2.9.19

AI25BTECH11003 - Bhavesh Gaikwad

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Question

Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$, $|\overrightarrow{b}| = 2$, and $|\overrightarrow{c}| = 3$. If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$, and $\overrightarrow{b}$ and $\overrightarrow{c}$ are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$.

Theoretical Solution

Given:

$$\|\mathbf{a}\| = 1,\ \|\mathbf{b}\| = 2,\ \|\mathbf{c}\| = 3 \tag{1}$$

$$\text{The projection of } \mathbf{b} \text{ along } \mathbf{a} = \mathbf{b}^T \frac{\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a} \tag{2}$$

$$\text{The projection of } \mathbf{c} \text{ along } \mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a} \tag{3}$$

$$\mathbf{b}^T \frac{\mathbf{a}}{\|\mathbf{a}\|}\mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{\|\mathbf{a}\|}\mathbf{a} \tag{4}$$

$$\text{Since, } \ \|\mathbf{a}\| = 1 \Rightarrow \quad \therefore \ \mathbf{b}^T\mathbf{a} = \mathbf{c}^T\mathbf{a} \tag{5}$$

Since **b** and **c** are perpendicular:

$$\mathbf{b}^T\mathbf{c} = 0 \tag{6}$$

Theoretical Solution

$$\text{Let } \mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c} \tag{7}$$

$$\|\mathbf{v}\|^2 = (3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})^T(3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}) \tag{8}$$

$$\|\mathbf{v}\|^2 = 9(\mathbf{a}^T\mathbf{a}) - 6(\mathbf{a}^T\mathbf{b}) + 6(\mathbf{a}^T\mathbf{c}) - 6(\mathbf{b}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) - 4(\mathbf{b}^T\mathbf{c}) + 6(\mathbf{c}^T\mathbf{a}) - 4(\mathbf{c}^T\mathbf{b} \tag{9}$$

$$\text{Since } \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \;\&\; \mathbf{a}^T\mathbf{c} = \mathbf{c}^T\mathbf{a} \tag{10}$$

$$\|\mathbf{v}\|^2 = 9(\mathbf{a}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) + 4(\mathbf{c}^T\mathbf{c}) - 12(\mathbf{a}^T\mathbf{b}) + 12(\mathbf{a}^T\mathbf{c}) - 8(\mathbf{b}^T\mathbf{c}) \tag{11}$$

Theoretical Solution

From Equation 1 & 6,

$$\mathbf{a}^T\mathbf{a} = \|\mathbf{a}\|^2 = 1,\ \mathbf{b}^T\mathbf{b} = \|\mathbf{b}\|^2 = 4,\ \mathbf{c}^T\mathbf{c} = \|\mathbf{c}\|^2 = 9,\ \mathbf{b}^T\mathbf{c} = 0 \tag{12}$$

$$\|\mathbf{v}\|^2 = 9 + 16 + 36 \tag{13}$$

$$\|\mathbf{v}\|^2 = 61 \quad \Rightarrow \quad \|\mathbf{v}\| = \sqrt{61} \tag{14}$$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}} \tag{15}$$

Python + C Code

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "libs/geofun.h"
#include "libs/matfun.h"

// Function to save vectors to file
void saveVectorsToFile(double **a, double **b, double **c, double
    **result, const char *filename) {
    FILE *fp = fopen(filename, "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return;
    }
```

Python + C Code

```
    fprintf(fp, "# Vector data for the problem\n");
    fprintf(fp, "# Format: vector_name x y z\n");
    fprintf(fp, "a %.6f %.6f %.6f\n", a[0][0], a[1][0], a[2][0]);
    fprintf(fp, "b %.6f %.6f %.6f\n", b[0][0], b[1][0], b[2][0]);
    fprintf(fp, "c %.6f %.6f %.6f\n", c[0][0], c[1][0], c[2][0]);
    fprintf(fp, "result %.6f %.6f %.6f\n", result[0][0], result
        [1][0], result[2][0]);
    fprintf(fp, "# Magnitude of result vector: %.6f\n", Matnorm(
        result, 3));

    fclose(fp);
}

// Function to compute 3a - 2b + 2c
double **computeResult(double **a, double **b, double **c) {
    double **scaled_a = Matscale(a, 3, 1, 3.0);
    double **scaled_b = Matscale(b, 3, 1, -2.0);
    double **scaled_c = Matscale(c, 3, 1, 2.0);
```

