha t}), (X_{mt}, Y_{mt}) (X', Y'), \text{index match}) \ldots.] = \text{get\_observations}(\ldots..)$



$$\underset{(\text{state})}{\mathbb{X}_t} = \overset{\text{size}:3}{\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix}} \xrightarrow[(N=1)]{\text{Add one landmark}} \overset{\text{size}: 3+2N}{\begin{bmatrix} x_t \\ y_t \\ \theta_t \\ \hline x_{0t} \\ y_{0t} \end{bmatrix}} \xrightarrow[(N=2)]{\text{Add one landmark}} \overset{\text{Size}: 3+2N}{\begin{bmatrix} x_t \\ y_t \\ \theta_t \\ x_{0t} \\ y_{0t} \\ \hline x_{1t} \\ y_{1t} \end{bmatrix}}$$

state after adding a landmark to the list of landmarks

state after adding another landmark to the list of landmarks

Grow inside For loop. Inside the for loop is also the correction step.

$$\Sigma_t = \Big[\quad \xrightarrow[\text{landmark}]{\text{Add one}} \quad \begin{array}{c|ccc|cc} & & 3 & & 2N & \\ \hline 3 & \sigma^2_{x_t} & \sigma_{x_t y_t} & \sigma_{x_t \theta_t} & 0 & 0 \\ & \sigma_{x_t y_t} & \sigma^2_{y_t} & \sigma_{y_t \theta_t} & 0 & 0 \\ & \sigma_{x_t \theta_t} & \sigma_{y_t \theta_t} & \sigma^2_{\theta_t} & 0 & 0 \\ \hline 2N & 0 & 0 & 0 & \sigma^2_{x_{0t}} & 0 \\ & 0 & 0 & 0 & 0 & \sigma^2_{y_{0t}} \end{array} \xrightarrow{\text{add a new landmark}}$$

N: of landmarks: 1

(N=1)

We assume that this new landmark is not correlated with the previous state.

We'll also assume that x_0 and y_0 of the new landmark are <u>uncorrelated</u>

Since we don't know anything about the initial position before our first measurement we will assume these values to be: $\sigma^2_{x_0} = \sigma^2_{y} = \infty$

N° of landmarks =2

(N=2)

$$\begin{array}{c} (3+2(N-1))\times(3+2(N-1)) \\ \underbrace{\begin{bmatrix} \sigma^2_{x_t} & \sigma_{x_t y_t} & \sigma_{x_t\theta_t} & 0 & 0 & 0 & 0 \\ \sigma_{x_t y_t} & \sigma^2_{y_t} & \sigma_{y_t\theta_t} & 0 & 0 & 0 & 0 \\ \sigma_{x_t\theta_t} & \sigma_{y_t\theta_t} & \sigma^2_{\theta_t} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{x0t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{y0t} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{x1t} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{y1t} \end{bmatrix}}_{3 \qquad\qquad 2N} \end{array} \begin{array}{l} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\}3 \\ \left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\}2N\text{, N is the number of found landmarks} \end{array}$$

Note: In the lecture the teacher uses $\underset{\smile}{r}$ and $\underset{\smile}{\alpha}$ as variables' names but I used $\underset{\smile}{D_t}$ and $\underset{\smile}{\phi_t}$ to prevent confusions with other variables

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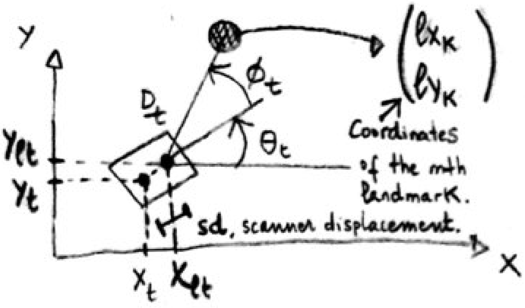
Observations



$$m_K := (\ell x_K, \ell y_K)$$

$$\begin{bmatrix} X_{\ell t} \\ Y_{\ell t} \end{bmatrix} = \begin{bmatrix} X_t \\ Y_t \end{bmatrix} + sd \begin{bmatrix} \cos\theta_t \\ \sin\theta_t \end{bmatrix}$$

$$D_{mt} = \sqrt{(X_{mt} - X_{\ell t})^2 + (Y_{mt} - Y_{\ell t})^2}$$

$$\phi_{mt} = \operatorname{atan}\left(\frac{Y_{mt} - Y_{\ell t}}{X_{mt} - X_{\ell t}}\right) - \theta_t$$

$$h_m(x_t, y_t, \theta_t, X_{mt}, Y_{mt}) = \begin{bmatrix} D_{mt}(X_t, X_{mt}, Y_{mt}) \\ \phi_{mt}(X_t, X_{mt}, Y_{mt}) \end{bmatrix}$$

$\underset{\smile}{m}$ is the landmark we are evaluating

$$\text{old } \underset{\in \mathbb{R}^{2\times3}}{H} = \overbrace{\begin{bmatrix} \frac{\partial D_t}{\partial x_t} & \frac{\partial D_t}{\partial y_t} & \frac{\partial D_t}{\partial \theta_t} \\ \frac{\partial \phi_t}{\partial x_t} & \frac{\partial \phi_t}{\partial y_t} & \frac{\partial \phi_t}{\partial \theta_t} \end{bmatrix}}^{3} \Bigg\} 2$$

In the previous lectures the landmarks' coordinates were constant, i.e, the landmark were fixed, so these coordinates weren't part of the $h(\cdot)$ function. Now, we have a different situation, our landmarks have become unknown as well, and so our function $H(\cdot)$ changes.

