4.7.25

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Question

Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

Theoretical Solution

According to the question,

$$\text{Equation of line } L_1\text{: } \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 4 \tag{1}$$

$$\implies n_1^\top \mathbf{x} = c_1$$

and

$$\text{Equation of line } L_2\text{: } \begin{pmatrix}4 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 10 \tag{2}$$

$$\implies n_2^\top \mathbf{x} = c_2$$

Formulae

The distance λ of a vector $\mathbf{P}$ from the line $\mathbf{n_2}^\top \mathbf{x} = c_2$ is given by ,

$$\lambda = \frac{\left|\mathbf{n_2}^\top \mathbf{P} - c_2\right|}{\left\|\mathbf{n_2}\right\|} \tag{3}$$

$$\lambda \left\| \mathbf{n_2} \right\| = \left| \mathbf{n_2}^\top \mathbf{P} - c_2 \right| \tag{4}$$

$$\implies \mathbf{n_2}^\top \mathbf{P} = c_2 \pm \lambda \left\| n_2 \right\| \tag{5}$$

Also, as $\mathbf{P}$ lies on line L_1,

$$\mathbf{n_1}^\top \mathbf{P} = c_1 \tag{6}$$

On putting eqns (5) and (6) in matrix form we will get,

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix}^\top \mathbf{P} = \begin{pmatrix} c_1 \\ c_2 \pm \lambda \left\| \mathbf{n_2} \right\| \end{pmatrix} \tag{7}$$

where,

$$\lambda = 1$$

Theoretical Solution

On substituting the values we will get,

$$\begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 4 \\ 10 \pm 5 \end{pmatrix} \tag{8}$$

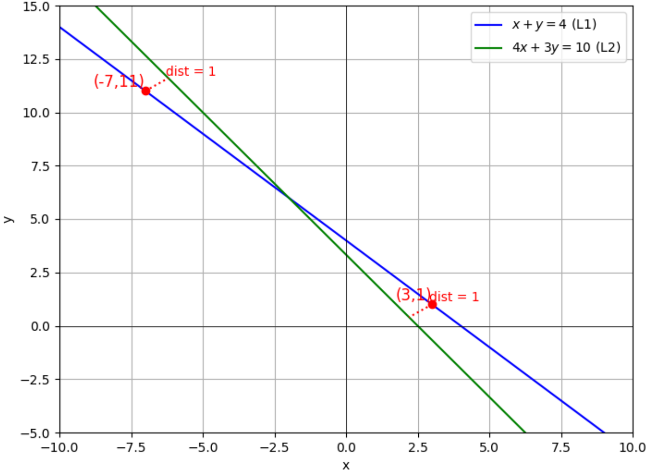
with the augmented matrix followed by row reduction

$$\begin{aligned} R_2 = R_2 - 4R_1 &\rightarrow \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & -1 & -6 \pm 5 \end{array}\right) \\ R_2 = -R_2 &\rightarrow \left(\begin{array}{cc|c} 1 & 1 & 4 \\ 0 & 1 & 6 \mp 5 \end{array}\right) \\ R_1 = R_1 - R_2 &\rightarrow \left(\begin{array}{cc|c} 1 & 0 & -2 \pm 5 \\ 0 & 1 & 6 \mp 5 \end{array}\right) \end{aligned} \tag{9}$$

Therefore the points on L_1 which lie at a unit distance from the line L_2 are

$$\mathbf{P} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } \mathbf{P} = \begin{pmatrix} -7 \\ 11 \end{pmatrix}$$

Plot



$x+y=4$ (L1)
$4x+3y=10$ (L2)
(-7,11)
dist = 1
(3,1)
dist = 1
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x