# Vehicle Dynamics second Assignment 2022/2023

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## Assignment description

The aim of this second assignment is the analysis of the steady state behaviour of a F-SAE formula type car, starting from the tyre data obtained fitting the tyre coefficient.

## 1 Test Performed

The original Simulink model has been slightly modified to perform two steady state turning test:

- Constant steering angle varying the pedal in input;
- Constant pedal varying the steering angle in input.

The system has been studied considering a lateral/turning steady state problem.

To define the speed test a constant value for steer has been selected, while the speed profile has been define as a parabola arm: the coefficients have been tweaked to avoid saturation within the given time frame. The requested pedal formula is:

$$req\_pedal = (0.1 * 100^{-}2) * clk^2 + 0.1$$

where $clk$ is the Time step size.
To define the steer test a constant value for speed has been selected while the requested steer is modeled as a piece-wise function:

$$f(x) = \begin{cases} req\_steer = 5 & clk \leq 10 \\ req\_steer = clk/2 & clk > 10 \end{cases}$$

After many attempts the test that better allows for the investigation of the SS phenomena have been chosen, and the initial conditions have been set as per the following table:

| **Steady state test initial conditions** | |
|---|---|
| $V_0$ | *0* |
| $T_0$ | *0* |
| Time step size | *1e-4* |
| Stop simulation time | *50* |

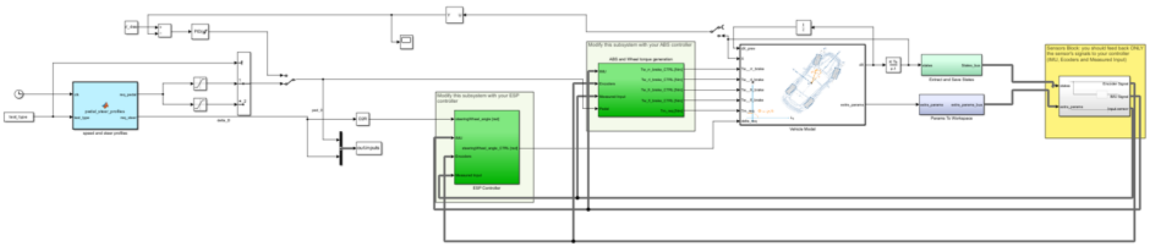
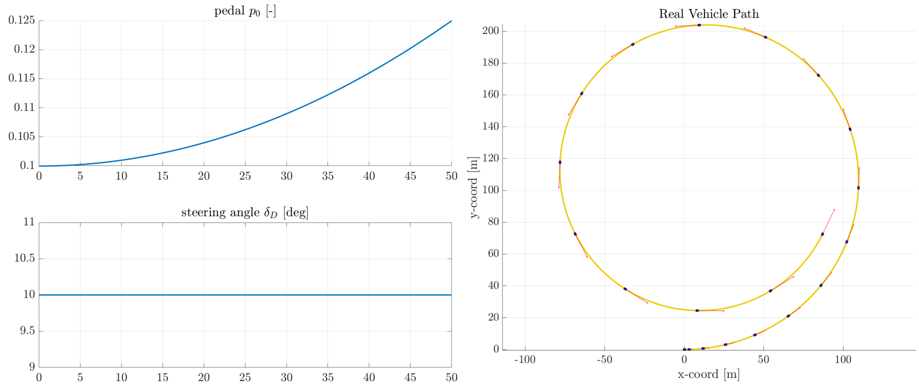


