EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:

Show that P(5,-3) is the point of trisection of the line segment that join the points A (7,-2) and B (1,-5).

Formula: D divides BC in the ratio k:1,

$$\mathbf{D} = \frac{k\mathbf{C} + \mathbf{B}}{k+1} \tag{1}$$

Solution:

Let
$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$
 (2)

Point P (Further to A, Ratio 2:1):

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \tag{3}$$

$$\implies \mathbf{P} = \begin{pmatrix} 7 & 1 \\ -2 & -5 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \tag{4}$$

$$\mathbf{P} = \begin{pmatrix} \frac{1 \times 1 + 2 \times 7}{3} \\ \frac{1 \times (-5) + 2 \times (-2)}{3} \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$
 (5)

Point Q (Nearer from A, Ratio 1:2):

$$\mathbf{Q} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$

$$\implies \mathbf{Q} = \begin{pmatrix} 7 & 1 \\ -2 & -5 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \tag{7}$$

(8)

$$\mathbf{Q} = \begin{pmatrix} \frac{1 \times 2 + 1 \times 7}{3} \\ \frac{2 \times (-5) + 1 \times (-2)}{3} \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$
 (9)

$$\mathbf{P} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \tag{10}$$

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Therefore, the point P(5,-39) is a point of trisection of the line joining the points A(7,-2) and B(1,-5).

Graph of the line segment AB with trisection points P and Q

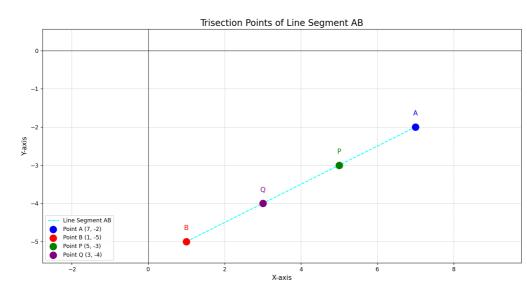


Fig. 0: Figure for 1.4.18