

1.6.29

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QUESTION

Show that the points $A = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, and $C = \begin{pmatrix} 3 \\ 8 \\ -11 \end{pmatrix}$ are collinear.

SOLUTION

To prove that the points are collinear, we examine whether the vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ are linearly dependent.

Step 1: Define the Points

Let the position vectors be:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \quad (0.1)$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad (0.2)$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 8 \\ -11 \end{pmatrix} \quad (0.3)$$

Step 2: Compute Direction Vectors

We compute the vectors from point A to B and A to C:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix} \quad (0.4)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ 8 \\ -11 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} \quad (0.5)$$

Step 3: Form the Matrix

We now construct a matrix M whose rows are the vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$M = \begin{pmatrix} -1 & -5 & 7 \\ 1 & 5 & -7 \end{pmatrix} \quad (0.6)$$

Step 4: Rank via Echelon Form

To determine the rank of matrix M , we reduce it to ****echelon form**** using row operations.

First, swap R_1 and R_2 to make the leading entry positive:

$$\begin{pmatrix} 1 & 5 & -7 \\ -1 & -5 & 7 \end{pmatrix} \quad (0.7)$$

Next, eliminate the first entry of R_2 :

$$R_2 \rightarrow R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 5 & -7 \\ 0 & 0 & 0 \end{pmatrix} \quad (0.8)$$

Step 5: Echelon Matrix

The resulting ****echelon form**** of matrix M is:

$$E = \begin{pmatrix} 1 & 5 & -7 \\ 0 & 0 & 0 \end{pmatrix} \quad (0.9)$$

Step 6: Rank of the Matrix

From equation (??), we observe that only one row is non-zero. Therefore, the rank of matrix M is:

$$\text{Rank}(M) = 1 \quad (0.10)$$

Step 7: Conclusion

Since the rank of the matrix formed by the vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ is 1, the vectors are linearly dependent. Hence, the points A , B , and C lie on the same line. **Therefore, the points are collinear.**

3D Plot of Points A, B, C with Labels

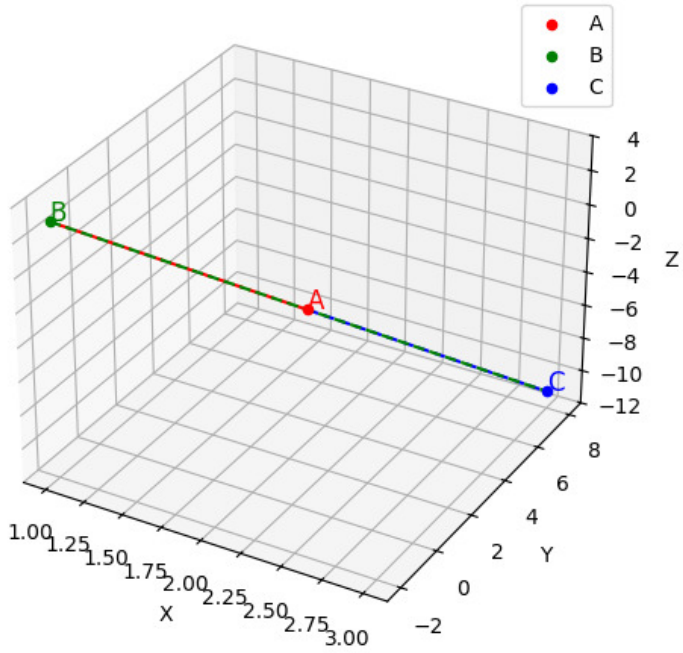


Fig. 0.1: 3D plot of points A, B, C showing collinearity