The Digital Image

Problems: Transmission interference, compression artifacts, spilling, scratches, sensor noise, bad contrast and resolution, motion blur

Pixel: Discrete samples of an continuous image function.

Charge Coupled Device (CCD)

Has an array of photosites (a bucket of electrical charge) that charge proportional to the incident light intensity during exposure. ADC happens line by line.

Bloooming: oversaturation of finite capacity photosites causes the vertical channels to "flood" (bright vertical line)

Bleeding/Smearing: While shifting down, the pixels above get some photons on bright spot with electronic shutters.

Dark Current: CCDs produce thermally generated charge they give non-zero output even in darkness (fluctuates randomly) due to spontaneous generation of electrons due to heat → cooling.

can be avoided by cooling, worse with age.

CMOS:

Same sensor elements as CCD, but each sensor has its own amplifier → faster readout, less power consumption, cheaper, more noise.

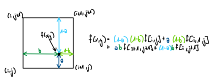
more noise, lower sensitivity

vs CCD cheaper, lower power, less sensitive, per pixel amplification random pixel access, no blooming, on chip integration

Sampling methods

Cartesian (grid), hexagonal, non-uniform

Quantization: Real valued function will get digital values (integers). A lossy process (original can-not be reconstructed). Simple version: equally spaced $2^b = \#bits$ levels



Bilinear Interpolation:

Resolution: Image resolution (cropping), geometric resolution (#pixels per area), radiometric resolution (#bits per pixel, color)

Image noise: commonly modeled by additive Gaussian noise: $I(x,y) = f(x,y) + c$, poisson noise (shot noise for low light, depends on signal & aperture time), multiplicative noise: $I = f + f \cdot c$, quantization errors, salt-and-pepper noise. SNR or peak SNR is used as an index of image quality $c \sim N(0, \sigma^2)$, $p(c) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{(c-\mu)^2}{2\sigma^2}\right)$, SNR: $S = \frac{F}{\sigma}$ where $F = \frac{1}{XY}\sum_{x=1}^{X}\sum_{y=1}^{Y} f(x,y)$.

Color cameras

Prism need 3 sensors and good alignment

Filter mosaic coat ☐ directly on sensor

Wheel multiple filters in front of same sensor

New CMOS sensor layers that absorb color at different depths → better quality

Image Segmentation

Complete segmentation

Finite set of non-overlapping regions that cover the whole image $I = \bigcup_{i=1}^{n} R_i$ and $R_i \cap R_j = \emptyset \; \forall i,j, i \neq j$

Thresholding: simple segmentation by comparing greylevel with a threshold to decide if in or out.

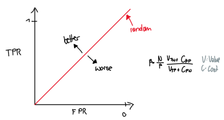
Chromakeying: when planning to segment, use special backgroundcolor. (Problems variations due to lighting, noise, ... mixed pixels (hard α-mask does not work)) $I_\alpha = |I - g| > T$

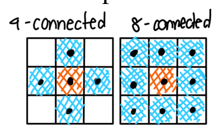
Receiver Operating Characteristic (ROC) analysis:

ROC curve characterizes performance of binary classifier Classification errors: False negative (FN), false positives (FP)

ROC curve plots TP fraction $\frac{TP}{TP+FN}$ vs FP fraction $\frac{FP}{FP+TN}$

Operating points: choose point with gradient





Pixel connectivity

also regions if x-connected

Connected component raster scanning: scanning row by row, if foreground & label if connected to other label, else give new label. (second pass to find equivalent labels)

Improve: when region found, follow border, then carry on (contour-based method)

Region growing

Start with seed point or region, add neighboring pixels that satisfy a criteria defining a region until we include no more pixels.

Seed region: by hand or automatically by conservative Thresholding

Inclusion criteria: greylevel thresholding, greylevel distribution model (include if $(I(x,y) - \mu^2)^2 < (n\sigma)^2$ and update μ and σ after each iteration) color or texture information

Snakes: active contour, a polygon and each point moves away from seed while criteria is met (can have smoothness constraint) Iteratively minimize enery function $E = E_{tension} + E_{stiffness} + E_{image}$

background subtraction

simple: $I_\alpha = |I - I_{bg}| < T$ better: $I_\alpha = \sqrt{(I - I_{bg})^T \Sigma^{-1} (I - I_{bg})}$ where Σ is the background pixel appearance covariance matrix, computed seperately for each pixel.

