2.4.22

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Question Find the equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Let,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix} \tag{1}$$

Given that the plane is a perpendicular bisector to the line joining points A and B. Since it is a perpendicular bisector to the line joining points A and B , the midpoint of the line joining points A and B lies on the plane.
Let the midpoint of points **A** and **B** be **X**. Then

$$||\mathbf{X} - \mathbf{A}||^2 = ||\mathbf{X} - \mathbf{B}||^2 \tag{2}$$

$$||\mathbf{X}||^2 + ||\mathbf{A}||^2 - 2||\mathbf{X}||\ ||\mathbf{A}|| = ||\mathbf{X}||^2 + ||\mathbf{B}||^2 - 2||\mathbf{X}||\ ||\mathbf{B}|| \tag{3}$$

$$||\mathbf{A}||^2 - 2||\mathbf{X}||\ ||\mathbf{A}|| = ||\mathbf{B}||^2 - 2||\mathbf{X}||\ ||\mathbf{B}|| \tag{4}$$

$$||\mathbf{A}||^2 - ||\mathbf{B}||^2 = 2\mathbf{X^T}\mathbf{A} - 2\mathbf{X^T}\mathbf{B} \tag{5}$$

$$||\mathbf{A}||^2 - ||\mathbf{B}||^2 = 2\mathbf{X^T}(\mathbf{A} - \mathbf{B}) \tag{6}$$

which can be written as :

$$||\mathbf{A}||^2 - ||\mathbf{B}||^2 = 2(\mathbf{A} - \mathbf{B})^T\mathbf{X} \tag{7}$$

$$(\mathbf{A} - \mathbf{B})^T\mathbf{X} = \frac{||\mathbf{A}||^2 - ||\mathbf{B}||^2}{2} \tag{8}$$

Now,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix} \tag{9}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ -2 \\ -4 \end{pmatrix} \tag{10}$$

And

$$\|\mathbf{A}\| = \sqrt{\mathbf{A^T A}} \tag{11}$$

$$\|\mathbf{A}\| = \sqrt{2(2) + 3(3) + 4(4)} \tag{12}$$

$$\|\mathbf{A}\| = \sqrt{29} \tag{13}$$

$$\|\mathbf{B}\| = \sqrt{\mathbf{B^T B}} \tag{14}$$

$$\|\mathbf{B}\| = \sqrt{4(4) + 5(5) + 8(8)} \tag{15}$$

$$\|\mathbf{B}\| = \sqrt{105} \tag{16}$$

Now substituting the respective value in Eq.8:

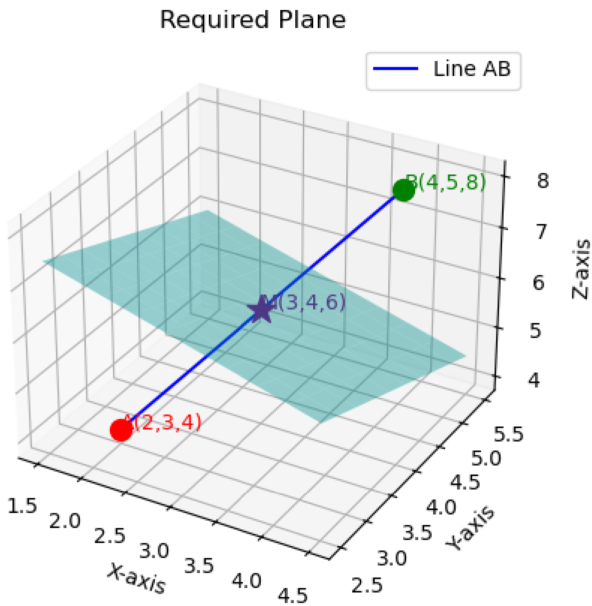
$$\begin{pmatrix} -2 & -2 & -4 \end{pmatrix} \mathbf{X} = \frac{29 - 105}{2} \tag{17}$$

$$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \mathbf{X} = 19 \tag{18}$$

Hence the equation of the plane is given by:

$$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \mathbf{X} = 19 \tag{19}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.


Required Plane
Line AB
B(4,5,8)
M(3,4,6)
A(2,3,4)
X-axis
Y-axis
Z-axis
1.5
2.0
2.5
3.0
3.5
4.0
4.5
2.5
3.0
3.5
4.0
4.5
5.0
5.5
4
5
6
7
8