MGT 6081 Final Project Report

Portfolio Simulation Surgery

Group 4

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GitHub Repo Link

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1. Introduction

This project focuses on constructing a diversified stock portfolio from a select group of 1-year historic stocks. These stocks are chosen for their significant influence on global market dynamics and their varied industry representations, ensuring a broad exposure to different economic sectors.

The theoretical foundation of this project is rooted in the principles of efficient frontier theory, portfolio optimization techniques, and various simulation methods. The primary objective is to develop a portfolio that offers the highest possible return for the least amount of variance. This retrospective analysis sets the foundation for the subsequent phase of the project, which involves simulating the portfolio's future performance under different financial models, including geometric Brownian motion (GBM), Merton's model, constant elasticity of variance (CEV), and the Heston model. We also introduced relevant options to hedge stock risks and enhance diversification of our portfolio, resulting in a positive-drifted Sharpe ratio.

2. Stock Selection and Data Collection

Our data collection process leveraged the historical market data from Yahoo Finance. The time range for the collected data spans from 2023/01 to 2024/04 encapsulating a broad spectrum of market conditions. The portfolio consists of six stocks from technology, e-commerce, automotive, healthcare, and financial services sectors, alongside two ETFs representing U.S. large-cap equities and emerging markets, plus a commodity ETF focused on gold. This historical data constitutes our training set, serving as the benchmark for our simulations and portfolio optimization exercises.

For the purpose of this project, we used data from 2023/01 to 2023/12 for training and 2024/01 to 2024/04 for testing. The period of 2013/01-2013/01 is used for the initial weights settlement and training of hyperparameters in the simulation models. We then used the simulated stock path to decide the updated weights, and performed a portfolio backtest based on the historical data of the test set (2024/01 to 2024/02). This allows us to compare the portfolio's past performance against more recent market behavior and assess the predictive power of our selected models. The preprocessing of the data involved the meticulous cleansing, normalization, and consolidation of stock prices from individual securities into a coherent dataset.

3. Portfolio Optimization

3.1 Historical Portfolio

We used the historical price data of the 8 securities from 2023-01 to 2023-12 to create an optimal portfolio that generates the highest return with the least variance. These returns then informed the average returns and covariance matrix for our assets, essential variables in the optimization algorithm. We positioned 86.26% in AAPL, reflecting our confidence in the technology sector's continued growth. A significant portion, 36.65%, is also held in AMZN, underscoring its expansive presence in e-commerce and cloud computing.

Our strategy includes a hedge component, as evidenced by a short position in EEM, equivalent to -100.00%. This is complemented by an 88.55% allocation to GLD, reflecting its status as a safe-haven asset amidst market uncertainties. Further diversification is achieved through a 64.97% investment in JPM, anchoring the portfolio with stable financial sector exposure, and a -59.83% short position in JNJ, which is indicative of a strategic counterbalance within the healthcare sector.

Our allocation also adjusts for market exposure with a -35.61% weighting in SPY, and a modest but optimistic 19.01% stake in TSLA, betting on the future of automotive and renewable energy.

This carefully calibrated portfolio represents a forward-thinking balance of growth, value, and defensive assets, designed to optimize returns against market volatility.The portfolio weights are as follows.

Table.1 Portfolio Weights

Ticker	AAPL	AMZN	EEM	GLD
Weight	0.86257	0.36653	-1.00000	0.88551
Ticker	JNJ	JPM	SPY	TSLA
Weight	-0.59833	0.64971	-0.35609	0.19009

In the quest to maximize the Sharpe ratio—our chosen metric for risk-adjusted returns—a constrained optimization technique was utilized, ensuring the sum of the asset weights equaled unity, reflecting a fully invested portfolio. We benchmarked our returns against the prevailing risk-free rate, derived from the current 3-month T-Bill rate. The expected annual return of our portfolio is 98.24%, with an annual volatility of 29.33% and a sharpe ratio of 3.17. The portfolio gives the highest expected return given a specific risk level. Notably, we found out that with satisfactory return, the Sharpe ratio still has room for enhancement. Next, we will simulate forward the portfolio into the future four months, utilizing historical data to judge simulated results, and deciding if there is any adjustment to the current portfolio.