Morphological operators

Logical transformations based on comparison of neighboring pixels

erode delete FG pixels with 8-connected BG pixels

dilate every BG pixels with 8-connected FG pixel make a FG pixel

Uses: smooth regions, remove noise and artifacts.

Image Filtering

Modify the pixels of an image based on some function of the local neighborhood of the pixels- If sum greater 1 get brighter, if smaller darker.

separable: if a kernel can be written as a product of two simpler filters → computationally faster (filter $P \times Q$, image $N \times M$: $(P+Q) * NM$ instead of $PQNM$)

shift invariant: Doing the same thing, applying the same function over all pixels (in the formula below if K does not depend on x, y)

linear: linear combination of neighbors can be written as: $I'(x,y) = \sum_{(i,j) \in \mathbb{N}(i,j)} K(x,y,i,j) I(x+i, y+j)$ TODO

Filter at edges: clip filter (black), wrap around, copy edge, reflect across edge, vary filter near edge

Correlation

$I' = K \circ I, I'(x,y) = \sum_{(i,j)\in\mathbb{N}(i,j)} K(i,j) I(x+i, y+j)$ e.g. template matching: search for best match by minimizing mean squared error or maximizing area correlation. (remove mean (from filter, from image) to avoid bias)

Convolution

$I' = K * I, I'(x,y) = \sum_{(i,j)\in\mathbb{N}(i,j)} K(i,j) I(x-i, y-j)$ if $K(i,j) = K(-i,-j) \implies corrolation = convolution$

Continuous: $(f*g)(t) = \int_{-\infty}^{\infty} f(\tilde{t}) g(t - \tilde{t}) d\tilde{t} = \int_{-\infty}^{\infty} f(t-\tilde{t}) g(\tilde{t}) dt$

Kernels

Box filter: all same values normalized to sum = 1

Gaussian Kernel: $K(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$ is separable, e.g. $\sigma = 1$

Rotationally symmetric, has single lobe, single lobe in frequency domain, simple relationship to σ easy to implement efficiently, neighbors decrease monotonically, no corruption from higher frequency.

Subtracting one from central element of low-pass filter gives a high-pass filter with inverted sign, because. $(f - \delta) * a = f * a - \delta * a = f * a - a = -(a - (f * a))$

Band pass filter: ▣ do LPF and HPF with cutoffs $D_{LP} < D_{HP}$

Band reject filter: ▣ do LPF and HPF with cutoffs $D_{LP} > D_{HP}$

Image sharpening: increases high frequency components to enhance edges: $I' = I + \alpha |K * I|$ K : high-pass filter, α: scalar $\in [0,1]$ds

Features

Edge Detection

How to tell if there is an edge? Local maxima of the first derivative and the zero crossing of the second derivative.

Edge detection filters:

Sobel:
$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, K_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Prewitt:
$K_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, K_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Roberts:
$K_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, K_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Gradient Magnitude:
$M(x,y) = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$

Gradient Angle:
$\alpha(x,y) = \tan^{-1}(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x})$

Laplacian operator

detect discontinuities by considering second derivative $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are discrete space approximations. Is isotropic(rotationally invariant), zero crossings make edge locations. Sensitive to fine de-

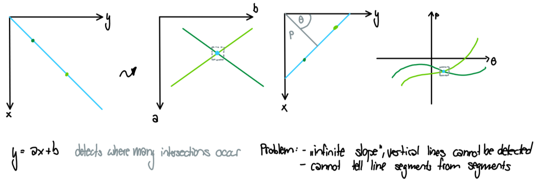
tails and noise (→ smoothing before applying).
blur image first (LoG)
Laplacian of Gaussian (LoG)**:** convolve gaussian blurring and laplacian operator in LoG operator (cheaper) $LoG(x,y) = -\frac{1}{\pi\sigma^4}(1-\frac{x^2+y^2}{2\sigma^2})e^{-\frac{x^2+y^2}{2\sigma^2}}$

Canny Edge Detector: 5 Steps

1. smooth image with a Gaussian filter
2. compute gradient magnitude and angle
3. apply non-maximum suppression to gradient magnitude image (Quantize edge normal to one of four directions: horizontal, +45°, vertical, -45°. If $M(x,y)$) smaller thn either of its neighbors in edge normal direction suppress, else keep
4. Double thresholding to detect strong and weak edge pixels
5. Reject weak edge pixels not connected to strong edge pixels

