UNIVERSITÀ DI TRENTO

Mechanical Vibrations

Analysis of a Beam-Cart System

(Project of May 2021)

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I. Introduction

The purpose of this report is to analyse a mechanical system (as shown in Figure 1) and its behavior once subjected to a periodic force generated by an electromagnetic shaker.
The system is composed of a cart that moves over a beam along a linear guide with non-negligible friction (modeled by a damper element). The cart is connected to the beam via two springs. Moreover, a disk is rigidly attached to the cart. The beam is grounded by two rods, screwed at its ends, which behave like two springs, allowing it to shake horizontally. Finally, a force sensor and two accelerometers, attached through an anti-vibration paste, are used to measure the respective data.
The analysis is divided into two parts:

- Single DoF: only the linear damped oscillator is considered, modeling the cart that moves on a grounded linear guide. This will allow us to estimate the values of the damping ratio and the damping and stiffness coefficients.
- Double DoF: the whole system (beam and cart) is considered, providing a more accurate model where the beam is not directly grounded anymore, but can oscillate.

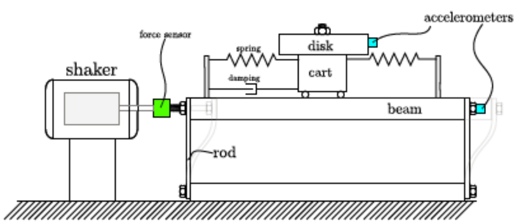

Figure 1: System sketch

II. Single DoF System

The first part of the analysis is performed on the linear damped oscillator modeled in Figure 2.

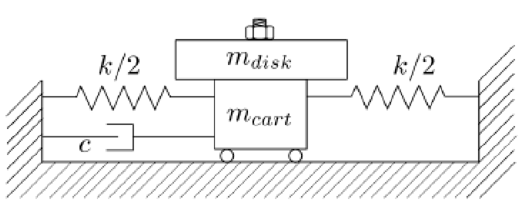

Figure 2: 1 DoF system

The cart is slowly moved away from the equilibrium position and suddenly released to produce the experimental data. This operation is performed 5 times, producing the following raw data:

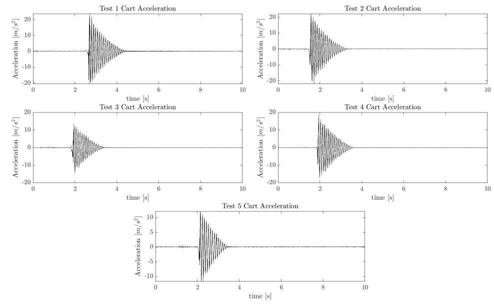

Figure 3: Raw Data

A. Signal Filtering

To be able to perform the logarithmic decrement method and estimate the value of the damping ratio, we need to clean the raw data filtering out the noise. Only then we can effectively estimate the positions of the peaks.
First we smooth out the noise with a fine moving average window (wide 0.0781% of the total data points). Secondly, we over-smooth the signal with a larger moving average window (wide 0.7812% of the total data points) to easily locate the peaks in time. Finally, we use the timestamps of the peaks of the over-smoothed signal to evaluate the smoothed signal, obtaining a good estimate of the values of the peaks of the raw data.
The following figures show the steps and the result of this process for each of the five tests:

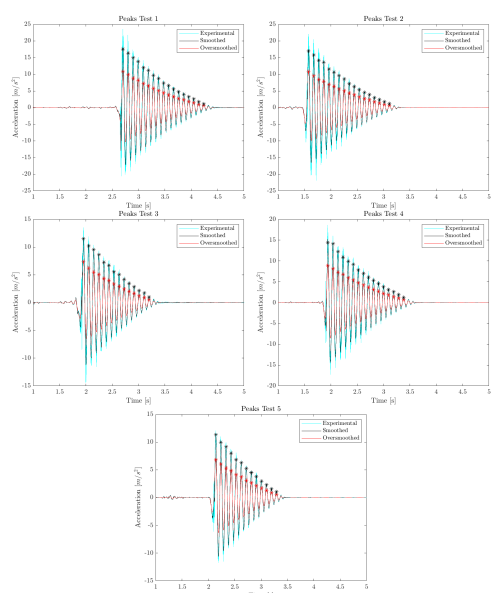

Figure 4: Double Filtered Cart Acceleration

For the purposes of avoiding background noise, only peaks with acceleration above $0.5\frac{m}{s^2}$ were considered.

