2.5.16

Shriyansh Chawda-EE25BTECH11052

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Question

Find the value of p for which the lines

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2} \quad \text{and} \quad \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular. (12, 2019)

Solution

Writing each line in symmetric form to read off direction vectors:

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \tag{1}$$

From

$$\frac{1-x}{3} = t, \; \frac{2y-14}{2p} = t, \; \frac{z-3}{2} = t, \tag{2}$$

we obtain

$$x = 1 - 3t, \quad y = 7 + pt, \quad z = 3 + 2t. \tag{3}$$

Hence,

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ p \\ 2 \end{pmatrix} \tag{4}$$

Solution

From

$$\frac{1-x}{3p} = s,\ \frac{y-5}{1} = s,\ \frac{6-z}{5} = s, \tag{5}$$

we obtain

$$x = 1 - 3ps, \quad y = 5 + s, \quad z = 6 - 5s. \tag{6}$$

Hence,

$$\mathbf{m_2} = \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix} \tag{7}$$

Solution

The lines are perpendicular when

$$\mathbf{m_1}^\top \mathbf{m_2} = 0 \tag{8}$$

Substituting,

$$\begin{pmatrix} -3 & p & 2 \end{pmatrix} \begin{pmatrix} -3p \\ 1 \\ -5 \end{pmatrix} = 0 \tag{9}$$

$$(-3)(-3p) + p \cdot 1 + 2(-5) = 0 \tag{10}$$

$$9p + p - 10 = 0 \tag{11}$$

$$10p - 10 = 0 \tag{12}$$

$$\implies p = 1 \tag{13}$$

Therefore,

$$\boxed{p = 1}$$

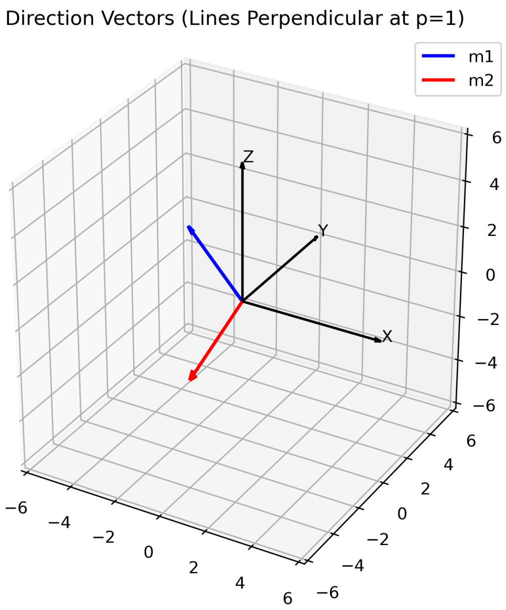

Figure: Plotting both lines