Hough Transform

Fitting a straight line to a set of edge



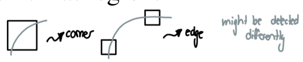
Alternative parameterization: $x\cos(\theta) + y\sin(\theta) = p$
Alternative parameterization: $x\cos(\theta) + y\sin(\theta) = p$
For circles: if r known: calculate circles with radius r around edge pixels → intersection of circles gives center.
where lots of them meet is the center of a circle.
else: use 3D hough transform with parameters (x_0, y_0, r)

Corner Detection

Edges are only well localized in one direction → detect corners.
Desirable properties: Accute localization, invariance against shift, rotation, scale, brightness change, robust against noise, high repeatability
Linear approximation for small $\Delta x \Delta y$: (Taylor) $f(x+\Delta x, y+\Delta y) \approx f(x,y) + f_x(x,y)\Delta x + f_y(x,y)\Delta y$

Local displacement sensitivity (Harris corners)

$S(\Delta x,\Delta y) = (\Delta x \Delta y)\sum_{x,y\in\text{window}} \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \approx$ SSD Find points where $\min \Delta^T M \Delta$ is large for $||\Delta|| = 1$ i. e. maximize the eigenvalues of M
$C(c,y) = \det(M) - k * trace(M)^2 = \lambda_1 * \lambda_2 + k * (\lambda_1 + \lambda_2)$ Harris cornerness: Measure of cornerness
Robustness of Harris corner detector: Invariant to brightness offset, invariant to shift and rotation but not to scaling! $\lambda_1 >> \lambda_2 \rightarrow$ edge, λ_1 and λ_2 large → corner, else → flat region.

not scale invariant: 

Overcome issues: look for strong DoG response or consider local maxima in position and scale space, Gaussian weighing.

Lowe's SIFT features

Look for strong responses of difference of Gaussians (DoG) filter, only look at local maxima in both position and scale.

DoG: $DoG(x,y) = \frac{1}{k} * e^{-\frac{x^2+y^2}{(k\sigma)^2}} - e^{-\frac{x^2+y^2}{\sigma^2}}$ e.g. $k = \sqrt{2}$
Orientation: create histogram of local gradient directions computed at selected scale, assign canonical orientation at peak of smoothed histogram. Get a SIFT descriptor (threshold image gradients are sampled over 16×16 array of locations in scale space) and do matching with these. Invariant to scale, rotation, illumination and viewpoint.

1 Fourier Transformation

Aliasing: Happens when undersampling e.g. taking every second pixel, else characteristic errors appear: typically small phenomena look bigger, fast phenomena look slower. (e.g. wagon wheels backwards in movies, checkerboards misrepresented)

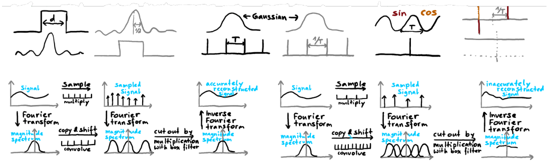
Fourier Transform

Represent function on a new basis with basis elements $e^{i2\pi(ux+vy)} = \cos(2\pi(ux+vy)) + i\sin(2\pi(ux+vy))$
$F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi fx}dx$, **2D:** $F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-2\pi i(ux,vy)}dxdy$
For images: transformed image → $F = U * f$ ← vectorized image, U: Fourier matrix
For discrete: $F(u,v) = \sum_{x=0}^{N-1}\sum_{y=0}^{M-1} f(x,y)e^{-2\pi i(\frac{ux}{N},\frac{vy}{M})}$
1D-periodic function: $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{i2\pi nt}{T}}$, $c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{\frac{-i2\pi nt}{T}}dt$