B. Oscillation Period and Damping Ratio

Knowing the peak accelerations and their timestamps, we can perform the logarithmic decrement method and calculate the rate δ of the free damped vibration decline.
We use the x_n and x_{n+m+1} peaks to avoid noise between adjacent peaks ($m = 6$) and discarding the first peak ($n = 2$):

$$\delta = \frac{1}{m} \log \left(\frac{x_n}{x_{n+m+1}} \right)$$

Which lets us compute the damping ratio:

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

Finally we obtain the pseudo period of the oscillation using the timestamps of the selected peaks:

$$T = \frac{t_{n+m} - t_n}{m}$$

Performing these calculations for each of the five tests, we get:

Test	ξ	$T[s]$
1	0.0210	0.0962
2	0.0210	0.0964
3	0.0234	0.0961
4	0.0220	0.0963
5	0.0255	0.0962

From which the mean values and standard deviations can be computed:

$$\begin{cases} \hat{T} = \overline{T} \pm \sigma_T \simeq 0.0963 \pm 0.0001 \, [s] \\ \hat{\xi} = \overline{\xi} \pm \sigma_\xi \simeq 0.0226 \pm 0.0019 \end{cases}$$

These results are susceptible to many factors, such as noise, smoothing severity, skipped cycles m between peaks for the calculation of δ and initial conditions in the laboratory experiments.
This, then, may warrant the consideration of the calculated uncertainty. We need to consider, though, that the same uncertainty is at least one order of magnitude smaller than the derived values despite all this sources of error. We could argue that the mean values is not significantly impacted, especially for the pseudo-period $\hat{T}$ and, therefore, we can take them as is.

C. Natural Frequency, Stiffness, Damping Coefficient

With the oscillation period and the damping ratio, the other parameters can be derived through the following equations:

$$\begin{cases} \omega_n = \frac{2\pi}{T\sqrt{1-\xi^2}} \\ k = \omega_n^2 (m_{cart} + m_{disk}) \\ c = 2\xi\omega_n (m_{cart} + m_{disk}) \end{cases}$$

Yielding the values:

Test	ω_n	K	C
1	65.3031	2198.3471	1.4155
2	65.2016	2191.5196	1.4092
3	65.3685	2202.7541	1.5793
4	65.2470	2194.5723	1.4772
5	65.3542	2201.7876	1.7192

Then, as done in the previous paragraph, their mean values and standard deviations are calculated:

$$\begin{cases} \hat{\omega_n} = \overline{\omega}_n \pm \sigma_{\omega_n} \simeq 65.29 \pm 0.07 \left[\frac{rad}{s}\right] \\ \hat{k} = \overline{k} \pm \sigma_k \simeq 2198 \pm 5 \left[\frac{N}{m}\right] \\ \hat{c} = \overline{c} \pm \sigma_c \simeq 1.52 \pm 0.13 \left[\frac{Ns}{m}\right] \end{cases}$$

III. Double DoF System

The second part of the analysis is performed on the entire system (Figure 5): the beam supporting the cart is connected to the frame via two rods being, therefore, able to move and bringing the number of DoF to two. A force is applied to the system by a shaker and measured by a force sensor. More precisely, it performs a frequency sweep between 5 and $30Hz$ for a total duration of $40s$. The data are collected with a sampling frequency of $6400Hz$.

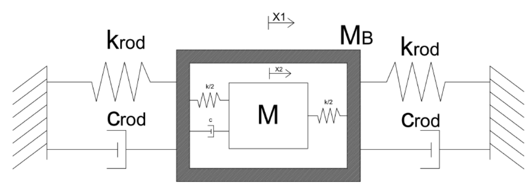

Figure 5: 2 DoF system

The following data were used to perform the analysis:

m_{cart}	0.3759	Mass of the cart $[kg]$
m_{disk}	0.1396	Mass of the disk $[kg]$
m_{beam}	4.7764	Mass of the beam $[kg]$
m_{shaker}	0.2	Moving mass of the shaker $[kg]$
l_{rod}	0.1	Length of the rod $[m]$
w_{rod}	0.025	Width of the rod $[m]$
t_{rod}	0.0015	Thickness of the rod $[m]$
ξ_{rod}	0.01	Damping Factor of the rod
l_{beam}	605	Length of the beam $[mm]$
w_{beam}	30	Width/Thickness of the beam $[mm]$
E	210	Young Modulus $[GPa]$
ρ	7850	Density $\left[\frac{kg}{m^3}\right]$

We need to also model the rods as shown in Figure 5 assuming it behaves like this:

Figure 6: Rod Model

We obtain the inertia and then calculate the stiffness through the equations:

$$I_{rod} = \frac{t_{rod}^3 w_{rod}}{12} \quad , \quad k_{rod} = \frac{12EI_{rod}}{l_{rod}^3}$$

At this point we can derive the equivalent stiffness and damping factor of the two rods:

$$k_{rod_{eq}} = 2k_{rod} = 35437.5 \left[\frac{N}{m}\right]$$

$$c_{rod_{eq}} = 2\xi_{rod}\sqrt{k_{rod_{eq}} m_{beam}} = 8.2283 \left[\frac{Ns}{m}\right]$$

A. Analytical Transfer Functions

$$\begin{bmatrix} m_{beam} & 0 \\ 0 & m_{tot} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_{rod_{eq}} + c & -c \\ -c & c \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{rod_{eq}} + k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f(t) \\ 0 \end{bmatrix}$$

