1.9.11

EE25BTECH11022 - sankeerthan

Question: if the distance between the points $(k, -2)$ and $(3, -6)$ is 10 units, find the positive value of k.

solution: Let the given points be

$$\mathbf{A} = \begin{pmatrix} k \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \tag{0.1}$$

The direction vector of the segment joining A and B is given by:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 - k \\ -6 - (-2) \end{pmatrix} = \begin{pmatrix} 3 - k \\ -4 \end{pmatrix} \tag{0.2}$$

The length of the segment is the magnitude of the direction vector:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 - k \\ -6 - (-2) \end{pmatrix} = \begin{pmatrix} 3 - k \\ -4 \end{pmatrix} \tag{0.3}$$

The distance between points **A** and **B** is given as, d = $\|\mathbf{B} - \mathbf{A}\|$ = 10

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})} \tag{0.4}$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = \|\mathbf{B} - \mathbf{A}\|^2 \, (\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = (10)^2 \tag{0.5}$$

$$100 = \begin{pmatrix} 3 - k & -4 \end{pmatrix} \begin{pmatrix} 3 - k \\ -4 \end{pmatrix} \tag{0.6}$$

$$100 = (3 - k) \times (3 - k) + (-4) \times (-4) \tag{0.7}$$

$$100 = (3 - k)^2 + 16 \tag{0.8}$$

$$(3 - k)^2 = 84 \tag{0.9}$$

$$3 - k = \pm \sqrt{84} \tag{0.10}$$

$$k = 3 + \sqrt{84}, 3 - \sqrt{84} \tag{0.11}$$
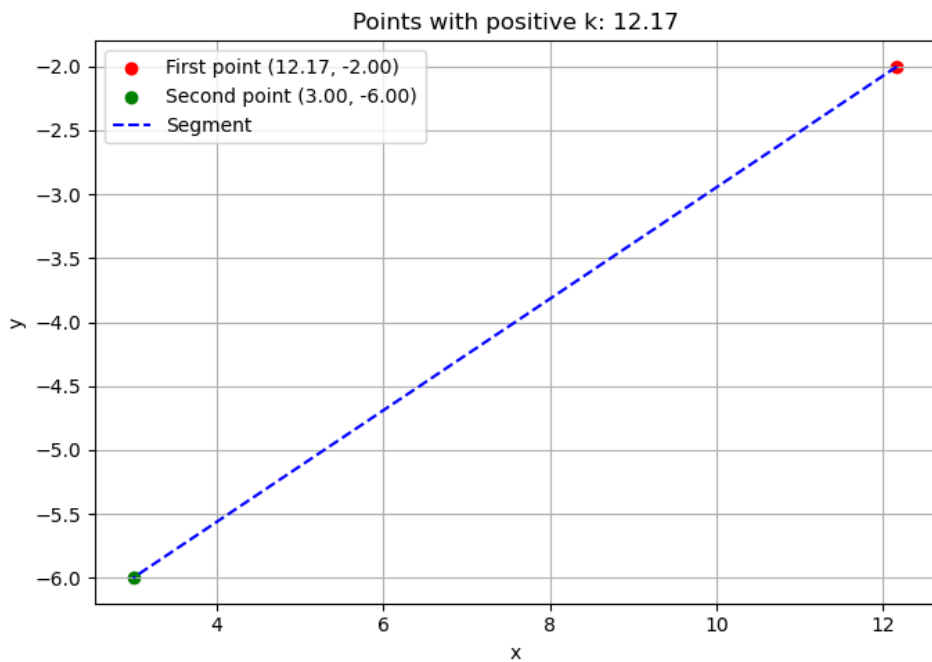
Therefore, the positive value of k is $3 + \sqrt{84} \approx 12.17$

Fig. 0.1: Plot of line segment **AB**