4.7.25

EE25BTECH11033 - Kavin

Question:
Find the points on the line $x+y=4$ which lie at a unit distance from the line $4x+3y=10$.

Solution:
According to the question,

$$\text{Equation of line } L_1\text{: } \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \tag{1}$$

$$\implies n_1^\top \mathbf{x} = c_1$$

and

$$\text{Equation of line } L_2\text{: } \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 10 \tag{2}$$

$$\implies n_2^\top \mathbf{x} = c_2$$

The distance λ of a vector $\mathbf{P}$ from the line $\mathbf{n_2}^\top \mathbf{x} = c_2$ is given by ,

$$\lambda = \frac{\left|\mathbf{n_2}^\top \mathbf{P} - c_2\right|}{\|\mathbf{n_2}\|} \tag{3}$$

$$\lambda \|\mathbf{n_2}\| = \left|\mathbf{n_2}^\top \mathbf{P} - c_2\right| \tag{4}$$

$$\implies \mathbf{n_2}^\top \mathbf{P} = c_2 \pm \lambda \|n_2\| \tag{5}$$

Also, as $\mathbf{P}$ lies on line L_1,

$$\mathbf{n_1}^\top \mathbf{P} = c_1 \tag{6}$$

On putting eqns (5) and (6) in matrix form we will get,

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix}^\top \mathbf{P} = \begin{pmatrix} c_1 \\ c_2 \pm \lambda \|\mathbf{n_2}\| \end{pmatrix} \tag{7}$$

where,

$$\lambda = 1$$

On substituting the values we will get,

$$\begin{pmatrix} 1 & 1 \\ 4 & 3 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 4 \\ 10 \pm 5 \end{pmatrix} \tag{8}$$

with the augmented matrix followed by row reduction

$$\xleftrightarrow{R_2=R_2-4R_1}\left(\begin{array}{cc|c}1 & 1 & 4\\ 0 & -1 & -6\pm 5\end{array}\right)\xleftrightarrow{R_2=-R_2}\left(\begin{array}{cc|c}1 & 1 & 4\\ 0 & 1 & 6\mp 5\end{array}\right)$$

$$\xleftrightarrow{R_1=R_1-R_2}\left(\begin{array}{cc|c}1 & 0 & -2\pm 5\\ 0 & 1 & 6\mp 5\end{array}\right)$$

Therefore the points on L_1 which lie at a unit distance from the line L_2 are ,

$$\mathbf{P}=\begin{pmatrix}3\\1\end{pmatrix} \text{ and } \mathbf{P}=\begin{pmatrix}-7\\11\end{pmatrix}$$