Table.2 Portfolio Performance

Expected annual return	98.24%
Annual volatility	29.33%
Sharpe Ratio	3.17
Current Portfolio Value	567.882

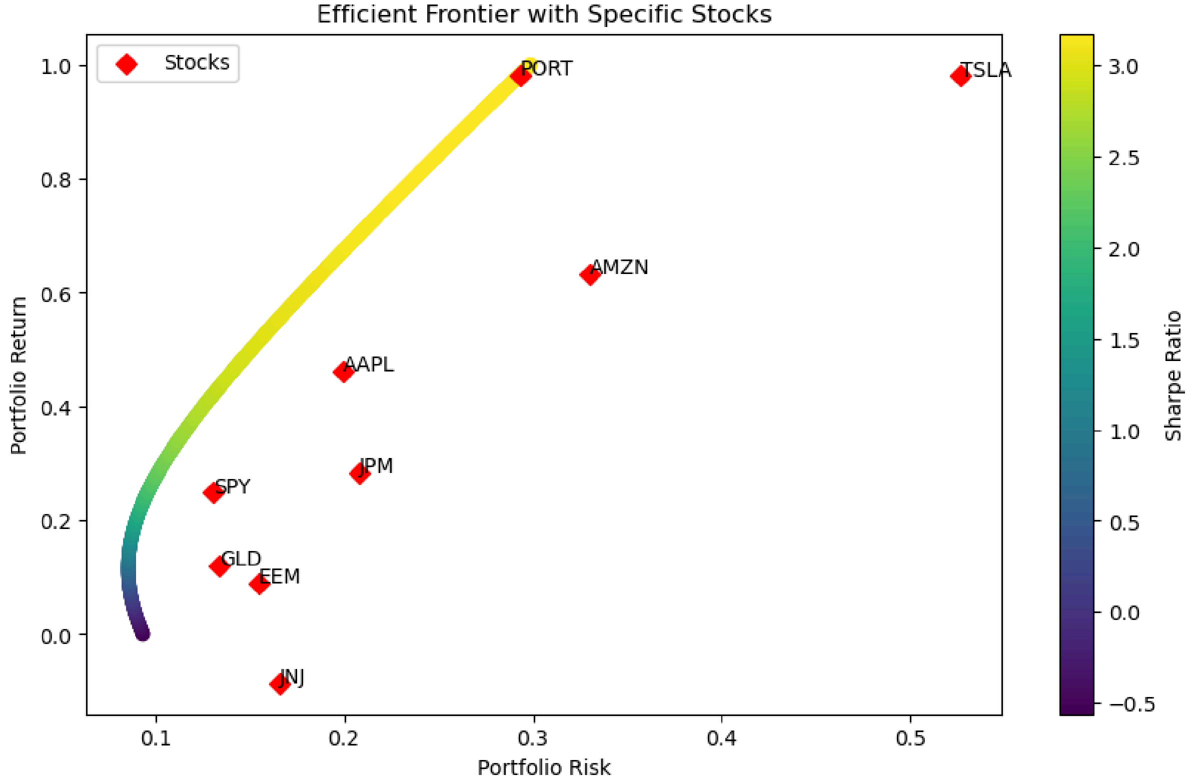
3.2 Efficient Frontier Visualization

The Efficient Frontier visualization serves as a compelling graphical representation of our portfolio optimization efforts. It is based on the optimal asset allocation strategy that we previously identified, aiming to maximize the Sharpe Ratio. The portfolio we derived, labeled 'PORT' in the visualization, is strategically placed on the frontier, indicating that it is one of the most efficient portfolios achievable given the set of selected stocks.

In the visualization, each individual stock from our optimized list is marked to show its risk-return profile. The 'PORT' point exhibits our calculated expected annual return and volatility derived from the optimal weights. These weights have been crafted not only to strive for high returns but also to manage and mitigate risk effectively.

