Matrices in Geometry - 1.9.26

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Problem Statement

Find the value of P, if the point $\mathbf{A}(0,2)$ is equidistant from point $\mathbf{B}(3,P)$ and $\mathbf{c}(p,5)$

Solution

Given: $A \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $B \begin{pmatrix} 3 \\ P \end{pmatrix}$ and a point $C \begin{pmatrix} P \\ 5 \end{pmatrix}$ such that **P** is equidistant from **A** and **B**.

$$\therefore \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{C}\| \tag{1}$$

$$\|\mathbf{A} - \mathbf{B}\|^2 = \|\mathbf{A} - \mathbf{C}\|^2$$
 (3)

$$(\mathbf{A} - \mathbf{B})^{\top} (\mathbf{A} - \mathbf{B}) = (\mathbf{A} - \mathbf{C})^{\top} (\mathbf{A} - \mathbf{C})$$
(4)

(5)

Solution

$$\mathbf{A}^{\mathsf{T}}\mathbf{A} - 2\mathbf{A}^{\mathsf{T}}\mathbf{B} + \mathbf{B}^{\mathsf{T}}\mathbf{B} = \mathbf{A}^{\mathsf{T}}\mathbf{A} - 2\mathbf{A}^{\mathsf{T}}\mathbf{C} + \mathbf{C}^{\mathsf{T}}\mathbf{C}$$
 (6)

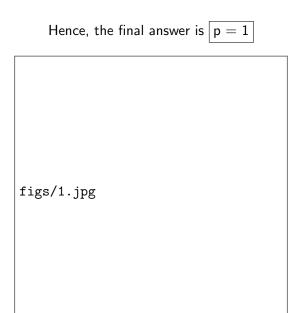
$$\|\mathbf{B}\|^2 - \|\mathbf{C}\|^2 = 2\mathbf{A}^{\top} (\mathbf{B} - \mathbf{C}) \tag{7}$$

$$\left\| \begin{pmatrix} 3 \\ P \end{pmatrix} \right\| - \left\| \begin{pmatrix} P \\ 5 \end{pmatrix} \right\| = 2 \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 3 - P \\ P - 5 \end{pmatrix} \tag{8}$$

$$9 + p^2 - p^2 - 25 = 2(0 + 2p - 10)$$
 (9)

$$-16 = 4p - 20 \implies 4p = 4 \implies p = 1 \tag{10}$$

Final Answer



(11)