Problem 2.10.52

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September 5, 2025

Problem

Let $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. A vector in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is

1. $4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
2. $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
3. $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
4. $4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Coplanarity

Given $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Let $\mathbf{r}$ be coplanar to $\mathbf{a}$ and $\mathbf{b}$

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \tag{1.1}$$

Simplify

Given the projection of $\mathbf{r}$ on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$

$$\frac{|\mathbf{r}^\top \mathbf{c}|}{\|\mathbf{c}\|} = \frac{1}{\sqrt{3}} \tag{1.2}$$

$$\begin{aligned}
\mathbf{r}^\top \mathbf{c} &= (\mathbf{a} + t\mathbf{b})^\top \mathbf{c} && (1.3)\\
&= \left(\mathbf{a}^\top + t\mathbf{b}^\top\right) c && (1.4)\\
&= \mathbf{a}^\top \mathbf{c} + t\left(\mathbf{b}^\top \mathbf{c}\right) && (1.5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c} &= t\left(\mathbf{b}^\top \mathbf{c}\right) && (1.6)\\
\implies t &= \frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} && (1.7)
\end{aligned}$$

Finding Values

$$\mathbf{r} = \mathbf{a} + \left(\frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}}\right)\mathbf{b} \tag{1.8}$$

$$\mathbf{r} = \mathbf{a} + \left(\frac{\pm\frac{\|\mathbf{c}\|}{\sqrt{3}} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}}\right)\mathbf{b} \tag{1.9}$$

$$\|\mathbf{c}\|^2 = \mathbf{c}^\top \mathbf{c} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{1.10}$$

$$= 1 + 1 + 1 = 3 \implies \|\mathbf{c}\| = \sqrt{3} \tag{1.11}$$

$$\mathbf{a}^\top \mathbf{c} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 1 + 2 - 1 = 2 \tag{1.12}$$

Conclusion

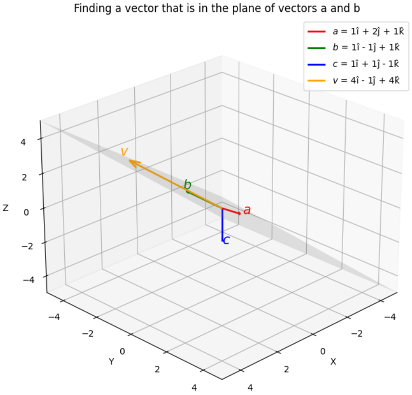
$$\mathbf{b}^\top \mathbf{c} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 1 - 1 - 1 = -1 \tag{1.13}$$

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \left(\frac{\pm 1 - 2}{-1}\right) \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{1.14}$$

$$\implies \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \implies \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ or } \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} \tag{1.15}$$

Hence Option(1) is the correct answer

Plot



Finding a vector that is in the plane of vectors a and b
a = 1î + 2ĵ + 1k̂
b = 1î - 1ĵ + 1k̂
c = 1î + 1ĵ - 1k̂
v = 4î - 1ĵ + 4k̂
v
b
a
c
Z
X
Y

Figure:

C Code

```c
void get_vectors(double* vector_data) {
    vector_data[0] = 1.0; vector_data[1] = 2.0; vector_data[2] = 1.0;
    vector_data[3] = 1.0; vector_data[4] = -1.0; vector_data[5] = 1.0;
    vector_data[6] = 1.0; vector_data[7] = 1.0; vector_data[8] = -1.0;
    vector_data[9] = 4.0; vector_data[10] = -1.0; vector_data[11] = 4.0;
}
```

Calling C Function

```
import ctypes
import numpy as np
def load_vectors_from_c():
    lib = ctypes.CDLL('./plane.so')
    # Define the C function signature
    get_vectors_c = lib.get_vectors
    get_vectors_c.argtypes = [ctypes.POINTER(ctypes.c_double)]
    get_vectors_c.restype = None

    vector_data = np.zeros(12, dtype=np.float64)
    get_vectors_c(vector_data.ctypes.data_as(ctypes.POINTER(
        ctypes.c_double)))
    # Reshape the data and return the individual vectors
    vectors_from_c = vector_data.reshape(4, 3)
    a, b, c, v = vectors_from_c[0], vectors_from_c[1],
        vectors_from_c[2], vectors_from_c[3]
    return a, b, c, v
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL

import sys
import matplotlib.pyplot as plt
import numpy as np
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
from call import load_vectors_from_c
hat_symbol = '\u0302'
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

a, b, c, v = load_vectors_from_c()
```

Python Code for Plotting

```
vx, vy, vz = v[0], v[1], v[2]
j_part_str = f"+ {vy}j{hat_symbol}" if vy >= 0 else f"- {abs(vy)}
    j{hat_symbol}"
k_part_str = f"+ {vz}k{hat_symbol}" if vz >= 0 else f"- {abs(vz)}
    k{hat_symbol}"
print(f"The vector in the plane of a and b is: {vx}i{hat_symbol}
    {j_part_str} {k_part_str}")

# Create a 3D plot
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
origin = [0, 0, 0]

# Plot vectors
vectors = {'a': (a, 'r'), 'b': (b, 'g'), 'c': (c, 'b'), 'v': (v,
    'orange')}
for name, (vec, color) in vectors.items():
    x, y, z = int(vec[0]), int(vec[1]), int(vec[2])
    j_part = f"+ {y}j{hat_symbol}" if y >= 0 else f"- {abs(y)}j{
        hat_symbol}"
```

Python Code for Plotting

```
    k_part = f"+ {z}k{hat_symbol}" if z >= 0 else f"- {abs(z)}k{hat_symbol}"
    label_text = f'${name}$ = {x}i{hat_symbol} {j_part} {k_part}'
  ax.quiver(origin[0], origin[1], origin[2], vec[0], vec[1], vec[2],
             color=color, label=label_text,
             arrow_length_ratio=0.1, linewidth=2)
    ax.text(vec[0]*1.1, vec[1]*1.1, vec[2]*1.1, f'${name}$', color=color, fontsize=15)
    # Plot the plane
x_plane = np.linspace(-5, 5, 10); y_plane = np.linspace(-5, 5, 10)
X, Y = np.meshgrid(x_plane, y_plane)
Z = X
ax.plot_surface(X, Y, Z, alpha=0.2, color='gray')
```

Python Code for Plotting

```
ax.view_init(elev=25, azim=45)
ax.set_xlabel('X'); ax.set_ylabel('Y'); ax.set_zlabel('Z')
ax.set_title('Finding a vector that is in the plane of vectors a
    and b')
ax.legend()
ax.grid(True)
ax.set_xlim([-5, 5]); ax.set_ylim([-5, 5]); ax.set_zlim([-5, 5])
```