2.9.19

AI25BTECH11003 - Bhavesh Gaikwad

Question: Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$, $|\overrightarrow{b}| = 2$, and $|\overrightarrow{c}| = 3$. If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$, and $\overrightarrow{b}$ and $\overrightarrow{c}$ are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$.

Solution:
Given:

$$||\mathbf{a}|| = 1,\ ||\mathbf{b}|| = 2,\ ||\mathbf{c}|| = 3 \tag{0.1}$$

$$\text{The projection of } \mathbf{b} \text{ along } \mathbf{a} = \mathbf{b}^T \frac{\mathbf{a}}{||\mathbf{a}||^2}\mathbf{a} \tag{0.2}$$

$$\text{The projection of } \mathbf{c} \text{ along } \mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{||\mathbf{a}||^2}\mathbf{a} \tag{0.3}$$

$$\mathbf{b}^T \frac{\mathbf{a}}{||\mathbf{a}||}\mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{||\mathbf{a}||}\mathbf{a} \tag{0.4}$$

$$\text{Since, } ||\mathbf{a}|| = 1 \Rightarrow \quad \therefore\ \mathbf{b}^T\mathbf{a} = \mathbf{c}^T\mathbf{a} \tag{0.5}$$

Since $\mathbf{b}$ and $\mathbf{c}$ are perpendicular:

$$\mathbf{b}^T\mathbf{c} = 0 \tag{0.6}$$

$$\text{Let } \mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c} \tag{0.7}$$

$$||\mathbf{v}||^2 = (3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})^T(3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}) \tag{0.8}$$

$$||\mathbf{v}||^2 = 9(\mathbf{a}^T\mathbf{a}) - 6(\mathbf{a}^T\mathbf{b}) + 6(\mathbf{a}^T\mathbf{c}) - 6(\mathbf{b}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) - 4(\mathbf{b}^T\mathbf{c}) + 6(\mathbf{c}^T\mathbf{a}) - 4(\mathbf{c}^T\mathbf{b}) + 4(\mathbf{c}^T\mathbf{c}) \tag{0.9}$$

$$\text{Since } \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a}\ \&\ \mathbf{a}^T\mathbf{c} = \mathbf{c}^T\mathbf{a} \tag{0.10}$$

$$||\mathbf{v}||^2 = 9(\mathbf{a}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) + 4(\mathbf{c}^T\mathbf{c}) - 12(\mathbf{a}^T\mathbf{b}) + 12(\mathbf{a}^T\mathbf{c}) - 8(\mathbf{b}^T\mathbf{c}) \tag{0.11}$$

From Equation 0.1 & 0.6,

$$\mathbf{a}^T\mathbf{a} = ||\mathbf{a}||^2 = 1,\ \mathbf{b}^T\mathbf{b} = ||\mathbf{b}||^2 = 4,\ \mathbf{c}^T\mathbf{c} = ||\mathbf{c}||^2 = 9,\ \mathbf{b}^T\mathbf{c} = 0 \tag{0.12}$$

$$\|\mathbf{v}\|^2 = 9 + 16 + 36 \tag{0.13}$$

$$\|\mathbf{v}\|^2 = 61 \quad \Rightarrow \quad \|\mathbf{v}\| = \sqrt{61} \tag{0.14}$$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}} \tag{0.15}$$

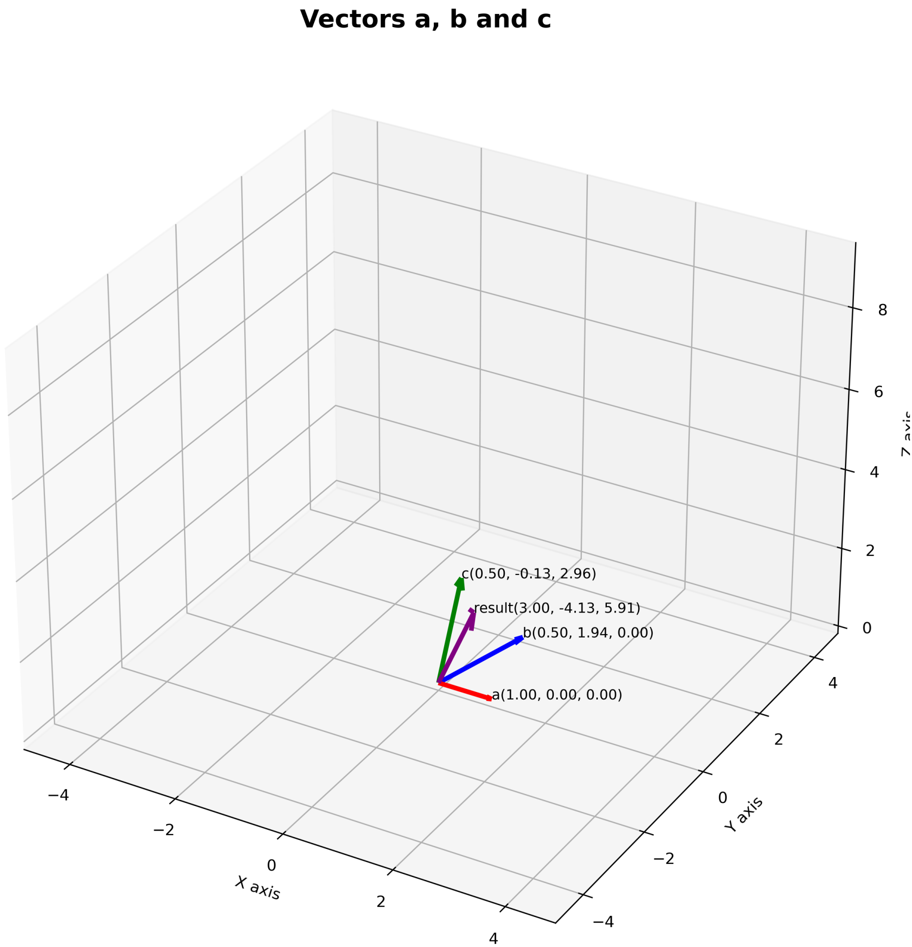

Fig. 0.1: Vector Representation