2.4.41

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Question

Problem

Determine whether the points $A(3, 6, 9)$, $B(10, 20, 30)$, $C(24, -41, 5)$ are the vertices of a right-angled triangle using matrices.

Process (Step 1)

Step 1: Represent points as vectors

$$\left(A\right) = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}, \quad \left(B\right) = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad \left(C\right) = \begin{bmatrix} 24 \\ -41 \\ 5 \end{bmatrix} \tag{1}$$

Step 2: Compute first difference vector

$$\left(B\right) - \left(A\right) = \begin{bmatrix} 10 - 3 \\ 20 - 6 \\ 30 - 9 \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix} \tag{2}$$

Step 2 (continued): Compute remaining difference vectors

$$\left(C\right) - \left(B\right) = \begin{bmatrix} 24 - 10 \\ -41 - 20 \\ 5 - 30 \end{bmatrix} = \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} \tag{3}$$

$$\left(C\right) - \left(A\right) = \begin{bmatrix} 24 - 3 \\ -41 - 6 \\ 5 - 9 \end{bmatrix} = \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix} \tag{4}$$

Step 3: Use dot product test

$$((X) - (Y)) \cdot ((Z) - (W)) = 0 \implies \text{Vectors are perpendicular} \tag{5}$$

Dot product results:

$$((B) - (A)) \cdot ((C) - (A)) = -595 \tag{6}$$

$$((B) - (A)) \cdot ((C) - (B)) = -1281 \tag{7}$$

$$((C) - (A)) \cdot ((C) - (B)) = 3261 \tag{8}$$

Since none are zero, the points **do not form a right-angled triangle**.

C Code

```c
#include <stdio.h>
int main() {
    double A[3] = {3, 6, 9};
    double B[3] = {10, 20, 30};
    double C[3] = {24, -41, 5};
    double dot1 = (B[0]-A[0])*(C[0]-A[0]) +
                  (B[1]-A[1])*(C[1]-A[1]) +
                  (B[2]-A[2])*(C[2]-A[2]);
    double dot2 = (B[0]-A[0])*(C[0]-B[0]) +
                  (B[1]-A[1])*(C[1]-B[1]) +
                  (B[2]-A[2])*(C[2]-B[2]);
    double dot3 = (C[0]-A[0])*(C[0]-B[0]) +
                  (C[1]-A[1])*(C[1]-B[1]) +
                  (C[2]-A[2])*(C[2]-B[2]);
    if(dot1==0 || dot2==0 || dot3==0)
        printf("Right-angled triangle\n");
    else
        printf("Not right-angled\n");
    return 0;
```

```python
import numpy as np

A = np.array([3, 6, 9])
B = np.array([10, 20, 30])
C = np.array([24, -41, 5])

# Dot products computed directly
dot1 = np.dot(B-A, C-A)
dot2 = np.dot(B-A, C-B)
dot3 = np.dot(C-A, C-B)

if dot1 == 0 or dot2 == 0 or dot3 == 0:
    print("Right-angled triangle")
else:
    print("Not right-angled")
```

Python Code (2/2: Plot)

```python
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')
ax.scatter(*A, color='black', s=80)
ax.text(A[0]+0.5, A[1]+0.5, A[2]+0.5, "A(3,6,9)")
ax.scatter(*B, color='blue', s=80)
ax.text(B[0]+0.5, B[1]+0.5, B[2]+0.5, "B(10,20,30)")

ax.scatter(*C, color='red', s=80)
ax.text(C[0]+0.5, C[1]+0.5, C[2]+0.5, "C(24,-41,5)")

ax.plot([A[0],B[0]], [A[1],B[1]], [A[2],B[2]], color='blue')
ax.plot([A[0],C[0]], [A[1],C[1]], [A[2],C[2]], color='green')
ax.plot([B[0],C[0]], [B[1],C[1]], [B[2],C[2]], color='red',
    linestyle='--')

plt.show()
```

Triangle formed by points A, B, C