

1.9.19

Josyula G S Avaneesh- EE25BTECH11030

Question Find the value of x for which the distance between the points $\mathbf{A}(x, 2)$ and $\mathbf{B}(9, 8)$ is 10 units.

Solution: Given details:

$$\mathbf{A} = \begin{pmatrix} x \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} 9 \\ 8 \end{pmatrix} \quad (2)$$

$$\|AB\| = 10 \quad (3)$$

Distance between 2 vectors \mathbf{A} and \mathbf{B} can be represented as:

$$\|AB\| = \sqrt{(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A})} \quad (4)$$

By substituting values:

$$\|AB\| = \sqrt{\begin{pmatrix} 9-x & 8-2 \end{pmatrix} \begin{pmatrix} 9-x \\ 8-2 \end{pmatrix}} = \sqrt{\begin{pmatrix} 9-x & 6 \end{pmatrix} \begin{pmatrix} 9-x \\ 6 \end{pmatrix}} \quad (5)$$

$$= \sqrt{(9-x)^2 + (6)^2} = \sqrt{(x^2 - 18x + 81) + 36} = \sqrt{x^2 - 18x + 117} \quad (6)$$

Now from equation (3) and (4), we can say that :

$$\sqrt{x^2 - 18x + 117} = 10 \quad (7)$$

Square on both sides

$$x^2 - 18x + 117 = 100 \quad (8)$$

$$x^2 - 18x + 17 = 0 \quad (9)$$

$$\text{On solving this we get, } x = 1 \text{ or } x = 17 \quad (10)$$

Final answer:

The values of x are 1 and 17. Therefore, the points $\mathbf{A}(x, 2)$ are $(1, 2)$ or $(17, 2)$.

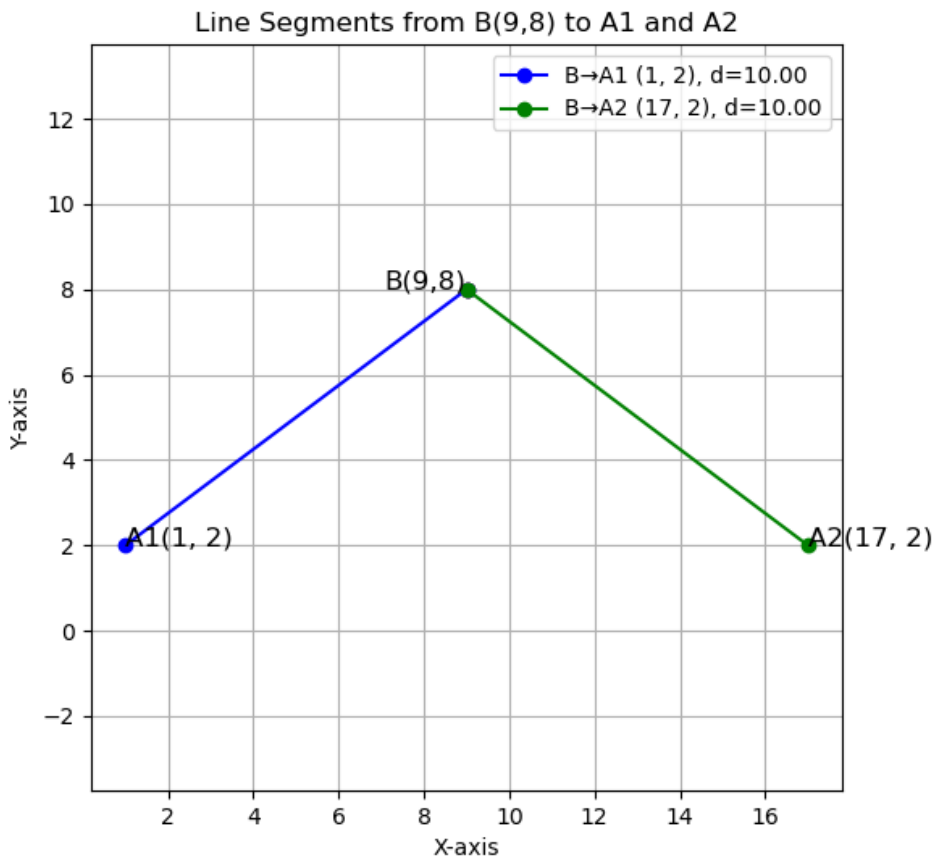


Fig. 0. distance between two points