The frontier curve itself is generated by simulating thousands of possible portfolio combinations, each with varying asset weights. It maps out the expected return against portfolio risk (standard deviation), and the resulting curve illustrates the trade-off between risk and return. Our portfolio's positioning towards the upper end of the frontier suggests a preference for higher returns, accompanied by a higher risk profile, which is consistent with an aggressive investment approach. The color gradient representing the Sharpe Ratio further enriches the analysis, with warmer colors indicating a higher ratio and, therefore, a more desirable risk-adjusted return. The 'PORT' position's color signifies its Sharpe Ratio, reflecting the efficiency of our risk-adjusted return relative to the other simulated portfolios.

Figure.1 Efficient Frontier with Stocks



4. Simulation Models Comparison and Selection

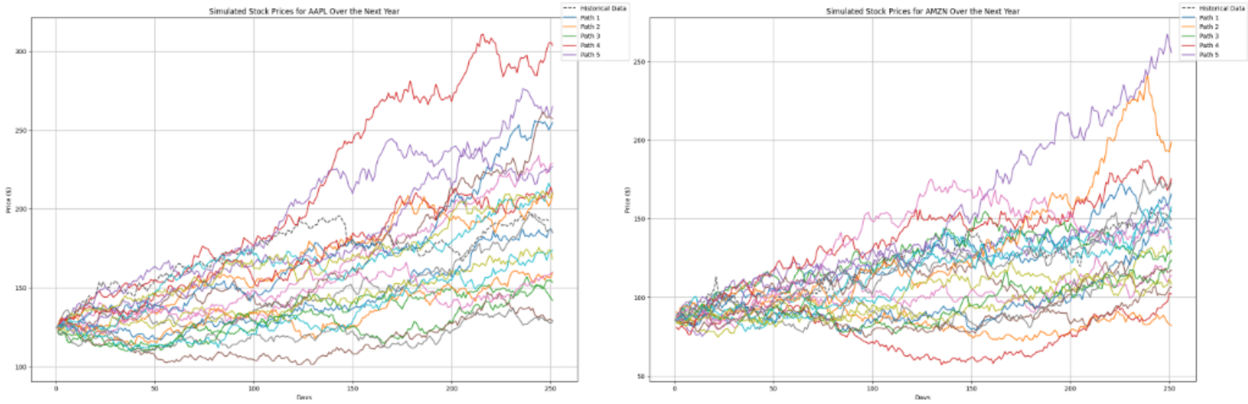
In this section, we will introduce the simulation model implemented in this project, and discuss the rationale behind the model selections of the specific stocks.

4.1 Model Comparison (2023/01 - 2023/12)

GBM is the simplest and most commonly used stock price model. It is suitable for short-term scenarios where market volatility is minimal and there are no significant news or events impacting the market, such as ordinary stock pricing and risk management.

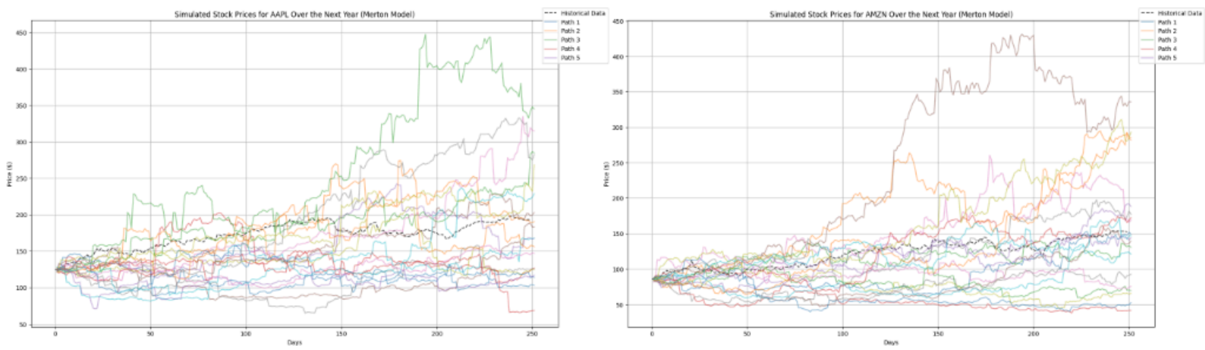
From the sample simulation, we can see the combination of mean drift and randomness emphasized by the GBM model. However, because it does not account for changes in volatility or jump behavior in stock prices, the GBM implication may lead to inaccuracies in more complex market environments.