Figure 1: Simulink model

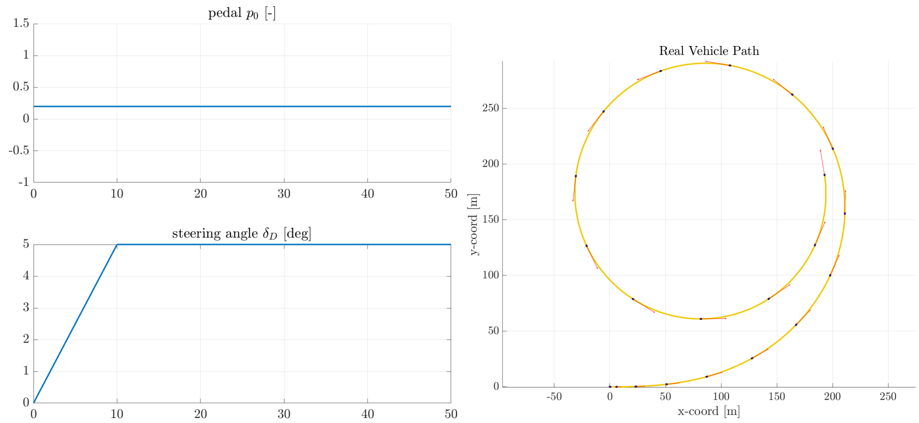
## 1.1 Speed test

Stating from an initial velocity of $0[km/h]$ it goes from $0.1[m/s]$ of pedal to $0.2[m/s]$ in 50 second, while the steering angle is kept constant at $10[deg]$ This way, varying slowly the speed with a constant steering angle, it's possible to study the vehicle after the transient in the SS configuration. The path shows clearly the over-steering tendency of the vehicle; in fact, plotting the body slip angle $\beta$ versus the acceleration $A_y$ it is clear that it is greatly decreasing as the acceleration increases.

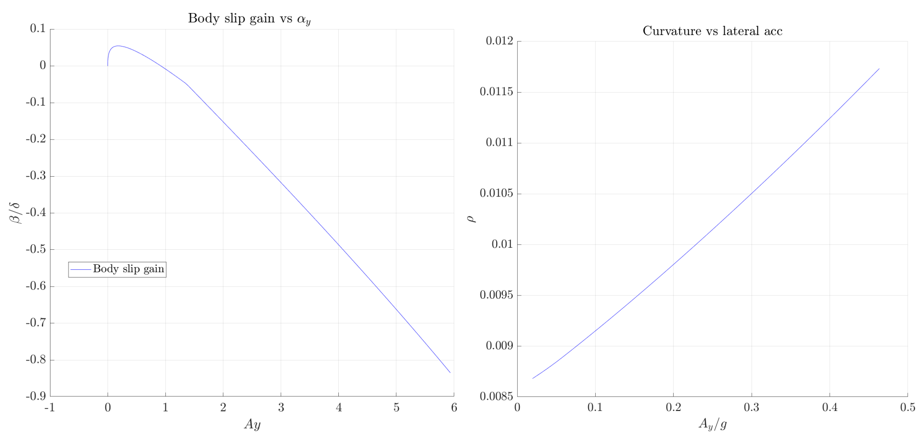



## 1.2 Steer Test

Starting from an initial velocity of $0[km/h]$ it reaches $5[deg]$ of input steering angle after 10 seconds, while the pedal is kept constant at $0.2[m/s]$. The test has been performed for 50 sec and its vehicle path presents the typical snail shape.




The vehicle clearly shows an over-steering behaviour since it is closing the trajectory and pointing inside of it. This is confirmed by plotting $\beta$. The same conclusions are obtained analysing the curvature plot $\rho\left[\frac{1}{m}\right]$ that shows how the curvature increases for an higher value of the lateral acceleration.



# 2 Data analysis and post processing

To understand the Steady State behaviour of the vehicle some information have been extrapolated from the model, given the following assumptions:

- two wheels drive
- front steering wheel
- open differential
- no significant aerodynamic force
- no ABS, ESP

## 2.1 Lateral load transfer

To derive the lateral load transfer as function of lateral acceleration, the *instantaneous* and then the *transient/elastic* term have been computed and added up. In Fig.2, it is possible to clearly distinguish the linear part of the curve, starting between $10sec$ and $30sec$. The simulation stops before the curve reaches the saturation.

$$\Delta Fz_r = ma_y * \left[\frac{Lfh_r r}{LW_r} + \frac{h_g s}{W_r}\frac{K_\phi r}{K_\phi r + K_\phi f}\right]$$

$$\Delta Fz_f = ma_y * \left[\frac{Lrh_r f}{LW_f} + \frac{h_g s}{W_f}\frac{K_\phi f}{K_\phi r + K_\phi f}\right]$$

