

Matrices in Geometry 2.4.34

EE25BTECH11037 - Divyansh

Question: What type of quadrilateral do the points **A** (2, -2), **B** (7, 3), **C** (11, -1) and **D** (6, -6) taken in that order, form?

Given: $\mathbf{A} \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\mathbf{C} \begin{pmatrix} 11 \\ -1 \end{pmatrix}$ and $\mathbf{D} \begin{pmatrix} 6 \\ -6 \end{pmatrix}$.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \quad (1)$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad (2)$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \quad (3)$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -4 \\ 4 \end{pmatrix} \quad (4)$$

$$\text{Checking opposite sides, } \mathbf{D} - \mathbf{C} = -(\mathbf{A} - \mathbf{B}) \text{ and } \mathbf{A} - \mathbf{D} = -(\mathbf{B} - \mathbf{C}) \quad (5)$$

Each pair of opposite sides are parallel and equal in length; this implies that the quadrilateral is a parallelogram.

Now, checking for right angle, we check for inner product.

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 5 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} = 0 \quad (6)$$

This implies that the angle at **B** is one right angle. A parallelogram with a right angle is a rectangle.

Checking for a square:

The given quadrilateral is a square if its diagonals are orthogonal,

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B}) = \begin{pmatrix} 9 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -9 \end{pmatrix} = -18 \quad (7)$$

We can see that $(\mathbf{C} - \mathbf{A})^\top (\mathbf{D} - \mathbf{B})$ is $\neq 0$, that is, the diagonals are not orthogonal and therefore the quadrilateral **ABCD** cannot be a square.

\therefore The quadrilateral **ABCD** is a rectangle.

Rectangle ABCD with Right Angles and Equal Opposite Sides

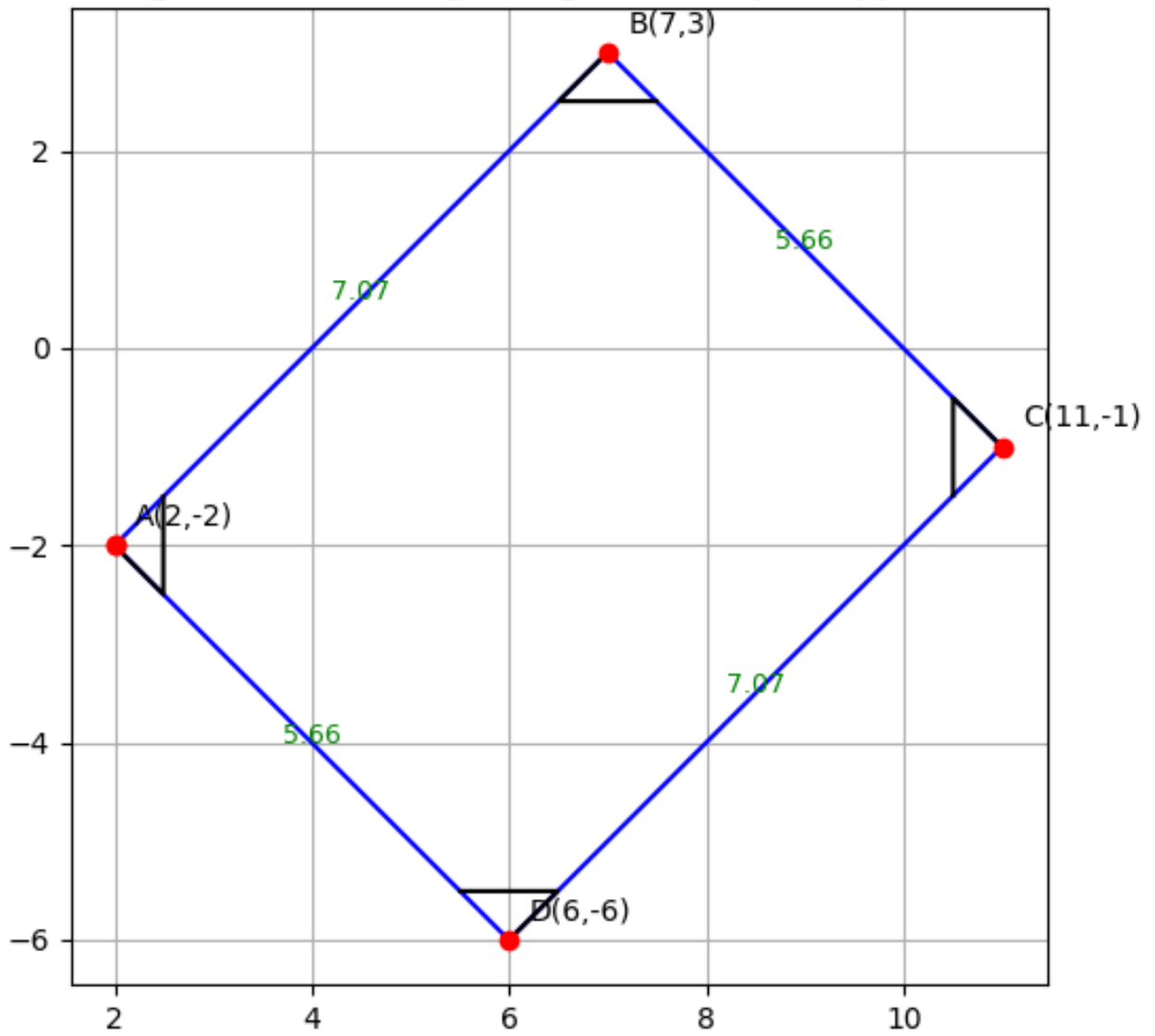


Fig. 1: Plot for 2.4.34