Formulating the equations of motion as above (where $m_{tot} = m_{cart} + m_{disk}$) and knowing that the Laplace transform of the acceleration is $A_i(s) = s^2 X_i(s)$, we can apply the Laplace Transform and derive the analytical transfer functions between the acceleration and the force of both the beam (G_{11}) and the cart (G_{21}):

$$G_a(s) = \begin{bmatrix} \frac{s^2(m_{tot}s^2+cs+k)}{det(s)} & \frac{s^2(cs+k)}{det(s)} \\ \frac{s^2(cs+k)}{det(s)} & \frac{s^2(m_{beam}s^2+(c_{rod_{eq}}+c)s+(k_{rod_{eq}}+k))}{det(s)} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$$with \quad det(s) = C_1 s^4 + C_2 s^3 + C_3 s^2 + C_4 s + C_5$$

$$where \begin{cases} C_1 = m_{tot} m_{beam} \\ C_2 = (c_{rod_{eq}} + c) m_{tot} + m_{beam} c \\ C_3 = (k_{rod_{eq}} + k) m_{tot} + c_{rod_{eq}} c + m_{beam} k \\ C_4 = k_{rod_{eq}} c + c_{rod_{eq}} k \\ C_5 = k_{rod_{eq}} k \end{cases}$$

It is important to consider that the stiffness and damping coefficient of the cart come with some uncertainty. Therefore, we can plot multiple curves using the maximum uncertainty in the interval $\pm 3\sigma_i$ ($i = k, c$) which, with the assumption of Gaussian noise, should contain about 99% of the outcomes.

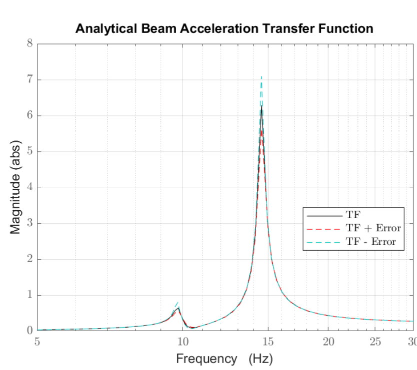

Figure 7: Beam Acceleration Analytical TF

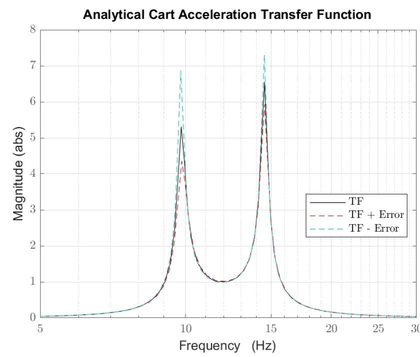

Figure 8: Cart Acceleration Analytical TF

The uncertainty introduces a negligible variation in terms of frequency and more prominent change in magnitude at the peaks.
In the plot of the beam acceleration TF we can identify a frequency at which the magnitude is almost null: $\approx 10.4[Hz]$. This means that the cart-beam system acts as a Frahm absorber, with the beam as the main system and the cart as the tunable (through its mass) auxiliary system.
In this case, in fact, the averaged resonant frequency of the auxiliary system (calculated in section II-C) is about $10.39[Hz]$.

B. Experimental Transfer Functions

The next step is to estimate the experimental transfer functions from the data of each of the five tests on the range $5 - 30[Hz]$:

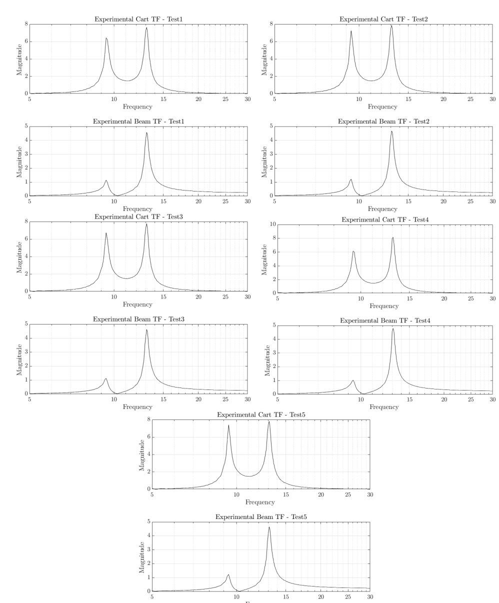

Figure 9: Cart-Beam Acceleration Experimental TFs

The mean experimental transfer function is then derived and plotted:

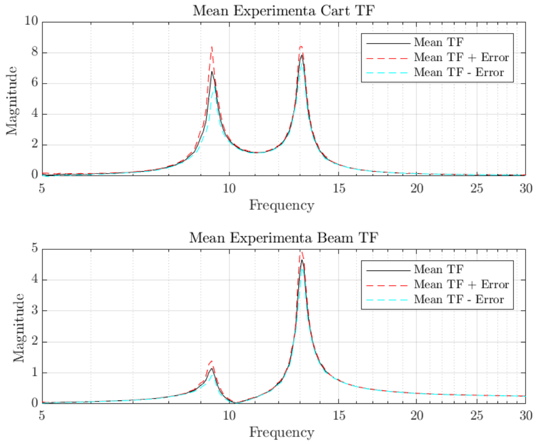

Figure 10: Cart-Beam Acceleration Mean Experimental TF

Again, as for the analytical TFs, the standard deviation across the five tests introduces a negligible frequency shift and a moderate difference in magnitude.

