AI25BTECH11021 - Abhiram Reddy N

Question:

For what value of p are the points (2, 1), (p, -1), and (-1, 3) collinear?

Solution

The points A, B, C are collinear if the vectors \overrightarrow{AB} and \overrightarrow{AC} are linearly dependent. This means one is a scalar multiple of the other.

We form the vectors:

$$\overrightarrow{AB} = (p-2, -1-1) = (p-2, -2)$$

 $\overrightarrow{AC} = (-1-2, 3-1) = (-3, 2)$

Create the matrix with these vectors as rows:

$$M = \begin{bmatrix} p-2 & -2 \\ -3 & 2 \end{bmatrix}$$

Perform row operations to put the matrix into echelon form.

$$R_1 = [p-2 -2]$$

Eliminate the first element of the second row:

$$R_2 \to R_2 + \frac{3}{p-2}R_1$$

$$R_2 = [-3, 2] + \frac{3}{p-2}[p-2, -2] = [-3+3, \quad 2 - \frac{6}{p-2}] = [0, \quad 2 - \frac{6}{p-2}]$$

For the vectors to be linearly dependent, the second row must be zero:

$$2 - \frac{6}{p - 2} = 0$$

Multiply both sides by p-2:

$$2(p-2) - 6 = 0$$
$$2p - 4 - 6 = 0$$
$$2p - 10 = 0$$
$$p = 5$$

VERIFICATION BY PLOTTING

The points become:

$$A = (2, 1), \quad B = (5, -1), \quad C = (-1, 3)$$

These points lie on the same straight line.

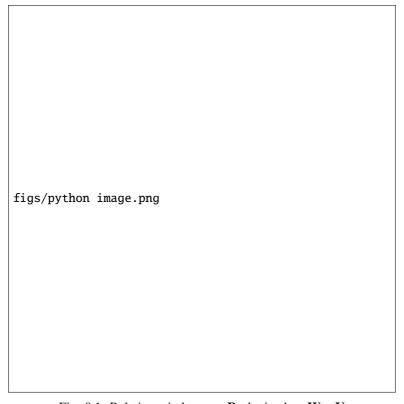


Fig. 0.1: Relative wind vector \mathbf{R} obtained as $\mathbf{W} - \mathbf{V}$