

Algorithm Lab

Week 2: Non-Comparison Sort

Description

Classical sorting algorithms are algorithms rearrange the input list to a certain order defined by the comparison function. For some special domain (in most case, subdomain of natural numbers), we can design special sorting algorithms that working without generalized comparison functions, respectively, non-comparison sorting algorithms.

*Instance: a list of non-negative integers $A = (a_1, a_2, \dots, a_n)$ that
for all $1 \leq i \leq n, 0 \leq i < r^d$ where r is radix and d is the position of
most significant digit.*

*Result: a list $B = (b_1, b_2, \dots, b_n)$ that $\{a_1, a_2, \dots, a_n\} = \{b_1, b_2, \dots, b_n\}$ and
for all $1 < i \leq n, b_{i-1} < b_i$.*

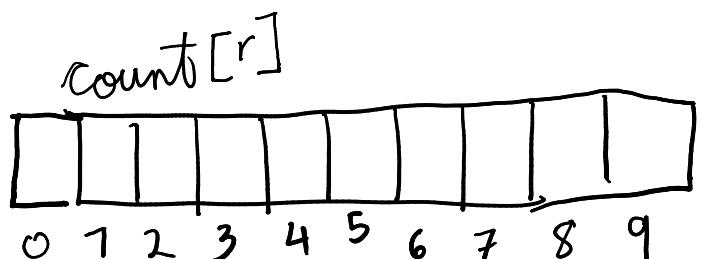
Algorithm Design

1. Set processing position to least significant order.
2. Rearrange the list by digit at processing position.
3. Set processing position to next digit.
4. Repeat step 2, 3 until finished the most significant digit rearrangement.

Implementation
 Language: C
 void sort_by_digit(int *A, int n, int r, int p)
 {
 int base = 1;
 while (p > 0)
 {
 --p;
 base *= r;
 }
 int count[r], B[n], m = 0;
 for (int j = 0; j < r; ++j) [count to zero]
 count[j] = 0;
 for (int i = 0; i < n; ++i) [count # of radix at base position]
 count[(A[i] / base) % r]++;
 for (int j = 0, psum = 0, sum = 0; j < r; ++j)
 {
 psum = sum;
 sum += count[j];
 count[j] = psum;
 }[Cumulative sum shifted 1 to the right]
 for (int i = 0; i < n; ++i) [Puts A[i] into B[order]]
 B[count[(A[i] / base) % r]++] = A[i];
 for (int i = 0; i < n; ++i)
 A[i] = B[i];
 } [after that increments the number stored in count]
 Copies B into A

```

    void radix_sort(int *A, int n, int r, int d)
    {
        for (int i = 0; i < d; ++i)
            sort_by_digit(A, n, r, i);
    }
  
```



position of digit to be sorted

Questions

- Function `sort_by_digit` is a stable counting sort. If we rewrite the function to an unstable version, can radix sort still work correctly?
- This radix implement is processing through least significant digit to most significant digit. Can't you design radix algorithm which start at most significant digit?
- Please design an algorithm to measure minimum usable d of list A and radix r .
- Please analysis the space complexity and the time complexity. (should be functions with arguments n, r, d)

1.) Yes as long as all the digit positions are sorted

2.) $\text{for } (i = d - 1; i \geq 0; i--)$

3.) $\text{int } r = 0, D = 0, X = 0, Y = 0;$
 $\text{for (int } i = 0; i < n; i++)\{$
 $x = \text{to_string}(A[i]).length$
 $\text{if } (x > D) D = X;$
 $\text{while } (x != 0)\{$
 $y = A[i] \% 10;$
 $\text{if } (y > r) r = y;$
 $x /= 10;$
 $\}$
}

D will be the number of digits of the longest number
 r will be the highest digit of all positions

4.) Time complexity $O(d(2r + 3n))$
space complexity $O(n + r)$