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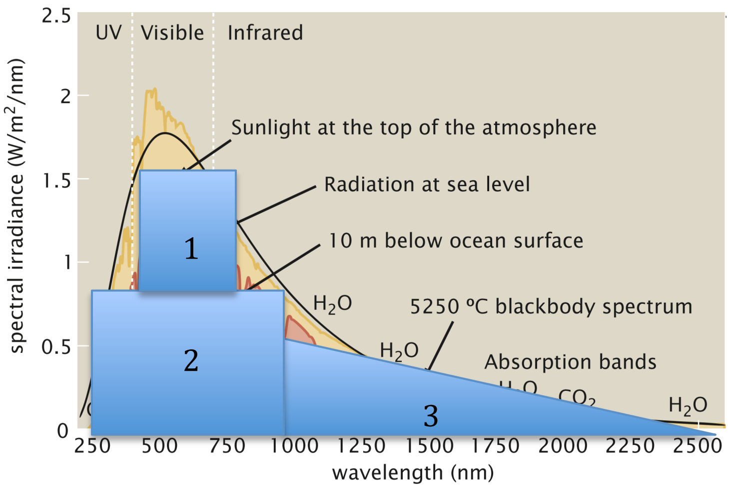
Section 2

Bi 1 HW 5

1a) The total amount of power per square meter available to humans across all wavelengths is the area under the black curve in the diagram, which will be approximated using the following shapes:



Therefore, the total area under the curve, and thus the total power per square meter, is approximately the area of the shapes 1, 2, and 3. This can be expressed as $A_{tot} = A_1 + A_2 + A_3 = b_1 * h_1 + b_2 * h_2 + \frac{1}{2} b_3 * h_3 = (250)(.75) + (750)(.75) + \frac{1}{2}(1500)(.5) = \mathbf{1100}\,\frac{\mathbf{W}}{\mathbf{m^2}}$. The UV light region is roughly triangular with a base of 400 nm, and a height of $\frac{1.5\frac{W}{m^2}}{nm}$, giving an area of $A = \frac{1}{2} * b * h = \frac{1}{2}(400)(1.5) = \mathbf{300}\,\frac{\mathbf{W}}{\mathbf{m^2}}$. The visible light region is roughly rectangular with a base of 300 nm and a height of $\frac{1.5\frac{W}{m^2}}{nm}$, giving an area of $A = b * h = (300) * (1.5) = \mathbf{500}\,\frac{\mathbf{W}}{\mathbf{m^2}}$. Lastly, the infrared light region is also roughly triangular with a base of 1800 nm and a height of $\frac{.5\frac{\mathrm{W}}{\mathrm{m}^2}}{\mathrm{nm}}$, giving an area of $A = \frac{1}{2} * b * h = \frac{1}{2}(1800)(.5) = \mathbf{500\,\frac{W}{m^2}}$.

1b) The U.S. household annual energy usage is about 10^4 kW $*$ hr, which gives us a power of $10^4\ \mathrm{kW} * \frac{\mathrm{hr}}{\mathrm{year}} * \frac{1\ \mathrm{year}}{365*24\ \mathrm{hours}} = 1\ \mathrm{kW} = 1000\ \mathrm{W}$. The solar panel area needed (assuming 100% efficiency) is thus $1000\ \mathrm{W} * \frac{1\mathrm{m}^2}{1100\ \mathrm{W}} = \mathbf{0.9\ m^2}$ for each household. However, a real solar panel would certainly have to be larger than this, since no solar panel is 100% efficient and not all of the solar energy absorbed can be converted directly into power.

1c) Assume that the porphyrin ring acts as a light absorbing disc of radius .5 nm, which implies an area of $A = \pi r^2 = \pi(5 * 10^{-10})^2 = 8 * 10^{-19}\ \mathrm{m}^2$. From 1a, we know that the power per square meter is $1100\,\frac{\mathrm{W}}{\mathrm{m}^2}$, so the power delivered to the ring is thus $1100\,\frac{\mathrm{W}}{\mathrm{m}^2} * 8 * 10^{-19}\ \mathrm{m}^2 = 9 * 10^{-16}\ \mathrm{W}$, or $9 * 10^{-16}$ J per second. Assume that the average photon coming in from the sun has wavelength of about 600 nm, and therefore that the average energy of these photons is about $E = \frac{ch}{\lambda} = \frac{(3.0*10^{8})(6.626*10^{-34})}{600*10^{-9}} = 3 * 10^{-19}\,\frac{\mathrm{J}}{\mathrm{photon}}$. Thus, the number of photons absorbed per second is roughly $9 * 10^{-16}\mathrm{J} * \frac{1\ \mathrm{photon}}{3*10^{-19}\ \mathrm{J}} = \mathbf{3 * 10^3\ photons}$.

