HW 2 – Business Analytics

This page provides guidelines for this homework assignment. You do not need to write anything in response to the items on this page. Questions start from the next page.

To solve the problem on the next page, you will need to do the following:

a. Use the Generalized Analytics Procedure (GAP) to set up your problem as follows:
   i. Define your model in words
      1. Identify the objective function in words
      2. Identify the random variables in words (none in this HW)
      3. Identify the decision variables in words
      4. Identify the constraints in words
   ii. Formulate your model mathematically
      1. Define the random variables (none in this HW)
      2. Define the decision variables
      3. Define the objective function in terms of decision variables
      4. Define the constraints in terms of the decision variables. Please include any non-negativity constraints in your formulation
b. Set up the problem in Excel and use Solver to find the optimal values of the decision variables. Ask Solver to create an Answer Report and a Sensitivity Report, if needed.
c. Answer the questions stated in the problem (in words).

Please submit only one file in PDF format with your write-up. Do not submit your Excel file. Your writeup must include the answer report and the sensitivity report generated by Excel Solver, whenever needed. If you make any additional assumptions, state them clearly.

Problem 1: Hyperloop

Elon Musk's newest venture is the Hyperloop service between Washington DC and Baltimore. The Hyperloop trains will offer customers a choice between coach and first-class tickets.
For the venture to comply with federal regulations, it must sell a minimum of 10 first-class tickets and a minimum of 10 coach tickets per trip. Currently the profit margin is $5 for each coach ticket and $8 for each first-class ticket. Due to safety reasons, the train total capacity is 50 travelers (excluding the crew). While first-class tickets are more profitable, first-class seats take up more space relative to coach seats. The overall length of the seating area of the train is 2400 inches. The seat pitch for the first class is 60 inches. The federally mandated seat pitch for the coach class is 30 inches.
Another consideration for deciding on the allocation of the seats is the weight capacity of the train. The allowed total passenger payload is 10000 lbs. It is also known that first class customers are, on average, heavier than coach customers. The typical weight of a first-class customer is 200lbs, while the typical weight of a coach customer is 150lbs.

a) How many tickets of each type should be sold in order to maximize profits?

i. Define the model in words

1. Identify the objective function: to find the number of each type of ticket to maximize profits
2. Identify the decision variables in words

The number of each type of ticket

3. Identify the constraints in words

The minimum number of each type of ticket is 10
The total capacity of the train is 50 travelers(tickets)
The overall length of the seating of the train is 2400 inches
The allowed total passenger payload is 10000lbs

ii. Formulate your model mathematically

1. Define the random variables (none in this HW)
2. Define the decision variables

Define m as the number of the first-class ticket, n as the number of the coach class ticket

3. Define the objective function in terms of decision variables

Let y be the profits we want to have
Since the profit margin is $5 for coach class, $8 for first-class
$y = 8m+5n$
we need to maximize y since we want to find how to maximize profits

4. Define the constraints in terms of the decision variables. Please include any non-negativity constraints in your formulation

1) $m \geq 10$, $n \geq 10$
2) $m+ n \leq 50$
3) $60\ m + 30\ n \leq 2400$

The overall length of the seating of the train is 2400 inches
The seat pitch for the first class is 60 inches. The federally mandated seat pitch for the coach class is 30 inches.

4) $200\ m + 150\ n \leq 10000$

The allowed total passenger payload is 10000 lbs.
The typical weight of a first-class customer is 200lbs, while the typical weight of a coach customer is 150lbs.

Answers:

To maximize profits, we need to sell 30 first class tickets and 20 coach class tickets. And the total profits are $ 340.

Decision Variables	first class	coach			
	m	n			
	30	20			
Information to Build Objective Function					
	profits				
	m	n			
	$8.00	$5.00			
Objective Function Calculation					
profits	$340.00				

Constraints

Constraint Name	*Coefficients*		*Conditions*		
	m	n	Left Hand	Operator	Right Hand
Min tickets(first class)	1	0	30	>=	10
Min tickets (coach class)	0	1	20	>=	10
Number of passengers	1	1	50	<=	50
Train length	60	30	2400	<=	2400
Allowed weight	200	150	9000	<=	10000

b)

If Elon Musk wants to increase the total capacity of the train without changing anything else, how much should he pay for an additional unit of increase in capacity?

If Elon Musk wants to improve one capacity without changing anything else, he needs to pay for an additional unit of increase in capacity for $2.

Microsoft Excel 16.69 Sensitivity Report
Worksheet: [HW2.xlsx]Q1
Report Created: 2023/2/1 16:21:27

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	m	30	0	8	2	3
C3	n	20	0	5	3	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D16	Min tickets(first class) Left Hand	30	0	10	20	1E+30
D17	Min tickets (coach class) Left Hand	20	0	10	10	1E+30
D18	Number of passengers Left Hand	50	2	50	10	5
D19	Train length Left Hand	2400	0.1	2400	300	600
D20	Allowed weight Left Hand	9000	0	10000	1E+30	1000

If the capacity has been increased from 50 to 60, does Elon Musk still have incentives to further increase the capacity? Please explain.

i. Define the model in words

1. Identify the objective function: to find the number of each type of ticket to maximize profits
2. Identify the decision variables in words

The number of each type of ticket

3. Identify the constraints in words

The minimum number of each type of ticket is 10
The total capacity of the train is 60 travelers(tickets)
The overall length of the seating of the train is 2400 inches
The allowed total passenger payload is 10000lbs

ii. Formulate your model mathematically

1. Define the random variables (none in this HW)
2. Define the decision variables

Define m as the number of the first-class ticket, n as the number of the coach class ticket

