1.5.32

EE25BTECH11045 - P.Navya Priya

Find the ratio in which the line segment joining the points $(1,-3)$ and $(4,5)$ is divided by X axis.

Solution: Let $\mathbf{A} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Consider a point $\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ on the X-axis. As the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are collinear The matrix $\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix}^\top$ has rank 1.

$$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix}^\top = \begin{pmatrix} 3 & x-1 \\ 8 & 3 \end{pmatrix}^\top \tag{1}$$

$$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix}^\top = \begin{pmatrix} 3 & 8 \\ x-1 & 3 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 3 & 8 \\ x-1 & 3 \end{pmatrix} \xleftrightarrow{R_2 = 8R_2 - 3R_1} \begin{pmatrix} 3 & 8 \\ 8(x-1)-9 & 0 \end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{8}{3} \\ 8x-17 & 0 \end{pmatrix} \tag{4}$$

$$\xleftrightarrow{R_2 \to R_2 - (8x-17)R_1} \begin{pmatrix} 1 & \frac{8}{3} \\ 0 & \frac{8(17-8x)}{3} \end{pmatrix} \tag{5}$$

To satisfy collinearity condition, the rank of above matrix should be 1. Hence,

$$\frac{8\,(17-8x)}{3} = 0 \tag{6}$$

$$x = 17/8 \tag{7}$$

Assume the ratio $\mathbf{B}$ divides $\mathbf{A}$ and $\mathbf{C}$ be k:1

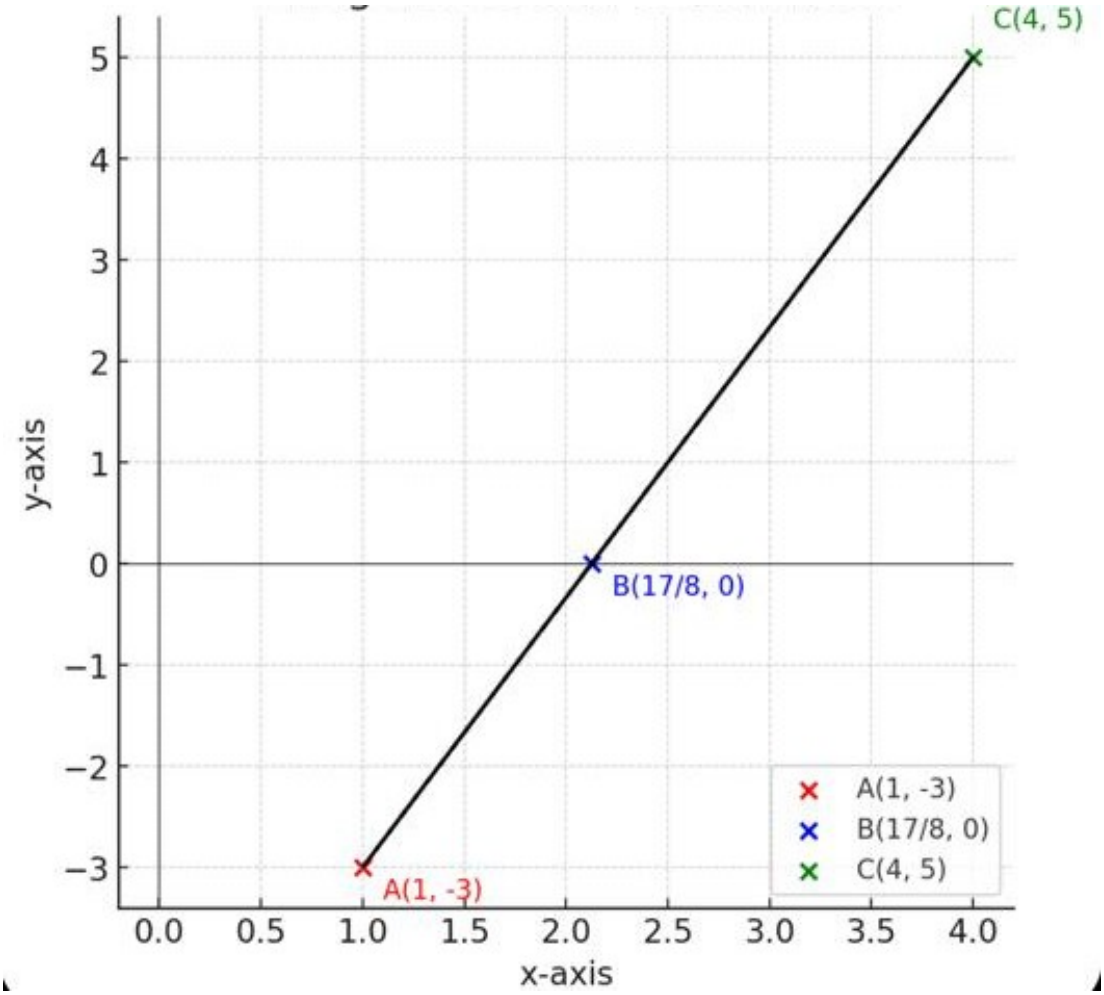
$$k = \frac{\left(\mathbf{A}-\mathbf{B}\right)^\top \left(\mathbf{B}-\mathbf{C}\right)}{\| \left(\mathbf{B}-\mathbf{C}\right) \|^2} \tag{8}$$

Substituting the values of $\mathbf{A}, \mathbf{B}\, and\, \mathbf{C}$

$$k = \frac{1095}{1825} \tag{9}$$

$$k = \frac{3}{5} \tag{10}$$

Hence the ratio is 3:5.



Plot of Intersection of AB by X-axis