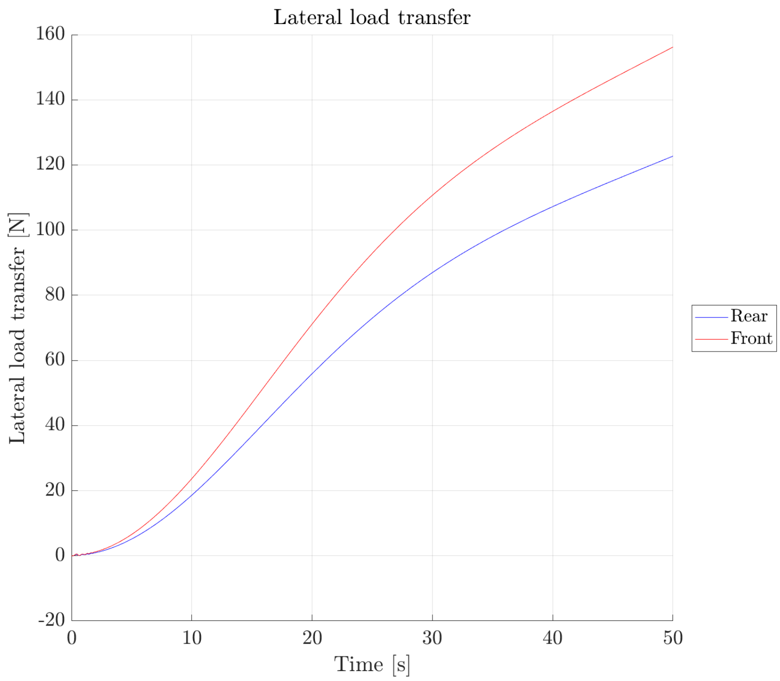



Figure 2: Lateral load transfer

## 2.2 Normalized axle characteristics

The Normalized axle characteristics have been computed considering the ratio between the Lateral forces $Fy_r$, $Fy_f$ (Totals of the forces felt by both tyres on the same axle) and the vertical forces in SS $Fz_0$ for both front and rear axles, plotted with respect to the slip angle. With this plot we can further confirm that the vehicle has an over-steering behaviour: the front axle slides less than the rear axle for same amount of load. For any load value the axle characteristic are both intersected just once; this means that only one Steady State solution exists and there are no unbalanced configurations in this range of values.
Changing the stiffness values by means, for example, of an anti roll bar, it is possible to see how the axle characteristics are modified. Increasing the front stiffness, also $\Delta F_y$ increases and so does the distance between the curve

$Fy_f$ and $Fy_r$. Therefore, the front must become less rigid if the $Fy_f$ curve needs to decrease; on the other hand, the rear should be more rigid to increase the $Fy_r$ curve.

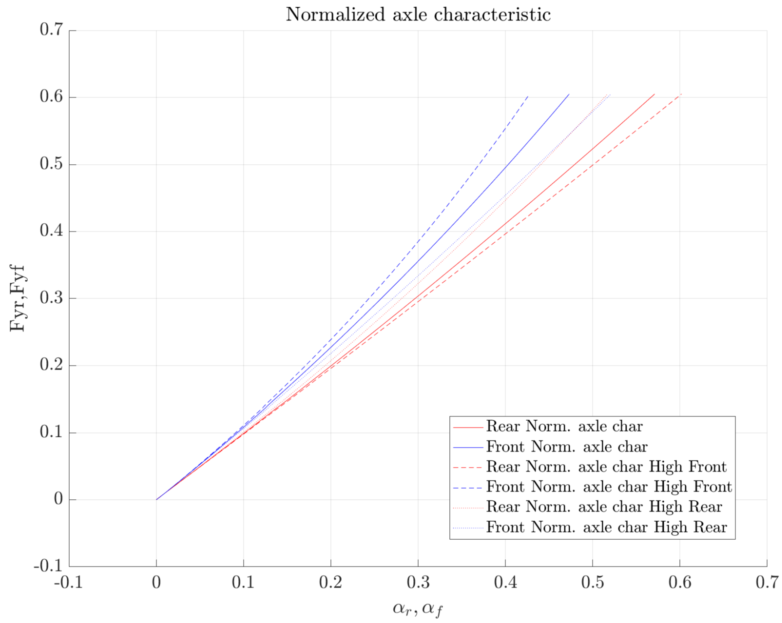



Figure 3: Normalized axle characteristics

## 2.3 Handling diagram

The handling diagram represents the difference between the actual curvature and the curvature that a vehicle needs if it has a neutral behaviour. We compute the handling curve considering the steering characteristics.