C. Analytical vs. Experimental Transfer Functions

Given that both formulations of the TFs are plotted on the same frequency range, we can compare them (11): in terms of magnitude there is a significant difference; in terms of frequency there is a non-negligible shift. Despite these differences, the zero of the beam TF is almost identical, preserving the Frahm absorber behaviour of the system.

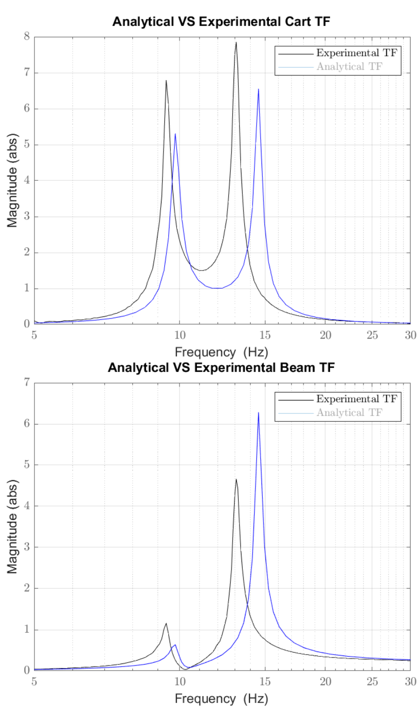

Figure 11: Cart-Beam Analytical VS Experimental TFs

D. Fitting Transfer Functions

Given the poor fit between analytical and experimental TFs, we can now consider the damping and the stiffness coefficients of both rods and springs as unknowns and try to optimize them making the analytical formulation fit the experimental data.
This is performed by minimizing the error between the two curves through a nonlinear programming solver. The initial guess plugged into the algorithm is the value of the unknowns used in the analytical formulation.
The optimized parameter found are:

Parameter	**Initial Guess**	**Fitting Beam TF**	**Fitting Cart TF**
$k_{rod_{eq}}\ \left[\frac{N}{m}\right]$	3.5438×10^4	2.6728×10^4	2.6697×10^4
$c_{rod_{eq}}\ \left[\frac{Ns}{m}\right]$	8.2283	3.1446	0.6795
$k_{cart}\ \left[\frac{N}{m}\right]$	2.1978×10^3	2.1563×10^3	2.1679×10^3
$c_{cart}\ \left[\frac{Ns}{m}\right]$	1.5201	2.1765	2.3934

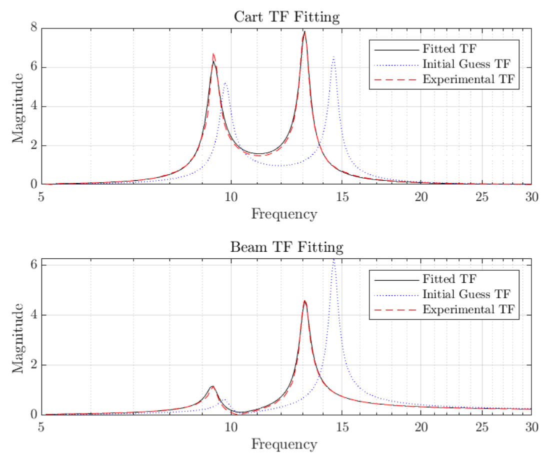

Figure 12: Cart-Beam TFs Fitting

In conclusion, we can say that the fitting was well performed since the parameters are comparable to the initial guess and the plot shows the curves to be almost identical.

References

[1] E. Dalla Ricca. Analysis of a beam-cart system: Laboratory experience. Technical report, University of Trento, 2021.

