

① Total probability of 2 dice = $6^2 = 6 \times 6 = 36$

Sum of even numbers and one of dice = 6 is,

(2, 6), (6, 2)

(4, 6), (6, 4)

(6, 6) = 5

$$P(E) = \frac{5}{36}$$

② Total probability = 36

Sum of numbers being less than 7 is,

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)

(2, 1), (2, 3), (2, 4), (2, 2)

(3, 1), (3, 2), (3, 3)

(4, 1), (4, 2)

(5, 1)

$$P(E) = \frac{15}{36} = \frac{5}{12}$$

③ Total probability = $2^3 = 2 \times 2 \times 2 = 8$

Given that, $P(A|B) = 7/8$ = at least one head

$P(A \cap B) = \text{at least 2 heads} = 4/8$

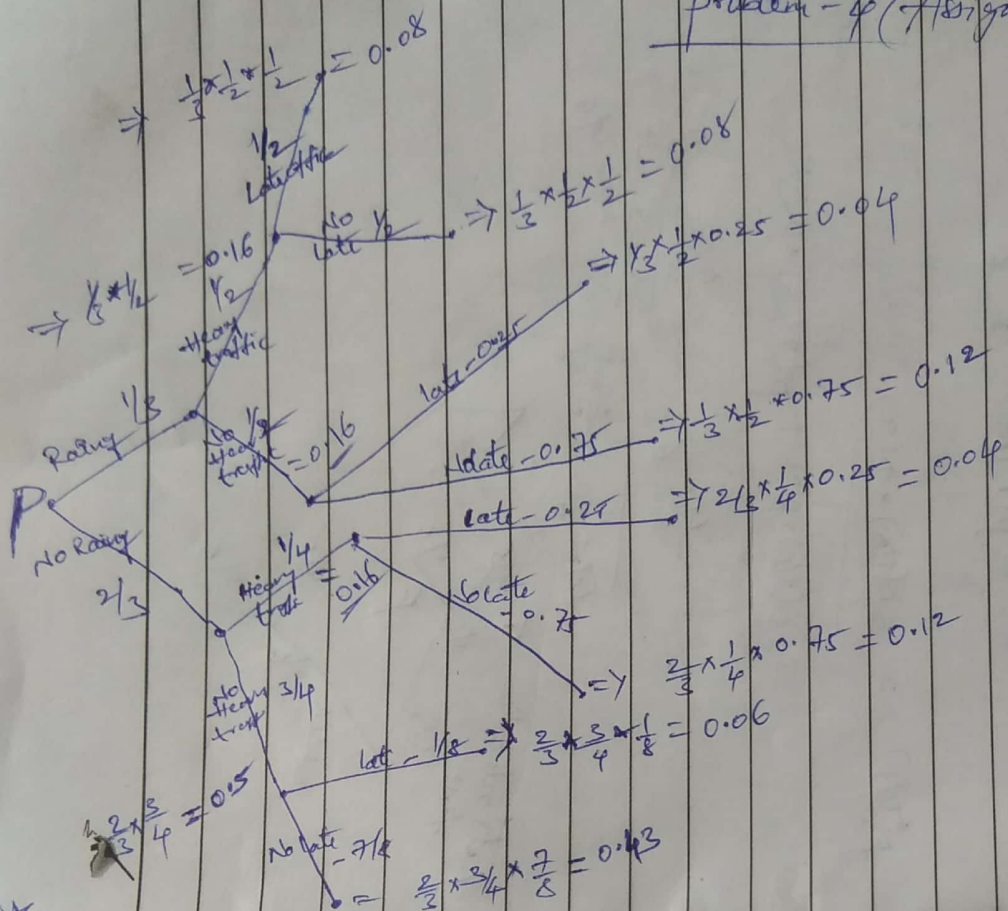
$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(B) = \frac{P(A \cap B)}{P(A|B)} = \frac{4/8}{7/8} = \frac{4}{7}$$

problem - 4 (Assignment)

M / T / W / T / F / S / S / U

Date

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(a) $p(\text{no rain} \cap \text{heavy traffic} \cap \text{no late}) = 0.12$

(b) $p(\text{late}) = 0.08 + 0.04 + 0.04 + 0.06 = 0.22$

(c) $p(\text{rained} / \text{late work}) = \frac{p(\text{rained} \cap \text{late work})}{p(\text{late work})} = \frac{0.08 + 0.04}{0.22} = 0.57$ (need to use 2 late work)

$$\begin{array}{r} 0.08 \\ 0.04 \\ \hline 0.12 \end{array}$$

⑤ 2 regular coins, 1 fake two-headed coin
($p(H) = 1$)

→ picking regular coin = $\frac{2}{3}$ (2 fair coins out of total 3 coins)
→ picking fake coin = $\frac{1}{3}$
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(a) $p(H)$?

