Chapter 3: Decision Analysis

3.1 Introduction

In decision analysis, decision–making scenarios are divided into the following three categories.

1. Decision making under certainty
2. Decision making under uncertainty
3. Decision making under risk

3.2 The Decision Table and Decision Making Under Certainty

Many decision analysis problems can be viewed as having three variables: decision alternatives, states of nature, and payoffs.

Decision alternatives are the various choices or options available to the decision maker in any given problem situation.

Example:

1. On most days, financial managers face the choices of whether to invest in blue chip stocks, bonds, commodities, certificates of deposit, money markets, annuities, and other investments.
2. Construction decision makers must decide whether to concentrate on one building job today, spread out workers and equipment to several jobs, or not work today.

A good decision maker identifies many options and effectively evaluates them.

States of nature are the occurrences of nature that can happen after a decision is made that can affect the outcome of the decision and over which the decision maker has little or no control.
These states of nature can be literally natural atmospheric and climatic conditions or they can be such things as the business climate, the political climate, the worker climate, or the condition of the marketplace, among many others.

Example:

1. The financial investor faces such states of nature as the prime interest rate, the condition of the stock market, the international monetary exchange rate, and so on.
2. A construction company is faced with such states of nature as the weather, wildcat strikes, equipment failure, absenteeism, and supplier inability to deliver on time.

States of nature are usually difficult to predict but are important to identify in the decision–making process.

The **payoffs** of a decision analysis problem are the benefits or rewards that result from selecting a particular decision alternative. Payoffs are usually given in terms of money.

In the financial investment industry, for example, the payoffs can be small, modest, or large, or the investment can result in a loss. Most business decisions involve taking some chances with personal or company money in one form or another. Because for–profit businesses are looking for a return on the money invested, the payoffs are extremely important for a successful manager. The trick is to determine which decision alternative to take in order to generate the greatest payoff. Suppose a CEO is examining various environmental decision alternatives. Positive payoffs could include increased market share, attracting and retaining quality employees, consumer appreciation, and governmental support. Negative payoffs might take the form of fines and penalties, lost market share, and lawsuit judgments.

3.2.1 Decision Table

The concepts of decision alternatives, states of nature, and payoffs can be examined jointly by using a **decision table,** or **payoff table**. The following table shows the structure of a decision table.

		State of Nature			
		s_1	s_2	$\cdots$	s_n
Decision Alternative	d_1	P_{11}	P_{12}	$\cdots$	P_{1n}
	d_2	P_{21}	P_{22}	$\cdots$	P_{2n}
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	d_m	P_{m1}	P_{m2}	$\cdots$	P_{mn}

where d_i are the various decision alternatives,

s_j are the states of nature,

P_{ij} are the various payoffs for each decision alternative under each state of nature.

Example 3.1

The investor is faced with the decision of where and how to invest $RM\,10{,}000$ under several possible states of nature.

The investor is considering four decision alternatives.

1. Invest in the stock market
2. Invest in the bond market
3. Invest in government certificates of deposit (CDs)
4. Invest in a mixture of stocks and bonds

Because the payoffs are in the future, the investor is unlikely to know ahead of time what the state of nature will be for the economy. However, the table delineates three possible states of the economy.

1. A stagnant economy
2. A slow–growth economy
3. A rapid–growth economy

The corresponding decision table is:

Yearly Payoffs on an investment of $RM\,10{,}000$

		State of Economy		
		Stagnant	Slow Growth	Rapid Growth
Investment Decision Alternative	Stocks	–500	700	2200
	Bonds	–100	600	900
	CDs	300	500	750
	Mixture	–200	650	1300

The decision table lists the payoffs for each possible investment decision under each possible state of the economy. Note that the largest payoff comes with a stock investment under a rapid–growth economic scenario, with a payoff of $RM\,2{,}200$ per year on an investment of RM 10,000. The lowest payoff occurs for a stock investment during stagnant economic times, with an annual loss of RM 500 on the RM 10,000 investment.

3.2.2 Decision Making Under Certainty

The most elementary of the decision–making scenarios is **decision making under certainty.**

In making decisions under certainty, *the states of nature are known*. The decision maker needs merely to examine the payoffs under different decision alternatives and select the alternative with the largest payoff.

