Matrices in Geometry - 2.4.34

EE25BTECH11037 Divyansh

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Problem Statement

What type of quadrilateral do the points **A** $(2, -2)$, **B** $(7, 3)$, **C** $(11, -1)$ and **D** $(6, -6)$ taken in that order, form?

Solution

Given: $\mathbf{A}\begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{B}\begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\mathbf{C}\begin{pmatrix} 11 \\ -1 \end{pmatrix}$ and $\mathbf{D}\begin{pmatrix} 6 \\ -6 \end{pmatrix}$.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{1}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{2}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \tag{3}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \tag{4}$$

Solution

$$\text{Checking opposite sides,} \quad \mathbf{D} - \mathbf{C} = -(\mathbf{A} - \mathbf{B}) \text{ and } \mathbf{A} - \mathbf{D} = -(\mathbf{B} - \mathbf{C}) \tag{5}$$

Each pair of opposite sides are parallel and equal in length; this implies that the quadrilateral is a parallelogram.
Now, checking for right angle, we check for inner product.

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 5 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} = 0 \tag{6}$$

Solution

This implies that the angle at $\mathbf{B}$ is one right angle. A parallelogram with a right angle is a rectangle.
Checking for a square:
The give quadrilateral is a square if its diagonals are orthogonal,

$$(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B}) = \begin{pmatrix}9 & 1\end{pmatrix}\begin{pmatrix}-1\\-9\end{pmatrix} = -18 \tag{7}$$

We can see that $(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B})$ is $\neq 0$, that is, the diagonals are not orthogonal and therefore the quadrilateral $\mathbf{ABCD}$ cannot be a square.
Therefore, the quadrilateral $\mathbf{ABCD}$ is a rectangle.

Final Answer

$$\therefore \text{ The quadrilateral } \textbf{ABCD} \text{ is a rectangle} \tag{8}$$