4.13.31

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Question

Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, line L has intercepts p and q. Then

1. $a^2 + b^2 = p^2 + q^2$
2. $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
3. $a^2 + p^2 = b^2 + q^2$
4. $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

According to the question,

$$\text{The equation of line}: \begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \mathbf{x} = 1 \tag{1}$$

Let the row coefficient vector of the original line be $\mathbf{m}$ and for the rotated line be $\mathbf{m}'$.

$$\mathbf{m}' = \mathbf{P}\mathbf{m} \tag{2}$$

where $\mathbf{P}$ is the rotation matrix

Equation

$$\|\mathbf{m}'\|^2 = \mathbf{m}'^\top \mathbf{m}' = \left(\mathbf{m}^\top \mathbf{P}^\top\right)(\mathbf{P}\mathbf{m}) = \mathbf{m}^\top \left(\mathbf{P}^\top \mathbf{P}\right) \mathbf{m} \tag{3}$$

Since $\mathbf{P}$ is an orthogonal matrix,

$$\therefore \mathbf{m}^\top \left(\mathbf{P}^\top \mathbf{P}\right) \mathbf{m} = \mathbf{m}^\top \mathbf{m} = \|m\|^2 \tag{4}$$

$$\implies \|\mathbf{m}'\|^2 = \|\mathbf{m}\|^2 \tag{5}$$

As $\mathbf{m}'$ is given by $\begin{pmatrix} \frac{1}{p} & \frac{1}{q} \end{pmatrix}^\top$,

$$\therefore \frac{1}{p^2} + \frac{1}{q^2} = \frac{1}{a^2} + \frac{1}{b^2} \tag{6}$$

C Code -Finding Equation of the plane

```c
#include <stdio.h>
#include <math.h>

void line_intercepts_after_rotation(double a, double b, double
    theta, double *p, double *q) {
    // vector [1/a, 1/b]
    double m[2] = {1.0/a, 1.0/b};
    // Rotation matrix
    double P[2][2] = {
        {cos(theta), -sin(theta)},
        {sin(theta), cos(theta)}
    };
    double m_new[2];
    m_new[0] = m[0]*P[0][0] + m[1]*P[1][0];
    m_new[1] = m[0]*P[0][1] + m[1]*P[1][1];
    *p = 1.0 / m_new[0];
    *q = 1.0 / m_new[1];
}
```

C Code -Finding Equation of the plane

```c
// Function to check options (returns index of true option)
int check_options(double a, double b, double p, double q, double
    eps) {
    double optA = fabs((a*a + b*b) - (p*p + q*q)) < eps;
    double optB = fabs((1.0/(a*a) + 1.0/(b*b)) - (1.0/(p*p) +
        1.0/(q*q))) < eps;
    double optC = fabs((a*a + p*p) - (b*b + q*q)) < eps;
    double optD = fabs((1.0/(a*a) + 1.0/(p*p)) - (1.0/(b*b) +
        1.0/(q*q))) < eps;

    if(optA) return 1;
    if(optB) return 2;
    if(optC) return 3;
    if(optD) return 4;
    return 0; // none true
}
```

Python+C code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")

# Load shared library
lib = ctypes.CDLL("./libline_rotation.so")

# Function signatures
lib.line_intercepts_after_rotation.argtypes = [
    ctypes.c_double, ctypes.c_double, ctypes.c_double,
    ctypes.POINTER(ctypes.c_double), ctypes.POINTER(ctypes.
        c_double)
]
lib.line_intercepts_after_rotation.restype = None
```

Python+C code

```
lib.check_options.argtypes = [
    ctypes.c_double, ctypes.c_double,
    ctypes.c_double, ctypes.c_double, ctypes.c_double
]
lib.check_options.restype = ctypes.c_int
# Parameters
a, b = 3.0, 4.0
theta = np.pi / 3
# Call C function to compute new intercepts
p, q = ctypes.c_double(), ctypes.c_double()
lib.line_intercepts_after_rotation(a, b, theta, ctypes.byref(p),
    ctypes.byref(q))
p, q = p.value, q.value
# Check which option is true
opt = lib.check_options(a, b, p, q, 1e-8)
print(f"Correct Option: {opt}")
```

Python+C code

```
# ========== PLOT ONLY IF OPTION B IS TRUE ==========
if opt == 2:
    # Original line
    x_vals = np.linspace(-1, max(a, 6), 400)
    y_vals = b * (1 - x_vals / a)
    # Original intercepts
    A = (a, 0)
    B = (0, b)
    # Rotated axes directions
    e1_new = np.array([np.cos(theta), np.sin(theta)])
    e2_new = np.array([-np.sin(theta), np.cos(theta)])
    # Transform new intercepts (rotated -> original coords)
    P = np.array([
        [np.cos(theta), -np.sin(theta)],
        [np.sin(theta), np.cos(theta)]
    ])
    Pp = P @ np.array([p, 0])
    Pq = P @ np.array([0, q])
```

Python+C code

```
# Plot
plt.figure(figsize=(7,7))
plt.axhline(0, color='gray', lw=1)
plt.axvline(0, color='gray', lw=1)
plt.plot(x_vals, y_vals, 'b', label="Original line")
plt.scatter(*A, color='blue')
plt.text(A[0], A[1]-0.3, f"A({a:.3f},0)", ha='center',
    fontsize=10, color="blue")
plt.scatter(*B, color='blue')
plt.text(B[0]-0.3, B[1], f"B(0,{b:.3f})", va='center',
    fontsize=10, color="blue")
t = np.linspace(-6, 6, 200)
plt.plot(t*e1_new[0], t*e1_new[1], 'r--', label="Rotated X'-
    axis")
plt.plot(t*e2_new[0], t*e2_new[1], 'r--', label="Rotated Y'-
    axis")
```

Python+C code