In *Ex. 3.1*, if it is known that the economy is going to be stagnant, the investor would select the decision alternative of CDs, yielding a payoff of RM 300. Indeed, each of the other three decision alternatives would result in a loss under stagnant economic conditions.

If it is known that the economy is going to have slow growth, the investor would choose stocks as an investment, resulting in a RM 700 payoff.

If the economy is certain to have rapid growth, the decision maker should opt for stocks, resulting in a payoff of RM 2,200.

Decision making under certainty is almost the trivial case.

3.3 Decision Making Under Uncertainty

Decision making under uncertainty occurs when it is unknown which states of nature will occur and the probability of a state of nature occurring is also unknown. Hence, the decision maker has virtually no information about which state of nature will occur, and he or she attempts to develop a strategy based on payoffs.
Several different approaches can be taken to making decisions under uncertainty.

3.3.1 Maximax Criterion

The **maximax criterion** approach is an optimistic approach in which the decision maker bases action on a notion that the best things will happen. The decision maker *isolates the maximum payoff under each decision alternative and then selects the decision alternative that produces the highest of these maximum payoffs*.

The name "maximax" means selecting the maximum overall payoff from the maximum payoffs of each decision alternative.

Example 3.2

Reconsider *Ex. 3.1*, determine which decision alternative to select and state the corresponding payoff by using the Maximax criterion.

3.3.2 Maximin Criterion

The **maximin criterion** approach to decision making is a pessimistic approach. The assumption is that the worst will happen and attempts must be made to minimize the damage. The decision maker starts by examining the payoffs under each decision alternative and selects the worst, or minimum, payoff that can occur under that decision. Then the decision maker *selects the maximum or best payoff of those minimums selected under each decision alternative*. Thus, the decision maker has maximized the minimums.

Example 3.3

Reconsider *Ex. 3.1*, determine which decision alternative to select and state the corresponding payoff by using the Maximin criterion.

3.3.3 Hurwicz Criterion

The Hurwicz criterion is an approach somewhere between the maximax and the maximin approaches. The **Hurwicz criterion** approach selects the maximum and the minimum payoff from each decision alternative. A value called alpha, α (not the same as the probability of a Type I error), which is between 0 and 1, is selected as a weight of optimism. The nearer alpha is to 1, the more optimistic is the decision maker. The use of alpha values near 0 implies a more pessimistic approach. The maximum payoff under each decision alternative is multiplied by alpha and the minimum payoff (pessimistic view) under each decision alternative is multiplied by $1-\alpha$ (weight of pessimism). These weighted products are summed for each decision alternative, resulting in a weighted value for each decision alternative. The maximum weighted value is selected, and the corresponding decision alternative is chosen.

Example 3.4

Reconsider *Ex. 3.1*,

(a) Suppose we are more optimistic than pessimistic and select $\alpha = 0.7$ for the weight of optimism. Determine which alternative will be selected and state the weighted payoff.

(b) Suppose we had been fairly pessimistic and chosen an alpha of 0.2, determine which alternative will be selected and state the weighted payoff.

3.3.4 Equally Likely Criterion

Equally likely criterion is also known as Laplace criterion. It is based on the simple principle that since probabilities of the state of nature are unknown, various events can be treated as equally likely. Under this assumption, the expected payoff for each act is computed first, followed by the mean of these expected payoff values. The decision maker compares the expected payoff and then selects the decision alternative that gives highest expected payoffs.

Example 3.5

Reconsider *Ex. 3.1*, determine which decision alternative to select and state the corresponding payoff by using the equally likely criterion.

3.3.5 Minimax Regret

The strategy of **minimax regret** is based on lost opportunity. Lost opportunity occurs when a decision maker loses out on some payoff or portion of a payoff because he or she chose the wrong decision alternative. For example, if a decision maker selects decision alternative d_i, which pays RM 200, and the selection of alternative d_j would have yielded RM 300, the opportunity loss is RM 100, where RM 300 – RM 200 = RM 100.

In analyzing decision–making situations under uncertainty, an analyst can transform a decision table (payoff table) into an **opportunity loss table,** which can be used to apply the minimax regret criterion.

After the opportunity loss table is determined, the decision maker examines the lost opportunity, or regret, under each decision, and selects the maximum regret for consideration. In making a decision based on a minimax regret criterion, the decision maker examines the maximum regret under each decision alternative and selects the minimum of these.