They are variables now

$$H_{mt} = \begin{array}{c|ccccccccccccccc}
 & \frac{\partial \cdot}{\partial x_t} & \frac{\partial \cdot}{\partial y_t} & \frac{\partial \cdot}{\partial \theta_t} & \frac{\partial \cdot}{\partial x_{0t}} & \frac{\partial \cdot}{\partial y_{0t}} & & & & & \frac{\partial \cdot}{\partial x_{mt}} & \frac{\partial \cdot}{\partial y_{mt}} & & & \\
\partial D_{mt} & \frac{\partial D_{mt}}{\partial x_t} & \frac{\partial D_{mt}}{\partial y_t} & \frac{\partial D_{mt}}{\partial \theta_t} & \overset{0}{\cancel{\frac{\partial D_{mt}}{\partial x_{0t}}}} & \frac{\partial D_{mt}}{\partial y_{0t}} & 0 & 0 & 0 & 0 & \frac{\partial D_{mt}}{\partial x_{mt}} & \frac{\partial D_{mt}}{\partial y_{mt}} & 0 & 0 & 0 \\
\partial \phi_{mt} & \frac{\partial \phi_{mt}}{\partial x_t} & \frac{\partial \phi_{mt}}{\partial y_t} & \frac{\partial \phi_{mt}}{\partial \theta_t} & \overset{0}{\cancel{\frac{\partial \phi_{mt}}{\partial x_{0t}}}} & \overset{0}{\cancel{\frac{\partial \phi_{mt}}{\partial y_{0t}}}} & 0 & 0 & 0 & 0 & \frac{\partial \phi_{mt}}{\partial x_{mt}} & \frac{\partial \phi_{mt}}{\partial y_{mt}} & 0 & 0 & 0 \\
 & 0 & 1 & 2 & \uparrow \text{index}=3 & 4 & \uparrow \text{index}=5 & & & & \uparrow \text{index} = 3+2\cdot\text{landmark\_index}_{(m)} & & & &
\end{array} \in \mathbb{R}^{2\times(3+2N)}$$

$$\underbrace{\quad 0 \quad 1 \quad 2 \quad}_{3} \qquad \underbrace{\quad 3 \quad 4 \quad 5 \quad \cdots \quad}_{2N}$$

landmark_index = 0 (m=0)

landmark_index = 1 (m=1)

(-1)

$$\boxed{\text{index} = 3 + 2 \cdot \text{Landmark\_index}_{(m)}}$$

$$2\times(3+2N) = 2\times(3+2+2(N-1))$$

$$\frac{\cdot}{\partial x_t} \quad \frac{\cdot}{\partial y_t} \quad \frac{\cdot}{\partial \theta_t} \qquad \frac{\cdot}{\partial x_{mt}} \quad \frac{\cdot}{\partial y_{mt}} \qquad \begin{matrix} 2(N-1) \\ \text{zeros} \end{matrix}$$

Remember

$$\frac{\partial D_{mt}}{\partial \theta_t} = \frac{sd}{\sqrt{q_{mt}}} \left((x_{mt} - x_{\ell t}) \sin\theta_t - (y_{mt} - y_{\ell t}) \cos\theta_t \right)$$

$$\frac{\partial \phi_{mt}}{\partial \theta_t} = \frac{-sd}{q_{tm}} \left((x_{mt} - x_{\ell t}) \cos\theta_t + (y_{mt} - y_{\ell t}) \sin\theta_t \right) - 1$$

$$z_t = \begin{bmatrix} \check{D}_t \\ \check{\phi}_t \end{bmatrix}, \quad h_{mt}(X_t, x_{mt}, y_{mt}) = \begin{bmatrix} D_{mt}(\ldots) \\ \phi_{mt}(\ldots) \end{bmatrix}$$

```
for i in len_control commands:
    control_command → Robot motion.
    prediction ( )  ──→ At this point we have : slam_ekf.specific_state = $\bar{\mu}_t$
                                                slam_ekf.covariance = $\bar{\Sigma}_t$
observations ← get_cylinders_from_scan
    for j=0 to num_observations : → Number of detected cylinders.
        if cylinder is not already registered as a landmark → add it → specific_state and covariance matrix grow [ ]→[ : ]  [ · ]→[
        correction step
            $k_t$ = slam-ekf.covariance · $H_{mt}^T$ ($H_{mt}$ · slam-ekf.cov · $H_{mt}^T$ + Q$)^{-1}$
            slam-ekf.specific_state = slam-ekf.specific_state + $k_t$ ($z_{mt}^{(j)}$ - h (slam-ekf.specific_state, $x_{mt}$, $y_{mt}$)))
            slam-ekf.Cov = (I - $k_t$ · $H_{mt}$) · slam-ekf.cov
```

