Matgeo Presentation - Problem 2.4.16

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Problem Statement

Verify the following:
(a) $(0, 7, -10), (1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.
(b) $(0, 7, 10), (-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right-angled triangle.

solution

Solution a

Property: In an isosceles triangle, the perpendicular bisector of a side passes through the opposite vertex.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix} \tag{0.1}$$

Midpoint of side AC:

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{C}}{2} = \frac{\begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix} + \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix}}{2} = \begin{pmatrix} 2 \\ 8 \\ -8 \end{pmatrix} \tag{0.2}$$

Direction vector of side AC:

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \tag{0.3}$$

solution

Vector from midpoint to B:

$$\mathbf{B} - \mathbf{M} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \\ -8 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \tag{0.4}$$

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{B} - \mathbf{M}) = \begin{pmatrix} 4 & 2 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = -4 - 4 + 8 = 0 \tag{0.5}$$

$\mathbf{B}$ lies on the perpendicular bisector of side AC.

$$\therefore AB = BC \implies \triangle ABC \text{ is isosceles.}$$

Solution b

Property: If two sides of a triangle are perpendicular, then the included angle is a right angle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 6 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ 9 \\ 6 \end{pmatrix} \tag{0.6}$$

solution

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}0\\7\\10\end{pmatrix}-\begin{pmatrix}-1\\6\\6\end{pmatrix}=\begin{pmatrix}1\\1\\4\end{pmatrix} \tag{0.7}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}-4\\9\\6\end{pmatrix}-\begin{pmatrix}-1\\6\\6\end{pmatrix}=\begin{pmatrix}-3\\3\\0\end{pmatrix} \tag{0.8}$$

$$(\mathbf{A}-\mathbf{B})^{\top}(\mathbf{C}-\mathbf{B})=\begin{pmatrix}1&1&4\end{pmatrix}\begin{pmatrix}-3\\3\\0\end{pmatrix}=-3+3+0=0 \tag{0.9}$$

$$\implies \mathbf{A}-\mathbf{B}\perp\mathbf{C}-\mathbf{B}\Rightarrow\triangle ABC \text{ is right-angled at } B.$$

C Source Code:gen point.c

```
#include <stdio.h>

// Function to write first set of points (isosceles triangle)
void generate_points_isosceles(const char *filename) {
    FILE *fp = fopen(filename, "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return;
    }

    double A[3] = {0, 7, -10};
    double B[3] = {1, 6, -6};
    double C[3] = {4, 9, -6};
```

C Source Code:gen point.c

```
    fprintf(fp, "%lf %lf %lf\n", A[0], A[1], A[2]);
    fprintf(fp, "%lf %lf %lf\n", B[0], B[1], B[2]);
    fprintf(fp, "%lf %lf %lf\n", C[0], C[1], C[2]);
    fclose(fp);
}

// Function to write second set of points (right-angled triang
void generate_points_right(const char *filename) {
    FILE *fp = fopen(filename, "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return;
    }
```

Python Script: solve triangle.py

```
import ctypes
import numpy as np

# Load shared object
lib = ctypes.CDLL("./gen_points.so")

# Call C functions
lib.generate_points_isosceles(b"iso_points.dat")
lib.generate_points_right(b"right_points.dat")

# Load points
iso_points = np.loadtxt("iso_points.dat")
right_points = np.loadtxt("right_points.dat")
```

Python Script: solve triangle.py

```
# Distance squared function
def dist2(P, Q):
    return np.sum((P - Q) ** 2)

# --------- Part (a) Isosceles Triangle -----------
A, B, C = iso_points
AB2 = dist2(A, B)
BC2 = dist2(B, C)
CA2 = dist2(C, A)

print("Isosceles Triangle Check:")
print("AB^2 =", AB2, "BC^2 =", BC2, "CA^2 =", CA2)
isosceles = (AB2 == BC2) or (BC2 == CA2) or (CA2 == AB2)
print("Isosceles:", isosceles)
print()
```

Python Script: solve triangle.py

```
# --------- Part (b) Right Angled Triangle ---------
P, Q, R = right_points
PQ2 = dist2(P, Q)
QR2 = dist2(Q, R)
RP2 = dist2(R, P)

print("Right Angled Triangle Check:")
print("PQ^2 =", PQ2, "QR^2 =", QR2, "RP^2 =", RP2)
right_angle = (PQ2 + QR2 == RP2) or (QR2 + RP2 == PQ2) or (RP2
print("Right Angled:", right_angle)
```