Example 3.6

Reconsider *Ex. 3.1*, determine the corresponding opportunity loss table. Determine also which alternative will be selected under minimax regret criterion.

		State of Economy		
		Stagnant	Slow Growth	Rapid Growth
Investment Decision Alternative	Stocks			
	Bonds			
	CDs			
	Mixture			

3.4 Decision Making Under Risk

Decision making under risk occurs when it is uncertain which states of nature will occur but the probability of each state of nature occurring has been determined. Using these probabilities, we can develop some additional decision–making strategies.

3.4.1 Expected Monetary Value (*EMV*)

One strategy that can be used in making decisions under risk is the expected monetary value (*EMV*) approach. A person who uses this approach is sometimes referred to as an *EMV* er. The expected monetary value of each decision alternative is calculated by multiplying the probability of each state of nature by the state's associated payoff and summing these products across the states of nature for each decision alternative, producing an expected monetary value for each decision alternative. The decision maker compares the expected monetary values for the decision alternatives and selects the alternative with the highest expected monetary value.

Example 3.7

Reconsider *Ex. 3.1*, suppose we determine that there is a 0.25 probability of a stagnant economy, a 0.45 probability of a slow–growth economy, and a .30 probability of a rapid–growth economy. The corresponding decision table can be given as:

		State of Economy		
		Stagnant (0.25)	Slow Growth (0.45)	Rapid Growth (0.3)
Investment Decision Alternative	Stocks	–500	700	2200
	Bonds	–100	600	900
	CDs	300	500	750
	Mixture	–200	650	1300

Compute the *EMV* for each alternative and determine which alternative will be selected.

The maximum of the expected monetary values is *RM* 850, which is produced from a stock investment. An *EMV* er chooses to invest in stocks on the basis of this information.

Decision Tree

A flowchart-like depiction of the decision process that includes various decision alternatives, various states of nature, and the payoffs.

Note:

1. Decision tree has a □ node to represent decision alternatives and a ○ node to represent states of nature.
2. If probabilities are available for states of nature, they are assigned to the line segment following the state of nature node symbol, ○.
3. Payoffs are displayed at the ends of the decision tree limbs.

Example 3.8
Draw a decision tree with EMV for *Ex. 3.7*.

The strategy of expected monetary value is based on a long–run average. If a decision maker could "play this game" over and over with the probabilities and payoffs remaining the same, he or she could *expect* to earn an average of $RM\,850$ in the long run by choosing to invest in stocks. The reality is that for any *one* occasion, the investor will earn payoffs of either $-RM\,500$, $RM\,700$, or $RM\,2{,}200$ on a stock investment, depending on which state of the economy occurs. The investor will not earn $RM\,850$ at any *one* time on this decision, but he or she could average a profit of $RM\,850$ if the investment continued through time. With an investment of this size, the investor will potentially have the chance to make this decision several times. Suppose, on the other hand, an investor has to decide whether to spend $RM\,5$ million to drill an oil well. Expected monetary values might not mean as much to the decision maker if he or she has only enough financial support to make this decision once.

3.4.2 Expected Opportunity Loss (*EOL*)

The approach is nearly identical to the EMV approach, except that a table (or matrix) of opportunity losses (or regrets) is used. The opportunity losses for each alternative are weighted by the probabilities of their respective states of nature and these products are summed across the states of nature. The alternative with the smallest expected loss is selected as the best choice.

Example 3.9
Reconsider *Ex. 3.6*, compute the *EOL* for each alternative and determine which alternative will be selected.

The EOL approach resulted in the same alternative as the EMV approach. The two methods always result in the same choice, because maximizing the payoffs (best EMV) is equivalent to minimizing the opportunity losses (lowest EOL).

3.4.3 Expected Value of Perfect Information (*EVPI*)

What is the value of knowing which state of nature will occur and when? The answer to such a question can provide insight into how much it is worth to pay for market or business research. The expected value of perfect information (*EVPI*) is the difference between the payoff that would occur if the decision maker knew which states of nature would occur and the expected monetary payoff from the best decision alternative when there is no information about the occurrence of the states of nature.

Expected Value of Perfect Information (*EVPI*)
= Expected Monetary Payoff with Perfect Information
– Expected Monetary Value without Information

Example 3.10

Compute the value of perfect information for the problems in *Ex. 3.6*.

