COMP 5112 Data Structures and Database Systems

Assignment 2

PolyU, Hong Kong

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Logistics: You should submit your solutions through Blackboard. The deadline is Monday Oct 5, 12:30 PM, right before our lectures. I will no accept submission from any other channels except Blackboard. You could only submit one pdf file that is smaller than 2MB. Pictures taken by phone will not be graded.

Problem 1 (3 point)
Assume that you have a binary search tree $\mathbb{T}$ with only one node 5. Now you insert a number sequence $\mathbb{S}$ in order into this binary search tree $\mathbb{T}$. $\mathbb{T}$ employs the usual ordering on natural numbers. Please draw the resultant tree for each of the following situations.

1. $\mathbb{S} = \{2, 6, 4, 9, 7, 3, 1\}$.
2. $\mathbb{S} = \{8, 3, 9, 2, 6, 1, 4\}$.
3. $\mathbb{S} = \{7, 2, 1, 8, 3, 6, 9\}$.

Problem 2 (5 point) For each of the five trees in Figure 1, is it a binary search tree? Please remove the root node of each valid binary search tree, and draw the resultant trees.

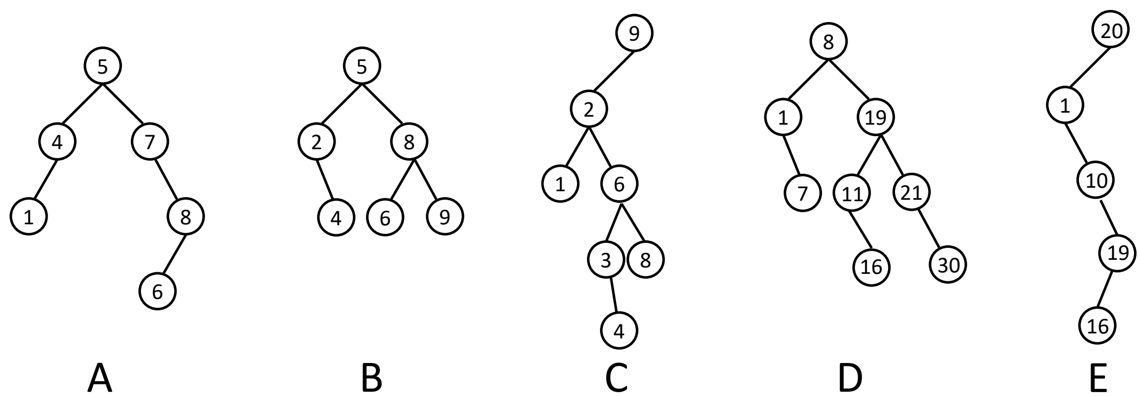

Figure 1: The five binary trees.

Problem 3 (1 point) When detecting a node k in a binary search tree, we would need to find the in-order successor of k, if the node k has both left and right children. Which of the following is true in this deletion operation?

1. The in-order successor of k is always a leaf node.

2. The in-order successor is always either a leaf node or a node with empty left child.

3. The in-order successor may be an ancestor of the node k.

4. The in-order successor is always either a leaf node or a node with empty right child.

Problem 4 (6 point) How to convert a binary tree to a binary search tree, while maintaining its original tree structure? What is the approach with the minimum time complexity in general cases? What is the time complexity of your best approach? Explain and justify your answer in detail by including the pseudocode of your best approach.

Problem 5 (2 point) If you were given four distinct keys, how many different binary search tree could you create by using these four distinct keys? Explain and justify your answer in detail.

Problem 6 (2 point) True or false? Explain and justify your answer in detail.

1. The in-order traversal of any binary search tree would be a sequence in an increasing order.

2. For each of the three tree traversals (in-order, pre-order, and postorder) that we learned in COMP5112, it is sufficient to construct the original binary search tree based on the traversal result.

Problem 7 (6 point) How to convert an unsorted array to a min heap (implemented by the array)? What is the approach with the minimum time complexity in general cases? What is the time complexity of your best approach? Explain and justify your answer in detail by including the pseudocode of your best approach. Test your pseudocode on an unsorted array $\{9, 2, 8, 5, 6, 1, 3\}$.

Problem 8 (1 point) Assume that we have a min heap stored in the array $\{\text{null}, 3, 4, 6, 9, 7\}$, what would the min heap be if we insert a new key 2?

Problem 9 (2 point) Suppose you have a hash table shown in Figure 2, implemented using linear probing. For the hash function, we are using the identity function modulo the length of the list, i.e., $h(x) = x \mod 9$. In which order could the elements have been added to the hash table? You should select all of the correct answer(s).

1. $\{9, 12, 14, 3, 4, 21, 18\}$.

2. $\{9, 14, 4, 18, 12, 3, 21\}$.

3. $\{12, 14, 3, 9, 4, 18, 21\}$.

4. $\{12, 3, 14, 18, 4, 9, 21\}$.

5. $\{12, 9, 18, 3, 14, 21, 4\}$.

0	1	2	3	4	5	6	7	8
9	18		12	3	14	4	21	

Figure 2: A hash table.

Problem 10 (2 point) Insert the following numbers in this order into an empty hash table of size 11 using chaining, $\{12, 44, 13, 88, 23, 94, 11, 39, 20, 16\}$. Use the hash function, $h(k) = (2k + 5) \mod 11$. What would be the resultant hash table looks like?

Problem 11 (2 point) Insert the following numbers in this order into an empty hash table of size 11 using quadratic probing, $\{12, 44, 13, 88, 23, 94, 11, 39, 20\}$. Use the hash function, $h(k, i) = (h_0(k) + i + i^2) \mod 11$, where $h_0(k) = k \mod 11$. What would be the resultant hash table looks like?

Problem 12 (2 point) What are the pros and cons of chaining, linear probing, and quadratic probing, when applying them to handle the collision issue in hash tables? (You have to use your own words. Copying from the slides would not get any credits.)

Problem 13 (2 point) Apply the merge sort and quick sort learned in COMP5112 to the number sequence $\{12, 44, 13, 88, 23, 94, 11, 39, 20, 16\}$, respectively. Explain the sorting process in detail.

Problem 14 (1 point) Among all the sorting algorithms learned in COMP5112, which one would give you the best performance, when applied to an array which is sorted or almost sorted (maximum 1 or two elements are misplaced)?

Problem 15 (1 point) You have an array of n elements. Suppose you implement the quick sort by always choosing the central element of the array as the pivot. What is the tightest upper bound for the worst-case performance? Explain and justify your answer in detail.

Problem 16 (4 point) What are the pros and cons of selection sort, insertion sort, merge sort, and quick sort?