$$\frac{\cdot}{\partial X_t}\ \frac{\cdot}{\partial Y_t}\ \frac{\cdot}{\partial \theta_t}$$

$$H_{mt}=\begin{array}{c|ccc|c|cc|c|}
 & \partial X_t & \partial Y_t & \partial \theta_t & & \partial X_{mt} & \partial Y_{mt} & \\
\partial D_{mt} & \frac{\partial D_{mt}}{\partial X_t} & \frac{\partial D_{mt}}{\partial Y_t} & \frac{\partial D_{mt}}{\partial \theta_t} & \cdots & \frac{\partial D_{mt}}{\partial X_{mt}} & \frac{\partial D_{mt}}{\partial Y_{mt}} & \cdots \\
\partial \phi_{mt} & \frac{\partial \phi_{mt}}{\partial X_t} & \frac{\partial \phi_{mt}}{\partial Y_t} & \frac{\partial \phi_{mt}}{\partial \theta_t} & \cdots & \frac{\partial \phi_{mt}}{\partial X_{mt}} & \frac{\partial \phi_{mt}}{\partial Y_{mt}} & \cdots
\end{array}$$

H must be $\mathbb{R}^{2\times(3+2N)=2\times(3+2+\underbrace{2(N-1)})}$

$$\frac{\cdot}{\partial X_{mt}}\ \frac{\cdot}{\partial Y_{mt}} \qquad 2(N-1) \text{ zeros}$$

and $\frac{\partial D_{mt}}{\partial X_{nt}}=0$

$\frac{\partial D_{mt}}{\partial Y_{nt}}=0$

$\frac{\partial \phi_{mt}}{\partial X_{nt}}=0$

$\frac{\partial \phi_{mt}}{\partial Y_{nt}}=0$

$m\neq n$

$$q_{mt}=(X_{mt}-X_{et})^2+(Y_{mt}-Y_{et})^2$$

$$D_{mt}=\sqrt{q_{mt}}$$

$$\phi_{mt}=\operatorname{atan}\left(\frac{Y_{mt}-Y_{et}}{X_{mt}-X_{lt}}\right)-\theta_t$$

$$\frac{\partial D_{mt}}{\partial X_{mt}}=\frac{\cancel{2}(X_{mt}-X_{lt})}{\cancel{2}\sqrt{q_{mt}}}=\frac{\Delta X_{met}}{\sqrt{q_{mt}}}=\frac{-\partial D_{mt}}{\partial X_t}$$

$$\frac{\partial D_{mt}}{\partial Y_{mt}}=\frac{(Y_{mt}-Y_{et})}{\sqrt{q_{mt}}}=\frac{\Delta Y_{met}}{\sqrt{q_{mt}}}=\frac{-\partial D_{mt}}{\partial Y_t}$$

$$\frac{d}{dx}(\operatorname{atan}x)=\frac{1}{1+x^2}$$

$$\frac{\partial \phi_{mt}}{\partial X_{mt}}=\frac{1}{1+\frac{(Y_{mt}-Y_{et})^2}{(X_{mt}-X_{et})^2}}\cdot\frac{-(Y_{mt}-Y_{et})}{(X_{mt}-X_{et})^2}=\frac{-(Y_{mt}-Y_{et})}{q_{mt}}=\frac{-\Delta Y_{met}}{q_{mt}}=\frac{-\partial \phi_{mt}}{\partial X_t}$$

$$\frac{\partial \phi_{mt}}{\partial Y_{mt}}=\frac{(X_{mt}-X_{lt})}{q_{mt}}=\frac{\Delta X_{met}}{q_{mt}}=\frac{-\partial \phi_{mt}}{\partial Y_{mt}}$$

So if we have an observation between the current state and the landmark $\underset{\smile}{m}$ we will have to compute:

$\frac{\partial D_{mt}}{\partial X_t}, \frac{\partial D_{mt}}{\partial Y_t}, \frac{\partial D_{mt}}{\partial \theta_t}, \frac{\partial \phi_{mt}}{\partial X_t}, \frac{\partial \phi_{mt}}{\partial Y_t}, \frac{\partial \phi_{mt}}{\partial \theta_t}$ and $\frac{\partial D_{mt}}{\partial X_{mt}}, \frac{\partial D_{mt}}{\partial Y_{mt}}, \frac{\partial \phi_{mt}}{\partial X_{mt}}, \frac{\partial \phi_{mt}}{\partial Y_{mt}}$

and $\frac{\partial D_{mt}}{\partial X_{nt}}=\frac{\partial D_{mt}}{\partial Y_{nt}}=\frac{\partial \phi_{mt}}{\partial X_{nt}}=\frac{\partial \phi_{mt}}{\partial Y_{nt}}=0 \quad \boxed{m\neq n}$

In each iteration of the inner foor loop the algorithm corrects the specific $\underset{(\mu_t)}{\text{state}}$ and the $\underset{(\Sigma_t)}{\text{covariance}}$, i.e, the components of μ_t and Σ_t are more accurate after each iteration of the inner for loop.