It would not be economically wise to spend more than *RM* 200 to obtain perfect information about these states of nature.

Example 3.11

A manufacturing company is faced with a capacity decision. Its present production facility is running at nearly maximum capacity. Management is considering the following three capacity decision alternatives.

1. No expansion
2. Add on to the present facility
3. Build a new facility

The managers believe that if a large increase occurs in demand for their product in the near future, they will need to build a new facility to compete and capitalize on more efficient technological and design advances. However, if demand does not increase, it might be more profitable to maintain the present facility and add no capacity. A third decision alternative is to add on to the present facility, which will suffice for a moderate increase in demand and will be cheaper than building an entirely new facility. A drawback of adding to the old facility is that if there is a large demand for the product, the company will be unable to capitalize on new technologies and efficiencies, which cannot be built into the old plant.

The following decision table shows the payoffs (in *RM* millions) for these three decision alternatives for four different possible states of demand for the company's product (less demand, same demand, moderate increase in demand, and large increase in demand). The values given in the parenthesis is the probability for the states of demand.

		State of Demand			
		Less (0.1)	No Change (0.25)	Moderate Increase (0.4)	Large Increase (0.25)
Capacity Decision	No Expansion	–3	2	3	6
	Add On	–40	–28	10	20
	Build a New Facility	–210	–145	–5	55

(a) Determine which decision alternative would be selected by the maximax criterion and the maximin criterion. State also the corresponding payoffs.

(b) Use $\alpha = 0.4$ and the Hurwicz criterion to determine which decision alternative would be selected.

(c) Calculate an opportunity loss table and determine which decision alternative would be selected by using the minimax regret criterion. State also the corresponding opportunity loss.

The opportunity loss table.

		State of Demand			
		Less	No Change	Moderate Increase	Large Increase
Capacity Decision	No Expansion				
	Add On				
	Build a New Facility				

(d) Compute the expected monetary values and determine which decision alternative would be selected. Compute the expected value of perfect information.

3.4.3 Revising Probabilities in Light or Sample Information

In this section we include the additional aspect of sample information in the decision analysis. If decision makers opt to purchase or in some other manner garner sample information, the probabilities of states of nature can be revised. The revisions can be incorporated into the decision–making process, resulting—one hopes—in a better decision.

Example 3.12

The *RM* 10,000 investment is reduced here to simpler terms. Only two decision alternatives are available to the investor: bonds and stocks. Only two states of the investment climate can occur: no growth or rapid growth.

The corresponding decision table is:
Yearly Payoffs on an investment of *RM* 10,000

		State of Economy	
		No Growth (0.65)	Rapid Growth (0.35)
Decision Alternative	Bonds	500	100
	Stocks	–200	1100

Compute the EMVs for each alternative and determine which alternative will be selected.

Suppose the investor has a chance to obtain some information from an economic expert about the future state of the investment economy. This expert does not have a perfect record of forecasting, but she has predicted a no–growth economy about 0.80 of the time when such a state actually occurred. She has been slightly less successful in predicting rapid–growth economies, with a .070 probability of success. The following table shows her success and failure rates in forecasting these two states of the economy.

	Actual State of the Economy	
	No Growth (s_1)	Rapid Growth (s_2)
Forecaster predicts no growth (F_1)	0.8	0.3
Forecaster predicts rapid growth (F_2)	0.2	0.7

Suppose the forecaster predicts no growth (F_1). The prior probabilities of the states of the economy are revised as shown below.

$$\begin{aligned} P(F_1) &= P(F_1 \cap s_1) + P(F_1 \cap s_2) \\ &= P(F_1 | s_1)P(s_1) + P(F_1 | s_2)P(s_2) \end{aligned}$$

$$P(s_1 | F_1) = \frac{P(F_1 \cap s_1)}{P(s_1)}$$

Revision based on a forecast of no growth (F_1)

State of Economy	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
No growth (s_1)	$P(s_1) = 0.65$	$P(F_1 \mid s_1) = 0.8$	$P(F_1 \cap s_1) = 0.52$	
Rapid growth (s_2)	$P(s_2) = 0.35$	$P(F_1 \mid s_2) = 0.3$	$P(F_1 \cap s_2) = 0.105$	

$P(F_1) = 0.625$

Revision based on a forecast of rapid growth (F_2)

State of Economy	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
No growth (s_1)	$P(s_1) = 0.65$	$P(F_2 \mid s_1) =$	$P(F_2 \cap s_1) =$	
Rapid growth (s_2)	$P(s_2) = 0.35$	$P(F_2 \mid s_2) =$	$P(F_2 \cap s_2) =$	

$P(F_2) =$

The payoffs are the same as in the decision table without information. However, the probabilities of no–growth and rapid–growth states have been revised. Multiplying the payoffs by these revised probabilities and summing them for each investment produces expected monetary values at the state–of–economy. The investor examines the expected monetary values and selects the investment with the highest value.
Determine the corresponding expected monetary values and make selections.

