# 4.11.11

Kavin B-EE25BTECH11033

September 05,2025

# Question

Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

# Theoretical Solution

Given the points,

$$\mathbf{A} = \begin{pmatrix} -2 \\ -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \tag{1}$$

Let the vector $\mathbf{P}$ be a point on the line $x - 3y = 0$ wihch divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$.

$$\mathbf{P} = \begin{pmatrix} 3k \\ k \end{pmatrix} , \tag{2}$$

The points $\mathbf{A}$, $\mathbf{P}$, $\mathbf{B}$ are collinear.

# Formulae

**Points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are defined to be collinear if**

$$\text{rank}\begin{pmatrix}\mathbf{P}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix} = 1 \tag{3}$$

# Theoretical Solution

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 3k+2 \\ k+5 \end{pmatrix} \tag{4}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} 3k+2 & 8 \\ k+5 & 8 \end{pmatrix} \tag{6}$$

$$R_2 \to R_2 - \tfrac{k+5}{3k+2} R_1 \implies \begin{pmatrix} 3k+2 & 8 \\ 0 & \frac{16k-24}{3k+2} \end{pmatrix}$$

For rank 1, the second row must be zero:

$$16k - 24 = 0 \implies k = 3/2 \tag{7}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 9/2 \\ 3/2 \end{pmatrix}$$

# Formulae

**Section formula for a vector P which divides the line formed by vectors A and B in the ratio k:1 is given by**

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{8}$$

$$k\left(\mathbf{P} - \mathbf{B}\right) = \mathbf{A} - \mathbf{P} \tag{9}$$

$$\implies k = \frac{\left(\mathbf{A} - \mathbf{P}\right)^{\top}\left(\mathbf{P} - \mathbf{B}\right)}{\left\|\mathbf{P} - \mathbf{B}\right\|^{2}} \tag{10}$$

# Theoretical Solution

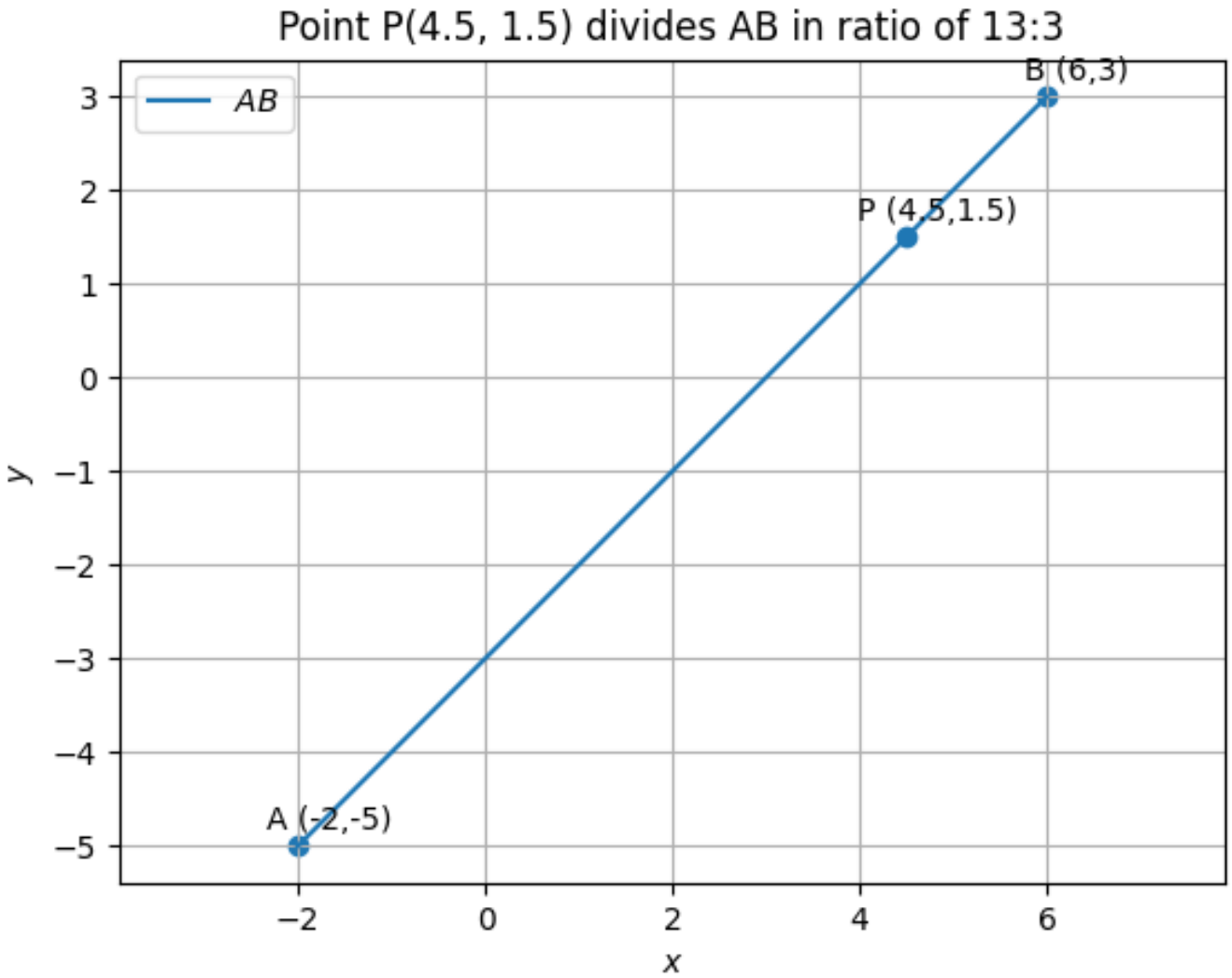
$$(\mathbf{A}-\mathbf{P})^{\top}(\mathbf{P}-\mathbf{B}) = \begin{pmatrix} -13/2 & -13/2 \end{pmatrix} \begin{pmatrix} -3/2 \\ -3/2 \end{pmatrix} = 39/2 \tag{11}$$

$$\|\mathbf{P}-\mathbf{B}\|^2 = \left( \sqrt{(-3/2)^2 + (-3/2)^2} \right)^2 = 9/2 \tag{12}$$

$$\implies k = 13/3 \tag{13}$$

Therefore the ratio in which $\mathbf{P}$ divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$ is $13:3$

# Plot


Point P(4.5, 1.5) divides AB in ratio of 13:3
AB
B (6,3)
P (4.5,1.5)
A (-2,-5)
3
2
1
0
−1
−2
−3
−4
−5
y
−2
0
2
4
6
x