Matgeo Presentation - Problem 3.2.4

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Question

Construct the triangle $BD'C'$ similar to $\triangle BDC$ with scale factor $\frac{4}{3}$.Draw the line segment $D'A'$ parallel to DA where A^prime lies on extended side BA.Is $A'BC'D'$ a parallelogram?

Description

Solution:

Vector	**Name**
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Vector **A**
$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vector **B**
$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$	Vector **C**
$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Vector **D**

Table: Variables Used

Solution

consider $\triangle$BDC.constructs a $\triangle \mathrm{BD}'C'$ with scale factor $\frac{4}{3}$.
This means

$$\triangle BD'C' \sim \triangle BDC. \tag{0.1}$$

$$\frac{\|\mathbf{D}' - \mathbf{B}\|}{\|\mathbf{D} - \mathbf{B}\|} = \frac{\|\mathbf{C}' - \mathbf{B}\|}{\|\mathbf{C} - \mathbf{B}\|} = \frac{\|\mathbf{C}' - \mathbf{D}'\|}{\|\mathbf{C} - \mathbf{D}\|} = \frac{4}{3}. \tag{0.2}$$

$$\mathbf{D}' = \mathbf{B} + \frac{4}{3}(\mathbf{D} - \mathbf{B}) \tag{0.3}$$

$$\mathbf{D}' = \begin{pmatrix} -4/3 \\ 4 \end{pmatrix} \tag{0.4}$$

$$\mathbf{C}' = \mathbf{B} + \frac{4}{3}(\mathbf{C} - \mathbf{B}) \tag{0.5}$$

$$\mathbf{C}' = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.6}$$

Construct $\mathbf{A}'$

Mark $\mathbf{D}'$ and $\mathbf{A}'$ parallel to $\mathbf{D} - \mathbf{A}$ with $\mathbf{A}'$ along the direction of $\mathbf{B} - \mathbf{A}$.

Solution

$$\mathbf{A}' - \mathbf{D}' = \lambda(\mathbf{A} - \mathbf{D}) \tag{0.7}$$

$$\mathbf{A}' = \begin{pmatrix} -4/3 \\ 4 \end{pmatrix} + \lambda(\begin{pmatrix} 0 \\ -3 \end{pmatrix}) \tag{0.8}$$

$$\implies \mathbf{A}' = \begin{pmatrix} -4/3 \\ 4 - 3\lambda \end{pmatrix} \tag{0.9}$$

$\mathbf{A}'$ lies on line through $\mathbf{B} - \mathbf{A}$ so,

$$\mathbf{A}' = \mathbf{B} + \mu(\mathbf{A} - \mathbf{B}) \tag{0.10}$$

$$\implies \mathbf{A}' = \begin{pmatrix} -4\mu \\ 0 \end{pmatrix} \tag{0.11}$$

$$\tag{0.12}$$

From equation (0.9) and (0.11)

$$\lambda = 4/3 \text{and} \mu = 1/3 \tag{0.13}$$

Solution

$$\implies \mathbf{A}' = \begin{pmatrix} -4/3 \\ 0 \end{pmatrix} \tag{0.14}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.15}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.16}$$

$$\implies \mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.17}$$

Solution

Check the parallelogram property of $A'BC'D'$

$$\mathbf{B} - \mathbf{A}' = -t(\mathbf{A} - \mathbf{B}) \tag{0.18}$$

$$\mathbf{D}' - \mathbf{C}' = k(\mathbf{C} - \mathbf{D}) \tag{0.19}$$

$$\text{From 0.17 } \mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.20}$$

$$\implies \mathbf{B} - \mathbf{A}' = -t(\mathbf{A} - \mathbf{B}) = t(\mathbf{C} - \mathbf{D}) = \frac{t}{k}\mathbf{D}' - \mathbf{C}' \tag{0.21}$$

$$\implies \mathbf{B} - \mathbf{A}' \parallel \mathbf{D}' - \mathbf{C}' \tag{0.22}$$

