

4.3.22

EE25BTECH11033 - Kevin

Question:

Find the ratio in which the line segment joining $\mathbf{A}(1, -5)$ and $\mathbf{B}(-4, 5)$ is divided by the X axis. Also find the coordinates of the point of division.

Solution:

Let the vector \mathbf{P} be the point on x -axis

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix}, \quad (1)$$

Given the points,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \quad (2)$$

The points \mathbf{A} , \mathbf{P} , \mathbf{B} are collinear.

Points \mathbf{A} , \mathbf{P} , \mathbf{B} are defined to be collinear if

$$\text{rank}(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = 1 \quad (3)$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} x - 1 \\ 5 \end{pmatrix} \quad (4)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \quad (5)$$

$$(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = \begin{pmatrix} x - 1 & -5 \\ 5 & 10 \end{pmatrix} \quad (6)$$

$$R_1 \leftrightarrow R_2 \implies \begin{pmatrix} 5 & 10 \\ x - 1 & -5 \end{pmatrix} \quad (7)$$

$$R_2 \rightarrow 2R_2 + R_1 \implies \begin{pmatrix} 5 & 10 \\ 2x + 3 & 0 \end{pmatrix} \quad (8)$$

$$R_2 \rightarrow R_2 - (2x + 3)R_1 \implies \begin{pmatrix} 1 & 2 \\ 0 & -4x - 6 \end{pmatrix} \quad (9)$$

For rank 1, the second row must be zero:

$$-4x - 6 = 0 \implies x = -3/2 \quad (10)$$

$$\therefore \mathbf{P} = \begin{pmatrix} -3/2 \\ 0 \end{pmatrix}$$

Section formula for a vector \mathbf{P} which divides the line formed by vectors \mathbf{A} and \mathbf{B} in the ratio $k:1$ is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (11)$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \quad (12)$$

$$\Rightarrow k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \quad (13)$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B}) = \begin{pmatrix} 5/2 & -5 \end{pmatrix} \begin{pmatrix} 5/2 \\ -5 \end{pmatrix} = 125/4 \quad (14)$$

$$\|\mathbf{P} - \mathbf{B}\|^2 = \left(\sqrt{(5/2)^2 + (-5)^2} \right)^2 = 125/4 \quad (15)$$

$$\Rightarrow k = 1 \quad (16)$$

Therefore the ratio in which \mathbf{P} divides the line segment joining the points \mathbf{A} and \mathbf{B} is $1 : 1$

See Fig. 0 ,

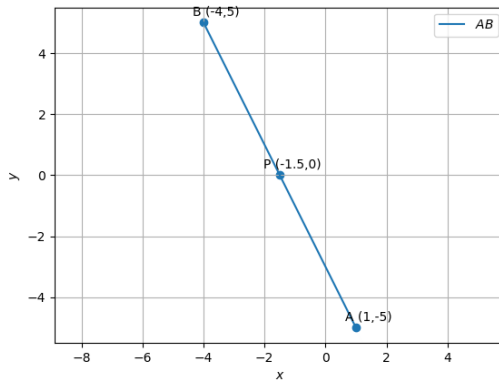


Fig. 0