UNIVERSITÀ DI TRENTO

Systems And Techniques for Digital Signal Processing

Exercise 12: Digital PWM Modulator

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I. Introduction

The aim of this project is to implement and analyse a *Digital PWM Modulator* preceded by a $\Sigma\Delta$ noise shaper of different orders. This work is based on the paper Quantization noise shaping in digital PWM converters [MNM08] as well as on several on-line articles and YouTube tutorials.

II. Objectives

Our objectives for this project are:

- Implement and plot the frequency response of a first, second and third-order sigma-delta noise shaper followed by a digital PWM modulator;
- Simulate the behavior of the various sigma-delta digital PWM modulators, comparing the various solutions and showing differences and improvements in terms of signal-to-noise-and-distortion ratio (SNDR) and total harmonic distortion (THD).

III. Filters

The $\Sigma\Delta$ noise shapers we have implemented are of order 1 through 3, plus a modified 3^{rd} order filter, as proposed in [MNM08]. The transfer funcions of these filters are shown in Figure 1:

Modulator	1^{st} Ord	2^{nd} Ord	3^{rd} Ord	3^{rd} Ord Modified
NTF(z)	$1\text{-}z^{-1}$	$(1\text{-}z^{-1})^2$	$(1\text{-}z^{-1})^3$	$(1\text{-}z^{-1})(1\text{-}Kz^{-1}+z^{-2})$

$K = 2\cos(2\pi f_o/f_s)$

Figure 1: Noise Transfer Functions of $\Sigma\Delta$ modulators

Using the Matlab solver, we have obtained the frequency response of each filter, as shown in Figure 2, and their Power Spectral Density via the built-in Matlab function *periodogram*, as shown in Figure 3.

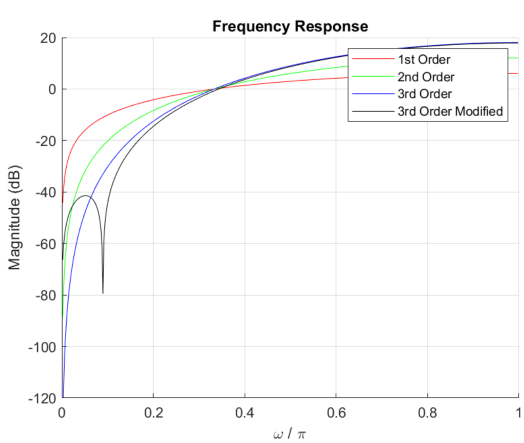

Figure 2: Frequency Response of $\Sigma\Delta$ modulators

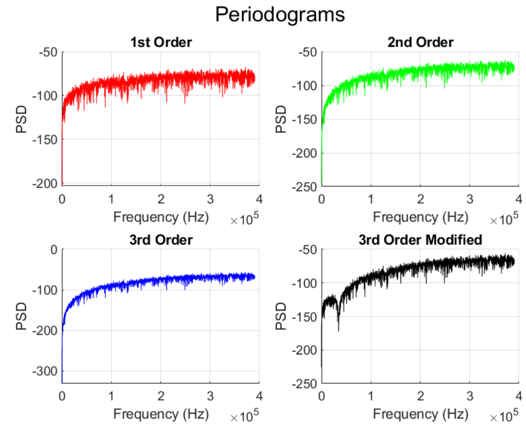

Figure 3: Power Spectral Density of $\Sigma\Delta$ modulators

It is clear that, moving from the 1^{st} to the 3^{rd} order, for the same frequency we have a lower magnitude. Furthermore, the 3^{rd} order modified filter introduces a peak which helps in keeping the magnitude at lower levels for an increased range of frequencies. This shows that the filter are, one more than the other, successful in shaping the noise and moving it to the higher frequencies.

