1.7.7

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Problem Statement

Find the value of p such that the points:

$$A(2,1), \quad B(p,-1), \quad C(-1,3)$$

are **collinear**, using matrices and echelon form.

Step 1: Vector Form

Two vectors from point A:

$$\overrightarrow{AB} = (p-2, -2), \qquad \overrightarrow{AC} = (-3, 2)$$

Construct a matrix:

$$M = p - 2 - 2 - 32$$

We will use row reduction (echelon form) to find when the rows are linearly dependent.

Step 2: Row Reduction

Let:

$$R_1 = [p-2 \quad -2], \quad R_2 = [-3 \quad 2]$$

Apply:

$$R_2 \to R_2 + \frac{3}{p-2} R_1$$

Compute:

$$R_2 = [0 \quad 2 - \frac{6}{p-2}]$$

Set the second row to zero:

$$2 - \frac{6}{p-2} = 0$$

$$2(p-2) = 6 \Rightarrow 2p - 4 = 6 \Rightarrow 2p = 10 \Rightarrow \boxed{p = 5}$$

Conclusion

Final Answer

$$\boxed{p = 5}$$

For $p = 5$, the points A, B, C are collinear since the row-reduced matrix becomes dependent (second row becomes zero).

C Code for echelon matrix

```c
#include <stdio.h>

void echelonForm(double matrix[2][2]) {
    // Assuming matrix is 2x2
    double factor;

    // Make the first element of second row zero by row operation
    if (matrix[0][0] == 0) {
        printf(Cannot perform elimination as pivot is zero.\n);
        return;
    }

    factor = matrix[1][0] / matrix[0][0];

    // Subtract factor * first row from second row
    matrix[1][0] = matrix[1][0] - factor * matrix[0][0];
    matrix[1][1] = matrix[1][1] - factor * matrix[0][1];
}
```

C Code for echelon matrix

```c
int main() {
    double p;
    printf(Enter value for p: );
    scanf(%lf, &p);

    // Create matrix with rows [p-2, -2] and [-3, 2]
    double matrix[2][2] = {
        {p - 2, -2},
        {-3, 2}
    };
```

C Code for echelon matrix

```
printf(Original matrix:\n);
for(int i=0; i<2; i++) {
    for(int j=0; j<2; j++) {
        printf(%8.3f , matrix[i][j]);
    }
    printf(\n);
}

echelonForm(matrix);

printf(\nMatrix after echelon form operation:\n);
for(int i=0; i<2; i++) {
    for(int j=0; j<2; j++) {
        printf(%8.3f , matrix[i][j]);
    }
    printf(\n);
}
```

Python Code for plot

```
import matplotlib.pyplot as plt

# Points
A = (2, 1)
B = (5, -1)
C = (-1, 3)

# Plot points
plt.scatter(*A, color='red', label='A(2,1)')
plt.scatter(*B, color='blue', label='B(5,-1)')
plt.scatter(*C, color='green', label='C(-1,3)')

# Plot line through A and C
x_values = [A[0], C[0]]
y_values = [A[1], C[1]]
plt.plot(x_values, y_values, 'k--', label='Line through A and C')
```

Python Code for plot

```
plt.legend()
plt.grid(True)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Collinear Points for p=5')

# Save the plot as an image file
plt.savefig('python_plot.png') # Saves to current directory

plt.show()
```

Plot

figs/python_plot.png