Figure.2 Sample output of GBM Simulation over AAPL and AMZN



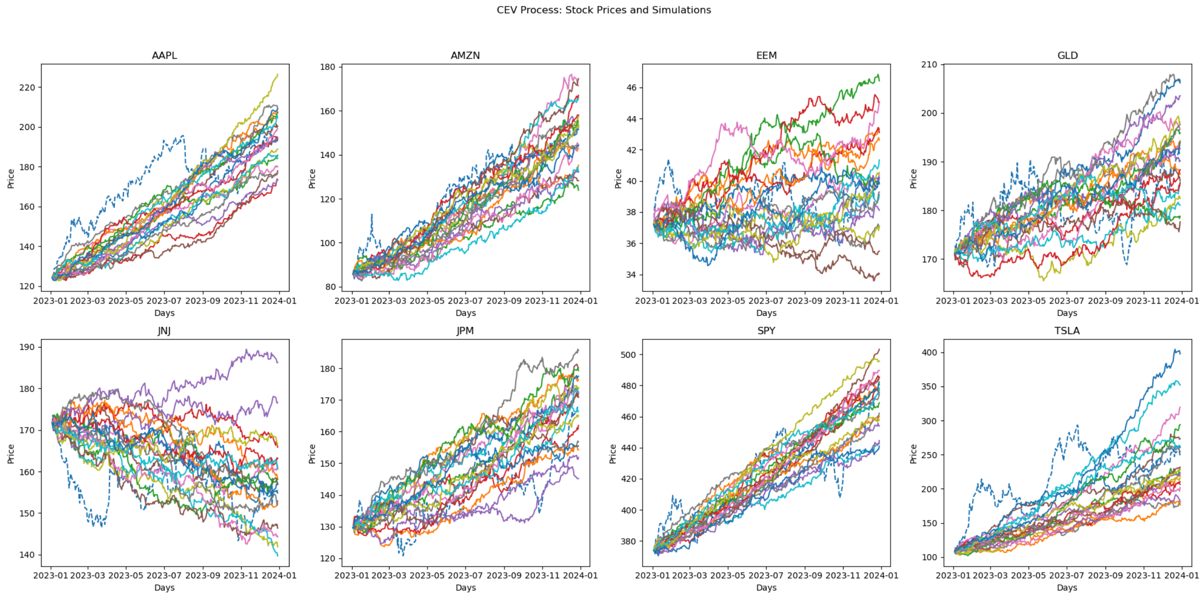
The Merton Jump Diffusion model, on the other hand, adds a jump component to GBM, allowing the model to simulate large and sudden changes in stock prices due to news or other information. However, the parameters for the jump process are difficult to estimate, and the model complexity is higher. It is ideal for markets that might be impacted by sudden news or events, such as financial crises or major corporate announcements.