[1]

```
%% Preliminary Commands
clc;
close all;
clear all;

currentFolder = strcat(pwd,'\Plots');

set(0,'defaulttextinterpreter','latex')
set(groot, 'defaultAxesTickLabelInterpreter','latex');
set(groot, 'defaultLegendInterpreter','latex');

%% Known Quantities
m_cart = 0.3759; % kg
m_disk = 0.1396; % kg
m_tot = m_cart + m_disk; % kg

n1 = 200;
n2 = 2000;

%% Experimantal Data Acquisition
OneDOF1 = importdata('Laboratory_Data\1dof_1.txt');
OneDOF2 = importdata('Laboratory_Data\1dof_2.txt');
OneDOF3 = importdata('Laboratory_Data\1dof_3.txt');
OneDOF4 = importdata('Laboratory_Data\1dof_4.txt');
OneDOF5 = importdata('Laboratory_Data\1dof_5.txt');

% Time is the same for all tests
time1 = OneDOF1(:,1);
% Prepare a matrix to store acceleration
data_size1 = size(OneDOF1,1);
acc_mat = zeros(data_size1,5);
for i=1:data_size1
    acc_mat(i,1) = OneDOF1(i,3);
    acc_mat(i,2) = OneDOF2(i,3);
    acc_mat(i,3) = OneDOF3(i,3);
    acc_mat(i,4) = OneDOF4(i,3);
    acc_mat(i,5) = OneDOF5(i,3);
end

n_test = 5;
% Plot the raw data
figure('Name','Raw Data')
for i=1:n_test
    plot(time1,acc_mat(:,i),'k');
    title(['Test ',num2str(i),' Cart Acceleration']);
    xlabel('time [s]');
    ylabel('Acceleration [$m/s^2$]');
    pbaspect([3 1 1])

    figure_name = strcat('\',num2str(i),'.Raw_Test_',num2str(i),'.png');
    exportgraphics(gcf,strcat(currentFolder,figure_name))
end

%% Logarithmic Decrement Technique
delta = zeros(n_test,1);
```

```
xi = zeros(n_test,1);
T = zeros(n_test,1);
omega_N = zeros(n_test,1);
K = zeros(n_test,1);
C = zeros(n_test,1);
PeakProminence = 0.4;

for i=1:n_test
    figure('Name','Smooth')
    plot(time1,acc_mat(:,i),'c');
    acc_len = length(acc_mat(:,i));
    hold on

    % Smoothed Signal
    span1 = n1/acc_len;
    acc_smooth1 = smooth(acc_mat(:,i),span1,'lowess');
    plot(time1,acc_smooth1,'k');

    % Oversmoothed Signal
    span2 = n2/acc_len;
    acc_smooth2 = smooth(acc_smooth1,span2,'lowess');
    plot(time1,acc_smooth2,'r');
    hold on;

    % Find the peak values and locations on the oversmoothed signal:
    [p_val,p_loc] = findpeaks(acc_smooth2,'MinPeakProminence',↙
PeakProminence,'MinPeakHeight',0.5);

    % Plot the peaks
    peak_time = time1(p_loc);
    scatter(peak_time,acc_smooth2(p_loc),'r','*');
    scatter(peak_time,acc_smooth1(p_loc),'k','*');
    title(['Peaks Test ', num2str(i)]);
    xlim([1 5]);
    xlabel('Time [s]');
    ylabel('Acceleration [$m/s^2$]');
    legend('Experimental','Smoothed','Oversmoothed');
    hold off

    figure_name = strcat('\',num2str(i+5),'.Peaks_Test_',num2str(i),'.png');
    exportgraphics(gcf,strcat(currentFolder,figure_name))

    % Compute the Damping Ratio through the Lograithmic Decrement method
    % delta = ln(x[1]/x[n+1])/n
    % xi = delta/(sqrt(4*(pi^2)+delta^2)

    % Neglect the first peak
    n = 2;
    % Avoid noise in adjacent peak by calculating delta over m cycles
    m = 6;

    % Logarithmic Decrement delta
    delta(i) = log(acc_smooth1(p_loc(n))/acc_smooth1(p_loc(n+1+m)))/m;
    % Damping Ratio xi
    xi(i) = delta(i)/(sqrt(4*pi^2 + (delta(i))^2));
```

```
    % Pseudo-Period T[n+1] - T[n]
    Tn = time1(p_loc(n));
    Tnp1 = time1(p_loc(n+m));
    T(i) = (Tnp1 - Tn)/m;
end

%% Mean Value and Standard Deviation
Mean_xi = mean(xi);
Sigma_xi = std(xi);

Mean_T = mean(T);
Sigma_T = std(T);

% Uncertainty on Damping Ratio is 1 order of magnitude smaller and should
%    be considered, but can be neglected since it does not have a significant
%    impact