IV. Waves

The second step was to generate the signal to give as input to the filters and choose a carrier wave:

- A square wave with $frequency = 5Hz$, $duration = 1s$ and $amplitude = 0.7$ was created and a white noise ($\mu = 0$, $\sigma = 0.05$) was added to it. Subsequently it was quantized with a 4-bit Quantizer. The result is shown in Figure 4;

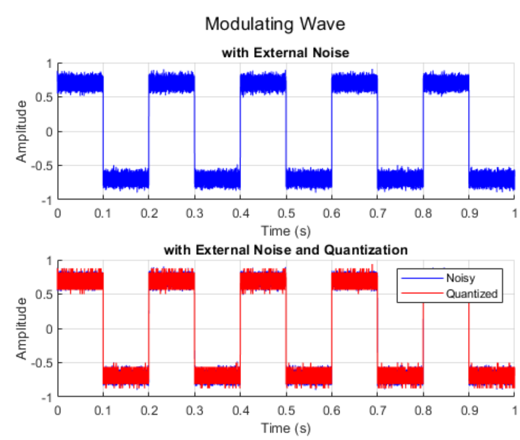

Figure 4: Noisy Square Wave

- Two waves were chosen as carriers: a *sine wave* and a *sawtooth wave*, quantized in the same manner as the input signal and generated with a frequency of $50Hz$. They are shown below in Figure 5.

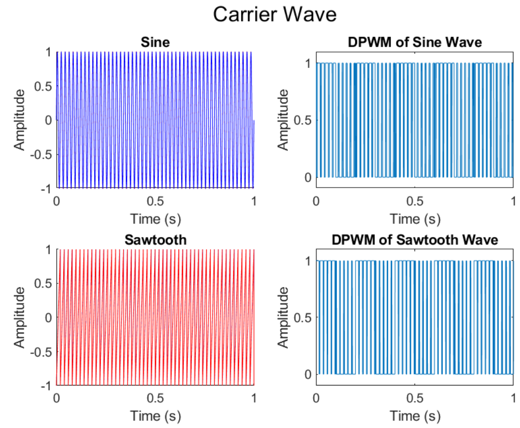

Figure 5: Modulating wave

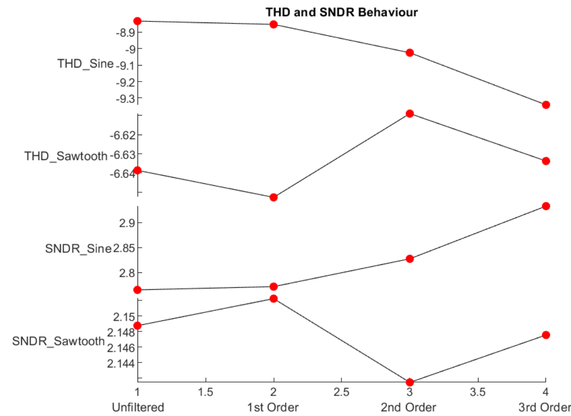

Figure 6: THD and SNR results

With both the carrier and the modulating wave ready, we can perform the digital pulse width modulation with a simple *if-else* condition:

- if $carrier < input \Rightarrow$ the output is 1;
- otherwise $\Rightarrow$ the output is 0.

An example is shown in Figure 5 for both the carrier waves.

V. Simulation

After defining the functions that apply the noise shaping to the input signal, we can simulate the system behaviour and analyse the output in terms of *Total Harmonic Distortion* (THD) and *Signal to Noise and Distortion Ratio* (SNDR). The THD was calculated using the built-in Matlab function, while the SNDR was found according to the formula:

$$SNDR = (10^{-\frac{SNR}{10}} + 10^{-\frac{THD}{10}})^{\frac{1}{2}}$$

Where SNR is the *Signal to Noise Ratio*.

VI. Results

The values of THD and SNDR of the output after each noise shaping filter, except the 3^{rd} modified which was not implemented, are shown in Figure 6. The spectral analysis, including the one of the unfiltered noise, of the output for both carrier waves is presented in Figures 7, 8, 9, 10.

