Matrices in Geometry 2.4.32

EE25BTECH11035 - Kushal B N

Question: The points $\mathbf{A}(-1,-2)$, $\mathbf{B}(4,3)$, $\mathbf{C}(2,5)$ and $\mathbf{D}(-3,0)$ in that order form a rectangle.
Given:
$\mathbf{A}\begin{pmatrix}-1\\-2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}4\\3\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}2\\5\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}-3\\0\end{pmatrix}$
Solution:

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}5\\5\end{pmatrix} \tag{1}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}-2\\2\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{C} = \begin{pmatrix}-5\\-5\end{pmatrix} \tag{3}$$

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix}2\\-2\end{pmatrix} \tag{4}$$

Checking opposite sides,

$$(\mathbf{B}-\mathbf{A}) = -(\mathbf{D}-\mathbf{C}) \tag{5}$$

Now, as each pair of opposite sides are parallel and equal in length, this means that the given points make up a parallelogram.
Checking for right angle, we need to check for inner product of the adjacent sides of the parallelogram.

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix}5 & 5\end{pmatrix}\begin{pmatrix}-2\\2\end{pmatrix} = 0 \tag{6}$$

This implies that the angle at B is a right angle. A parallelogram with a right angle is a rectangle.
Checking for square:
The given quadrilateral will be a square if its diagonals are orthogonal.

$$(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B}) = \begin{pmatrix}3 & 7\end{pmatrix}\begin{pmatrix}-7\\-3\end{pmatrix} = -42 \tag{7}$$

From (7), we can see that $(\mathbf{C}-\mathbf{A})^\top(\mathbf{D}-\mathbf{B}) \neq 0$, that is the diagonals are not orthogonal and hence the given quadrilateral cannot be a square.

$\therefore$ The quadrilateral **ABCD** is a rectangle.

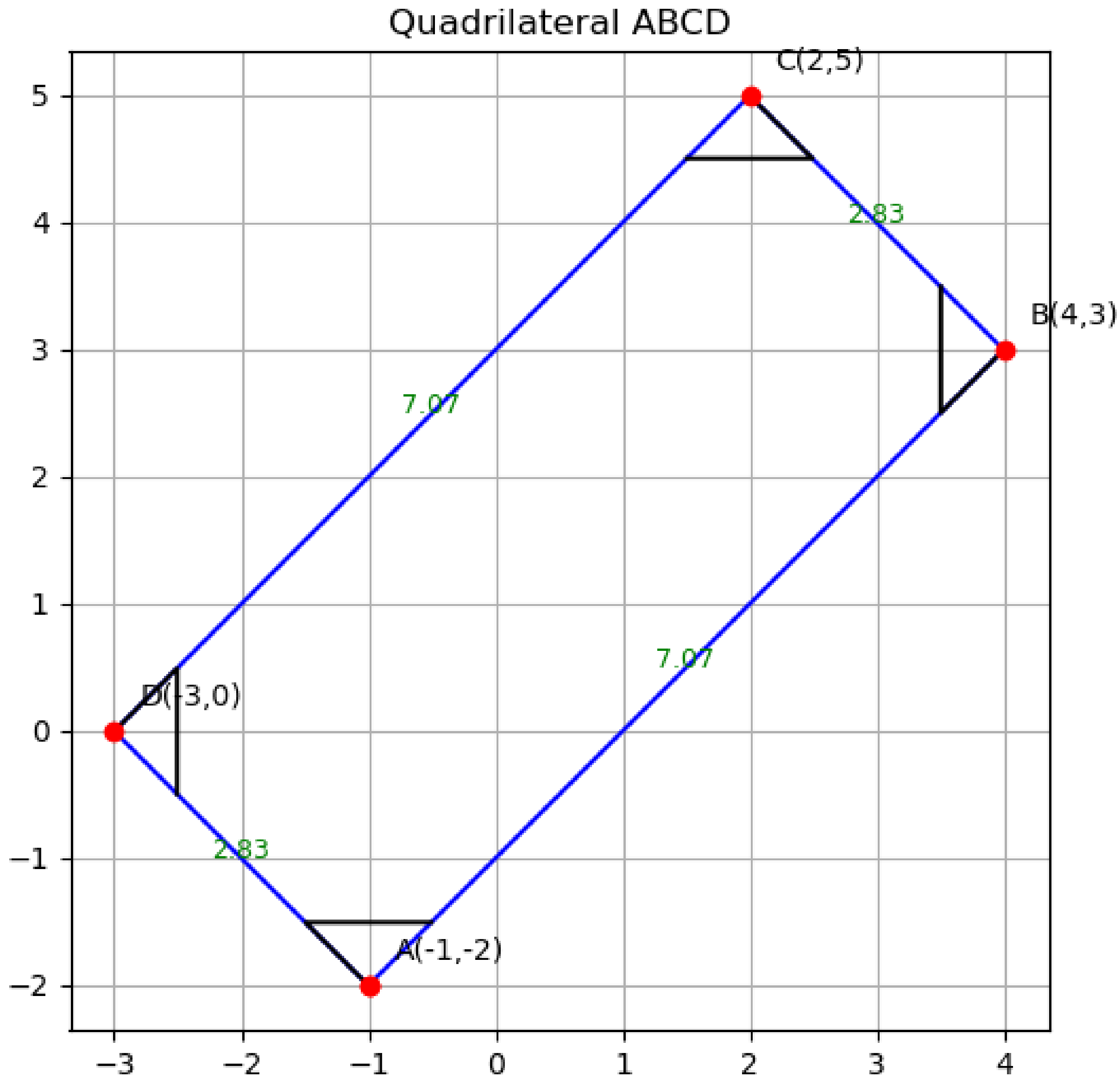

Fig. 1: Plot for 2.4.32