Python Script: plot triangle.py

```
import sys
sys.path.insert(0,  '/home/dhanush-sagar/matgeo/codes/CoordGeo
import numpy as np
import matplotlib.pyplot as plt


# Local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

# Load both sets of points
iso_points = np.loadtxt("iso_points.dat")
right_points = np.loadtxt("right_points.dat")

# ---- Plot Isosceles Triangle ----
fig1 = plt.figure()
```

Python Script: plot triangle.py

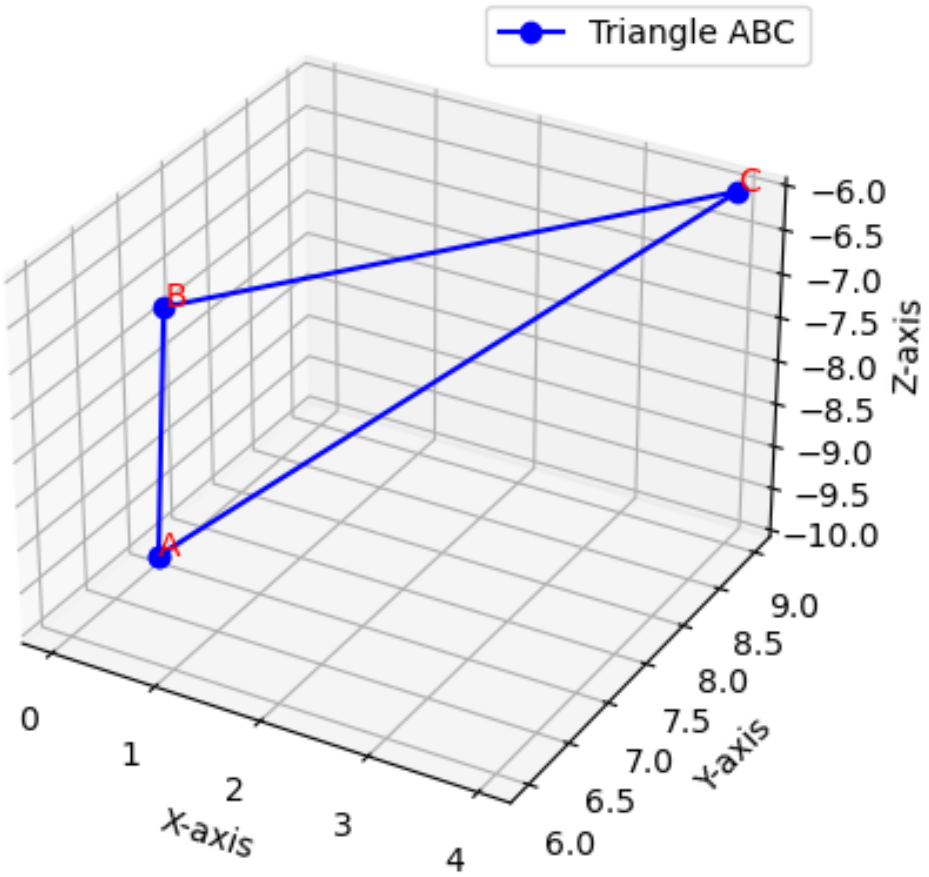
```
ax1 = fig1.add_subplot(111, projection='3d')
A, B, C = iso_points
tri_iso = np.vstack((A, B, C, A))
ax1.plot(tri_iso[:,0], tri_iso[:,1], tri_iso[:,2], 'b-o', labe
ax1.text(A[0], A[1], A[2], "A", color='red')
ax1.text(B[0], B[1], B[2], "B", color='red')
ax1.text(C[0], C[1], C[2], "C", color='red')
ax1.set_title("Isosceles Triangle")
ax1.set_xlabel("X-axis")
ax1.set_ylabel("Y-axis")
ax1.set_zlabel("Z-axis")
plt.legend()
plt.savefig("isosceles_triangle.png")
plt.show()

# ---- Plot Right-Angled Triangle ----
fig2 = plt.figure()
```

