

# Matrices in Geometry 1.9.24

EE25BTECH11035 - Kushal B N

**Question:** The x-coordinate of a point  $\mathbf{P}$  is twice its y-coordinate. If  $\mathbf{P}$  is equidistant from the points  $\mathbf{Q}(2, -5)$  and  $\mathbf{R}(-3, 6)$ , find the coordinates of  $\mathbf{P}$ .

**Given:**

$$\mathbf{P}\begin{pmatrix} 2k \\ k \end{pmatrix}, \mathbf{Q}\begin{pmatrix} 2 \\ -5 \end{pmatrix}, \mathbf{R}\begin{pmatrix} -3 \\ 6 \end{pmatrix}.$$

Distances  $PQ = PR$

So their norms must be equal and also the square of their norms.

$$\|\mathbf{Q} - \mathbf{P}\|^2 = \|\mathbf{R} - \mathbf{P}\|^2 \quad (1)$$

$$\|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{Q} + \|\mathbf{Q}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{R} + \|\mathbf{R}\|^2 \quad (2)$$

$$\Rightarrow \frac{\|\mathbf{Q}\|^2 - \|\mathbf{R}\|^2}{2} = \mathbf{P}^\top (\mathbf{Q} - \mathbf{R}) \quad (3)$$

$$\|\mathbf{Q}\|^2 = \begin{pmatrix} 2 & -5 \end{pmatrix} \begin{pmatrix} 2 \\ -5 \end{pmatrix} = 4 + 25 = 29 \quad (4)$$

$$\|\mathbf{R}\|^2 = \begin{pmatrix} -3 & 6 \end{pmatrix} \begin{pmatrix} -3 \\ 6 \end{pmatrix} = 9 + 36 = 45 \quad (5)$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 5 \\ -11 \end{pmatrix} \quad (6)$$

$$\Rightarrow -8 = \begin{pmatrix} 2k & k \end{pmatrix} \begin{pmatrix} 5 \\ -11 \end{pmatrix} \quad (7)$$

$$-8 = 10k - 11k = -k \quad (8)$$

$$\Rightarrow \boxed{k = 8} \quad (9)$$

**Final Answer:** The coordinates of point  $\mathbf{P} = \begin{pmatrix} 16 \\ 8 \end{pmatrix}$ .

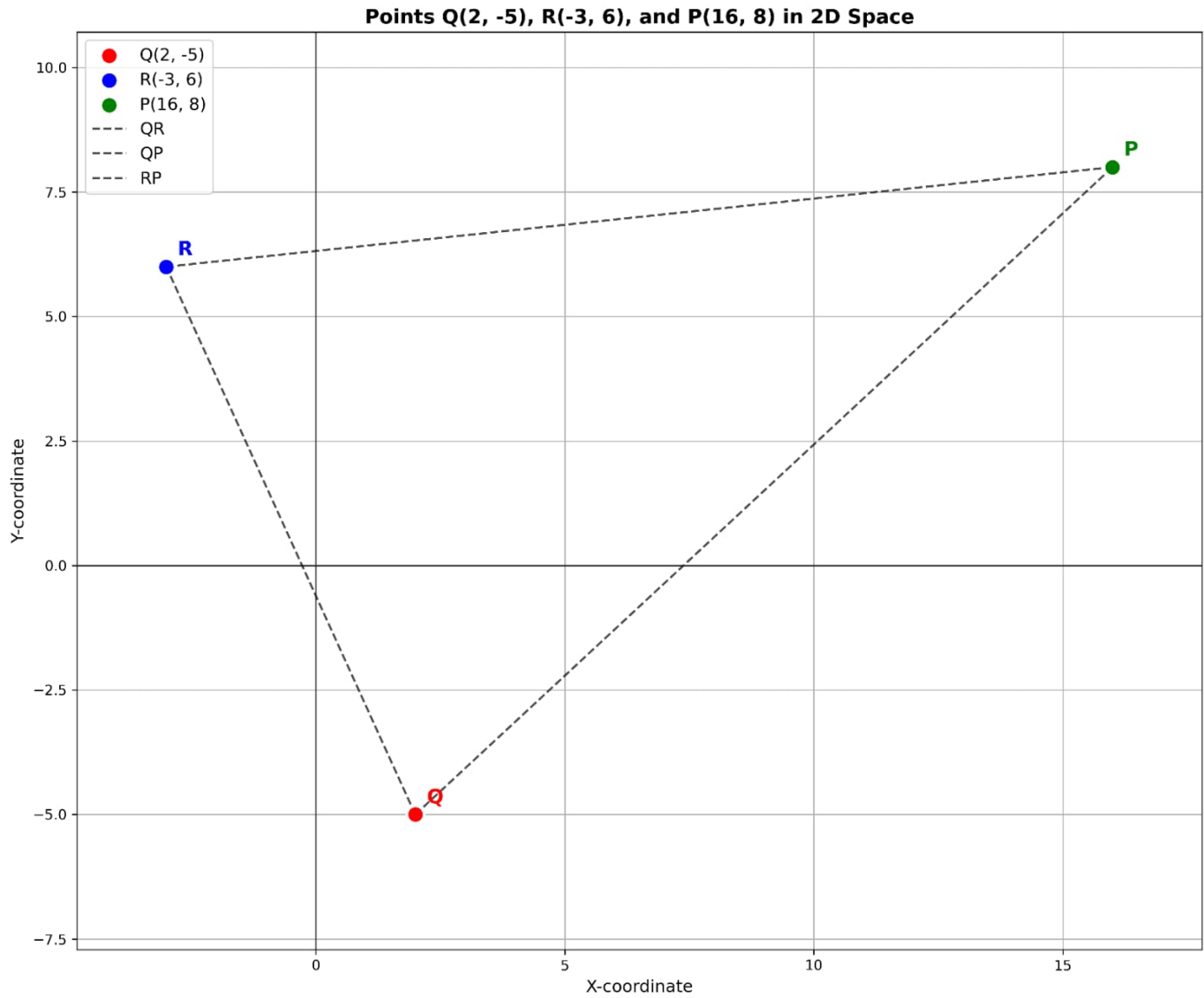


Fig. 1: Plot of the three points in 2-D Space