By construction of A'

$$\mathbf{D}' - \mathbf{A}' \parallel \mathbf{D} - \mathbf{A} \tag{0.23}$$

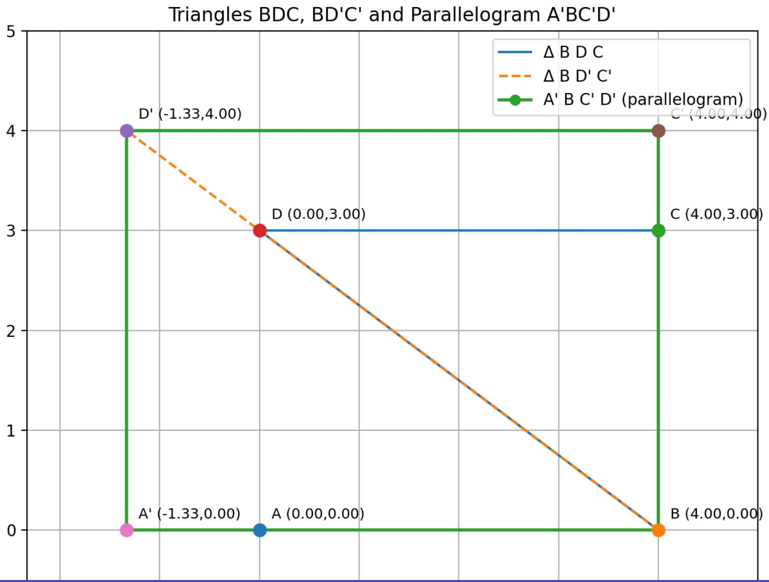
$$\mathbf{D} - \mathbf{A} \parallel \mathbf{C} - \mathbf{B} \tag{0.24}$$

$$\mathbf{C} - \mathbf{B} \parallel \mathbf{C}' - \mathbf{B} \tag{0.25}$$

$$\implies \mathbf{D}' - \mathbf{A}' \parallel \mathbf{C}' - \mathbf{B} \tag{0.26}$$

Conclusion and plot

$\implies A'BC'D'$ a parallelogram



C Code: triangle.c

```
/* triangle.c
   - writes points to "triangle.dat"
   - computes A' so that D'A' || DA and A' lies on extended BA
   - checks whether A' B C' D' is a parallelogram
*/
#include <stdio.h>
#include <math.h>

typedef struct { double x, y; } Point;

int areParallel(Point p1, Point p2, Point q1, Point q2) {
    double dx1 = p2.x - p1.x, dy1 = p2.y - p1.y;
    double dx2 = q2.x - q1.x, dy2 = q2.y - q1.y;
    return fabs(dy1 * dx2 - dy2 * dx1) < 1e-8;
}

int main(void) {
    FILE *fp = fopen("triangle.dat", "w");
    if (!fp) {
        perror("fopen");
        return 1;
    }

    /* Choose ABCD to be a parallelogram (so final shape will be a parallelogram).
       Example: rectangle/parallelogram with A=(0,0), B=(4,0), D=(0,3).
       Then C = B + D - A = (4,3).
    */
    Point A = {0.0, 0.0};
    Point B = {4.0, 0.0};
    Point D = {0.0, 3.0};
    Point C = { B.x + D.x - A.x, B.y + D.y - A.y }; /* ensures ABCD is parallelogram */

    double k = 4.0 / 3.0;
```