Properties of Fourier transform

Linearity: $F(ax(t)+by(t)) = aX(t)+bY(t)$
Time Shift: $F(x(t\pm t_0)) = X(t)e^{\pm i2\pi f t_0}$
Frequency Shift: $F(e^{i2\pi f_0 t}x(t)) = X(f-f_0)$
Scaling: $F(x(at)) = \frac{1}{|a|}X\left(\frac{f}{a}\right)$
Convolution: $F(x(t)*y(t)) = X(f)\cdot Y(f)$
Duality: $F(X(t)) \longleftrightarrow x(-f)$
Sampling:
A sampling function $s(t)$ which is an impulse train with period T and its Fourier transform $S(f)$:
$S(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$
$S(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f-\frac{n}{T})$ where $\delta(*)$ Dir.-delt. f.
A continuous signal can be sampled by multiplying with $s(t)$: $x_s(t) = x(t)s(t)$To compute the Fourier Transform of $x_s(t)$, we can use the convolution theorem:
$F(x_s(t)) = X(t) * S(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f-\frac{n}{T}) * X(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(f-\frac{n}{T})$
Sampling in 2D:$sample_{2D}(f(x,y)) = \sum_{i=\infty}^{\infty}\sum_{j=\infty}^{\infty} f(x,y) * \delta(x-i, x-j) = f(x,y)\sum_{i=\infty}^{\infty}\sum_{j=\infty}^{\infty}\delta(x-i,x-j)$
DFT:
$F(f(x,y))(\sum_{i=\infty}^{\infty}\sum_{j=\infty}^{\infty}\delta(x-i,x-j))F(f(x,y)) * F(\sum_{i=\infty}^{\infty}\sum_{j=\infty}^{\infty}\delta(x-i,x-j)) = \sum_{i=\infty}^{\infty}\sum_{j=\infty}^{\infty} F(u-i, v-j)$
Dirac Delta Function:
$\int_{-\infty}^{\infty}\delta(t)dt = 1$ and $\delta(t) = \begin{cases} 0 & \text{for } x \neq 0 \\ und. & \text{for } x = 0 \end{cases}$
Sifting Property:
$\int_{-\infty}^{\infty} f(t)\cdot\delta(x-a)dx = f(a)$
Dirac Comb:
$\text{III}_T(x) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$,
sampling = product with this
TODO!
Box Filter: $\text{rect}(t) = \begin{cases} 1, & \text{if } |t| \leq \frac{1}{2} \\ 0, & \text{otherwise.} \end{cases}$
Triangle Filter: $\text{tri}(t) = \begin{cases} 1-|t|, & \text{if } |t| \leq 1 \\ 0, & \text{otherwise.} \end{cases}$

Fourier transform of important functions



Nyquist Sampling theorem

The sampling frequency must be at least twice the highest frequency $w_s \geq 2w$ If not the case: band limit before with low-pass filter. Perfect reconstruction: $sinc(x) = \frac{sin(\pi x)}{\pi x}$
Why should this hold? Function $f(t)$, sampling function $S_{\Delta t}(t)$ with sampling frequency w_s. Fourier transform of the sampled function can be derived as

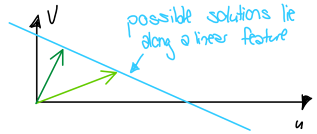
$$\begin{aligned}\tilde{F}(u) &= F(f(t)\cdot S_{\Delta t}(t)) \\ &= F(u) * S_{\Delta t}(w) \\ &= \int_{-\infty}^{\infty} F(\tilde{t})S_{\Delta t}(w-\tilde{t})\,d\tilde{t} \\ &= \int_{-\infty}^{\infty} F(\tilde{t})\frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta(w-\tilde{t}-\frac{n}{\Delta T})\,d\tilde{t} \\ &= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F(w-nw_s).\end{aligned}$$

If we want to reconstruct the signal $f(t)$ from F and $S_{\Delta t}$, $F(w)$ cannot overlap with its neighbors $F(w-w_s)$ and $F(w+w_s)$. Thus, w_s should be larger than w_n. Highest frequency of $f(t)$.
Image restoration problem: $f(x) \rightarrow h(x) \rightarrow g(x) \rightarrow \tilde{h}(x) \rightarrow f(x)$
The "inverse" kernel $\tilde{h}(x)$ should compensate $h(x)$.
May be determined by: $F(\tilde{h})(u,v)\cdot F(h(u,v)) = 1$
Problems: Convolution with kernel k may cancel out some frequencies & noise amplification.
Avoid: Regularization: $F(\tilde{h})(u,v) = \frac{F(h)}{|F(h)|^2+\varepsilon}$ avoid singularities

2 Optical Flow

Apparent motion of brightness patterns use extracted feature points and commpute their velocity vectors projection of 3D velocity vectors on I
Problem: cannot distingish motion from changing lighting! also estimate observed projected motion field normal flow not always well defined
Key assumptions:
Brightness constancy: Proection of the same point looks

the same in every frame.
Small motion: Points do not move far
Spatial coherence: Points move like their neighbors
Brightness constancy constraint: $I(x,y,t-1) = I(x+u,y+v,t)I = Intensity$
Small motion $\rightarrow$ can linearize with Taylor expansion:
$I(x,y,t-1) \approx I(x,y,t-1) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \frac{\partial I}{\partial t} \approx 0$ or shorthand $I_x \cdot u + I_y \cdot v + I_t \approx 0$
move $I-t$ on one side, vectorize unknowns. For LK, sum up over a window of pixes
Apature problem: 1 equation, 2 unknowns cannot determine exact location, take normal flow.