% Uncertainty on Period is very small and can be neglected

%% Natural Frequency, Stiffness, Damping Coefficient
for i=1:n_test
    % Natural Frequency
    omega_N(i) = (2*pi)/(T(i)*sqrt(1 - (xi(i))^2));

    % Stiffness
    K(i) = omega_N(i)^2*m_tot;

    % Damping Coefficient
    C(i) = 2*xi(i)*m_tot*omega_N(i);
end

% Mean Value and Standard Deviation
Mean_omega_N = mean(omega_N);
Sigma_omega_N = std(omega_N);

Mean_K = mean(K);
Sigma_K = std(K);

Mean_C = mean(C);
Sigma_C = std(C);

%% 2 DoF System
% Known Quantities
m_cart = 0.3759; % kg
m_disk = 0.1396; % kg
m_beam = 4.7764; % kg
m_shaker = 0.2000; % kg
l_rod = 0.1; % m
w_rod = 0.025; % m
t_rod = 0.0015; % m
C_rod = 0.01;
l_beam = 605; % mm
w_beam = 30; % mm
E = 210; % GPa
rho = 7850; % kg/m^3
```

```
fs = 6400; % Hz

% Choose a linear model for the beams and define the equivalent system
m_rod = rho*l_rod*w_rod*t_rod; % kg
I = (t_rod^3)*w_rod/12; % kg*m^2
K_rod_eq = 2*12*(E*10^9)*I/(l_rod^3);
C_rod_eq = 2*C_rod*sqrt(K_rod_eq*m_beam);

m1 = m_beam;
c1 = C_rod_eq;
k1 = K_rod_eq;

m2 = m_tot;
c2 = Mean_C;
c2_UP = c2 + 3*Sigma_C;
c2_DOWN = c2 - 3*Sigma_C;
C2 = [c2_DOWN,c2,c2_UP];
k2 = Mean_K;
k2_UP = k2 + 3*Sigma_K;
k2_DOWN = k2 - 3*Sigma_K;
K2 = [k2_DOWN,k2,k2_UP];

%% Compute the Analytical Transfer Functions between Cart/Beam Acceleration and Force
% The TF for the acceleration is G(s)*s^2

% Cart Acceleration VS Force
G_Cart_Numerator = zeros(3,5);
G_Cart_Denominator = zeros(3,5);

for i=1:3
    tmp_Num = [0,C2(i),K2(i),0,0];
    tmp_Den = [m1*m2, ...
               m1*C2(i) + m2*(c1 + C2(i)), ...
               m1*K2(i) + c1*C2(i) + m2*(k1 + K2(i)), ...
               K2(i)*c1 + k1*C2(i), ...
               k1*K2(i)];

    G_Cart_Numerator(i,:) = tmp_Num;
    G_Cart_Denominator(i,:) = tmp_Den;
end

G_Cart_DOWN = tf(G_Cart_Numerator(1,:),G_Cart_Denominator(1,:));
G_Cart = tf(G_Cart_Numerator(2,:),G_Cart_Denominator(2,:));
G_Cart_UP = tf(G_Cart_Numerator(3,:),G_Cart_Denominator(3,:));

figure('Name','Analytical Cart Acceleration Transfer Function')
k = bodeplot(G_Cart,'k');
setoptions↙
(k,'FreqUnits','Hz','FreqScale','log','PhaseVisible','off','MagUnits','abs');
hold on
grid on
bodeplot(G_Cart_UP,'r--');
bodeplot(G_Cart_DOWN,'c--');
title('Analytical Cart Acceleration Transfer Function');
xlabel('Frequency ');
```

```
ylabel('Magnitude ');
xlim([5 30]);
hold off
legend('TF','TF + Error','TF - Error');

exportgraphics(gcf,strcat(currentFolder,'\11.Cart_Analytical_TF.png'))

% Beam Acceleration VS Force
G_Beam_Numerator = zeros(3,5);
G_Beam_Denominator = zeros(3,5);

for i=1:3
    tmp_Num = [m2,C2(i),K2(i),0,0];
    tmp_Den = [m1*m2, ...
               m1*C2(i) + m2*(c1 + C2(i)), ...
               m1*K2(i) + c1*C2(i) + m2*(k1 + K2(i)), ...
               K2(i)*c1 + k1*C2(i), ...
               k1*K2(i)];

    G_Beam_Numerator(i,:) = tmp_Num;
    G_Beam_Denominator(i,:) = tmp_Den;
end

G_Beam_DOWN = tf(G_Beam_Numerator(1,:),G_Beam_Denominator(1,:));
G_Beam = tf(G_Beam_Numerator(2,:),G_Beam_Denominator(2,:));
G_Beam_UP = tf(G_Beam_Numerator(3,:),G_Beam_Denominator(3,:));

figure('Name','Analytical Beam Acceleration Transfer Function')
k = bodeplot(G_Beam,'k');
setoptions↙
(k,'FreqUnits','Hz','FreqScale','log','PhaseVisible','off','MagUnits','abs');
hold on
grid on
bodeplot(G_Beam_UP,'r--');
bodeplot(G_Beam_DOWN,'c--');
title('Analytical Beam Acceleration Transfer Function');
xlabel('Frequency ');
ylabel('Magnitude ');
xlim([5 30]);
hold off
legend('TF','TF + Error','TF - Error');

exportgraphics(gcf,strcat(currentFolder,'\12.Beam_Analytical_TF.png'))