Figure.3 Sample output of Merton Jump Diffusion over AAPL and AMZN



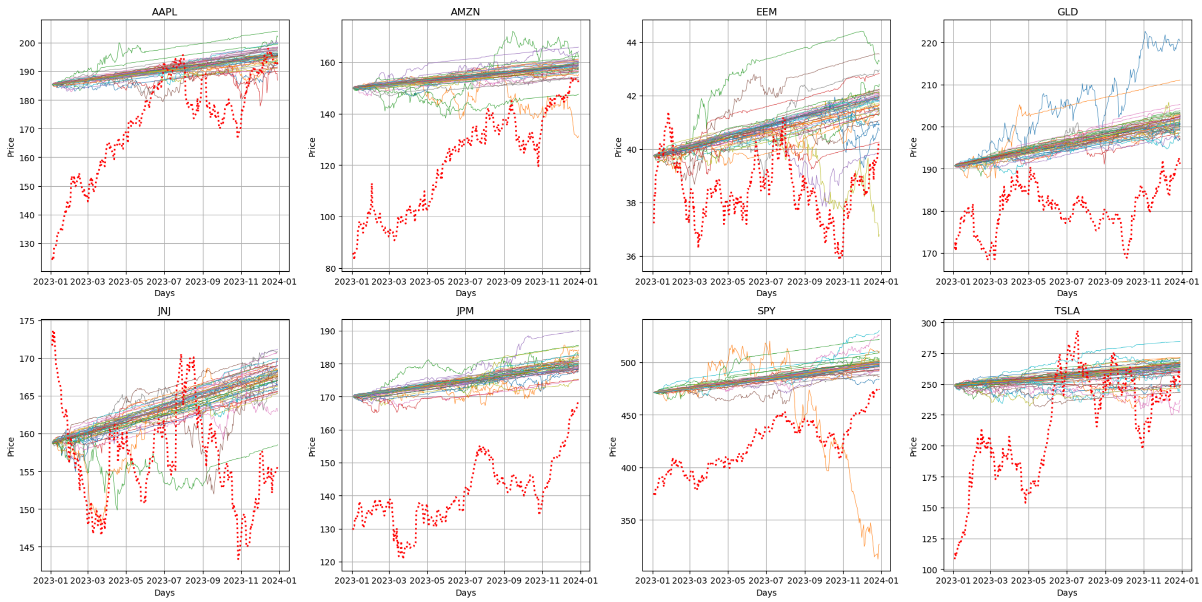
The CEV accounts for the correlation between volatility and the level of stock prices (volatility is positively related to some power of the stock price), better capturing the high volatility of low-priced stocks. While it is more complex than GBM, with more difficult parameter estimation and numerical implementation. When stock prices exhibit non-constant volatility, particularly under extreme market conditions, the CEV becomes crucial for pricing financial derivatives and assessing risk. During our implementation of CEV, while we tried to optimize the power coefficient, we found that the result does not converge. That could be possibly related to the training sample size. Through empirical tuning and comparison, we take the mean of estimated gamma in different iterations to generate the result. In the graph, the dash line presents the real historical data.

Figure.4 Sample output of CEV Model over All Stocks



The Heston model allows volatility itself to vary over time following a stochastic process (stochastic volatility model). It better models the uncertainty and dynamics of market volatility, including the volatility smile. On the other hand, it is more mathematically and computationally complex, requiring sophisticated numerical methods like Monte Carlo simulation. The Heston model is highly suitable for options pricing, particularly when volatility is unstable.

Figure.5 Sample output of Heston Model over All Stocks



4.2 Model Selection (2024/01 - 2024/04)

For the purpose of modeling the stock prices under different market conditions and assessing the performance of financial derivatives, we applied specific models to our selected stocks based on their individual characteristics and historical price behaviors.

GBM is applied to AAPL, AMZN, and JNJ, owing to the general assumption that these stocks follow a path that can be modeled by standard Brownian motion with a constant drift and volatility, suitable for stocks with relatively stable price movements.

The Merton model is selected for TSLA, which we considered as a hot stock with frequent dramastic announcements, including the recent layoff news that caused a sharp decrease in stock value. Therefore, we believe the model is perfect to reflect the stock's propensity for sudden jumps or drops in price due to innovation breakthroughs or other significant company-specific events.

The CEV model is utilized for JPM and SPY as it accounts for the observed tendency of volatility to fluctuate with the level of the underlying asset price, which is pertinent for financial and large-cap equity assets. JPM is a large investment bank, and SPY is an outstanding market index. We believe an adjustment to the stochastic term is necessary for simulation.

The Heston model is designated for GLD and EEM. This model allows for a stochastic volatility process, providing a more flexible framework to capture the dynamic risk profile of commodities like gold and the diverse and potentially volatile nature of emerging market equities.