$$\frac{\delta_h}{\tau} - \rho L = -(\alpha_r(a_y) - \alpha_f(a_y))$$

It is also possible to compute the analytical under-steering gradient as the derivative of $\Delta\alpha$:

$$K_{us} = \frac{d(\Delta\alpha)}{dA_y}$$

So, assuming a linear model, we get:

$$A_y K_{us} = -(\alpha_r - \alpha_f)$$

Then the Handling curve can be fitted defining a polynomial whit the linear coefficient $K_{us}$ and higher order coefficients $K_{us_i}$ (which are needed to fit the non-linear part ).

$$\frac{\delta_h}{\tau} - \rho L = K_{us} * A_y + K_{us_1} * A_y^2 + K_{us_2} * A_y^3 ...$$

The **polyfit** function has been used in two different instances to extrapolate the coefficients of the linearization and of the fitting of the handling curve: it has been provided with 2 or more points, depending on the order of the wanted polynomial. A $3^{rd}$ order polynomial has been used to perform the fitting. The plot shows the comparison between the original, linearized and fitted curve.
It is also possible to compare the theoretical $K_{us}$, calculated as

$$K_{us_{theor}} = -\frac{m}{L^2} * (\frac{L_f}{k_s r}\frac{L_r}{K_s f}) = -5.76x10^{-4}$$

with respect to the fitted $K_{us}$ derived from the first term of the polynomial.

$$K_{us_{fit}} = -9.4x10^{-3}$$

The two values are different due to approximation and the presence of the transient part that perturbs the fitted value.

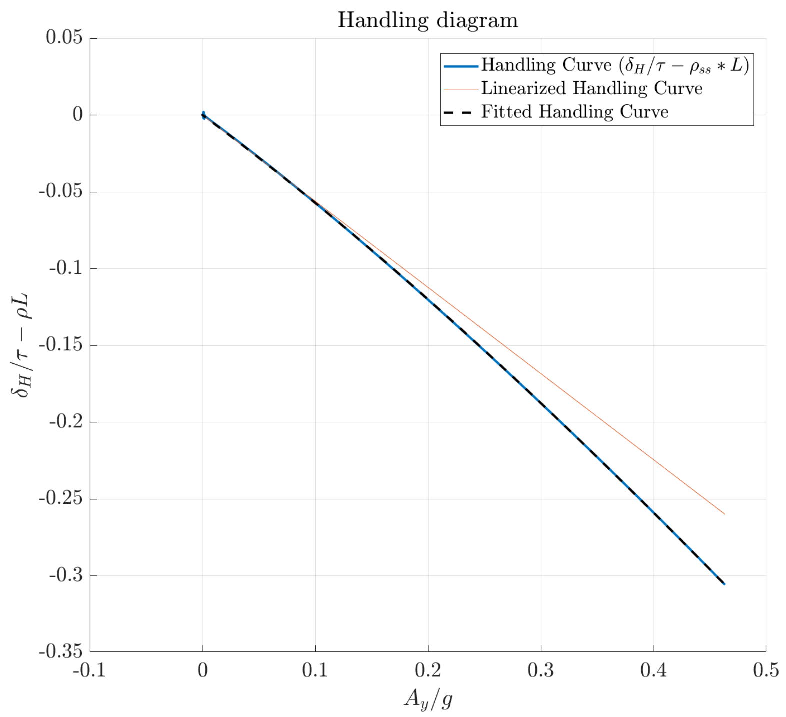



Figure 4: Handling Diagram

## 2.4 Yaw rate gain

The yaw rate gain describes how yaw rate, normalized with respect to the steering angle, changes when varying the velocity. Since the vehicle has an over-steering behavior, it is possible to visualize the critical velocity

$$u_{cr} = \sqrt{\frac{1}{K_{us}}} \approx 95[km/h]$$

that brings the vehicle to instability: after reaching the critical velocity it is necessary to decrease the steering angle.