%% Experimental Data Acquisition and TF Estimation
TwoDOF1 = importdata('Laboratory_Data\2dof_1.txt');
TwoDOF2 = importdata('Laboratory_Data\2dof_2.txt');
TwoDOF3 = importdata('Laboratory_Data\2dof_3.txt');
TwoDOF4 = importdata('Laboratory_Data\2dof_4.txt');
TwoDOF5 = importdata('Laboratory_Data\2dof_5.txt');

% Time is the same for all tests
time2 = TwoDOF1(:,1);
% Prepare a matrix to store acceleration
data_size2 = size(TwoDOF1,1);
```

```
force_mat = zeros(data_size2,5);
acc_cart_mat = zeros(data_size2,5);
acc_beam_mat = zeros(data_size2,5);
for i=1:data_size2
    force_mat(i,1) = TwoDOF1(i,2);
    force_mat(i,2) = TwoDOF2(i,2);
    force_mat(i,3) = TwoDOF3(i,2);
    force_mat(i,4) = TwoDOF4(i,2);
    force_mat(i,5) = TwoDOF5(i,2);

    acc_cart_mat(i,1) = TwoDOF1(i,3);
    acc_cart_mat(i,2) = TwoDOF2(i,3);
    acc_cart_mat(i,3) = TwoDOF3(i,3);
    acc_cart_mat(i,4) = TwoDOF4(i,3);
    acc_cart_mat(i,5) = TwoDOF5(i,3);

    acc_beam_mat(i,1) = TwoDOF1(i,4);
    acc_beam_mat(i,2) = TwoDOF2(i,4);
    acc_beam_mat(i,3) = TwoDOF3(i,4);
    acc_beam_mat(i,4) = TwoDOF4(i,4);
    acc_beam_mat(i,5) = TwoDOF5(i,4);
end

% Prepare matrices to store TFs data
TF_dim = 32769;
TF_Cart_Mag = zeros(TF_dim,n_test);
TF_Cart = zeros(TF_dim,n_test);
TF_Beam_Mag = zeros(TF_dim,n_test);
TF_Beam = zeros(TF_dim,n_test);

for i=1:n_test
    [tmp_TF_Cart,Freq_Cart] = tfestimate(force_mat(:,i),acc_cart_mat(:,i),[],[],[],↙
fs);
    TF_Cart(:,i) = tmp_TF_Cart;
    TF_Cart_Mag(:,i) = abs(tmp_TF_Cart);

    [tmp_TF_Beam,Freq_Beam] = tfestimate(force_mat(:,i),acc_beam_mat(:,i),[],[],[],↙
fs);
    TF_Beam(:,i) = tmp_TF_Beam;
    TF_Beam_Mag(:,i) = abs(tmp_TF_Beam);

    % Plot the raw data
    figure('Name','Experimental Cart/Beam Acceleration Transfer Functions')
    nexttile
    plot(Freq_Cart,TF_Cart_Mag(:,i),'k');
    grid on
    title(strcat('Experimental Cart TF - Test', num2str(i)));
    xlabel('Frequency ');
    ylabel('Magnitude ');
    xlim([5 30]);
    xscale log;

    nexttile
    plot(Freq_Beam,TF_Beam_Mag(:,i),'k');
    grid on
```

```
    title(strcat('Experimental Beam TF - Test', num2str(i)));
    xlabel('Frequency ');
    ylabel('Magnitude ');
    xlim([5 30]);
    xscale log;

    figure_name = strcat('\',num2str(i+12),'.Cart-Beam_Experimental_TF_Test_',num2str↙
(i),'.png');
    exportgraphics(gcf,strcat(currentFolder,figure_name))
end

%% Compute the Mean Experimental Transfer Function and Its Standard Deviation
TF_Cart_Mag_Transposed = TF_Cart_Mag';
Mean_TF_Cart = mean(TF_Cart_Mag_Transposed)';
Sigma_TF_Cart = std(TF_Cart_Mag_Transposed)';

TF_Beam_Mag_Transposed = TF_Beam_Mag';
Mean_TF_Beam = mean(TF_Beam_Mag_Transposed)';
Sigma_TF_Beam = std(TF_Beam_Mag_Transposed)';

figure('Name','Mean Cart/Beam Acceleration Transfer Function')
nexttile
plot(Freq_Cart,Mean_TF_Cart,'k');
title('Mean Experimenta Cart TF');
xlabel('Frequency ');
ylabel('Magnitude ');
xlim([5 30]);
xscale log;
%yscale log;
hold on
grid on
plot(Freq_Cart,Mean_TF_Cart+3*Sigma_TF_Cart,'r--');
plot(Freq_Cart,Mean_TF_Cart-3*Sigma_TF_Cart,'c--');
hold off
legend('Mean TF','Mean TF + Error','Mean TF - Error');

nexttile
plot(Freq_Beam,Mean_TF_Beam,'k');
title('Mean Experimenta Beam TF');
xlabel('Frequency ');
ylabel('Magnitude ');
xlim([5 30]);
xscale log;
%yscale log;
hold on
grid on
plot(Freq_Beam,Mean_TF_Beam+3*Sigma_TF_Beam,'r--');
plot(Freq_Beam,Mean_TF_Beam-3*Sigma_TF_Beam,'c--');
hold off
legend('Mean TF','Mean TF + Error','Mean TF - Error');

