Question:

Let **P** and **Q** be the points of trisection of the line segment that join the points **A** (2,-2) and **B** (-7,4) such that **P** is closer to **A**. Find the coordinates of **P** and **Q**.

Formula : D divides BC in the ratio k:1,

$$\mathbf{D} = \frac{k\mathbf{C} + \mathbf{B}}{k+1} \tag{1}$$

Solution:

Let
$$\mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -7 \\ 4 \end{pmatrix}$$
 (2)

Point P (Further to A, Ratio 2:1):

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \tag{3}$$

$$\implies \mathbf{P} = \begin{pmatrix} 2 & -7 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \tag{4}$$

$$\mathbf{P} = \begin{pmatrix} \frac{1 \times (-7) + 2 \times 2}{3} \\ \frac{1 \times 4 + 2 \times (-2)}{3} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$
 (5)

Point Q (Nearer from A, Ratio 1:2):

$$\mathbf{Q} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \tag{6}$$

$$\implies \mathbf{Q} = \begin{pmatrix} 2 & -7 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix} \tag{7}$$

(8)

$$\mathbf{Q} = \begin{pmatrix} \frac{2\times(-7)+1\times2}{3} \\ \frac{2\times4+1\times(-2)}{3} \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$
 (9)

$$\mathbf{P} = (-1, 0) \qquad \mathbf{Q} = (-4, 2) \tag{10}$$

Graph of the line segment AB with trisection points P and Q

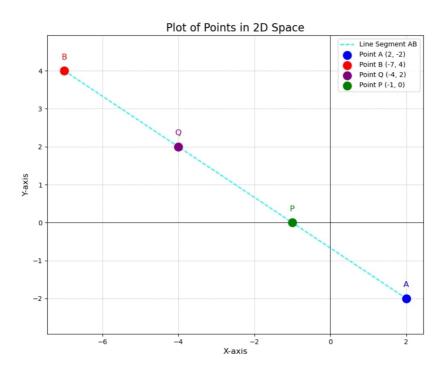


Fig. 0: Figure for 1.5.26