Matrices in Geometry - 1.9.27

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Problem Statement

Find the value of P, if the point **A** $(0, 2)$ is equidistant from point **B** $(3, P)$ and **c** $(p, 5)$

Solution

Given: $\mathbf{A}\begin{pmatrix}0\\2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}3\\P\end{pmatrix}$ and a point $\mathbf{C}\begin{pmatrix}P\\5\end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \qquad (1)$$

$$\text{On squaring both the sides, we get} \qquad (2)$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \qquad (3)$$

$$\left(\mathbf{A} - \mathbf{B}\right)^\top \left(\mathbf{A} - \mathbf{B}\right) = \left(\mathbf{A} - \mathbf{C}\right)^\top \left(\mathbf{A} - \mathbf{C}\right) \qquad (4)$$

$$(5)$$

Solution

$$\mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{B} = \mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{C} + \mathbf{C}^\top \mathbf{C} \tag{6}$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{A}^\top \left(\mathbf{B} - \mathbf{C}\right) \tag{7}$$

$$\left\| \begin{pmatrix} 3 \\ P \end{pmatrix} \right\| - \left\| \begin{pmatrix} P \\ 5 \end{pmatrix} \right\| = 2 \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 3 - P \\ P - 5 \end{pmatrix} \tag{8}$$

$$9 + p^2 - p^2 - 25 = 2\left(0 + 2p - 10\right) \tag{9}$$

$$-16 = 4p - 20 \implies 4p = 4 \implies p = 1 \tag{10}$$

Final Answer

Hence, the final answer is $\boxed{p = 1}$ (11)

Points A, B, C with A Equidistant from B and C