exportgraphics(gcf,strcat(currentFolder,'\18.Mean_Cart-Beam_Experimental_TF.png'))

%% Compare Analytical and Experimental TFs
figure('Name','Analytical VS Experimental Cart Acceleration Transfer Function')
```

```
p1 = plot(Freq_Cart,Mean_TF_Cart,'k');
xscale log;
hold on
p2 = bodeplot(G_Cart,'b');
setoptions(p2,'FreqUnits','Hz','PhaseVisible','off', 'MagUnits',↙
'abs','FreqScale','log');
title('Analytical VS Experimental Cart TF');
xlim([5 30]);
h = [p1;findobj(gcf,'type','line')];
legend(h,'Experimental TF','Analytical TF');
grid on
hold off

exportgraphics(gcf,strcat(currentFolder,'\19.Analytical_VS_Experimental_Cart_TF.png'))

figure('Name','Analytical VS Experimental Beam Acceleration Transfer Function')
p1 = plot(Freq_Beam,Mean_TF_Beam,'k');
xscale log;
hold on
p2 = bodeplot(G_Beam,'b');
setoptions(p2,'FreqUnits','Hz','PhaseVisible','off', 'MagUnits',↙
'abs','FreqScale','log');
title('Analytical VS Experimental Beam TF');
xlim([5 30]);
h = [p1;findobj(gcf,'type','line')];
legend(h,'Experimental TF','Analytical TF');
grid on
hold off

exportgraphics(gcf,strcat(currentFolder,'\20.Analytical_VS_Experimental_Beam_TF.png'))

%% Fitting
% Assume the Spring Stiffness, the Rod Stiffness, the Damping Factor of
% the linear guide and the Damping Factor of the rods are unknowns.

% Select frequencies from 5 to 30 Hz
range = find(Freq_Beam>5 & Freq_Beam<30);
Freq_Range = Freq_Beam(range);

TF_Beam_Range = Mean_TF_Beam(range);
TF_Cart_Range = Mean_TF_Cart(range);

s = sqrt(-1)*Freq_Range*2*pi;

% Redefine the TFs with the new unknowns
G_Cart_Fit = @(k_beam,c_beam,k_cart,c_cart) ...
             (c_cart.*s.^3 + k_cart.*s.^2)./ ...
             ((m1*m2).*s.^4 + ...
              (m1*c_cart + m2*(c_beam + c_cart)).*s.^3 + ...
              (m1*k_cart + m2*(k_beam + k_cart) + c_beam*c_cart).*s.^2 + ...
              (k_cart*c_beam + k_beam*c_cart).*s + ...
              k_beam*k_cart);
G_Beam_Fit = @(k_beam,c_beam,k_cart,c_cart) ...
             (m2.*s.^4 + c_cart.*s.^3 + k_cart.*s.^2)./ ...
             ((m1*m2).*s.^4 + ...
```

```
               (m1*c_cart + m2*(c_beam + c_cart)).*s.^3 + ...
               (m1*k_cart + m2*(k_beam + k_cart) + c_beam*c_cart).*s.^2 + ...
               (k_cart*c_beam + k_beam*c_cart).*s + ...
               k_beam*k_cart);

% Define the Error
err_Cart = @(x) rms(TF_Cart_Range - abs(G_Cart_Fit(x(1),x(2),x(3),x(4))));
err_Beam = @(x) rms(TF_Beam_Range - abs(G_Beam_Fit(x(1),x(2),x(3),x(4))));

% Define the Initial Guess
x0 = [k1,c1,k2,c2];

% From the initial guess minimize the error
options = optimset('MaxFunEvals', 10000);
x_opt_Cart = fminsearch(err_Cart,x0,options);
x_opt_Beam = fminsearch(err_Beam,x0,options);

figure('Name','Fitted Transfer Function')
nexttile
plot(Freq_Range,abs(G_Cart_Fit(x_opt_Cart(1),x_opt_Cart(2),x_opt_Cart(3),x_opt_Cart
(4))),'k');
hold on
grid on
plot(Freq_Range,abs(G_Cart_Fit(x0(1),x0(2),x0(3),x0(4))),'b:');
plot(Freq_Range,TF_Cart_Range,'r--');
title('Cart TF Fitting')
xlabel('Frequency ');
ylabel('Magnitude ');
xlim([5 30]);
xscale log;
legend('Fitted TF','Initial Guess TF','Experimental TF')
hold off

nexttile
plot(Freq_Range,abs(G_Beam_Fit(x_opt_Beam(1),x_opt_Beam(2),x_opt_Beam(3),x_opt_Beam
(4))),'k');
hold on
grid on
plot(Freq_Range,abs(G_Beam_Fit(x0(1),x0(2),x0(3),x0(4))),'b:');
plot(Freq_Range,TF_Beam_Range,'r--');
title('Beam TF Fitting')
xlabel('Frequency ');
ylabel('Magnitude ');
xlim([5 30]);
xscale log;
legend('Fitted TF','Initial Guess TF','Experimental TF')
hold off

exportgraphics(gcf,strcat(currentFolder,'\21.Fitting.png'))
```