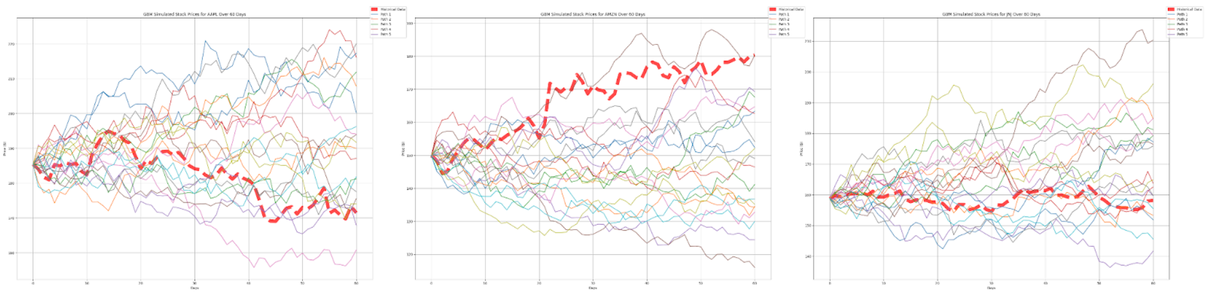
By assigning these models to each stock based on their expected behavior and the type of market conditions they face, we aim to achieve a more accurate simulation of price movements and a better assessment of risk for our portfolio.

5. Optimal Model Discussion

5.1 Separate Simulation

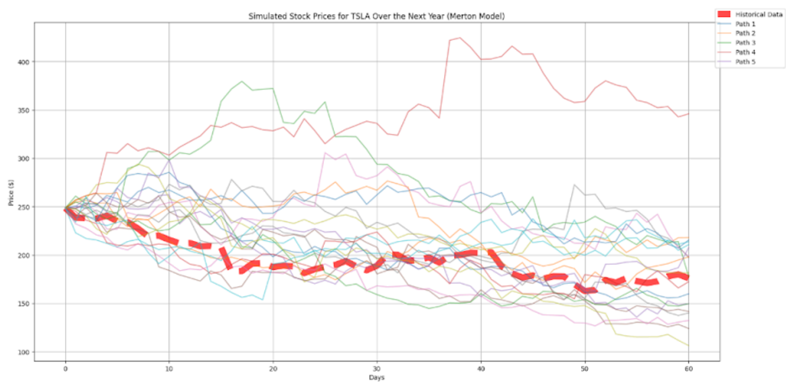
The GBM simulations focused on AAPL, AMZN, and JNJ. Historical data was used to estimate the annual mean return μ and annual volatility σ. A series of simulated paths were generated for each stock, using Brownian motion to account for the randomness in price movements and allowing for the visualization of a range of potential future scenarios. The red dash line shows the real historical data.

Figure.6 GBM Simulation for AAPL, AMZN and JNJ



The Merton model was applied to TSLA to include random jumps. Parameters for the Merton model included the same mean and volatility used in GBM, with additional parameters for the jump intensity λ, mean jump size (μ_{jump}), and jump volatility (σ_{jump}). Similar to GBM, the Merton model produced a set of potential price paths.

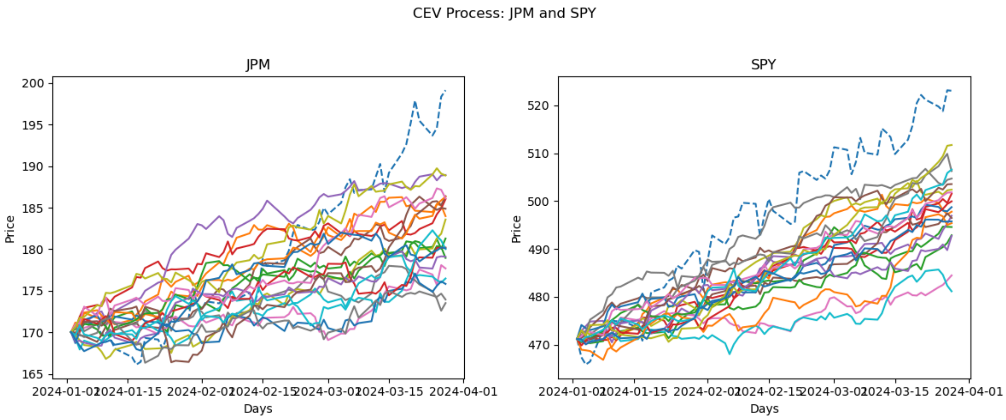
Figure.7 Merton Simulation for TSLA



The simulation of CEV model starts with parameters estimation, including the stock's drift, volatility and the elasticity parameter from $dS_t = \mu S_t dt + \sigma S_t^{\gamma} dW_t$.