1d) Since the ring only absorbs with about 1% efficiency, we would only expect to see $(3 * 10^3)(.01) =$ **30 photons** actually absorbed per second.

1e) The energy associated with a 680 nm photon is $E = \frac{ch}{\lambda} = \frac{(3.0*10^8)(6.626*10^{-34})}{680*10^{-9}} =$ $\mathbf{2.9 * 10^{-19} \frac{J}{photon}}$. If the molecule absorbs 30 photons every second, then the average energy absorbed is 30 photons $* \frac{2.9*10^{-19} J}{\text{photon}} = \mathbf{9 * 10^{-18}\ J}$ per second. This is equivalent to $9 * 10^{-18}\ \text{J} * \frac{1 k_b T}{4.11*10^{-21}\ \text{J}} = \mathbf{2 * 10^3\ k_b T}$.

1f) The energy of a 680 nm photon in k_bT units is $2.9 * 10^{-19} \frac{\text{J}}{\text{photon}} * \frac{1 k_b T}{4.11*10^{-21}\ \text{J}} =$ $71 \frac{k_b T}{\text{photon}}$. The energy required to split water into protons and O_2 is 125 k_bT, which means that $125\ k_b T * \frac{1\ \text{photon}}{71\ k_b T} = \mathbf{1.7\ photons}$ are required. During ATP hydrolysis, 20 k_bT is required, and therefore $20\ k_b T * \frac{1\ \text{photon}}{71\ k_b T} = \mathbf{0.28\ photons}$ worth of energy are needed. For glucose, we know that the molecule has a molar mass of $180.15 \frac{\text{g}}{\text{mol}}$, and therefore 1 g of glucose corresponds to $1\text{g} * \frac{1\ \text{mol}}{180.15\ \text{g}} * \frac{6.022*10^{23}\ \text{molecules}}{1\ \text{mol}} = 3 *$ 10^{21} molecules. If this process releases, $16\ \text{kJ} * \frac{1 k_b T}{4.11*10^{-24}\ \text{kJ}} = 4 * 10^{24}\ k_b T$, then the energy per molecule is $\frac{4*10^{24}\ k_b T}{3*10^{21}\ \text{molecules}} = 10^3\ k_b T$. This corresponds to $10^3\ k_b T *$ $\frac{1\ \text{photon}}{71\ k_b T} = \mathbf{10\ photons}$ worth of energy.

2a) Using the equation $\Delta G_{elec} = q\Delta V$, we have $\Delta G_{elec} = (1.6 * 10^{-19}\ \text{C}) * \left(0.1 \frac{\text{J}}{\text{C}}\right) =$ $1.6 * 10^{-20}$ J, or equivalently $1.6 * 10^{-20}\ \text{J} * \frac{1 k_b T}{4.11*10^{-21}\ \text{J}} = \mathbf{3.9\ k_b T}$.

2b) $\Delta G_{chem} = \Delta G_{out} + \Delta G_{in} = \boldsymbol{-T\Delta S_{out} - T\Delta S_{in}}$

2c) There are Ω possible positions to be occupied, and N protons that will occupy some of them. Therefore, the number of possible ways for the spaces to be chosen is $\binom{\Omega}{N} = \frac{\Omega!}{N!(\Omega-\mathrm{N})!} \approx \frac{\Omega^{\mathrm{N}}}{N!}$ for $\Omega \gg \mathrm{N}$, and thus we have $\boldsymbol{W(\mathrm{N}, \Omega) = \frac{\Omega^{\mathrm{N}}}{N!}}$.

2d) $\Delta S_{out} = \Delta k_b \ln(W) = k_b \ln(W_f) - k_b \ln(W_i) = \mathrm{k_b} \ln\left(\frac{\Omega^{\mathrm{N}-1}}{(\mathrm{N}-1)!}\right) - \mathrm{k_b} \ln\left(\frac{\Omega^{\mathrm{N}}}{\mathrm{N}!}\right) =$

$k_b \ln\left(\frac{\Omega^{\mathrm{N}-1}}{\Omega^{\mathrm{N}}} * \frac{\mathrm{N}!}{(\mathrm{N}-1)!}\right) = k_b \ln(\Omega^{-1} * \mathrm{N}) = \boldsymbol{k_b \ln\left(\frac{\mathrm{N}}{\Omega}\right)}$

