

4.12.8

AI25BTECH11003 - Bhavesh Gaikwad

Question: Distance of the point (α, β, γ) from y-axis is

- a) β
- b) $|\beta|$
- c) $|\beta + \gamma|$
- d) $\sqrt{\alpha^2 + \gamma^2}$

Solution:

Let $\mathbf{A} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

Let \mathbf{B} be an arbitrary point on the y-axis.

$$\text{Equation of y-axis: } \mathbf{r} = t\mathbf{e}_2 \text{ OR } \mathbf{r} = t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (0.1)$$

$$\therefore \mathbf{B} = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} \quad (0.2)$$

$$\text{For minimum distance from y-axis: } (\mathbf{A} - \mathbf{B}) \text{ should be perpendicular to } \mathbf{e}_2 \quad (0.3)$$

OR

$$(\mathbf{A} - \mathbf{B})^T \mathbf{e}_2 = 0 \Rightarrow \begin{pmatrix} \alpha \\ \beta - t \\ \gamma \end{pmatrix}^T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \quad (0.4)$$

Therefore, from Equation 0.4,

$$t = \beta \quad (0.5)$$

Therefore the distance between y-axis and \mathbf{A} is:

$$\|\mathbf{B} - \mathbf{A}\| = \left\| \begin{pmatrix} \alpha \\ 0 \\ \gamma \end{pmatrix} \right\| = \sqrt{\alpha^2 + \gamma^2} \quad (0.6)$$

Therefore, Option D is Correct.

(0.7)

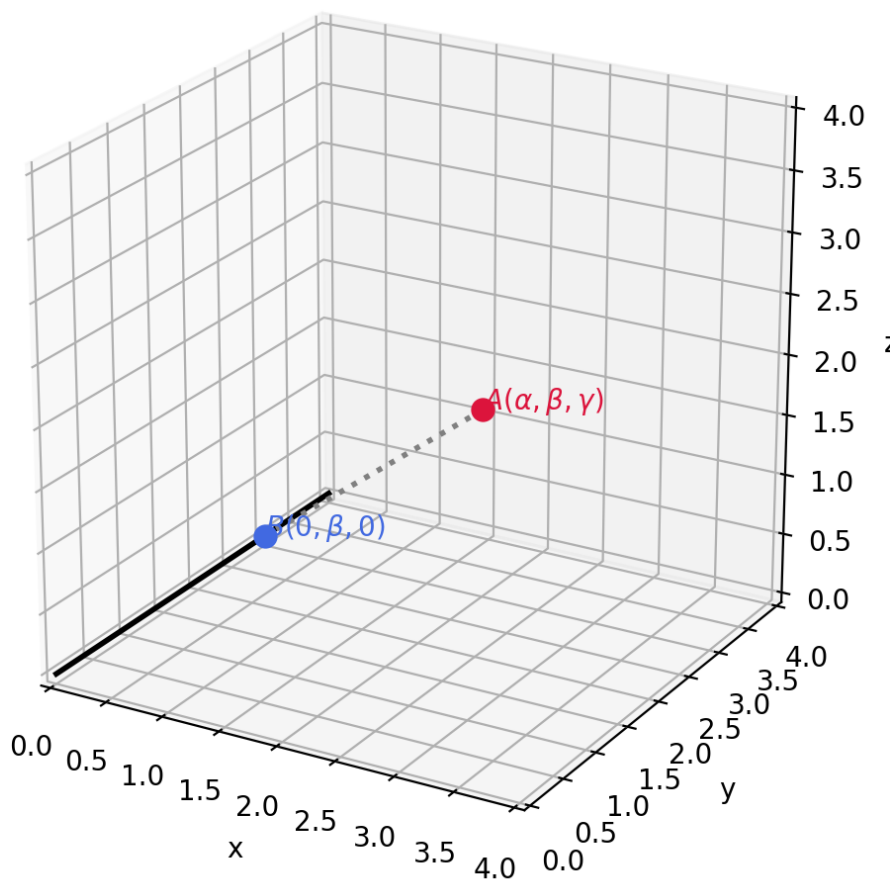


Fig. 0.1: Plane