Add additional smoothness constraint:
$e_s = \int\int (u_x^2 + u_y^2 + v_x^2 + v_y^2) dxdy \; close \approx parallel$
Besides OF constraint:
$e_c = \int\int (I_x u + I_y v + I_t)^2 dxdy$ Minimize $e_s + \lambda e_c$

Lukas-Kanade

Assume same displacement for $N \times M$ window $\rightarrow$ linear least squares problem:
$\begin{bmatrix} I_x(x_1,y_1) & I_y(x_1,y_1) \\ \vdots & \vdots \\ I_x(x_{NM},y_{NM}) & I_y(x_{NM},y_{NM}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(x_1,y_1) \\ \vdots \\ I_t(x_{NM},y_{NM}) \end{bmatrix} \implies$
$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_y I_x & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$
When solvable? $A^T A$ invertible, eigenvalues λ_1, λ_2 large, $\frac{\lambda_1}{\lambda_2}$ small
Errors: motion is large(r than a pixel)
$\rightarrow$ iterative refinement and coarse-to-fine estimation.
A point does not move like its neighbors
$\rightarrow$ motion segmentation.
Brightness constancy does not hold:
$\rightarrow$ exhaustive neighborhood search with normalized corrolation.
KLT feature tracker: to find patches where LSE well-behaved $\rightarrow$ LK-flow
Iterative refinement: Estimate velocity, warp using estimate, refine,...
Coarse-toFine Estimation: Image Pyramid. Start small, compute OF, rescale, take larger and initialize with last estimate
Applications: Image stabilization (get flow between two frames and warp image using same OF for all pixels s.st. OF close to 0) frame interpolation, video compression, object tracking, motion segmentation

Affine motion: $I_x(a_1 + a_2 x + a_3 y) + I_y(a_4 + a_5 x + a_6 y) + I_t \approx 0$

SSD tracking: For large displacements: match template against each pixel in small area around, match measure can be (normalized) correlation or SSD choose max. as match (sub-pixel also possible)
Bayesan Optical Flow: Some low-level motion illusions can be explained by adding an underlying model to LK-tracking e.g. brightness constancy with noise.

3 Video Compression

Interlaced video format: 2 temporally shifted half images
$\rightarrow$ increase frequency, decrease spatial resolution $\rightarrow$ not progressive
Lossy video compression: take advantage of redundancy spatial correlation between pixels, temporal correlation between frames
$\rightarrow$ basically drop perceptually unimportant details
with optical flow: Encode optical flow based on previous frame can cause blocking artifacts, does not work well for lots of movemen, fast movement and scene changes.
If temporal redundancy fails $\rightarrow$ use motion-compensated prediction
Types of coded frames:
I-Frame: Intra-coded frame, coded independently of all others
P-Frame: Predictively coded frame, based on previously coded frame
B-Frame: Bi-directionally coded frame, based on previous & future

Block-Matching Motion Estimation:

Is a type of temporal redundancy reduction
Motion Estimation Algorithm ME

1. Partition frame into blocks (e.g. 16×16 pixels)
2. For each block, find the best matching block in reference frame

Metrics for best match: sum of differences or squared sum of diff.
Candidate blocks: All blocks in e.g. 32×32 pixel area
Search strategies: Full search, partial (fast) search
Motion Compensation Algorithm MC Use the best matching of reference frame as prediction of blocks in current frame
$\rightarrow$ gives motion vectors & MC prediciotn error or residual (encode with conventionl image coder)
Motion Vector: relative horizontal & vertical offsets of a given block from one frame to another
Not limited to integer-pixel offsets, can use half-pixel ME to capture sub-pixel motion.
Half-pixel ME (coarse-fine) algorithm:

1. Coarse step: find best integer move
2. Fine step: refine by spatial interpolation and best-matching

Advantages and disadvanages
+ good, robust performance, one MV per block $\rightarrow$ useful for compression, simple periodic structure (GoP)
- assumes translational motion (fails for complex motion)
$\rightarrow$ codes these frames/blocks without prediction produces blocking artifacts
MPEG-GoP IBBPBBPBBI dependencies between frames

Scalable Video Coding:

Decompose video into multiple layers of prioritized importance: e.g.
temporal scalability: Include B-frames or not
spatial scalability: Base resolution + upsampling difference
SNR scalability: Base with coarse quatizer + fine quantizer
Benefits: Adapting to different bandwidths, facilitates error resiliency by identifying more and less important bits.