$\Delta S_{in} = \Delta k_b \ln(W) = k_b \ln(W_f) - k_b \ln(W_i) = \mathrm{k_b} \ln\left(\frac{\Omega^{\mathrm{M}+1}}{(\mathrm{M}+1)!}\right) - \mathrm{k_b} \ln\left(\frac{\Omega^{\mathrm{M}}}{\mathrm{M}!}\right) =$

$k_b \ln\left(\frac{\Omega^{\mathrm{M}+1}}{\Omega^{\mathrm{M}}} * \frac{\mathrm{M}!}{(\mathrm{M}+1)!}\right) = k_b \ln\left(\Omega * \frac{1}{\mathrm{M}+1}\right) \approx \boldsymbol{k_b \ln\left(\frac{\Omega}{M}\right)}$ ($M + 1 \approx M$ since $M \gg 1$).

2e) $\Delta G_{chem} = -T(\Delta S_{out} + \Delta S_{in})$

$\Delta G_{chem} = -T\left(k_b \ln\left(\frac{\mathrm{N}}{\Omega}\right) + k_b \ln\left(\frac{\Omega}{\mathrm{M}}\right)\right) = -Tk_b \ln\left(\frac{N}{\Omega} * \frac{\Omega}{M}\right) = -Tk_b \ln\left(\frac{N}{M}\right)$

$\Delta G_{chem} = Tk_b \ln\left(\frac{M}{N}\right) = Tk_b \ln\left(\frac{c_{in}\Omega}{c_{out}\Omega}\right) = \boldsymbol{Tk_b \ln\left(\frac{c_{in}}{c_{out}}\right)}$

2f)

$\Delta G_{chem} = Tk_b \ln\left(\frac{c_{in}}{c_{out}}\right) =$

$Tk_b \ln\left(\frac{10^{-pH_{in}}}{10^{-pH_{out}}}\right) = Tk_b \ln(10^{pH_{out}-pH_{in}}) = \boldsymbol{Tk_b \Delta pH \ln(10)}$

Thus, $\Delta G_{tot} = \Delta G_{elec} + \Delta G_{chem} = \boldsymbol{(q\Delta V + \Delta pH \ln(10))k_b T}$

2g) $\Delta G_{tot} = (q\Delta V + \Delta pH \ln(10))k_b T = (3.9 + (1)\ln(10))k_b T = \mathbf{6.2}\ \boldsymbol{k_b T}$

3a) $E = \tau\theta$

$20 = \tau\left(\frac{2\pi}{3}\right) \rightarrow \boldsymbol{\tau} = \mathbf{10}\ \frac{\mathbf{k_b T}}{\mathbf{rad}}$

3b) $\Delta G_{tot} = (3.9 + \Delta pH \ln(10))k_b T = \tau\theta = \tau(\frac{\pi}{6})$

$$\boldsymbol{\tau = \frac{6}{\pi}(3.9 + \Delta pH \ln(10))k_b T}$$

3c) $9.54\ k_b T = \frac{6}{\pi}(3.9 + \Delta pH \ln(10))k_b T \rightarrow \Delta \boldsymbol{pH} = \mathbf{.48}$

3d) From 2g, the free energy change of a photon moving across the membrane is

$6.2\ k_b T$ when $\Delta pH \approx 1.0$. Since it takes $20\ k_b T$ to generate 1 ATP, we need

$20\ k_b T * \frac{1 \text{ photon}}{6.2\ k_b T} = 3.18$ protons $\approx$ **4 protons** to power the production of 1 ATP.

3e) $E = \frac{ch}{\lambda}$

Purple bacteria reaction center: $E = \frac{(3.0*10^{8})(6.626*10^{-34})}{960*10^{-9}} = 2.1 * 10^{-19}\text{J} * \frac{1 k_b T}{4.11*10^{-21}\text{J}} =$

$\mathbf{50.}\ \boldsymbol{k_b T}$

Photosystem 2 of cyanobacteria:

$$E = \frac{(3.0*10^{8})(6.626*10^{-34})}{680*10^{-9}} = 2.9 * 10^{-19}\text{J} * \frac{1 k_b T}{4.11*10^{-21}\text{J}} = \boldsymbol{71\ k_b T}$$

3f) Since both systems have identical outputs, their input energies can be compared to determine the relative efficiencies of each. Since the purple bacteria reaction center only inputs $50.\, k_b T$, while the cyanobacteria photosystem 2 inputs 71 $k_b T$, the purple bacteria reaction center is $\frac{71}{50} = \mathbf{1.4}$ **times as efficient** as the cyanobacteria photosystem 2.