

1.9.29

EE25BTECH11041 - Naman Kumar

Question:

Find the value of y for which the distance between the points $\mathbf{A}(3, -1)$ and $\mathbf{B}(11, y)$ is 10 units.

Solution:

Distance Between any two vectors =

$$D = \|\mathbf{A} - \mathbf{B}\| \quad (1)$$

Where

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{B})} \quad (2)$$

Given,

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 11 \\ y \end{pmatrix} \quad (3)$$

and distance, $D = 10$

Now using values of Both \mathbf{A} and \mathbf{B} ,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 - 11 \\ -1 - y \end{pmatrix} \quad (4)$$

Next

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 3 - 11 & -1 - y \end{pmatrix} \begin{pmatrix} 3 - 11 \\ -1 - y \end{pmatrix} \quad (5)$$

Putting values in (2)

$$\sqrt{(3 - 11)^2 + (-1 - y)^2} = 10 \quad (6)$$

$$(3 - 11)^2 + (-1 - y)^2 = 100 \quad (7)$$

$$64 + 1 + y^2 + 2y = 100 \quad (8)$$

$$y^2 + 2y - 35 = 0 \quad (9)$$

$$(y + 7)(y - 5) = 0 \quad (10)$$

$$y = -7 \text{ or } 5 \quad (11)$$

Therefore, there are two possible values of $\mathbf{B}(11, 5)$ or $\mathbf{B}(11, -7)$

