

2.10.52

EE25BTECH11023 - Venkata Sai

Question:

Let $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. A vector in the plane of \mathbf{a} and \mathbf{b} whose projection on \mathbf{c} is $\frac{1}{\sqrt{3}}$, is

1) $4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

2) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

3) $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

4) $4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Solution:

Given

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (1)$$

Let \mathbf{r} be coplanar to \mathbf{a} and \mathbf{b}

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \quad (2)$$

Given the projection of \mathbf{r} on \mathbf{c} is $\frac{1}{\sqrt{3}}$

$$\frac{|\mathbf{r}^\top \mathbf{c}|}{\|\mathbf{c}\|} = \frac{1}{\sqrt{3}} \quad (3)$$

$$\Rightarrow \mathbf{r}^\top \mathbf{c} = \pm \frac{\|\mathbf{c}\|}{\sqrt{3}} \quad (4)$$

$$\mathbf{r}^\top \mathbf{c} = (\mathbf{a} + t\mathbf{b})^\top \mathbf{c} \quad (5)$$

$$= (\mathbf{a}^\top + t\mathbf{b}^\top) \mathbf{c} \quad (6)$$

$$= \mathbf{a}^\top \mathbf{c} + t(\mathbf{b}^\top \mathbf{c}) \quad (7)$$

$$\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c} = t(\mathbf{b}^\top \mathbf{c}) \quad (8)$$

$$\Rightarrow t = \frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} \quad (9)$$

From (2)

$$\mathbf{r} = \mathbf{a} + \left(\frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} \right) \mathbf{b} \quad (10)$$

$$\mathbf{r} = \mathbf{a} + \left(\frac{\pm \frac{\|\mathbf{c}\|}{\sqrt{3}} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} \right) \mathbf{b} \quad (11)$$

$$\|\mathbf{c}\|^2 = \mathbf{c}^T \mathbf{c} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (12)$$

$$= 1 + 1 + 1 = 3 \implies \|\mathbf{c}\| = \sqrt{3} \quad (13)$$

$$\mathbf{a}^T \mathbf{c} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 1 + 2 - 1 = 2 \quad (14)$$

$$\mathbf{b}^T \mathbf{c} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 1 - 1 - 1 = -1 \quad (15)$$

$$(16)$$

Substitute (13),(14),(15) in (11)

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} \pm 1 - 2 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (17)$$

$$\implies \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \implies \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \text{ or } \begin{pmatrix} 4 \\ -1 \\ 4 \end{pmatrix} \quad (18)$$

Hence Option(1) is the correct answer

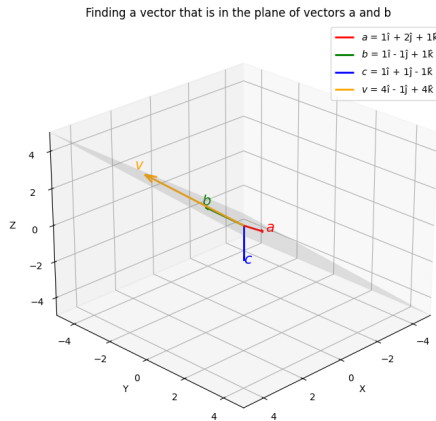


Fig. 4.1