3. Define the objective function in terms of decision variables

Let y be the profits we want to have
Since the profit margin is $5 for coach class, $8 for first-class
y= 8m+5n
we need to maximize y since we want to find how to maximize profits

4. Define the constraints in terms of the decision variables. Please include any non-negativity constraints in your formulation

1) $m \geq 10$, $n \geq 10$
2) $m+ n \leq 60$
3) $60 m + 30 n \leq 2400$
The overall length of the seating of the train is 2400 inches
The seat pitch for the first class is 60 inches. The federally mandated seat pitch for the coach class is 30 inches.
4) $200 m + 150 n \leq 10000$
The allowed total passenger payload is 10000 lbs.
The typical weight of a first-class customer is 200lbs, while the typical weight of a coach customer is 150lbs.

Decision Variables	first class	coach			
	m	n			
	20	40			
Information to Build Objective Function					
	profits				
	m	n			
	$8.00	$5.00			
Objective Function Calculation					
profits	$360.00				

Constraints					
Constraint Name	*Coefficients*		*Conditions*		
	m	n	Left Hand	Operator	Right Hand
Min tickets(first class)	1	0	20	>=	10
Min tickets (coach class)	0	1	40	>=	10
Number of passengers	1	1	60	<=	60
Train length	60	30	2400	<=	2400
Allowed weight	200	150	10000	<=	10000

Microsoft Excel 16.69 Sensitivity Report
Worksheet: [HW2.xlsx]Q2
Report Created: 2023/2/1 16:27:25

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	m	20	0	8	2	1.333333333
C3	n	40	0	5	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D16	Min tickets(first class) Left Hand	20	0	10	10	1E+30
D17	Min tickets (coach class) Left Hand	40	0	10	30	1E+30
D18	Number of passengers Left Hand	60	0	60	1E+30	0
D19	Train length Left Hand	2400	0.066666667	2400	450	0
D20	Allowed weight Left Hand	10000	0.02	10000	0	1500

If it increases the capacity from 50 to 60, it shows the total profits has increased from $340 to $360. While there is no reduced cost and shadow cost for capacity, so he has no motivation after increasing the capacity to 60. Also, from the above figure(when the capacity is set at 50), we know that the allowable range for capacity is [45, 60], so for Musk, there is no motivation.

c) Due to an unprecedented outbreak of a novel infectious disease, the Hyperloop must either redesign all train cars to follow the CDC guidelines for social distancing or shut down all operations. The full redesign would reduce the length of the seating area of each train by 50%. Other design elements remain unchanged. Given that Elon Musk is unwilling to increase prices, how many tickets of each type should be sold in order to maximize profits? To gain more profits under this new design, should Elon Musk argue to reduce the minimum number of first-class and coach tickets to be sold per trip, as required by the federal regulations? Please explain.

i. Define the model in words

1. Identify the objective function: to find if the seating area reduced, how many tickets of each type should be sold in order to maximize profits.
2. Identify the decision variables in words

The number of each type of ticket

3. Identify the constraints in words

The total capacity of the train is 50 travelers(tickets)
The overall length of the seating of the train is 1200 inches
The allowed total passenger payload is 10000 lbs

ii. Formulate your model mathematically

1. Define the random variables (none in this HW)
2. Define the decision variables

Define m as the number of the first-class ticket, n as the number of the coach class ticket

3. Define the objective function in terms of decision variables

Let y be the profits we want to have

Since the profit margin is $5 for coach class, $8 for first-class

$y = 8m+5n$

we need to maximize y since we want to find how to maximize profits

4. Define the constraints in terms of the decision variables. Please include any non-negativity constraints in your formulation

1) $m+ n \leq 50$

2) $60 m + 30 n \leq 1200$

The overall length of the seating of the train is 1200 inches, reduced by 50%.

The seat pitch for the first class is 60 inches. The federally mandated seat pitch for the coach class is 30 inches.

3) $200 m + 150 n \leq 10000$

The allowed total passenger payload is 10000 lbs.

The typical weight of a first-class customer is 200lbs, while the typical weight of a coach customer is 150lbs.

Decision Variables	first class	coach			
	m	**n**			
	0	40			
Information to Build Objective Function					
	profits				
	m	**n**			
	$8.00	$5.00			
Objective Function Calculation					
profits	$200.00				

Constraints					
Constraint Name	*Coefficients*		*Conditions*		
	m	n	Left Hand	Operator	Right Hand
Number of passengers	1	1	40	<=	60
Train length	60	30	1200	<=	1200
Allowed weight	200	150	6000	<=	10000

a) reduce the minimum sold tickets

Decision Variables	first class	coach			
	m	**n**			
	10	20			
Information to Build Objective Function					
	profits				
	m	**n**			
	$8.00	$5.00			
Objective Function Calculation					
profits	$180.00				

Constraints					
Constraint Name	*Coefficients*		*Conditions*		
	m	n	Left Hand	Operator	Right Hand
Min tickets(first class)	1	0	10	>=	10
Min tickets (coach class)	0	1	20	>=	10
Number of passengers	1	1	30	<=	60
Train length	60	30	1200	<=	1200
Allowed weight	200	150	5000	<=	10000

b) do not reduce the minimum sold tickets

Compared a) and b), we know if we reduced the minimum sold tickets, we can get a higher profit. Then we need to sell zero first-class tickets and 20 coach tickets.