Optimal values of γ for each stock were determined through minimizing the L2 norm error between market prices and simulated prices. The values of γ were found to vary for each stock, with values such as 0.237 for JPM and 0.775 for SPY, indicating differences in the degree of leverage effect among the different securities. With the parameters established, the simulation of stock price paths was conducted accounting for the non-constant volatility by adjusting for the elasticity parameter.

Figure.8 CEV Simulation for JPM and SPY

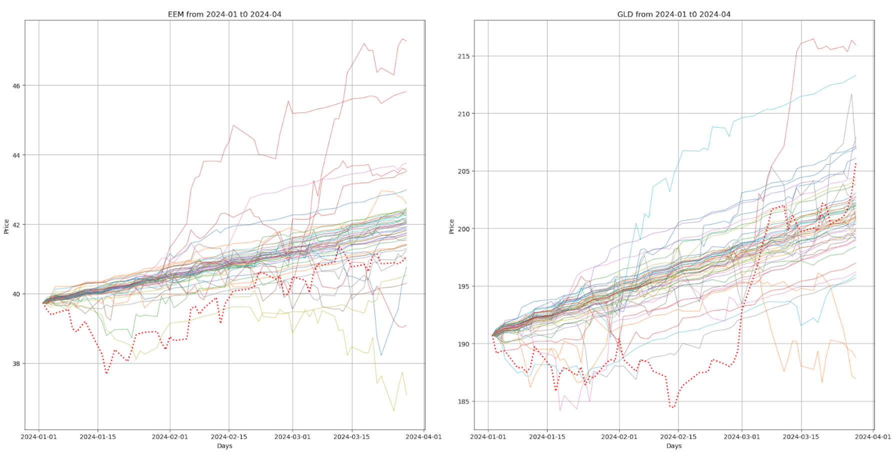


The analysis revealed variances in the simulation results, with some simulations exhibiting higher volatility, suggesting sensitivity to the initial conditions and parameter choices. As we found that gamma optimization is not convergent with the given data, we have already taken the mean of gamma over different iterations to reduce potential biases.

The initial parameters of the Heston model, including the initial stock price (S0), initial variance (V0), mean reversion rate (kappa), long-term variance (theta), volatility of volatility (sigma), and correlation between stock and variance (rho), were established from historical data. This process involved optimizing the model parameters to minimize the discrepancy between historical and model-implied variances, with an L2 regularization term added to prevent overfitting.

A rolling window of 30 days was used to calculate historical variances, which then served as reference points for the optimization procedure. The simulation utilized the Euler-Maruyama method to generate paths for the variance process, followed by the simulation of stock price paths. A large number of paths were simulated to capture a range of possible future scenarios.

Figure.9 Heston Simulation for EEM and GLD



From the visualization above, the simulation processes fit the real historical data reasonably. For the next step, we chose a simulation path of each stock to update the stock weights, and test the portfolio performance over the given historical data.

5.2 Portfolio with Simulation Price

After simulating future stock prices using various models, we embarked on re-optimizing the portfolio based on this simulated data. The intention was to identify an optimal allocation of assets that could potentially yield the highest return for the least amount of risk, as reflected by the Sharpe ratio, from 2024/01 to 2024/04.

The mean returns and covariance matrix from the simulated returns were used to recalibrate the portfolio optimization model. Using these metrics and considering the risk-free rate of 5.245%, the optimization sought to maximize the Sharpe ratio to achieve the most efficient risk-adjusted return.