The overall expected monetary value of the opportunity is

EMV for Opportunity =

3.4.4 Expected Value of Sample Information (*EVSI*)

The preceding calculations for the investment example show that the expected monetary value of the opportunity is $RM\,513.84$ with sample information, but it is only $RM\,360$ without sample information. Using the sample information appears to profit the decision maker.

Apparent Profit of Using Sample Information = $RM\,513.84$ – $RM\,360$ = $RM\,153.84$

How much did this sample information cost?
If the sample information is not free, less than $RM\,153.84$, the decision maker is gained by using it.

How much is it worth to use sample information?
Obviously, the decision maker should not pay more than $RM\,153.84$ for sample information because an expected $RM\,360$ can be earned without the information.

In general, the expected value of sample information ($EVSI$) is worth no more than the difference between the expected monetary value with the information and the expected monetary value without the information.

Expected Value of Sample Information ($EVSI$)
= Expected Monetary Value with Information
– Expected Monetary Value without Information

Suppose the decision maker had to pay $RM\,100$ for the forecaster's prediction. The expected monetary value of the decision with information is reduced from $RM\,513.84$ to $RM\,413.84$, which is still superior to the $RM\,360$, expected monetary value without sample information.

Example 3.13

In *Ex. 3.11*, the decision makers were faced with the opportunity to increase capacity to meet a possible increase in product demand. Here we reduced the decision alternatives and states of nature and altered the payoffs and probabilities. The decision table is given below.

		State of Demand		
		Less (0.2)	No Change (0.3)	Large Increase (0.5)
Decision Alternative	No Expansion	–3	2	6
	New Facility	–50	–25	65

The decision makers can buy information about the states of demand for $RM\,5$ (recall that amounts are in RM millions). The state–of–demand forecaster has historically not been accurate 100% of the time. The probabilities that the forecaster will predict a particular state of demand under the actual states of demand are given in the table below.

		Actual State of Demand		
		Less	No Change	Large Increase
Forecast	Less	0.75	0.10	0.05
	No Change	0.20	0.80	0.30
	Large Increase	0.05	0.10	0.65

Calculate the expected value of sampling information for this problem.

Solution

Revision based on a forecast of Less Demand (F_1)

State of Demand	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
Less (s_1)	$P(s_1) =$	$P(F_1 \mid s_1) =$	$P(F_1 \cap s_1) =$	
No Change (s_2)	$P(s_2) =$	$P(F_1 \mid s_2) =$	$P(F_1 \cap s_2) =$	
Large Increase (s_3)	$P(s_3) =$	$P(F_1 \mid s_3) =$	$P(F_1 \cap s_3) =$	

$P(F_1) =$

Revision based on a forecast of No Change in Demand (F_2)

State of Demand	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
Less (s_1)	$P(s_1) =$	$P(F_2 \mid s_1) =$	$P(F_2 \cap s_1) =$	
No Change (s_2)	$P(s_2) =$	$P(F_2 \mid s_2) =$	$P(F_2 \cap s_2) =$	
Large Increase (s_3)	$P(s_3) =$	$P(F_2 \mid s_3) =$	$P(F_2 \cap s_3) =$	

$P(F_2) =$

Revision based on a forecast of Large Increase in Demand (F_3)

State of Demand	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Revised Probabilities
Less (s_1)	$P(s_1) =$	$P(F_3 \mid s_1) =$	$P(F_3 \cap s_1) =$	
No Change (s_2)	$P(s_2) =$	$P(F_3 \mid s_2) =$	$P(F_3 \cap s_2) =$	
Large Increase (s_3)	$P(s_3) =$	$P(F_3 \mid s_3) =$	$P(F_3 \cap s_3) =$	

$P(F_3) =$