Matrices in Geometry - 2.4.32

EE25BTECH11035 Kushal B N

Sep, 2025

Problem Statement

The points **A** $(-1, -2)$, **B** $(4, 3)$, **C** $(2, 5)$ and **D** $(-3, 0)$ in that order form a rectangle.

Solution

Given $\mathbf{A}\begin{pmatrix}-1\\-2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}4\\3\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}2\\5\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}-3\\0\end{pmatrix}$.

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}5\\5\end{pmatrix} \tag{1}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}-2\\2\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{C}=\begin{pmatrix}-5\\-5\end{pmatrix} \tag{3}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}2\\-2\end{pmatrix} \tag{4}$$

Solution

Checking opposite sides,

$$(\mathbf{B}-\mathbf{A}) = -(\mathbf{D}-\mathbf{C}) \tag{5}$$

Now, as each pair of opposite sides are parallel and equal in length, this means that the given points make up a parallelogram.
Checking for right angle, we need to check for inner product of the adjacent sides of the parallelogram.

$$(\mathbf{B}-\mathbf{A})^\top (\mathbf{C}-\mathbf{B}) = \begin{pmatrix} 5 & 5 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 0 \tag{6}$$

This implies that the angle at B is a right angle. A parallelogram with a right angle is a rectangle.

Solution

Checking for square:
The given quadrilateral will be a square if its diagonals are orthogonal.

$$(\mathbf{C}-\mathbf{A})^{\top}(\mathbf{D}-\mathbf{B}) = \begin{pmatrix} 3 & 7 \end{pmatrix} \begin{pmatrix} -7 \\ -3 \end{pmatrix} = -42 \tag{7}$$

From (7), we can see that $(\mathbf{C}-\mathbf{A})^{\top}(\mathbf{D}-\mathbf{B}) \neq 0$, that is the diagonals are not orthogonal and hence the given quadrilateral cannot be a square. Hence, it is proved that the given points **ABCD** in that order form a rectangle.

Conclusion

∴ The quadrilateral **ABCD** is a rectangle.