C Code: triangle.c

```
/* BD'C' similar to BDC with scale factor k: D' = B + k*(D - B), C' = B + k*(C - B) */
Point Dp = { B.x + k * (D.x - B.x), B.y + k * (D.y - B.y) };
Point Cp = { B.x + k * (C.x - B.x), B.y + k * (C.y - B.y) };

/* Solve for t where A' = B + t*(A - B) and D'A' || DA.
   Derivation:
     Let v = A - D.
     Let u(t) = (B - D') + t*(A - B). (u = A' - D')
     Parallel condition: u.x * v.y - u.y * v.x = 0
   => t = [ (B.y - D'.y)*v.x - (B.x - D'.x)*v.y ]
          / [ (A.x - B.x)*v.y - (A.y - B.y)*v.x ]
*/
double vx = A.x - D.x;
double vy = A.y - D.y;
double numerator = (B.y - Dp.y) * vx - (B.x - Dp.x) * vy;
double denominator = (A.x - B.x) * vy - (A.y - B.y) * vx;

if (fabs(denominator) < 1e-12) {
    fprintf(stderr, "Denominator␣~␣0:␣can't␣find␣unique␣A'␣(degenerate␣configuration)\n");
    fclose(fp);
    return 1;
}

double t = numerator / denominator;
Point Ap = { B.x + t * (A.x - B.x), B.y + t * (A.y - B.y) };

/* Write coordinates */
fprintf(fp, "A␣␣=␣(%.6f,␣%.6f)\n", A.x, A.y);
fprintf(fp, "B␣␣=␣(%.6f,␣%.6f)\n", B.x, B.y);
fprintf(fp, "C␣␣=␣(%.6f,␣%.6f)\n", C.x, C.y);
fprintf(fp, "D␣␣=␣(%.6f,␣%.6f)\n", D.x, D.y);
fprintf(fp, "D'␣=␣(%.6f,␣%.6f)\n", Dp.x, Dp.y);
fprintf(fp, "C'␣=␣(%.6f,␣%.6f)\n", Cp.x, Cp.y);
```

C Code: triangle.c

```
    fprintf(fp, "A'␣=␣(%.6f,␣%.6f)\n", Ap.x, Ap.y);

    /* Check parallelogram: opposite sides parallel */
    int cond1 = areParallel(Ap, B, Dp, Cp); /* A'B || D'C' */
    int cond2 = areParallel(Ap, Dp, B, Cp); /* A'D' || B C' */

    if (cond1 && cond2) {
        fprintf(fp, "\nA'BC'D'␣is␣a␣parallelogram.\n");
        printf("A'BC'D'␣is␣a␣parallelogram.\n");
    } else {
        fprintf(fp, "\nA'BC'D'␣is␣NOT␣a␣parallelogram.\n");
        printf("A'BC'D'␣is␣NOT␣a␣parallelogram.\n");
    }

    fclose(fp);
    return 0;
}
```

Python: plot.py

```
 import numpy as np
import matplotlib.pyplot as plt

# --- Given vertices
A = np.array([0.0, 0.0])
B = np.array([4.0, 0.0])
C = np.array([4.0, 3.0])
D = np.array([0.0, 3.0])

k = 4.0 / 3.0 # scale factor

# --- Compute C' and D' (scaled triangle BDC)
Dp = B + k * (D - B) # D'
Cp = B + k * (C - B) # C'

# --- A' must be such that A'BC'D' is a parallelogram
# In a parallelogram: A' = B + D' - C'
Ap = B + Dp - Cp

# --- Collect points
points = {"A":A, "B":B, "C":C, "D":D, "D'":Dp, "C'":Cp, "A'":Ap}

# --- Print coordinates
print("Corrected coordinates:")
for name, p in points.items():
    print(f"{name:3} = ({p[0]:.6f}, {p[1]:.6f})")

# --- Plotting
plt.figure(figsize=(8,6))

# Original quadrilateral ABCD
plt.plot([A[0],B[0],C[0],D[0],A[0]], [A[1],B[1],C[1],D[1],A[1]],
        'b-', linewidth=1.5, label="ABCD")
```

Python: plot.py

```
# Triangle B-D’-C’
plt.plot([B[0],Dp[0],Cp[0],B[0]], [B[1],Dp[1],Cp[1],B[1]],
        ’r--’, linewidth=1.5, label="␣B␣D’␣C’")

# Parallelogram A’-B-C’-D’
plt.plot([Ap[0],B[0],Cp[0],Dp[0],Ap[0]], [Ap[1],B[1],Cp[1],Dp[1],Ap[1]],
        ’g-o’, linewidth=2, label="Parallelogram␣A’␣B␣C’␣D’")

# Labels
for name, p in points.items():
    plt.scatter(p[0], p[1], s=60, zorder=5)
    plt.text(p[0]+0.1, p[1]+0.1, f"{name}␣({p[0]:.2f},{p[1]:.2f})", fontsize=9)

plt.gca().set_aspect(’equal’, adjustable=’box’)
plt.grid(True)
plt.legend()
plt.title("Parallelogram␣A’␣B␣C’␣D’␣with␣scaled␣triangle␣BD’C’")
plt.show()
```