QUANTUM GAME THEORY WITH JULIA: A COMPUTATIONAL ANALYSIS

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OBJECTIVES

Quantum Game Theory is an emerging inter-disciplinary field concerning Quantum Physics and Game Theory. My presentation will be on using Julia to simulate various quantum game theory models, that will be of special interest to the community. All the codes that have been prepared in Julia are present here: https://github.com/indrag49/QGameTheory-Julia. The quantum game theoretic models that have been simulated are:

1. Quantum Spin Flip Game
2. Quantum Prisoner's Dilemma
3. Quantum Hawk and Dove Game
4. Quantum Newcomb's Game
5. Quantum Battle Of The Sexes Game

The objective of this presentation is to introduce Quantum Game Theory by simulating the models with Julia and understand more complex cases.

INTRODUCTION

Quantum game theory [1] has become an exhilarating field of study that makes use of quantum manipulations to model the interplay between participating agents. These agents as a result apply quantum strategies instead of the classical versions as studied in classical game theory. Quantum game theory has found intriguing applications in several fields like population biology and market economics. A study on quantum Evolutionary Stable Strategies (QESS) was carried out by A. Iqbal and A. H. Toor, where they applied quantum game theory concepts on the original work on ESS by J. Maynard Smith and G. R. Price. Now, for the programming language part, Julia is fast and dynamically typed giving a feeling of a scripting language like Python. It provides its users with extensive visualization interfaces and toolboxes. It is good at performing tasks related to scientific computing and as a result, makes it an adequate software for this project.

QUANTUM SPIN FLIP GAME

Let us consider two players Alice (A) and Bob (B), and two spin states, 'up' (u) = $|0\rangle$ and 'down' (d) = $|1\rangle$. Let Alice set the initial state to be 'down', so, $|\psi\rangle = |1\rangle$ and Bob plays H,

$$Hd = H\,|1\rangle = \frac{u-d}{\sqrt{2}} \tag{1}$$

Now, Alice plays σ_x,

$$\sigma_x(Hd) = \frac{d-u}{\sqrt{2}} \tag{2}$$

or she plays I,

$$I(Hd) = \frac{u-d}{\sqrt{2}} \tag{3}$$

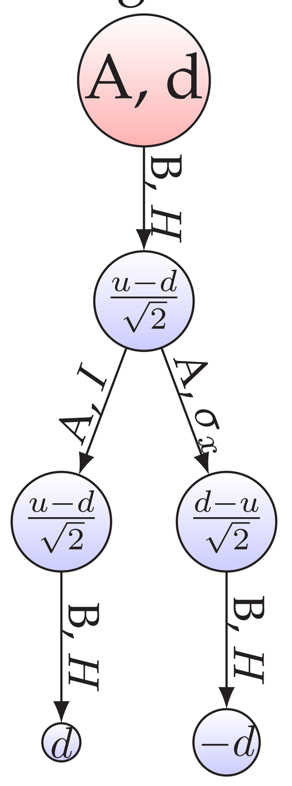
After this, Bob again plays the H operation,

$$H(\sigma_x(Hd)) = -d \tag{4}$$

or,

$$H(I(H(d)) = d \tag{5}$$

The game tree:



Here, I is the $2X2$ identity operator, H is the Hadamard gate and σ_x is the Pauli-X gate. The Julia code that simulates the above game tree is `QuantumPennyFlip.jl`. The final quantum state of the game is then measured with the code `QuantumMeasure.jl` that generates the given probability distribution plot, showing that Alice always wins in this game.

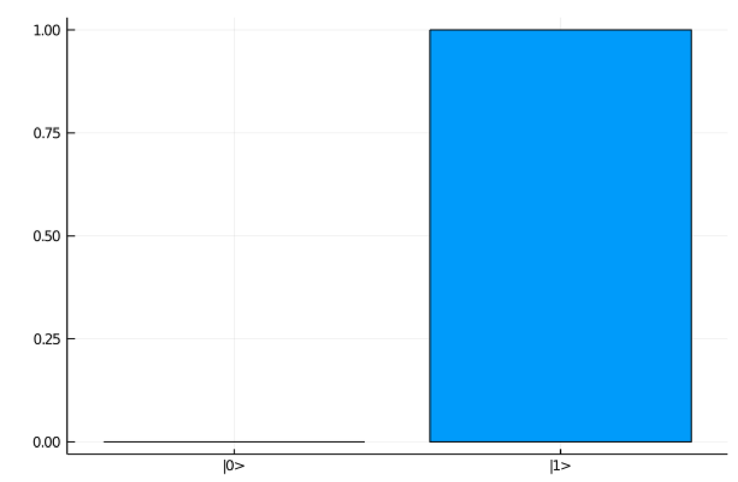


Figure 1: Figure caption

QUANTUM PRISONER'S DILEMMA

The Quantum Prisoner's Dilemma is simulated with `QuantumPrisonersDilemma.jl` which allows us to take in the strategies by Alice and Bob, and also the payoffs of the players corresponding to the choices available to them, i.e, w, x, y and z. The payoffs follow the inequality $z > w > x > y$. For a simple case, where Alice plays the Hadamard operator (H) and Bob plays the Pauli-Z gate, and $w = 3, x = 1, y = 0, z = 5$, the expected payoff to Alice is 1.5 and the expected payoff to Bob is 4. The final $4X4$ matrix that is generated, sampling moves from the list $\{I, \sigma_x, H, \sigma_z\}$ for both the players, is given below. If the equilibrium of the payoff matrix, that reflects the players' rationalities, is calculated, it can be seen that one can escape the dilemma in the quantum version of the game.

Alice/Bob	I	σ_x	H	σ_z
I	$(3,3)$	$(0,5)$	$(0.5,3)$	$(1,1)$
σ_x	$(5,0)$	$(1,1)$	$(0.5,3)$	$(0,5)$
H	$(3,0.5)$	$(3,0.5)$	$(2.25,2.25)$	$(1.5,4)$
σ_z	$(1,1)$	$(5,0)$	$(4,1.5)$	$(3,3)$

CONCLUSION

The remaining quantum game theoretic models that has been handeled in this project, are simulated with `QuantumNewComb.jl` for Quantum Newcomb's game, `QuantumHawkDove.jl` for Quantum Hawk and Dove game and `QuantumBattleOftheSexes.jl` for Quantum Battle of the sexes game. All these codes are available in the link provided under the **Objectives** section. An R package named `QGameTheory` [2] has been also developed, that helps in simulating this quantum game theoretic models and is available at the CRAN repository.

REFERENCES

References

[1] J.O.Grabbe. *An introduction to quantum game theory*, 2005.

[2] Indranil Ghosh. *QGameTheory, CRAN*, 2020.

FUTURE RESEARCH

Quantum versions of few more games like Monty Hall Problem, Parrondo's Game and Two Person Duels is to be implemented in Julia. Understanding these basic codes will help the practitioners to design more complex quantum game theoretic models and perform further analyses and apply in fields like population biology or market economics.

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