2.7.16

Aditya Appana - EE25BTECH11004

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Question

Find $|\mathbf{a} \times \mathbf{b}|$ if $\mathbf{a} = (2\hat{i} + \hat{j} + 3\hat{k})$ and $\mathbf{b} = (3\hat{i} + 5\hat{j} - 2\hat{k})$.

Given Information

The vectors are

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \tag{2}$$

Formula

To calculate the cross-product of the two vectors a and b, we use the following determinant:

$$\begin{pmatrix} \left|\mathbf{A}_{23}\mathbf{B}_{23}\right| \\ \left|\mathbf{A}_{31}\mathbf{B}_{31}\right| \\ \left|\mathbf{A}_{12}\mathbf{B}_{12}\right| \end{pmatrix}$$

Where $\mathrm{X}_{ij} = \begin{pmatrix} \mathrm{x}_i \\ \mathrm{x}_j \end{pmatrix}$.

Solution

Expanding the determinants, we get:

$$\begin{pmatrix} ((-2)-15) \\ ((-4)-9) \\ (10-3) \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} -17 \\ 13 \\ 7 \end{pmatrix} \tag{4}$$

We need to find the norm of this vector, which is done by:

$$\sqrt{17^2 + 13^2 + 7^2} \tag{5}$$

$$= 22.516660498395403 \tag{6}$$

Python Code

```
import numpy as np
import math
import matplotlib.pyplot as plt
import numpy.linalg as LA

vecA = np.array([2,1,3])
vecB = np.array([3,5,-2])

crossprod = np.cross(vecA,vecB)
print(crossprod)

mod = np.linalg.norm(crossprod)
print(mod)
```

Python Code

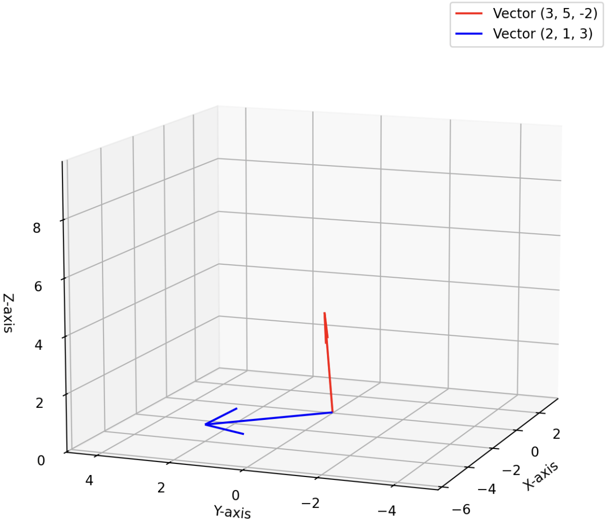
```python
fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')
origin = np.array([0, 0, 0])

ax.quiver(*origin, *vecA, color='r', label='Vector (3, 5, -2)')
ax.quiver(*origin, *vecB, color='b', label='Vector (2, 1, 3)')

max_val = np.max(np.abs(np.concatenate((vecA, vecB))))
ax.set_xlim([-max_val, max_val])
ax.set_ylim([-max_val, max_val])
ax.set_zlim([-max_val, max_val])
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('3D Vectors')

ax.legend()
ax.grid(True)
plt.show()
```

Plot



C Code

```c
#include<stdio.h>
#include<math.h>

int crossprod(float a0, float a1, float a2, float b0, float b1,
    float b2){

float modA = sqrt(pow(a0,2)+pow(a1,2)+pow(a2,2));
float modB = sqrt(pow(b0,2)+pow(b1,2)+pow(b2,2));

float dotprod = a0*b0 + a1*b1 + a2*b2;

float mod = sqrt(pow(modA,2)*pow(modB,2) - pow(dotprod,2));

return mod;
}
```

Python and C Code

```
import numpy as np
import ctypes
import matplotlib.pyplot as plt
c_lib=ctypes.CDLL('./4c.so')

c_lib.crossprod.argtypes = [ctypes.c_float, ctypes.c_float,ctypes
    .c_float, ctypes.c_float, ctypes.c_float, ctypes.c_float]
c_lib.crossprod.restype = ctypes.c_float
vecA = np.array([2,1,3])
vecB = np.array([3,5,-2])

mod = c_lib.crossprod(
    ctypes.c_float(vecA[0]),
    ctypes.c_float(vecA[1]),
    ctypes.c_float(vecA[2]),
    ctypes.c_float(vecB[0]),
    ctypes.c_float(vecB[1]),
    ctypes.c_float(vecB[2]))
```

Python and C Code

```
print(mod)
vecA = np.array([2,1,3]).reshape(-1,1)
vecB = np.array([3,5,-2]).reshape(-1,1)

fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')

origin = np.array([0, 0, 0])

ax.quiver(*origin, *vecA, color='r', label='Vector (3, 5, -2)')
ax.quiver(*origin, *vecB, color='b', label='Vector (2, 1, 3)')

max_val = np.max(np.abs(np.concatenate((vecA, vecB))))
```

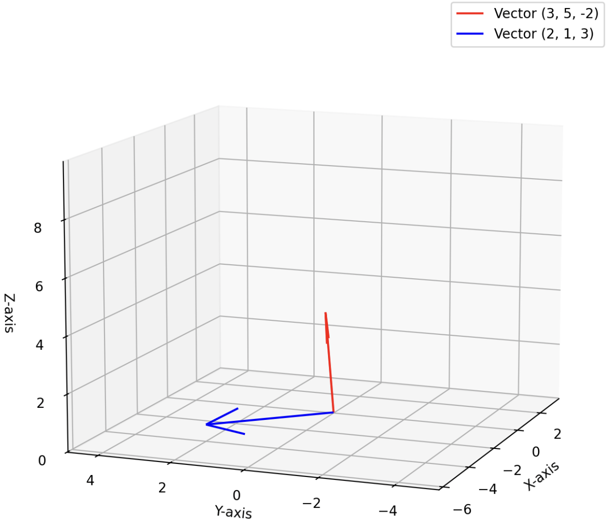
Python and C Code

```python
ax.set_xlim([-max_val, max_val])
ax.set_ylim([-max_val, max_val])
ax.set_zlim([-max_val, max_val])

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Two 3D Vectors')

ax.legend()
ax.grid(True)
plt.show()
```

Plot


Vector (3, 5, -2)
Vector (2, 1, 3)
Z-axis
Y-axis
X-axis