```
# New intercepts (P, Q)
  plt.scatter(*Pp, color='green')
  plt.text(Pp[0], Pp[1]-0.2, f"P({p:.3f},0)", color='green', ha
      ='center', fontsize=10)

  plt.scatter(*Pq, color='green')
  plt.text(Pq[0]-0.2, Pq[1], f"Q(0,{q:.3f})", color='green', va
      ='center', fontsize=10)

  plt.axis("equal")
  plt.legend(loc="upper right")
  plt.title("Graph for the True Relation:\n1/a + 1/b = 1/p + 1/
      q")
  plt.savefig("/home/user/Matrix/Matgeo_assignments/4.13.31/
      figs/Figure_1")
  plt.show()
```

Python code

```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")

def line_intercepts_after_rotation(a, b, theta):
    m = np.array([1/a, 1/b])
    P = np.array([
        [np.cos(theta), -np.sin(theta)],
        [np.sin(theta), np.cos(theta)]
    ])
    m_new = m @ P
    return 1/m_new[0], 1/m_new[1] # p, q
```

Python code

```python
# Parameters
a, b = 3, 4
theta = np.pi/3
# New intercepts
p, q = line_intercepts_after_rotation(a, b, theta)
# ========== BOOLEAN CHECK SECTION ==========
eps = 1e-8

optA = abs((a**2 + b**2) - (p**2 + q**2)) < eps
optB = abs((1/a**2 + 1/b**2) - (1/p**2 + 1/q**2)) < eps
optC = abs((a**2 + p**2) - (b**2 + q**2)) < eps
optD = abs((1/a**2 + 1/p**2) - (1/b**2 + 1/q**2)) < eps

print(f"Option A : {optA}")
print(f"Option B : {optB}")
print(f"Option C : {optC}")
print(f"Option D : {optD}")
```

```
if optB: # Only plot when option B is satisfied
    # Original line
    x_vals = np.linspace(-1, max(a, 6), 400)
    y_vals = b*(1 - x_vals/a)
    # Original intercepts
    A = (a,0)
    B = (0,b)
    # Rotated axes directions
    e1_new = np.array([np.cos(theta), np.sin(theta)])
    e2_new = np.array([-np.sin(theta), np.cos(theta)])
    # Transform new intercepts (in rotated coords) back to old
        coords
    P = np.array([[np.cos(theta), -np.sin(theta)],
                  [np.sin(theta), np.cos(theta)]])
    Pp = P @ np.array([p,0]) # (p,0) in rotated coords
    Pq = P @ np.array([0,q]) # (0,q) in rotated coords
```

Python code

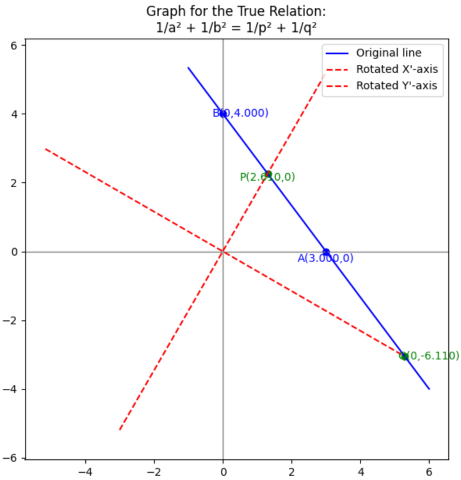
```python
# Plot
plt.figure(figsize=(7,7))
plt.axhline(0, color='gray', lw=1)
plt.axvline(0, color='gray', lw=1)
# Original line
plt.plot(x_vals, y_vals, 'b', label="Original line")
# Original intercepts
plt.scatter(*A, color='blue')
plt.scatter(*B, color='blue')
plt.text(A[0], A[1]-0.3, f"A({a},0)", ha='center')
plt.text(B[0]-0.3, B[1], f"B(0,{b})", va='center')
# Draw rotated axes as extended lines
t = np.linspace(-6, 6, 200)
plt.plot(t*e1_new[0], t*e1_new[1], 'r--', label="Rotated X'-
    axis")
plt.plot(t*e2_new[0], t*e2_new[1], 'r--', label="Rotated Y'-
    axis")
```

Python code

```python
# New intercepts on rotated axes
  plt.scatter(*Pp, color='green')
  plt.scatter(*Pq, color='green')
  plt.text(Pp[0], Pp[1]-0.2, f"P({p:.2f},0)", color='green', ha='center')
  plt.text(Pq[0]-0.2, Pq[1], f"Q(0,{q:.2f})", color='green', va='center')

  plt.axis("equal")
  plt.legend()
  plt.title("Graph for the True Relation: 1/a + 1/b = 1/p + 1/q")
  plt.savefig("/home/user/Matrix/Matgeo_assignments/4.13.31/figs/Figure_1")
  plt.show()
```

Plot


Graph for the True Relation:
1/a² + 1/b² = 1/p² + 1/q²
Original line
Rotated X'-axis
Rotated Y'-axis
B(0,4.000)
P(2.610,0)
A(3.000,0)
Q(0,-6.110)
6
4
2
0
−2
−4
−6
−4
−2
0
2
4
6