```c
      double **temp = Matadd(scaled_a, scaled_b, 3, 1);
    double **result = Matadd(temp, scaled_c, 3, 1);

    freeMat(scaled_a, 3);
    freeMat(scaled_b, 3);
    freeMat(scaled_c, 3);
    freeMat(temp, 3);

    return result;
}

// Function exported for Python to use
double compute_vector_magnitude(double *a, double *b, double *c)
    {
    // Create matrix representations
    double **vec_a = createMat(3, 1);
    double **vec_b = createMat(3, 1);
    double **vec_c = createMat(3, 1);
```

Python + C Code

```c
    for (int i = 0; i < 3; i++) {
        vec_a[i][0] = a[i];
        vec_b[i][0] = b[i];
        vec_c[i][0] = c[i];
    }

    double **result = computeResult(vec_a, vec_b, vec_c);
    double magnitude = Matnorm(result, 3);

    freeMat(vec_a, 3);
    freeMat(vec_b, 3);
    freeMat(vec_c, 3);
    freeMat(result, 3);

    return magnitude;
}
```

Python + C Code

```c
int main() {
    printf("Solving: Find |3a - 2b + 2c| where |a|=1, |b|=2, |c|=3\n");
    printf("Conditions: proj_a(b) = proj_a(c) and b L c\n\n");

    // Create specific vectors that satisfy the given conditions
    // Let a = (1, 0, 0) - unit vector along x-axis
    double **a = createMat(3, 1);
    a[0][0] = 1.0; a[1][0] = 0.0; a[2][0] = 0.0;

    // For b and c to have equal projections along a and be perpendicular:
    // b*a = c*a and b*c = 0
    // Choose vectors that satisfy these conditions properly:

    double k = 0.5; // projection value
```

Python + C Code

```
 // b = (k, sqrt(4-k^2), 0)
double **b = createMat(3, 1);
b[0][0] = k;
b[1][0] = sqrt(4 - k*k);
b[2][0] = 0.0;

// c = (k, m, n) where b*c = 0 and |c| = 3
// From b*c = 0: k*k + m*sqrt(4-k^2) = 0
// So m = -k*k/sqrt(4-k^2)
double **c = createMat(3, 1);
c[0][0] = k;
c[1][0] = -k*k / sqrt(4 - k*k);

// n^2 = 9 - k^2 - m^2
double m = c[1][0];
double n_squared = 9 - k*k - m*m;
c[2][0] = sqrt(n_squared);
```

Python + C Code

```
        printf("Vector a: ");
    printMat(a, 3, 1);
    printf("Magnitude of a: %.6f\n\n", Matnorm(a, 3));

    printf("Vector b: ");
    printMat(b, 3, 1);
    printf("Magnitude of b: %.6f\n\n", Matnorm(b, 3));

    printf("Vector c: ");
    printMat(c, 3, 1);
    printf("Magnitude of c: %.6f\n\n", Matnorm(c, 3));

    // Verify conditions
    printf("Verification of conditions:\n");
    printf("b*a = %.6f\n", Matdot(b, a, 3));
    printf("c*a = %.6f\n", Matdot(c, a, 3));
    printf("b*c = %.6f\n", Matdot(b, c, 3));
    printf("Equal projections: %s\n", (fabs(Matdot(b, a, 3) -
```

```c
Matdot(c, a, 3)) < 1e-10) ? "Yes" : "No");
  printf("Perpendicular: %s\n", (fabs(Matdot(b, c, 3)) < 1e-10)
       ? "Yes" : "No");
  printf("\n");

  // Compute 3a - 2b + 2c
  double **result = computeResult(a, b, c);

  printf("Result vector (3a - 2b + 2c): ");
  printMat(result, 3, 1);

  double magnitude = Matnorm(result, 3);
  printf("Magnitude |3a - 2b + 2c| = %.6f\n", magnitude);
  printf("Theoretical value = sqrt(61) = %.6f\n", sqrt(61));
  printf("Error: %.10f\n\n", fabs(magnitude - sqrt(61)));
```

Python + C Code

```c
    // Save vectors to file
    saveVectorsToFile(a, b, c, result, "vectors.dat");
    printf("Vectors saved to vectors.dat\n");

    // Clean up
    freeMat(a, 3);
    freeMat(b, 3);
    freeMat(c, 3);
    freeMat(result, 3);

    return 0;
}
```

Python + C Code

```python
#!/usr/bin/env python3

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import ctypes
import os
import sys

def load_vectors_from_file(filename):
    """Load vectors from the data file created by C program"""
    vectors = {}
    try:
        with open(filename, 'r') as f:
            lines = f.readlines()
        for line in lines:
            line = line.strip()
            if line.startswith('#') or not line:
```

Python + C Code

