2.4.43

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Question:
Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Solution:
Let the two given lines be L_1 and L_2.

$$L_1 : \frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$$
$$L_2 : \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$$

The direction vector of a line in the form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ are

$$\mathbf{d} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Let the direction vector of lines L_1 and L_2 be $\mathbf{d_1}$ and $\mathbf{d_2}$. From the equations of the lines L_1 and L_2,

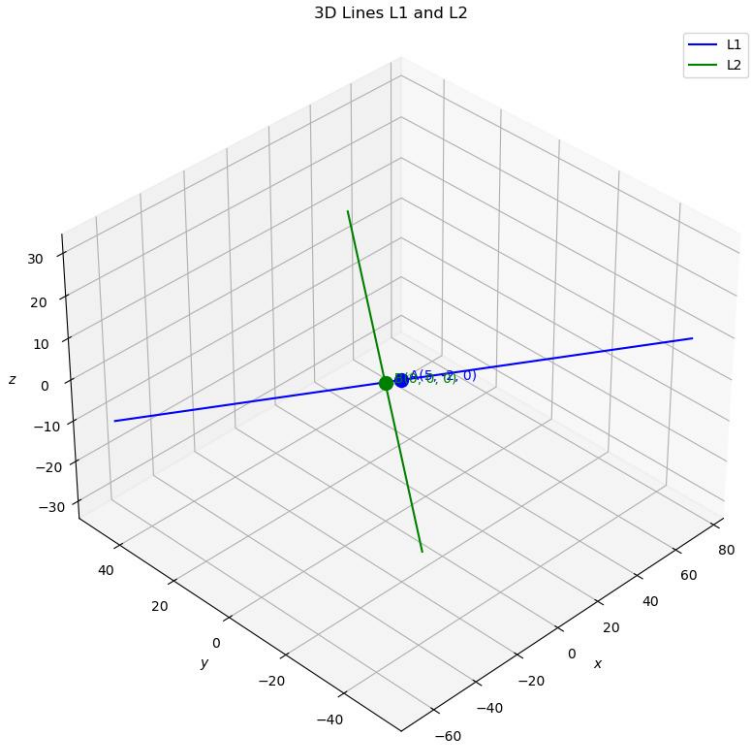
$$\mathbf{d_1} = \begin{pmatrix} 7 \\ -5 \\ 1 \end{pmatrix}, \mathbf{d_2} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{1}$$

For the two lines to be perpendicular, the inner product or dot product of their direction vectors must be zero.

$$\mathbf{d_1}^\top \mathbf{d_2} = 0 \tag{2}$$

$$\mathbf{d_1}^\top \mathbf{d_2} = \begin{pmatrix} 7 & -5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{3}$$
$$= (7)(1) + (-5)(2) + (1)(3) \tag{4}$$
$$= 7 - 10 + 3 \tag{5}$$
$$= 0 \tag{6}$$

$\because$ The dot product of the direction vectors of the two lines is 0
$\implies$ The lines are **perpendicular** to each other.



Lines L_1 and L_2