2.4.16

EE25BTECH11021 - Dhanush Sagar

Question

Verify the following:

(a) $(0, 7, -10), (1, 6, -6)$ and $(4, 9, -6)$ are the vertices of an isosceles triangle.

(b) $(0, 7, 10), (-1, 6, 6)$ and $(-4, 9, 6)$ are the vertices of a right-angled triangle.

Solution a

Property: In an isosceles triangle, the perpendicular bisector of a side passes through the opposite vertex.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix} \tag{0.1}$$

Midpoint of side AC:

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix} + \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix}}{2} = \begin{pmatrix} 2 \\ 8 \\ -8 \end{pmatrix} \tag{0.2}$$

Direction vector of side AC:

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \tag{0.3}$$

Vector from midpoint to B:

$$\mathbf{B} - \mathbf{M} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \\ -8 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \tag{0.4}$$

$$(\mathbf{C} - \mathbf{A})^{\top}(\mathbf{B} - \mathbf{M}) = \begin{pmatrix} 4 & 2 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = -4 - 4 + 8 = 0 \tag{0.5}$$

$\mathbf{B}$ lies on the perpendicular bisector of side AC.

$$\therefore AB = BC \implies \triangle ABC \text{ is isosceles.}$$

Solution b

Property: If two sides of a triangle are perpendicular, then the included angle is a right

angle.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 6 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ 9 \\ 6 \end{pmatrix} \tag{0.6}$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 0 \\ 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \tag{0.7}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} -4 \\ 9 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} \tag{0.8}$$

$$(\mathbf{A} - \mathbf{B})^{\top}(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \\ 0 \end{pmatrix} = -3 + 3 + 0 = 0 \tag{0.9}$$

$$\implies \mathbf{A} - \mathbf{B} \perp \mathbf{C} - \mathbf{B} \Rightarrow \triangle ABC \text{ is right-angled at } B.$$

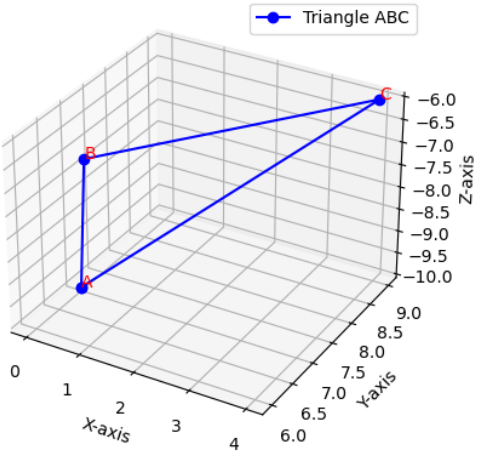

Fig. 0.1: isosceles triangle(a)

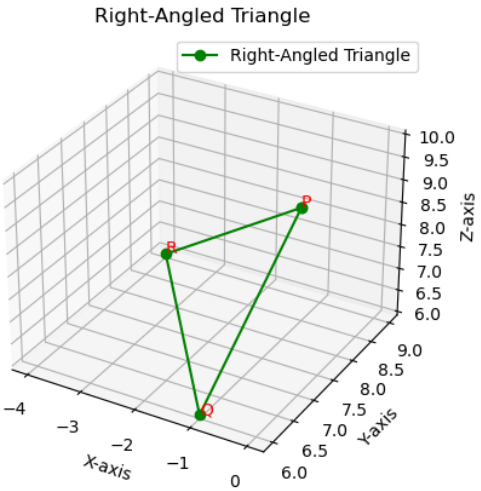
Right-Angled Triangle
Right-Angled Triangle
P
R
Q
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
Z-axis
9.0
8.5
8.0
7.5
7.0
6.5
6.0
Y-axis
−4
−3
−2
−1
0
X-axis

Fig. 0.2