A1:

a/ $O(n)$: because I need to shake hands with one by one

b/ $O(n^2)$: 1st person does with $(n-1)$ people

2nd person does with $(n-2)$ people

. . . . .

$(n-1)^{th}$ person does with only 1 person

$\Rightarrow (n-1) + (n-2) + \ldots + 1 = \dfrac{n(n-1)}{2}$

c/ $O(n)$: because we need to climb one by one

d/ $O(1)$: we only need to press the button

e/ $O(n)$: when I am at ground, and want to go to the highest floor, the lift has to travel through each floor

f/ $O(n)$: assume that the book has n pages

I read twice $\Rightarrow$ I need to read $2n$ pages

$\Rightarrow O(n)$

$$1, \ \log_2(n), \ n, \ n\log_2(n), \ n^2, \ n^3, \ 2^n, \ 3^n, \ n!$$

A3:

a) $f(n) = 3n + 4 \Rightarrow O(n)$

b) $f(n) = 1 + 2n + n(1 + 3n + 5)$

$\qquad = 3n^2 + 8n + 1$

$\qquad \Rightarrow O(n^2)$

c) $f(n) = 2(n + 100) + 3n\dfrac{(n+100)(n+99)}{2} + 9n$

$\qquad \Rightarrow O(n^3)$

d) $f(n) = 2\dfrac{n}{2} + 2\dfrac{n(n+1)}{2} + \dfrac{n(n+1)}{2}(3n+1)$

$\qquad \Rightarrow O(n^3)$

e) $f(n) = 3 \cdot 1000n + 3\dfrac{1000n(1000n-1)}{2}$

$\qquad \Rightarrow O(n^2)$

A4:

a) $5 + 0.001n^3 + 0.025n$

$\Rightarrow$ highest growth-rate : n^3 $\Rightarrow$ $O(n^3)$

b) $500n + 100n^{1.5} + 50n \log_{10} n$

$\Rightarrow$ highest growth-rate : $n^{1.5}$ $\Rightarrow$ $O(n^{1.5})$

c) $100n + 0.01n^2$

$\Rightarrow$ highest growth-rate : n^2 $\Rightarrow$ $O(n^2)$

d) $2n + n^{0.5} + 0.5n^{1.25}$

$\Rightarrow$ highest growth-rate : $n^{1.25}$ $\Rightarrow$ $O(n^{1.25})$

e) $0.3n + 5n^{1.5} + 2.5n^{1.75}$

$\Rightarrow$ highest growth-rate : $n^{1.75}$ $\Rightarrow$ $O(n^{1.75})$

f) $0.01n + 100n^2$

$\Rightarrow$ highest growth-rate : n^2 $\Rightarrow$ $O(n^2)$

g) $n \log_3 n + n \log_2 n$

$\Rightarrow$ highest growth-rate : $n \log_2 n$ $\Rightarrow$ $O(n \log n)$

h/ $0.01 n^2 \log_2 n + n(\log_2 n)^2$

=> highest growth-rate: $n^2 \log_2 n$ => $O(n^2 \log n)$

i/ $2\log_2 n + \log_5 n$

=> highest growth-rate: $\log_2 n$ => $O(\log n)$

A5:

$$\begin{cases} T(n) = 1 \quad, \quad n \leq 0 \\ T(n) = T(n-2) + T(n/2) \quad, \quad n > 0 \end{cases}$$

A6:

a/ $f(n) = 2^{n+1}$ is $O(2^n)$

=> Correct. Because $2^{n+1} = 2 \cdot 2^n$

=> The component has the highest growth·rate
is 2^n => $O(2^n)$

b/ $f(n) = 2^{2n}$ is $O(2^n)$

=> Incorrect. Because $2^{2n} = 4^n$

$\Rightarrow$ The component has the highest growth rate

is 4^n $\Rightarrow$ $O(4^n)$