4.11.11

EE25BTECH11033 - Kavin

Question:
Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

Solution:
Given the points,

$$\mathbf{A} = \begin{pmatrix} -2 \\ -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \tag{1}$$

Let the vector $\mathbf{P}$ be a point on the line $x - 3y = 0$ wihch divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$.

$$\mathbf{P} = \begin{pmatrix} 3k \\ k \end{pmatrix}, \tag{2}$$

The points $\mathbf{A}$, $\mathbf{P}$, $\mathbf{B}$ are collinear.
Points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = 1 \tag{3}$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 3k + 2 \\ k + 5 \end{pmatrix} \tag{4}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} 3k + 2 & 8 \\ k + 5 & 8 \end{pmatrix} \tag{6}$$

$$R_2 \to R_2 - \tfrac{k+5}{3k+2} R_1 \implies \begin{pmatrix} 3k + 2 & 8 \\ 0 & \frac{16k-24}{3k+2} \end{pmatrix}$$

For rank 1, the second row must be zero:

$$16k - 24 = 0 \implies k = 3/2 \tag{7}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 9/2 \\ 3/2 \end{pmatrix}$$

Section formula for a vector $\mathbf{P}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio k:1 is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{8}$$

$$k(\mathbf{P}-\mathbf{B}) = \mathbf{A}-\mathbf{P} \tag{9}$$

$$\implies k = \frac{(\mathbf{A}-\mathbf{P})^\top(\mathbf{P}-\mathbf{B})}{\|\mathbf{P}-\mathbf{B}\|^2} \tag{10}$$

$$(\mathbf{A}-\mathbf{P})^\top(\mathbf{P}-\mathbf{B}) = \begin{pmatrix}-13/2 & -13/2\end{pmatrix}\begin{pmatrix}-3/2\\-3/2\end{pmatrix} = 39/2 \tag{11}$$

$$\|\mathbf{P}-\mathbf{B}\|^2 = \left(\sqrt{(-3/2)^2+(-3/2)^2}\right)^2 = 9/2 \tag{12}$$

$$\implies k = 13/3 \tag{13}$$

Therefore the ratio in which **P** divides the line segment joining the points **A** and **B** is $13:3$