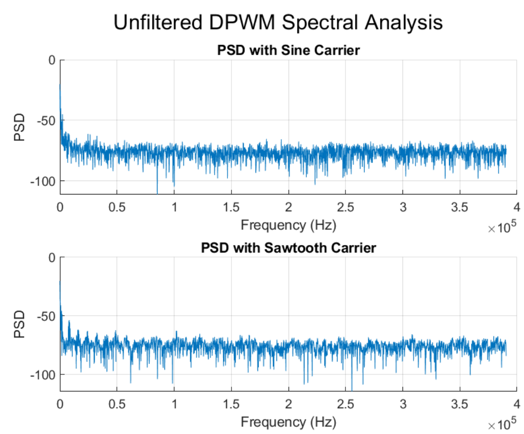

Figure 7: Unfiltered DPWM Spectral Analysis

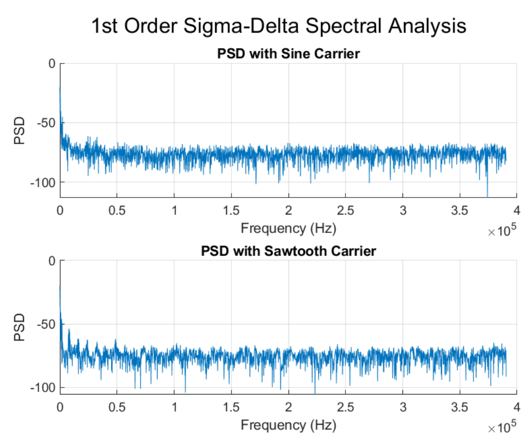

Figure 8: 1^{st} order $\Sigma\Delta$ Spectral Analysis

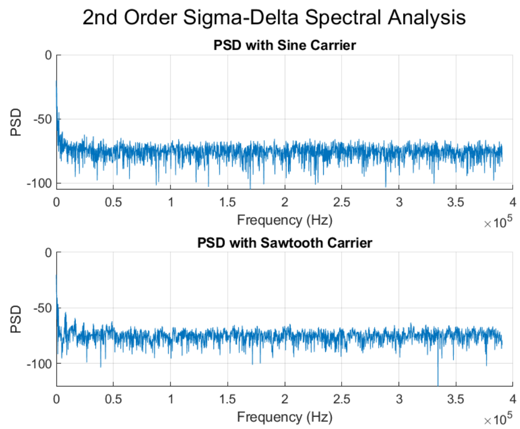

Figure 9: 2^{nd} order $\Sigma\Delta$ Spectral Analysis

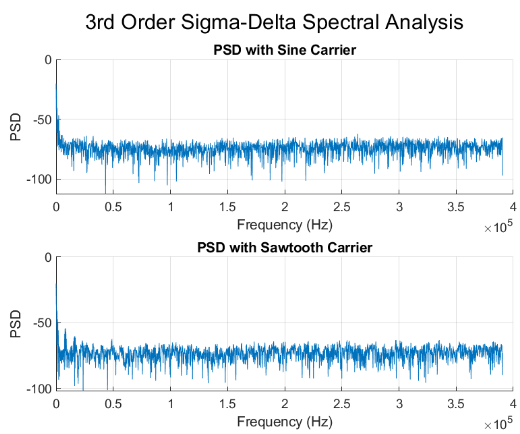

Figure 10: 3^{rd} order $\Sigma\Delta$ Spectral Analysis

VII. Conclusions

The most interesting result is the difference between the trends of THD and SNDR for the sine and the sawtooth carriers: while the output with the sine carrier shows a consistent improvement (THD trending negatively, SNDR trending positively), the one with the sawtooth carrier exhibits an oscillating behaviour (but only in the second or third decimal point) with the values for the 3^{rd} order $\Sigma\Delta$ just slightly worse than the ones of the unfiltered output, which is a counter-intuitive conclusion. This could mean the sawtooth is not an appropriate carrier, but more likely it is due to the fact that the Matlab functions used in the simulations were built for sinusoidal waves. Furthermore, the improvement in the THD and SNDR values of the output obtained using the sine wave as carrier, is not vary large: only a few decimal points. This could be traced back to the chosen characteristics of the waves or to the method used to calculate the PSD.

References

[MNM08] E. Alarcon M. Norris, L. M. Platon and D. Maksimovic. Quantization noise shaping in digital pwm converters. *IEEE Power Electronics Specialists Conference*, 2008.