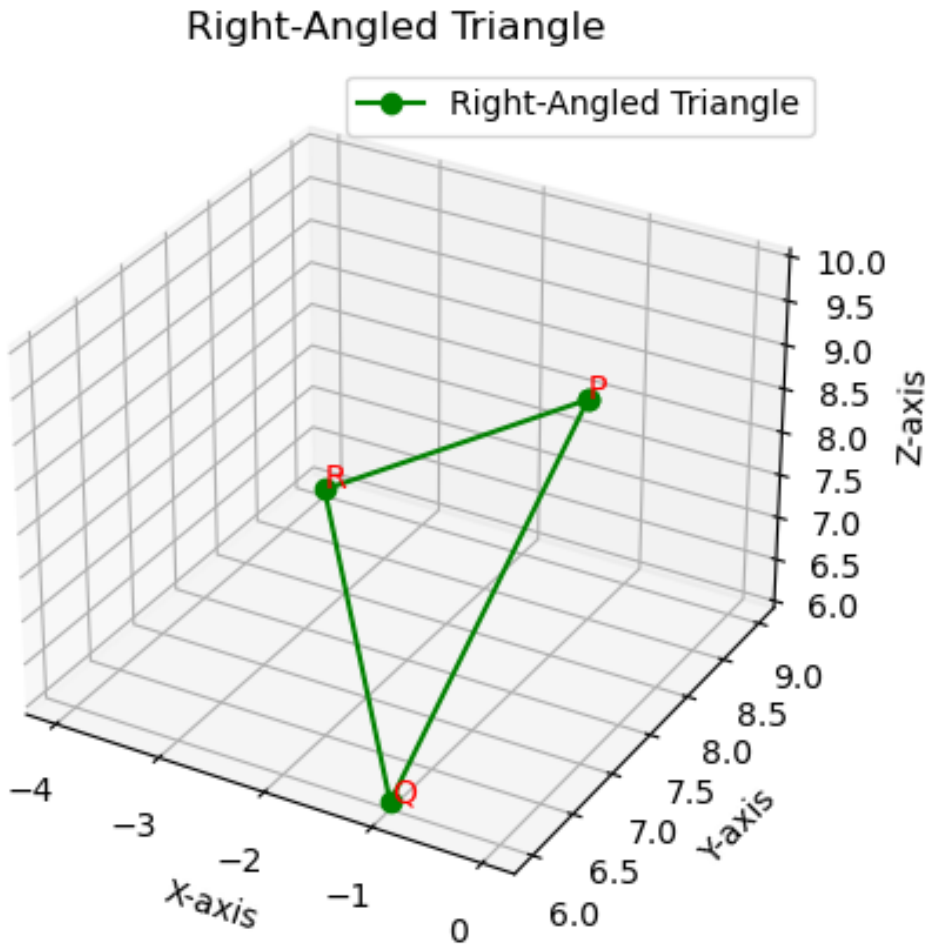
Python Script: plot triangle.py

```
ax2 = fig2.add_subplot(111, projection='3d')
P, Q, R = right_points
tri_right = np.vstack((P, Q, R, P))
ax2.plot(tri_right[:,0], tri_right[:,1], tri_right[:,2], 'g-o'
ax2.text(P[0], P[1], P[2], "P", color='red')
ax2.text(Q[0], Q[1], Q[2], "Q", color='red')
ax2.text(R[0], R[1], R[2], "R", color='red')
ax2.set_title("Right-Angled Triangle")
ax2.set_xlabel("X-axis")
ax2.set_ylabel("Y-axis")
ax2.set_zlabel("Z-axis")
plt.legend()
plt.savefig("right_triangle.png")
plt.show()
```

Result Plot


Triangle ABC
A
B
C
X-axis
Y-axis
Z-axis
0
1
2
3
4
6.0
6.5
7.0
7.5
8.0
8.5
9.0
−6.0
−6.5
−7.0
−7.5
−8.0
−8.5
−9.0
−9.5
−10.0

Result Plot


Right-Angled Triangle
Right-Angled Triangle
P
R
Q
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
Z-axis
9.0
8.5
8.0
7.5
7.0
6.5
6.0
Y-axis
−4
−3
−2
−1
0
X-axis