Question-1.9.11

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Question

if the distance between the points $(k, -2)$ and $(3, -6)$ is 10 units, find the positive value of k.

Solution

Let the given points be

$$\mathbf{A} = \begin{pmatrix} k \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \tag{1}$$

The direction vector of the segment joining A and B is given by:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-k \\ -6-(-2) \end{pmatrix} = \begin{pmatrix} 3-k \\ -4 \end{pmatrix} \tag{2}$$

The length of the segment is the magnitude of the direction vector:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-k \\ -6-(-2) \end{pmatrix} = \begin{pmatrix} 3-k \\ -4 \end{pmatrix} \tag{3}$$

The distance between points $\mathbf{A}$ and $\mathbf{B}$ is given as, $\mathrm{d} = \|\mathbf{B} - \mathbf{A}\| = 10$

Solution

$$\|\mathbf{B}-\mathbf{A}\| = \sqrt{(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{B}-\mathbf{A})} \tag{4}$$

$$(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{B}-\mathbf{A}) = \|\mathbf{B}-\mathbf{A}\|^2 (\mathbf{B}-\mathbf{A})^{\top}(\mathbf{B}-\mathbf{A}) = (10)^2 \tag{5}$$

$$100 = \begin{pmatrix} 3-k & -4 \end{pmatrix} \begin{pmatrix} 3-k \\ -4 \end{pmatrix} \tag{6}$$

