

Matrices in Geometry - 1.9.26

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Aug, 2025

Problem Statement

Find the value of P , if the point $\mathbf{A}(0, 2)$ is equidistant from point $\mathbf{B}(3, P)$ and $\mathbf{C}(p, 5)$

Solution

Given: $\mathbf{A} \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} 3 \\ P \end{pmatrix}$ and a point $\mathbf{C} \begin{pmatrix} P \\ 5 \end{pmatrix}$ such that \mathbf{P} is equidistant from \mathbf{A} and \mathbf{B} .

$$\therefore \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (1)$$

On squaring both the sides, we get (2)

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \quad (3)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{C})^\top (\mathbf{A} - \mathbf{C}) \quad (4)$$

$$(5)$$

Solution

$$\mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{B} = \mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{C} + \mathbf{C}^\top \mathbf{C} \quad (6)$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{A}^\top (\mathbf{B} - \mathbf{C}) \quad (7)$$

$$\left\| \begin{pmatrix} 3 \\ P \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} P \\ 5 \end{pmatrix} \right\|^2 = 2 \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 3 - P \\ P - 5 \end{pmatrix} \quad (8)$$

$$9 + P^2 - P^2 - 25 = 2(0 + 2P - 10) \quad (9)$$

$$-16 = 4P - 20 \implies 4P = 4 \implies P = 1 \quad (10)$$

Final Answer

Hence, the final answer is $p = 1$

(11)

figs/1.jpg