

Matrices in Geometry 1.9.27

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EE25BTECH11038 - GNANTHIK LUCKY

Question: Find the value of P, if the point **A** (0, 2) is equidistant from point **B** (3, P) and **C** (p, 5)

Given: **A** $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$, **B** $\begin{pmatrix} 3 \\ P \end{pmatrix}$ and a point **C** $\begin{pmatrix} P \\ 5 \end{pmatrix}$ such that **P** is equidistant from **A** and **B**.

$$\therefore \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \quad (1)$$

$$\text{On squaring both the sides, we get} \quad (2)$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2 \quad (3)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{C})^\top (\mathbf{A} - \mathbf{C}) \quad (4)$$

$$\mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{B} + \mathbf{B}^\top \mathbf{B} = \mathbf{A}^\top \mathbf{A} - 2\mathbf{A}^\top \mathbf{C} + \mathbf{C}^\top \mathbf{C} \quad (5)$$

$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{A}^\top (\mathbf{B} - \mathbf{C}) \quad (6)$$

$$\left\| \begin{pmatrix} 3 \\ P \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} P \\ 5 \end{pmatrix} \right\|^2 = 2 \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 3 - P \\ P - 5 \end{pmatrix} \quad (7)$$

$$9 + P^2 - P^2 - 25 = 2(0 + 2P - 10) \quad (8)$$

$$-16 = 4P - 20 \implies 4P = 4 \implies P = 1 \quad (9)$$

$$\text{Hence, the final answer is } \boxed{P = 1} \quad (10)$$

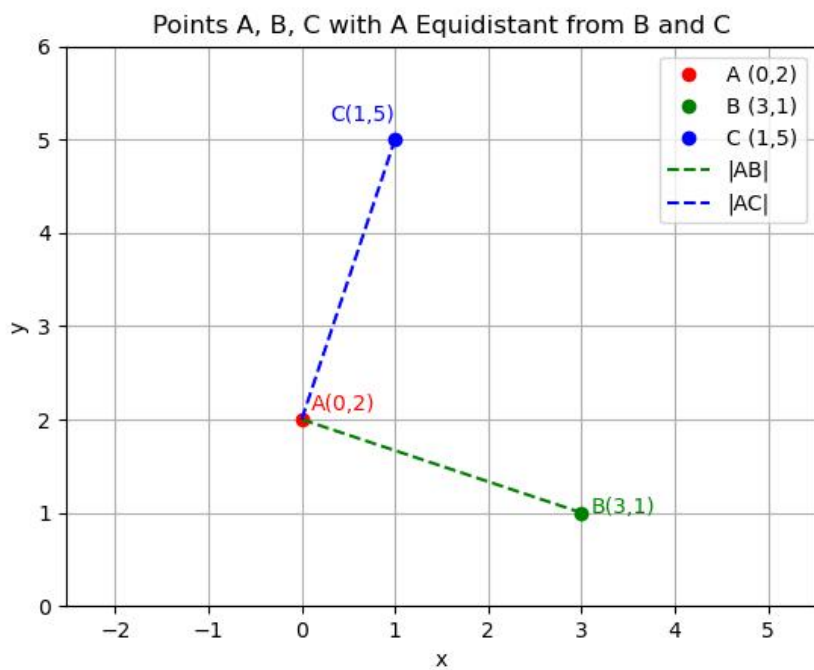


Fig. 1: Plot for 1.9.27