MatGeo Assignment 1.6.22

AI25BTECH11007

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Question

Show that the points A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2) are collinear.

Solution

Let

$$\mathbf{A} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 0 \\ \frac{1}{3} \\ 2 \end{pmatrix}. \tag{1}$$

Form the difference (direction) vectors

$$\overrightarrow{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -1-2 \\ 2-(-3) \\ 1-4 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix}, \tag{2}$$

$$\overrightarrow{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 0-2 \\ \frac{1}{3}-(-3) \\ 2-4 \end{pmatrix} = \begin{pmatrix} -2 \\ \frac{10}{3} \\ -2 \end{pmatrix}. \tag{3}$$

Build the 3×2 matrix whose columns are $\overrightarrow{AB}$ and $\overrightarrow{AC}$:

$$\mathbf{M} = \begin{pmatrix} \overrightarrow{AB} & \overrightarrow{AC} \end{pmatrix} = \begin{pmatrix} -3 & -2 \\ 5 & \frac{10}{3} \\ -3 & -2 \end{pmatrix}. \tag{4}$$

We consider the matrix

$$\mathbf{M} = \begin{pmatrix} -3 & -2 \\ 5 & \frac{10}{3} \\ -3 & -2 \end{pmatrix}. \tag{5}$$

Perform row operations to reduce to echelon form:

$$M = \begin{pmatrix} -3 & -2 \\ 5 & \frac{10}{3} \\ -3 & -2 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 5 & \frac{10}{3} \\ -3 & -2 \\ -3 & -2 \end{pmatrix}$$

$$\xrightarrow{R_2 \to 5R_2 + 3R_1,\ \ R_3 \to 5R_3 + 3R_1} \begin{pmatrix} 5 & \frac{10}{3} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Thus, the echelon form of M has only $\mathbf{1}$ non-zero row.

$$\text{rank}(M) = 1. \tag{6}$$

Since the rank of the matrix is 1, the two direction vectors are linearly dependent, and hence the points A, B, C are collinear.

Plot

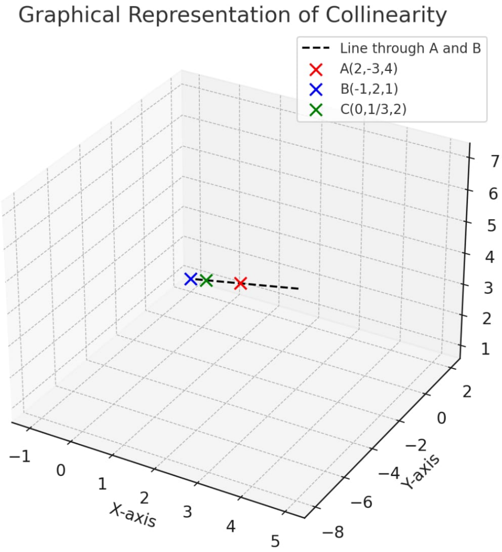

Figure: Image Visual

Conclusion

As the rank of the matrix M is 1, the three given points are collinear.