Matrices in Geometry - 1.9.24

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Problem Statement

The x-coordinate of a point **P** is twice is y-coordinate. If **P** is equidistant from the points **Q**$(2, -5)$ and **R**$(-3, 6)$, find the coordiantes of **P**.

Solution

Given $\mathbf{P}\begin{pmatrix} 2k \\ k \end{pmatrix}$, $\mathbf{Q}\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $\mathbf{R}\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

Distances $PQ = PR$

So their norms must be equal and also the square of their norms.

$$\|\mathbf{Q} - \mathbf{P}\|^2 = \|\mathbf{R} - \mathbf{P}\|^2 \tag{1}$$

$$\|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{Q} + \|\mathbf{Q}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{R} + \|\mathbf{R}\|^2 \tag{2}$$

$$\implies \frac{\|\mathbf{Q}\|^2 - \|\mathbf{R}\|^2}{2} = \mathbf{P}^\top(\mathbf{Q} - \mathbf{R}) \tag{3}$$

Solution

$$\|\mathbf{Q}\|^2 = \begin{pmatrix} 2 & -5 \end{pmatrix} \begin{pmatrix} 2 \\ -5 \end{pmatrix} = 4 + 25 = 29 \tag{4}$$

$$\|\mathbf{R}\|^2 = \begin{pmatrix} -3 & 6 \end{pmatrix} \begin{pmatrix} -3 \\ 6 \end{pmatrix} = 9 + 36 = 45 \tag{5}$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 5 \\ -11 \end{pmatrix} \tag{6}$$

$$\implies -8 = \begin{pmatrix} 2k & k \end{pmatrix} \begin{pmatrix} 5 \\ -11 \end{pmatrix} \tag{7}$$

$$-8 = 10k - 11k = -k \tag{8}$$

$$\implies \boxed{k = 8} \tag{9}$$

Final Answer

$$\boxed{k = 8}$$

The coordinates of point $P = \begin{pmatrix} 16 \\ 8 \end{pmatrix}$.

Points Q(2, -5), R(-3, 6), and P(16, 8) in 2D Space