MatGeo Assignment 1.11.9

AI25BTECH11007

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Question

If

$$\mathbf{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{i} - 2\hat{j} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.

Solution

A vector perpendicular to both **a** and **b** is given by

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}.$$

$$\mathbf{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}.$$

Expanding,

$$\mathbf{n} = \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}.$$

The magnitude is

$$|\mathbf{n}| = \sqrt{0^2 + 19^2 + 19^2} = 19\sqrt{2}.$$
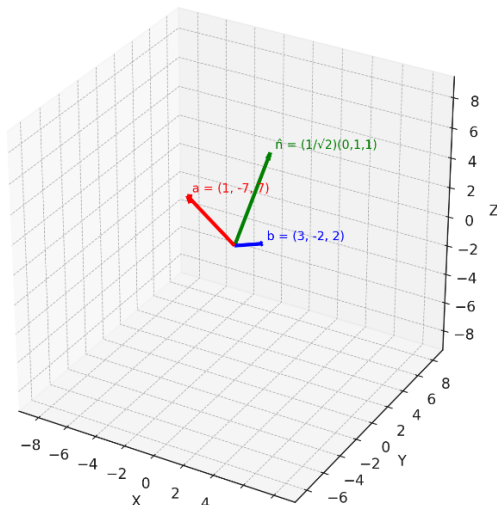
Hence the required unit vector is

$$\hat{n} = \frac{\mathbf{n}}{|\mathbf{n}|} = \frac{1}{19\sqrt{2}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

Therefore, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}),$$

or its negative.

Vectors a (red), b (blue), and unit normal n̂ (green)

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