

1.2.29

AI25BTECH11021 - Abhiram Reddy N

Question:

For what value of p are the points $(2, 1)$, $(p, -1)$, and $(-1, 3)$ collinear?

Solution

The points A, B, C are collinear if the vectors \overrightarrow{AB} and \overrightarrow{AC} are linearly dependent. This means one is a scalar multiple of the other.

We form the vectors:

$$\overrightarrow{AB} = (p - 2, -1 - 1) = (p - 2, -2)$$

$$\overrightarrow{AC} = (-1 - 2, 3 - 1) = (-3, 2)$$

Create the matrix with these vectors as rows:

$$M = \begin{bmatrix} p - 2 & -2 \\ -3 & 2 \end{bmatrix}$$

Perform row operations to put the matrix into echelon form.

$$R_1 = [p - 2 \quad -2]$$

Eliminate the first element of the second row:

$$R_2 \rightarrow R_2 + \frac{3}{p - 2} R_1$$

$$R_2 = [-3, 2] + \frac{3}{p - 2} [p - 2, -2] = [-3 + 3, \quad 2 - \frac{6}{p - 2}] = [0, \quad 2 - \frac{6}{p - 2}]$$

For the vectors to be linearly dependent, the second row must be zero:

$$2 - \frac{6}{p - 2} = 0$$

Multiply both sides by $p - 2$:

$$2(p - 2) - 6 = 0$$

$$2p - 4 - 6 = 0$$

$$2p - 10 = 0$$


$$\boxed{p = 5}$$

VERIFICATION BY PLOTTING

The points become:

$$A = (2, 1), \quad B = (5, -1), \quad C = (-1, 3)$$

These points lie on the same straight line.



`figs/python image.png`

Fig. 0.1: Relative wind vector \mathbf{R} obtained as $\mathbf{W} - \mathbf{V}$