4 Unitary Transforms

Vectorization: interpret image as vector row-by-ow: $I = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$
linear image processing: can be written as $\vec{g} = H\vec{f}$
Image collection (IC): $F = [f_1, f_2 \ldots f_n]$
Autocorrelation matrix $Rff = \frac{F \cdot F^H}{N}$ its Eigenvector with largest Eigenvalue is direction of largest variance among pictures.
Unitary transform: for transform A iff $A^H = A^{-1}$
if real-valued $\rightarrow$ orthonormalevery unitary transform is a rotation + sign flip, length conserved
Energy conservation: $||\vec{C}||^2 = \vec{C}^H C = \vec{f}^H A^H A f = ||\vec{f}||^2$

Karhunen-Loeve Transform

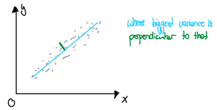
Same as PCA. Order by decreasing eigenvalues
Energy concentration property: no other unitary transform packs as much energy in the first J coefficients (for arbitrary J) and mean squared approximation error by choosing only first J coefficients is minimized.
Optimal energy concentratioin of KLT consider truncated coefficient vector $\vec{b} = I_J\vec{c}$ (I_J: identity matrix with first J columns) Energy in first J coefficients for an arbitrary transform A: $E = Tr(R_{bb}) = Tr(I_J R_{cc} I_J) = Tr(I_J A R_{ff} A^H I_J) = \sum_{k=0} J = 1 a_k^T R_{ff} a_k^*$ where a_k^T is $k-th$ row of A. Lagrangian cost function to enforce unit-length basis vectors: $L = E + \sum_{k=0}^{J-1} \lambda_k (1 - a_k^T a_k^*) = \sum_{k=0}^{J-1} a_k^T R_{ff} a_k^* + \sum_{k=0}^{J-1} \lambda_k (1 - a_k^T a_k^*)$
Differentiating L with respect to a_j: $R_{ff} a_j^* = \lambda_i a_j^* \quad \forall_j < J$ necessary condition

Simple recognition

SSD between images, best match wins very expensive, since need to correlate with every image

Principle Component analysis PCA

Steps: standardize data, get Eigenvectors and values from covariance matrix or do SVD, sort Eigenvalues and vectors in descending order get j largest components, construct projection matrix from selected j Eigenvectors transform dataset by multiplying with projection matrix.



Uses of PCA: lossy compression by keeping only the most important k components. Face recognition eigenfaces and face detection.

Eigenspace matching

Do PCA with mean subtraction and get closest rank-k approximation of database images (eignfaces)
For a new query: normalize, subtract mean (of database) project to subspace then do similarity matching with eigenfaces.

4.1 Fischerfaces:

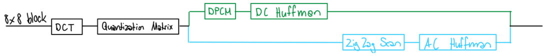
Find directions where ratio between / within individual variance is maximized. Linearly project to basis where dimension with good signal: noise ratio is maximized.

$$W_{\text{opt}} = \underset{W}{\operatorname{argmax}} \frac{\det(WR_BW^H)}{\det(WR_WW^H)}, R_b = \sum R_B \sum_{i=1} cN_i(\vec{\mu}_i - \vec{\mu})(\vec{\mu}_i - \vec{\mu})^H, R_W = \sum_{i=1}^{c} \sum_{\Gamma_l \in Class} (\Gamma_l - \mu_i)(\Gamma_l - \mu_i)^H$$

Fischer linear discriminant analysis (LDA)**:** maximize between class scatter, while minimizing within less scatter

JPEG Compression

Divide image into 8×8 block:



DC: First coefficient (general intensity)

ZigZag: 

Quantization Table: Divide by this value, round to nearest integer

Why DCT? Compared to DFT uses only real values, easier to compute

5 Pyramids and Wavelets

Scale-space representations

From an original signal $f(x)$ generate a parametric family of signals $f^+(x)$ where fine-scale information is successively suppressed e.g. successive smoothing or image pyramids (smooth & downsample)

Applications: Search for correspondence (look at coarse scale, then refine with finer scale) edge tracking coarse to fine estimation control of detail and computational cost (e.g. textures)

Example: CMU face detection: need different scales for template to match.

6 Textures

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7 Augmented Reality

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8 Drawing Triangles

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9 Transforms

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10 Transform Geometry and Textures

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11 Graphics Pipeline

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12 Rendering Equation

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13 Rigging, FK and IK

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14 Physically-based Animation

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15 Partial Differential Equations

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16 Animation

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