2.5.16

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:
Find the value of p for which the lines are perpendicular.

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2} \qquad \text{and} \qquad \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

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Solution:
Writing each line in symmetric form to read off direction vectors.

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \tag{1}$$

From

$$\frac{1-x}{3} = t, \ \frac{2y-14}{2p} = t, \ \frac{z-3}{2} = t, \tag{2}$$

we obtain

$$x = 1-3t, \quad y = 7+pt, \quad z = 3+2t. \tag{3}$$

Hence, the direction vector is

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ p \\ 2 \end{pmatrix} \tag{4}$$

From

$$\frac{1-x}{3p} = s, \ \frac{y-5}{1} = s, \ \frac{6-z}{5} = s, \tag{5}$$

we obtain

$$x = 1-3ps, \quad y = 5+s, \quad z = 6-5s. \tag{6}$$

Hence, the direction vector is

$$\mathbf{m_2} = \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix} \tag{7}$$

The lines are perpendicular when

$$\mathbf{m_1}^\top \mathbf{m_2} = 0 \tag{8}$$

Substituting from (4) and (7),

$$\begin{pmatrix} -3 & p & 2 \end{pmatrix} \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix} = 0 \tag{9}$$

$$(-3)(-3p) + p \cdot 1 + 2(-5) = 0 \tag{10}$$

$$9p + p - 10 = 0 \tag{11}$$

$$10p - 10 = 0 \tag{12}$$

$$\implies p = 1 \tag{13}$$

Therefore, the required value is

$$\boxed{p = 1}$$

The plot of the two lines are show in the plot below :

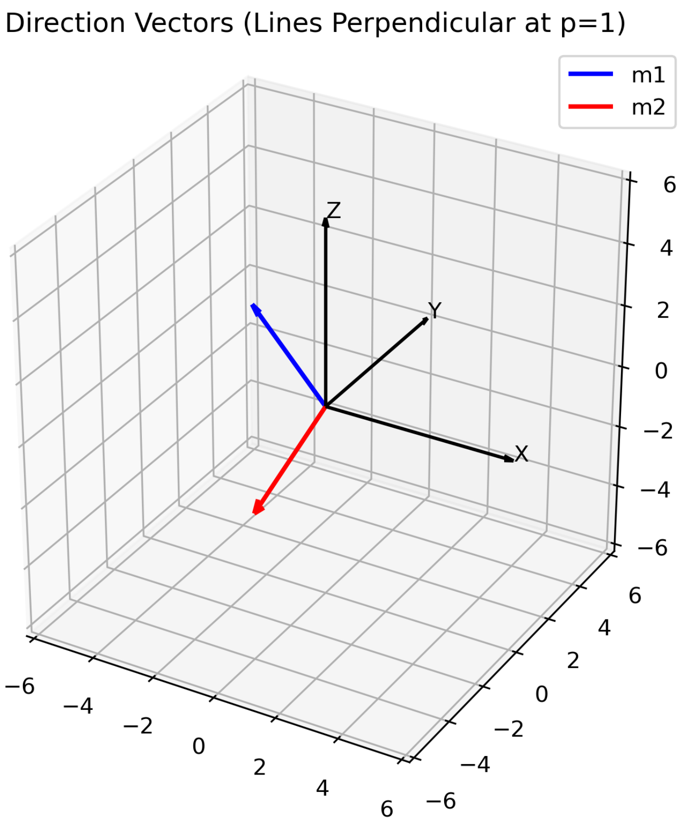

Fig. 1