$$p(\text{regular coin} \cap \text{head}) = \frac{2}{3} \times \frac{1}{2}$$

$$p(\text{fake coin and head}) = \frac{1}{3} \times 1$$

$$p(\text{total heads}) = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

⑥ probability of two-headed coin

Bayes' theorem

$$p\left(\frac{\text{head}}{\text{2-headed coin}}\right) = 1 \text{ and } p\left(\frac{\text{2-headed coin}}{\text{pick}}\right) = \frac{1}{3}$$

$$p\left(\frac{\text{2-headed coin}}{\text{head}}\right) = \frac{p\left(\frac{\text{head}}{\text{2-headed coin}}\right) \times p\left(\frac{\text{2-headed coin}}{\text{pick}}\right)}{p(\text{head})} \rightarrow \text{from (a)}$$

$$= \frac{1 \times \frac{1}{3}}{\frac{2}{3}} = \frac{1 \times \frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

⑥ $p(A) = p(\text{coffee}) = \frac{70}{100} = 0.70$

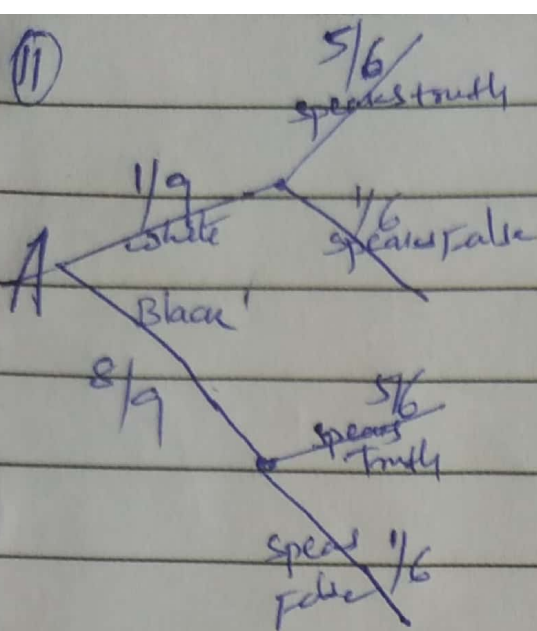
$$p(B) = p(\text{cake}) = \frac{40}{100} = 0.40$$

$$p(A \cap B) = p(\text{coffee and cake}) = \frac{20}{100} = 0.20$$

Given that,

$$p(A/B) = \frac{p(A \cap B)}{p(B)} = \frac{0.20}{0.40} = \frac{20}{40} = \frac{1}{2} = 0.5$$

(11)



$P(\text{speaks truth and white})$

$$P(A \cap B) = \frac{5}{6} \times \frac{1}{9}$$

$$P(\text{speaks truth} / \text{white}) \times P(\text{white})$$

$$P(\text{white drawn} / A \text{ spoke truth})$$

$$= \frac{P(\text{truth} / \text{white}) \times P(\text{white})}{P(\text{spoke truth})}$$

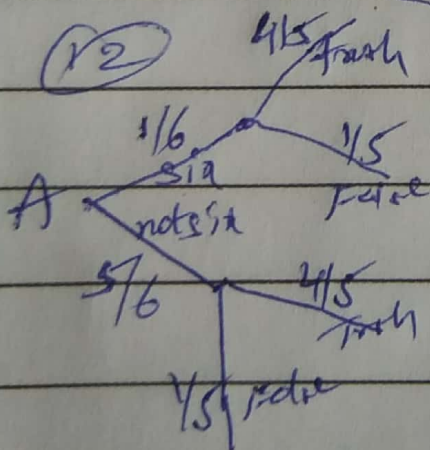
\downarrow
 A speaks truth and white $\quad \quad \quad A$ didn't speak truth and not white

$$= \frac{\frac{5}{6} \times \frac{1}{9}}{(\frac{5}{6} \times \frac{1}{9}) + (\frac{8}{9} \times \frac{1}{6})}$$

$$= \frac{5}{5+8} = \frac{5}{13}$$

$$= \frac{(\frac{5}{6} \times \frac{1}{9}) + (\frac{8}{9} \times \frac{1}{6})}{5+8} = \frac{5}{13}$$

(12)



$$P(\text{truth} / \text{sin}) = P(A \text{ says truth and sin})$$

$$= P(A \cap B) = \frac{1}{6} \times \frac{4}{5}$$

$$P(\text{actually sin} / \text{spoke truth}) = \frac{1}{6} \times \frac{4}{5}$$

$$= \frac{(\frac{1}{6} \times \frac{4}{5}) + (\frac{5}{6} \times \frac{1}{5})}{4+5}$$

$$= \frac{4}{4+5} = \frac{4}{9}$$