2.3.7

EE25BTECH11004 - Aditya Appana

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Question

Find the acute angle between the planes $\mathbf{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$ and $\mathbf{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$

Solution

Let the normal vectors be

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \tag{1}$$

$$\mathbf{n_2} = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \tag{2}$$

The formula to calculate the angle between the two planes is

$$\theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{\mathbf{n_1}^T \mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right) = \sin^{-1}\left(\frac{\mathbf{n_1}^T \mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|}\right)$$
(3)

Substituting n_1, n_2 in this formula:

$$\theta = \sin^{-1} \left(\frac{\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}^T \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}}{\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \| \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} |} \right) = \sin^{-1} \left(\frac{19}{|3||7|} \right) = \sin^{-1} \left(\frac{11}{21} \right)$$
(4)

This is 31.58906757233914°

Plot of the planes

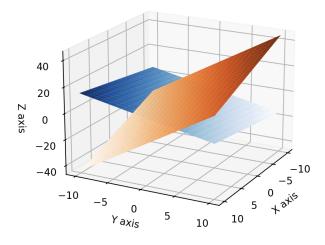


Figure 1: Plot

Plot of the planes

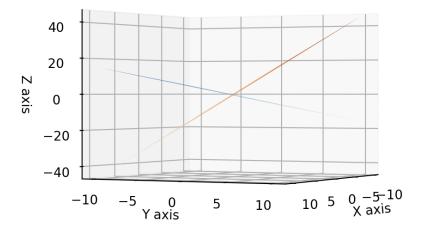


Figure 2: Plot