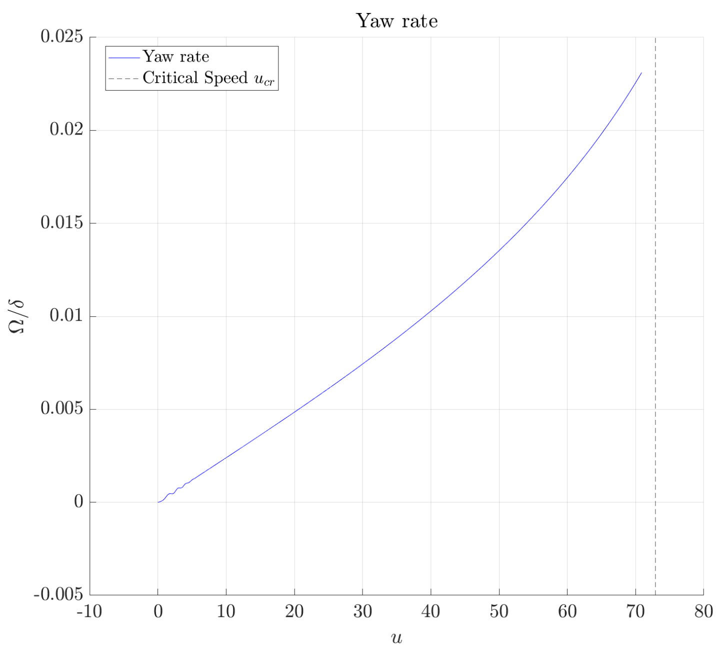
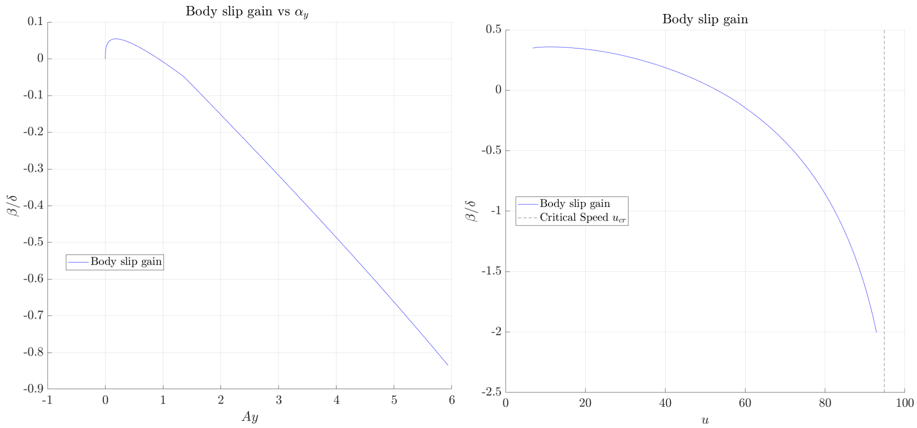



Figure 5: Yaw Rate

## 2.5 Beta gain

The body slip gain has been plotted with respect to $u$ and $A_y$. From Fig.6 it is possible to see that $\frac{\beta}{u}$ describes a parabola: initially positive, it becomes negative until it reaches the critical speed $u_{cr}$. From Fig.6 it is possible to see the the vehicle initially points outside the trajectory with a positive $\beta$; given its negative tendency, the vehicle ends up pointing inside the trajectory and does so faster since it is over-steering.




# 3 Vehicle model variations

We can now explore how the variation of

- Camber
- Toe
- Stiffness

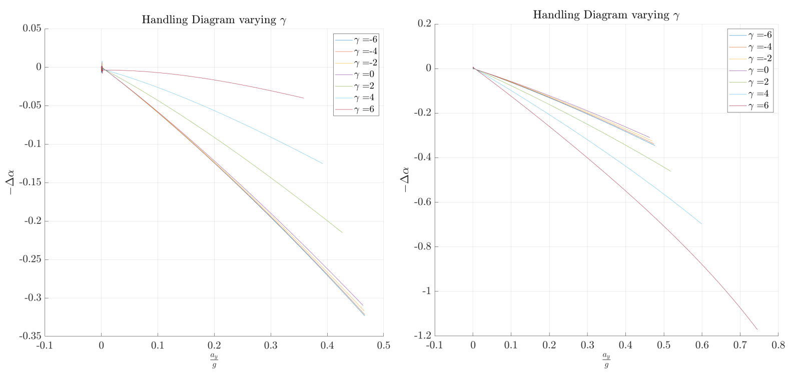
affects vehicle behaviour.

