Matrices in Geometry - 1.9.27

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September 8, 2025

Problem Statement

Find the value of P, if the point $\mathbf{A}\,(0, 2)$ is equidistant from point $\mathbf{B}\,(3, P)$ and $\mathbf{c}\,(p, 5)$

Solution

Given: $\mathbf{A}\begin{pmatrix}0\\2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}3\\P\end{pmatrix}$ and a point $\mathbf{C}\begin{pmatrix}P\\5\end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{A}-\mathbf{B}\| = \|\mathbf{A}-\mathbf{C}\| \tag{1}$$

On squaring both the sides, we get (2)

$$\|\mathbf{A}-\mathbf{B}\|^2 = \|\mathbf{A}-\mathbf{C}\|^2 \tag{3}$$

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{A}-\mathbf{B}) = (\mathbf{A}-\mathbf{C})^\top(\mathbf{A}-\mathbf{C}) \tag{4}$$

(5)

Solution

$$\mathbf{A}^\top\mathbf{A} - 2\mathbf{A}^\top\mathbf{B} + \mathbf{B}^\top\mathbf{B} = \mathbf{A}^\top\mathbf{A} - 2\mathbf{A}^\top\mathbf{C} + \mathbf{C}^\top\mathbf{C} \tag{6}$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{A}^\top(\mathbf{B} - \mathbf{C}) \tag{7}$$

$$\left\|\begin{pmatrix}3\\P\end{pmatrix}\right\| - \left\|\begin{pmatrix}P\\5\end{pmatrix}\right\| = 2\begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}3-P\\P-5\end{pmatrix} \tag{8}$$

$$9 + p^2 - p^2 - 25 = 2(0 + 2p - 10) \tag{9}$$

$$-16 = 4p - 20 \implies 4p = 4 \implies p = 1 \tag{10}$$

Final Answer

$$\text{Hence, the final answer is } \boxed{p = 1} \tag{11}$$