

1.7.9

AI25BTECH11023 - Pratik R

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Question:

If the points $\vec{A} = (k + 1, 2k)$, $\vec{B} = (3k, 2k + 3)$, and $\vec{C} = (5k - 1, 5k)$ are collinear, then find the value of k .

Solution:

Given that

$$\vec{A} = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}, \vec{B} = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}, \vec{C} = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix}.$$

\vec{A} , \vec{B} and \vec{C} are collinear therefore

$$\text{rank} \begin{pmatrix} \vec{B} - \vec{A} & \vec{C} - \vec{A} \end{pmatrix} = 1 \quad (0.1)$$

$$(\vec{B} - \vec{A}) = \begin{pmatrix} 2k-1 \\ 3 \end{pmatrix} \quad (0.2)$$

$$(\vec{C} - \vec{A}) = \begin{pmatrix} 4k-2 \\ 3k \end{pmatrix} \quad (0.3)$$

$$\begin{pmatrix} \vec{B} - \vec{A} & \vec{C} - \vec{A} \end{pmatrix}^T = \begin{pmatrix} 2k - 1 & 3 \\ 4k - 2 & 3k \end{pmatrix} \quad (0.4)$$

$R_2 \rightarrow R_2 - (R_1 \times 2) :$

$$= \begin{pmatrix} 2k - 1 & 3 \\ 0 & 3k - 6 \end{pmatrix} \quad (0.5)$$

For rank = 1,

$$3k - 6 = 0 \implies k = 2 \quad (0.6)$$

Hence , the value of k is 2.

A, B and C are collinear