These new sets of weights reflected a shift from the previously held positions, indicating a potential change in the investment strategy based on simulated market dynamics. The new portfolio allocation and comparison with original allocation was as follows:

Table.3 Portfolio Weights Comparison

Ticker	AAPL	AMZN	EEM	GLD
Simulated Weight	0.01711	-0.03410	0.01908	0.4438
Original Weight	0.86257	0.36653	-1.00000	0.88551
Ticker	JNJ	JPM	SPY	TSLA
Simulated Weight	-0.01318	0.0802	0.4884	-0.00135
Original Weight	-0.59833	0.64971	-0.35609	0.19009

The simulated weight for GLDmarks a significant pivot, escalating from an original weight of 0.8855 to a simulated 0.4438, making it a prominent player in the adjusted portfolio. In sharp contrast, AAPL sees its influence in the portfolio sharply curtailed, with its weight plummeting from 0.86257 to 0.01711. The SPY also commands attention with its simulated weight soaring to 0.4884 from a negative original weight, reflecting a bullish stance in the simulation. These noteworthy adjustments indicate a dynamic strategy shift, with the simulated weights suggesting a tactical move towards assets with potential for better performance in the current market climate.

After calculating the optimal weights using simulated data, we test the portfolio performance on the real data from 2024/01 to 2024/04. The expected annual return on the newly optimized portfolio was calculated to be 38.69% with a volatility of 8.21%. A remarkably high Sharpe ratio of 4.07 was derived, indicating an exceptionally efficient portfolio compared to the theoretical risk-free investment, especially over a short-term horizon.

Table.4 Portfolio Results Comparison

Metrics	Simulated Portfolio	Original Portfolio
Expected annual return	38.69%	20.09%
Annual volatility	8.21%	22.39%
Sharpe Ratio	4.07	0.66
Current Portfolio Value	351.5371	241.8220

6. Option-Hedged Stock Portfolio

In this section, we will combine options into the portfolio to hedge risks and enhance diversification. The inclusion of SPY options presents us with versatile instruments that can serve multiple strategic functions, such as portfolio protection, income generation, and capitalizing on market volatility. Since the majority of securities we choose are included in S&P 500, we expect the option to bring positive influences.

Table.5 Option-Involved Portfolio Weights

Ticker	AAPL	AMZN	EEM	GLD	JNJ
Simulated Weight	-0.26370	0.07619	-0.36896	0.33179	-0.142838
Ticker	JPM	SPY	TSLA	^SPY PUT	
Simulated Weight	0.545535	1	-0.17945	0.001441	

Table.6 Option-Hedged Portfolio Results Comparison

Metrics	Option-Involved Portfolio	Simulated Portfolio
Expected annualized return	41.99%	38.69%
Annualized volatility	11.40%	8.21%
Sharpe Ratio	4.79	4.07

The portfolio that includes option hedging typically displays better performance metrics, boasting both a higher return and an improved Sharpe ratio. With the incorporation of SPY put options, we observe a strategic weight shift toward the SPY Index, a move rationalized by their inherent correlation. The put options serve not only as a hedge but also potentially enhance the return profile due to this correlation dynamic.

To further refine our portfolio, we could consider the integration of a broader range of options, such as those tied to the NASDAQ 100 and S&P 500 indices, as well as options on the S&P 500 Volatility Index. These additional instruments can serve to fine-tune our risk management. Moreover, incorporating Russell 2000 Index Options could introduce a desirable layer of diversification, tapping into the growth potential of small-cap stocks while maintaining a stance on risk exposure.

7. Closing Remarks

In summary, the selection of various strategies has led to a marked enhancement in our portfolio's Sharpe ratio. We have selected and tailored our models to align with the specific pricing trends and volatility patterns of individual stocks, leading to a successful re-allocation of stocks and significant Sharpe ratio improvement. We found that the portfolio weights greatly differ during different time spans. Moreover, we included options to hedge risks and enhance diversification of our portfolio. Our portfolio aims to shed light on portfolio allocation and option-involved strategies implementation.

Appendix: github repository:

https://github.com/Go1vf/Portfolio-Simulation-Surgery