

2.4.41

EE25BTECH11044 - Sai Hasini Pappula

Question:

Determine whether the points $A(3, 6, 9)$, $B(10, 20, 30)$, $C(24, -41, 5)$ are the vertices of a right-angled triangle using matrices.

Solution:

$$(A) = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}, \quad (B) = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad (C) = \begin{bmatrix} 24 \\ -41 \\ 5 \end{bmatrix}. \quad (1)$$

$$(B)-(A) = \begin{bmatrix} 10-3 \\ 20-6 \\ 30-9 \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix}, \quad (C)-(B) = \begin{bmatrix} 24-10 \\ -41-20 \\ 5-30 \end{bmatrix} = \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix}, \quad (C)-(A) = \begin{bmatrix} 24-3 \\ -41-6 \\ 5-9 \end{bmatrix} = \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix}. \quad (2)$$

$$((B)-(A))^T((C)-(A)) = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix} = 7 \cdot 21 + 14 \cdot (-47) + 21 \cdot (-4) = -595 \neq 0. \quad (3)$$

$$((B)-(A))^T((C)-(B)) = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 7 \cdot 14 + 14 \cdot (-61) + 21 \cdot (-25) = -1281 \neq 0. \quad (4)$$

$$((C)-(A))^T((C)-(B)) = \begin{bmatrix} 21 & -47 & -4 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 21 \cdot 14 + (-47) \cdot (-61) + (-4) \cdot (-25) = 3261 \neq 0. \quad (5)$$

Since none of the products is zero, no angle of the triangle is 90° .

Conclusion:

The points A , B , and C do **not** form a right-angled triangle.

Triangle formed by points A, B, C

