3.2.4

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Question:
Construct the triangle BD′C′ similar to △BDC with scale factor $\frac{4}{3}$.Draw the line segment D′A′ parallel to DA where A^{p}rime lies on extended side BA.Is A′BC′D′ a parallelogram?
solution

Vector	Name
$\begin{pmatrix}0\\0\end{pmatrix}$	Vector **A**
$\begin{pmatrix}4\\0\end{pmatrix}$	Vector **B**
$\begin{pmatrix}4\\3\end{pmatrix}$	Vector **C**
$\begin{pmatrix}0\\3\end{pmatrix}$	Vector **D**

TABLE 0: Variables Used

consider △BDC.constructs a △BD′C' with scale factor $\frac{4}{3}$.
This means

$$\triangle BD'C' \sim \triangle BDC. \tag{1}$$

$$\frac{\|\mathbf{D}' - \mathbf{B}\|}{\|\mathbf{D} - \mathbf{B}\|} = \frac{\|\mathbf{C}' - \mathbf{B}\|}{\|\mathbf{C} - \mathbf{B}\|} = \frac{\|\mathbf{C}' - \mathbf{D}'\|}{\|\mathbf{C} - \mathbf{D}\|} = \frac{4}{3}. \tag{2}$$

$$\mathbf{D}' = \mathbf{B} + \frac{4}{3}(\mathbf{D} - \mathbf{B}) \tag{3}$$

$$\mathbf{D}' = \begin{pmatrix}-4/3\\4\end{pmatrix} \tag{4}$$

$$\mathbf{C}' = \mathbf{B} + \frac{4}{3}(\mathbf{C} - \mathbf{B}) \tag{5}$$

$$\mathbf{C}' = \begin{pmatrix}4\\4\end{pmatrix} \tag{6}$$

Construct A′
Mark **D′** and **A′** parallel to **D** − **A** with **A′** along the direction of **B** − **A**.

$$\mathbf{A}' - \mathbf{D}' = \lambda(\mathbf{A} - \mathbf{D}) \tag{7}$$

$$\mathbf{A}' = \begin{pmatrix}-4/3\\4\end{pmatrix} + \lambda(\begin{pmatrix}0\\-3\end{pmatrix}) \tag{8}$$

$$\implies \mathbf{A}' = \begin{pmatrix}-4/3\\4 - 3\lambda\end{pmatrix} \tag{9}$$

A′ lies on line through **B** − **A** so,

$$\mathbf{A}' = \mathbf{B} + \mu(\mathbf{A} - \mathbf{B}) \tag{10}$$

$$\implies \mathbf{A}' = \begin{pmatrix}-4\mu\\0\end{pmatrix} \tag{11}$$

$$\tag{12}$$

From equation (9) and (11)

$$\lambda = 4/3 \text{and} \mu = 1/3 \tag{13}$$

$$\implies \mathbf{A}' = \begin{pmatrix}-4/3\\0\end{pmatrix} \tag{14}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} \tag{15}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} \tag{16}$$

$$\implies \mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{17}$$

Check the parallelogram property of A′BC′D′

$$\mathbf{B} - \mathbf{A}' = -t(\mathbf{A} - \mathbf{B}) \tag{18}$$
$$\mathbf{D}' - \mathbf{C}' = k(\mathbf{C} - \mathbf{D}) \tag{19}$$
$$\text{From Equation (17) } \mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{20}$$
$$\implies \mathbf{B} - \mathbf{A}' = -t(\mathbf{A} - \mathbf{B}) = t(\mathbf{C} - \mathbf{D}) = \frac{t}{k}\mathbf{D}' - \mathbf{C}' \tag{21}$$
$$\implies \mathbf{B} - \mathbf{A}' \parallel \mathbf{D}' - \mathbf{C}' \tag{22}$$

By construction of A′

$$\mathbf{D}' - \mathbf{A}' \parallel \mathbf{D} - \mathbf{A} \tag{23}$$
$$\mathbf{D} - \mathbf{A} \parallel \mathbf{C} - \mathbf{B} \tag{24}$$
$$\mathbf{C} - \mathbf{B} \parallel \mathbf{C}' - \mathbf{B} \tag{25}$$
$$\implies \mathbf{D}' - \mathbf{A}' \parallel \mathbf{C}' - \mathbf{B} \tag{26}$$

$\implies$ A′BC′D′ a parallelogram