

4.11.4

EE25BTECH11026-Harsha

Question:

Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\mathbf{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\mathbf{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

Solution:

Let us solve the given question theoretically and then verify the solution computationally.

According to the question,

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{n}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad c_1 = 4 \quad c_2 = -5 \quad (0.1)$$

The equation of intersection of planes is given by

$$\mathbf{n}_1^\top \mathbf{x} - c_1 + \lambda (\mathbf{n}_2^\top \mathbf{x} - c_2) = 0 \quad (0.2)$$

$$\Rightarrow (\mathbf{n}_1^\top + \lambda \mathbf{n}_2^\top) \mathbf{x} = c_1 + \lambda c_2 \quad (0.3)$$

Let the direction vector of the plane perpendicular to intersection of planes be \mathbf{n}_3

$$\therefore (\mathbf{n}_1^\top + \lambda \mathbf{n}_2^\top) \mathbf{n}_3 = 0 \quad (0.4)$$

$$\Rightarrow \lambda = -\frac{\mathbf{n}_1^\top \mathbf{n}_3}{\mathbf{n}_2^\top \mathbf{n}_3} \quad (0.5)$$

$$\therefore \lambda = -\frac{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}}{\begin{pmatrix} 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}} = \frac{7}{19} \quad (0.6)$$

$$\Rightarrow \text{equation of the plane: } (33 \quad 45 \quad 50) \mathbf{x} = 41 \quad (0.7)$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.

Required Plane through Intersection, \perp to Given Plane