```
continue
            parts = line.split()
            if len(parts) >= 4:
                name = parts[0]
                x, y, z = float(parts[1]), float(parts[2]), float(
                    parts[3])
                vectors[name] = np.array([x, y, z])
        return vectors
    except FileNotFoundError:
        print(f"Error: {filename} not found. Please run the C
            program first.")
        return None
    except Exception as e:
        print(f"Error reading file: {e}")
        return None
```

Python + C Code

```python
def load_shared_library():
    """Load the shared library created by C program"""
    try:
        lib_path = "./vectors.so"
        if not os.path.exists(lib_path):
            print("Error: vectors.so not found. Please compile the C program first.")
            print("Run: gcc -fPIC -shared -o vectors.so vector_solution.c -lm")
            return None
        lib = ctypes.CDLL(lib_path)
        lib.compute_vector_magnitude.argtypes = [
            ctypes.POINTER(ctypes.c_double),
            ctypes.POINTER(ctypes.c_double),
            ctypes.POINTER(ctypes.c_double)
        ]
        lib.compute_vector_magnitude.restype = ctypes.c_double
        return lib
```

```
    except Exception as e:
        print(f"Error loading shared library: {e}")
        return None

def create_vector_visualization(vectors):
    """Create 3D visualization of the vectors"""
    fig = plt.figure(figsize=(12, 9))
    ax = fig.add_subplot(111, projection='3d')

    # Colors for different vectors
    colors = {'a': 'red', 'b': 'blue', 'c': 'green', 'result': '
        purple'}
    labels = {'a': 'Vector a', 'b': 'Vector b', 'c': 'Vector c',
        'result': '3a - 2b + 2c'}
    origin = np.array([0, 0, 0])
```

Python + C Code

```
    # Plot vectors
for name, vector in vectors.items():
    if name in colors:
        ax.quiver(origin[0], origin[1], origin[2],
                  vector[0], vector[1], vector[2],
                  color=colors[name], arrow_length_ratio=0.1,
                  linewidth=3, label=labels[name])
        ax.text(vector[0], vector[1], vector[2],
                f'{name}({vector[0]:.2f}, {vector[1]:.2f}, {
                    vector[2]:.2f})',
                fontsize=9)

# Set equal aspect ratio and labels
max_range = max([np.linalg.norm(v) for v in vectors.values()
    ]) * 1.2
ax.set_xlim([-max_range/2, max_range/2])
ax.set_ylim([-max_range/2, max_range/2])
ax.set_zlim([0, max_range])
```

```
        ax.set_xlabel('X axis')
    ax.set_ylabel('Y axis')
    ax.set_zlabel('Z axis')

    # Remove legend
    # ax.legend()

    ax.grid(True)

    # Add title as requested
    plt.title('Vectors a, b and c', fontsize=16, fontweight='bold
        ')

    # Save with requested filename, without bottom-left info box
    plt.tight_layout()
    plt.savefig('fig1.png', dpi=300, bbox_inches='tight')
    plt.show()
```

Python + C Code

```python
 def verify_calculations_with_library(vectors, lib):
    """Verify calculations using the C shared library"""
    if lib is None:
        print("Shared library not available, skipping verification.")
        return
    try:
        a_array = (ctypes.c_double * 3)(*vectors['a'])
        b_array = (ctypes.c_double * 3)(*vectors['b'])
        c_array = (ctypes.c_double * 3)(*vectors['c'])
        magnitude_from_c = lib.compute_vector_magnitude(a_array, b_array, c_array)
        print(f"Magnitude from C library: {magnitude_from_c:.6f}")
        print(f"Magnitude from Python: {np.linalg.norm(vectors['result']):.6f}")
        print(f"Theoretical value (root61): {np.sqrt(61):.6f}")
```

Python + C Code

```python
        except Exception as e:
            print(f"Error calling C function: {e}")

def main():
    print("Loading vectors from vectors.dat...")
    vectors = load_vectors_from_file("vectors.dat")
    if vectors is None:
        print("Please ensure you have run the C program first to
            generate vectors.dat")
        return

    lib = load_shared_library()
    verify_calculations_with_library(vectors, lib)
    create_vector_visualization(vectors)

if __name__ == "__main__":
    main()
```

Vector Representation