$$100 = (3-k)\times(3-k) + (-4)\times(-4) \tag{7}$$

$$100 = (3-k)^2 + 16 \tag{8}$$

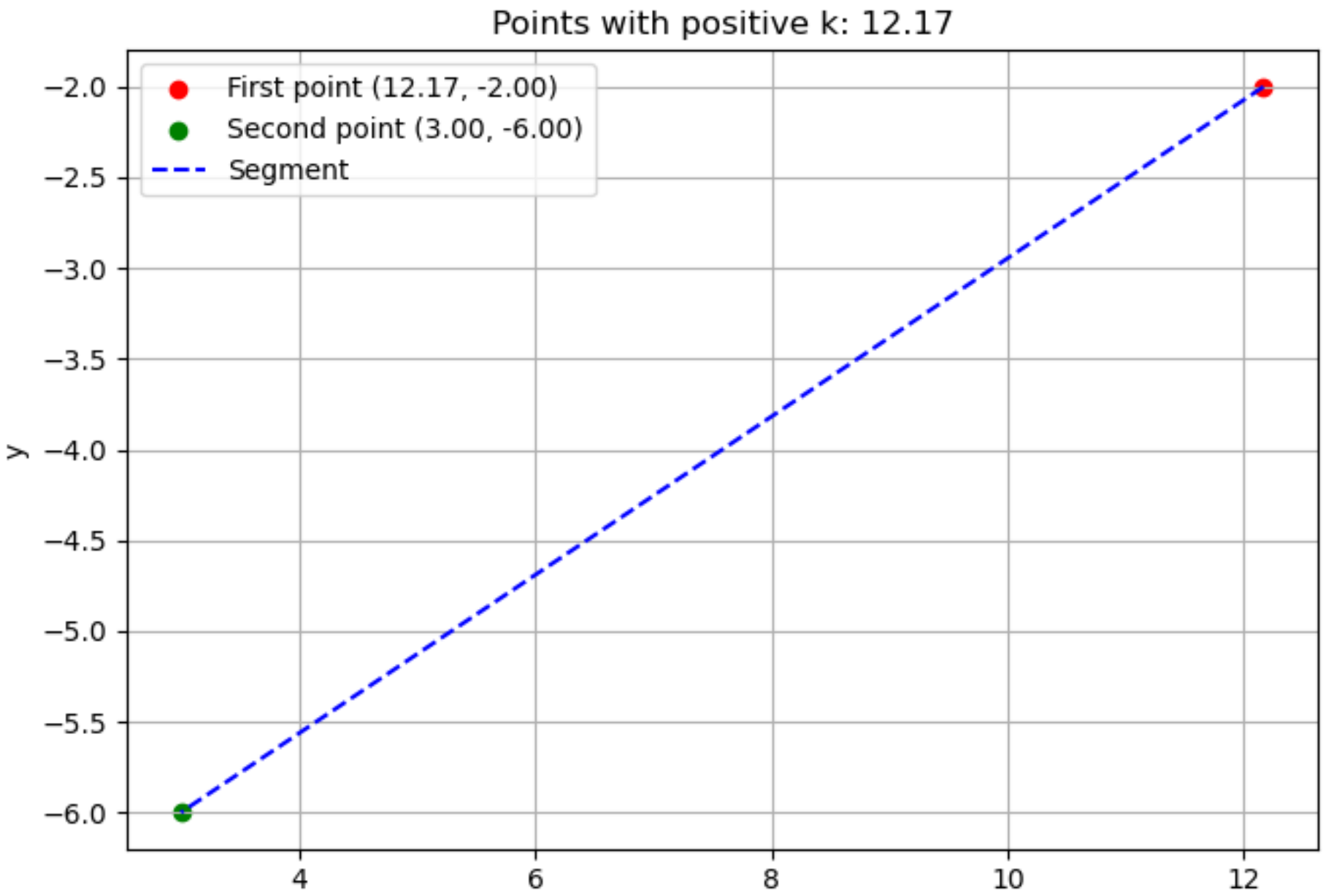
$$(3-k)^2 = 84 \tag{9}$$

$$3-k = \pm\sqrt{84} \tag{10}$$

$$k = 3+\sqrt{84}, 3-\sqrt{84} \tag{11}$$

Therefore, the positive value of k is $3+\sqrt{84} \approx 12.17$

graph


Points with positive k: 12.17
First point (12.17, -2.00)
Second point (3.00, -6.00)
Segment
y
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
4
6
8
10
12

C-Code

```c
#include <stdio.h>
#include <math.h>

// Fills k and both point arrays.
void find_k_and_points(double *k, double pt1[2], double pt2[2]) {
    *k = 3 + sqrt(84);
    pt1[0] = *k; pt1[1] = -2;
    pt2[0] = 3; pt2[1] = -6;
}

int main() {
    double k, pt1[2], pt2[2];
    find_k_and_points(&k, pt1, pt2);
    printf("Positive value of k: %.6f\n", k);
    printf("First point: (%.6f, %.6f)\n", pt1[0], pt1[1]);
    printf("Second point: (%.6f, %.6f)\n", pt2[0], pt2[1]);
    return 0;
}
```

Python-Code

```
# Code by GVV Sharma
# Modified for Problem Solution
# Released under GNU GPL
# Calculating area enclosed between curves
import ctypes
import numpy as np

# Load compiled shared library
lib = ctypes.CDLL('./code.so')

# Set function argument and return types
lib.find_k_and_points.argtypes = [
    ctypes.POINTER(ctypes.c_double), # double *k
    np.ctypeslib.ndpointer(dtype=np.double, ndim=1, flags='
        C_CONTIGUOUS'), # pt1[2]
```

Python-Code

```
np.ctypeslib.ndpointer(dtype=np.double, ndim=1, flags='
    C_CONTIGUOUS') # pt2[2]
]
lib.find_k_and_points.restype = None

def get_points():
    k = ctypes.c_double()
    pt1 = np.zeros(2, dtype=np.double)
    pt2 = np.zeros(2, dtype=np.double)
    lib.find_k_and_points(ctypes.byref(k), pt1, pt2)
    return k.value, pt1, pt2
```

Python-Code

```
import matplotlib.pyplot as plt
import numpy as np
from call import get_points

k, pt1, pt2 = get_points()

plt.figure(figsize=(7,5))
plt.scatter(pt1[0], pt1[1], color='red', label=f'First point ({
    pt1[0]:.2f}, {pt1[1]:.2f})')
plt.scatter(pt2[0], pt2[1], color='green', label=f'Second point
    ({pt2[0]:.2f}, {pt2[1]:.2f})')
```

Python-Code

```
plt.plot([pt1[0], pt2[0]], [pt1[1], pt2[1]], 'b--', label='
    Segment')
plt.title(f'Points with positive k: {k:.2f}')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc='best')
plt.grid(True)
plt.tight_layout()
plt.show()
```