4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Solution:
Given:

$$\mathbf{P} = \begin{pmatrix} 6 \\ 5 \\ 9 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix} \tag{0.1}$$

Let **n** be the perpendicular vector to plane.

$$\mathbf{n} = (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} |\mathbf{A_{23}} & \mathbf{B_{23}}| \\ |\mathbf{A_{31}} & \mathbf{B_{31}}| \\ |\mathbf{A_{12}} & \mathbf{B_{12}}| \end{pmatrix} = \begin{pmatrix} 12 \\ -16 \\ 12 \end{pmatrix} \tag{0.2}$$

OR

$$\mathbf{n} = \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix} \tag{0.3}$$

if $\mathbf{n} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ then the equation of the plane would be

$$\alpha(x) + \beta(y) + \gamma(z) = k, \text{ Where k is a constant} \tag{0.4}$$

From Equation 0.3 and 0.4,

$$\alpha = 3, \beta = -4, \gamma = 3 \tag{0.5}$$

$$\therefore \text{ The equation of the plane will be } 3x - 4y + 3z = k. \tag{0.6}$$

Putting Coordinates of **A** in equation 0.6 to get k,

$$k = 3(3) - 4(-1) + 3(2) \quad \Rightarrow k = 19 \tag{0.7}$$

$$\therefore\ 3x - 4y + 3z = 19 \tag{0.8}$$

Let **L** be the line perpendicular to plane and passing through **P**.
Let **Q** be a position vector of a point on the plane and the line **L**

$$\text{From Equation 0.3, } \Rightarrow \mathbf{L} = \mathbf{P} + t\mathbf{n} \quad \Rightarrow \mathbf{L} = \begin{pmatrix}6\\5\\9\end{pmatrix} + t\begin{pmatrix}3\\-4\\3\end{pmatrix} \tag{0.9}$$

Therefore from Equation 0.9, $\mathbf{Q} = \begin{pmatrix}6+3t\\5-4t\\9+3t\end{pmatrix}$

Putting co-ordinates of **Q** in equation of plane (from equation 0.8).

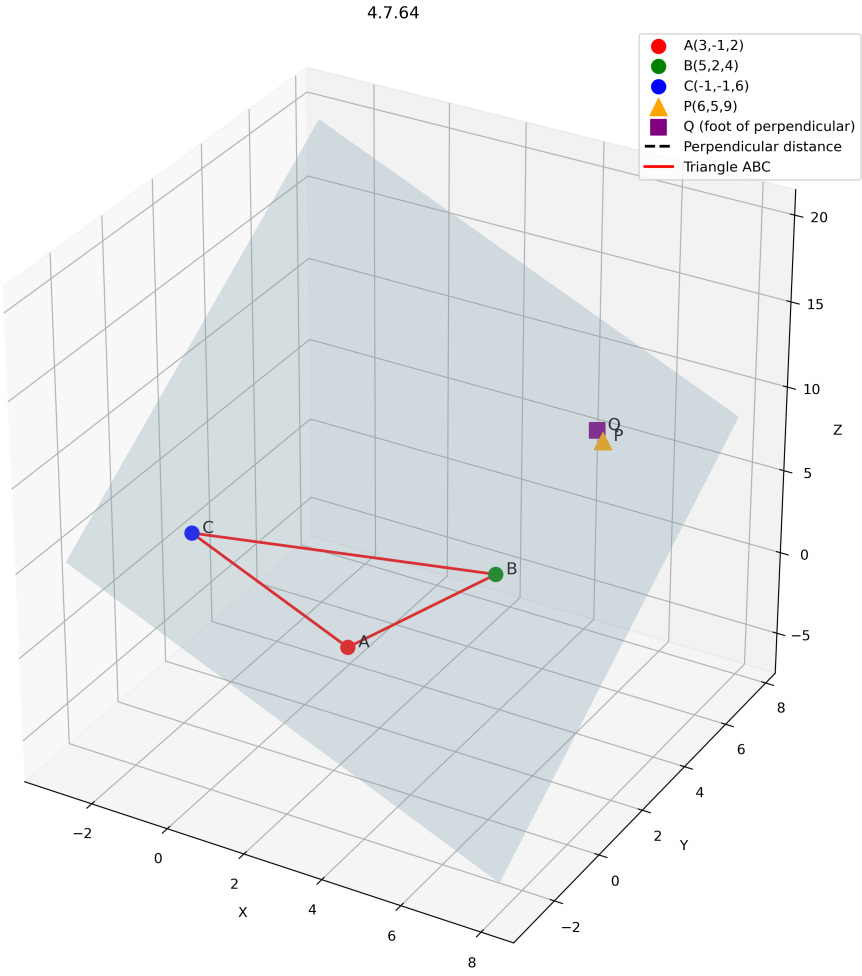
$$3(6+3t) - 4(5-4t) + 3(9+3t) = 19 \quad \Rightarrow t = -\frac{3}{17} \tag{0.10}$$

$$\therefore\ \mathbf{Q} = \begin{pmatrix}93/17\\97/17\\144/17\end{pmatrix} \tag{0.11}$$

The Distance between the plane and **P** is $\|\mathbf{P} - \mathbf{Q}\|$

$$\|\mathbf{P} - \mathbf{Q}\| = \left\|\begin{pmatrix}9/17\\-12/17\\9/17\end{pmatrix}\right\| = \frac{3\sqrt{34}}{17} \tag{0.12}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P} \textit{ is } \frac{3\sqrt{34}}{17} \textit{ units.}} \tag{0.13}$$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A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
P(6,5,9)
Q (foot of perpendicular)
Perpendicular distance
Triangle ABC
A
B
C
P
Q
X
Y
Z

Fig. 0.1: Plane