## 3.1 Variable Camber

The camber angle changes according to the variation of the lateral forces and, consequently, with $\Delta Fy$. Camber is useful to react against rolling due to suspension stiffness but, since it changes widely the vehicle behaviour, it is necessary to choose it wisely.

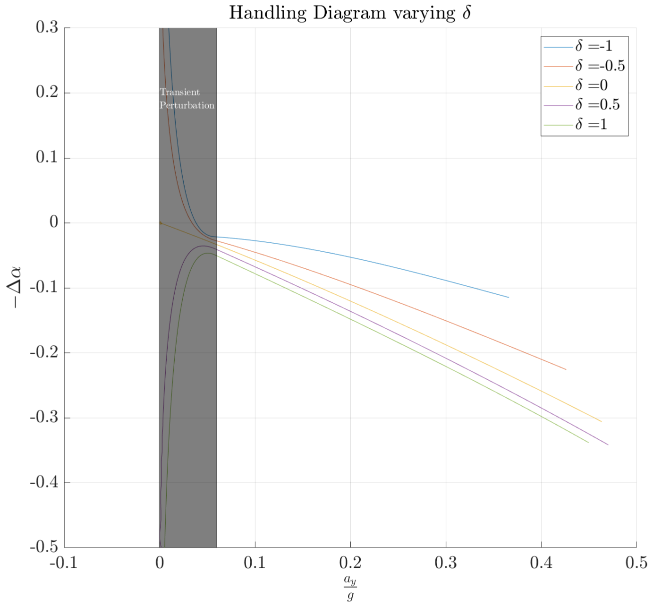
A range of values between $-6[deg]$ and $6[deg]$, with a step of $2[deg]$, has been applied to one axle per time. The output shows that the characteristics of the handling diagram are similar, only the slope changes: the vehicle shows an over-steering behaviour for negative value of camber; while a positive camber makes the vehicle more under-steering. The opposite happens varying $\gamma$ for rear tyres (negative angle $\Rightarrow$ more under-steering; positive angle $\Rightarrow$ more over-steering).

Even if the slope and the overall behaviour is in accordance with what was expected for camber changes, the magnitude of variation between positive and negative value is different, probably due to a problem during computation.



## 3.2 Variable Toe angle

Toe angle changes the vehicle behaviour and also changes during motion. To study the effect that an initial toe angle generates on a vehicle, a set of values between $1[deg]$ and $-1[deg]$ with step size of $0.5[deg]$ has been chosen, considering that a high toe value should be always avoided since it has great influence on handling and tyre deterioration. The output shows that, for positive values of toe, the vehicle becomes more over-steering; the opposite happens for negative values of toe. Due to the transient, the initial part of the plot has been discarded and counterbalanced; data become reliable after a value of normalized acceleration around 0.08.




## 3.3 Variable Stiffness

The suspension rolling stiffness variation (by means of anti-roll bar or spring setting) changes the lateral load transfer and the axle cornering stiffness. To study the vehicle's behaviour the stiffness ratio $\epsilon_\phi$ has been introduced. It represents the amount of load that is transferred from front to rear axle and it is defined as:

$$\epsilon_\phi = \frac{K_{\phi_f}}{K_{\phi_f} + K_{\phi_r}}$$

It directly affects the lateral load transfer $\Delta F_y$ and changes the axle characteristics and the overall behaviour, including the over-steering/under-steering tendency.
The ratio $\epsilon_\phi$ has been varied from 0.2 to 0.8 and the results show that, for a high value of front stiffness and given the same acceleration, $-\Delta\alpha$ is bigger, showing an under-steering behaviour. On the other hand, if $\epsilon_\phi$ is low the vehicle presents an over-steering tendency.