1.

a)

记录:

To compute the projection, I apply the camera matrix to the homogeneous coordinates of point. (X, Y, Z) are (X, Y, Z, 1) and projection is given by dividing the firt two coordinates by the third.

$$P \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 21 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -21 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ -21 \end{pmatrix}$$

$$P \cdot \begin{pmatrix} 4 \\ 5 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 39 \\ -4 \end{pmatrix} = \begin{pmatrix} -\frac{5}{2} \\ -\frac{39}{4} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{5}{2} \\ -\frac{39}{4} \end{pmatrix}$$

b)

camera center is the null space of P.

$$\begin{cases} x + z = 0 \\ 2y + 4z + 5w = 0 \\ -z + 2w = 0 \end{cases} \qquad \begin{cases} x = -2t \\ y = -\frac{13}{2}t \\ z = 2t \\ w = t \end{cases} \qquad C = \begin{pmatrix} -2 \\ -\frac{13}{2} \\ 2 \end{pmatrix}$$

$$P = [A \;\; t] \quad \det(A) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & -1 \end{vmatrix} = -2 < 0$$

$$R_3 = \frac{\mathrm{sign}(\det(A))}{\|A_3\|} A_3 = -1 \cdot (0 \; 0 \; -1) = (0 \; 0 \; 1)$$

The principal axis is R_3 (0 0 1).

c)

记录:

The vanishing points are the last row equal to 0.

So, 3D point $\mathbf{X} = (X, Y, Z)$.

$$P\mathbf{X} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 4 & 5 \\ 0 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

$$\begin{cases} X + Z = x \\ 2Y + 4Z + 5 = y \\ -Z + 2 = 0 \end{cases} \qquad \begin{cases} X = x - 2 \\ Y = \frac{y-13}{2} \\ Z = 2 \end{cases}$$

So, the set of 3D project onto vanishing points follows $Z = 2$ in $\mathbb{P}^2$

d)

The set of 3D points project onto vanishing points represents a line in 3D that is parallel to the principal axis and passes the camera center. The line is at infinity in the direction of principal axis and any point on this line can be moved arbitrarily far away from the camera. Because the projection of a line at infinty is a point at infinity in the same direction as the line.

2.

a)

$l_1 = (0, 1, 1) \quad l_2 = (1, 0, 1)$

$$\begin{cases} l_1^T X = 0 \\ l_2^T X = 0 \end{cases} \Rightarrow \begin{cases} y + z = 0 \\ x + z = 0 \end{cases} \qquad \begin{cases} x = -t \\ y = -t \\ z = t \end{cases}$$

$\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ is the representative of our intersection point.

b)

$$\tilde{l}_1 = HL_1 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$\tilde{l}_2 = HL_2 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$$\begin{cases} \tilde{l}_1 X = 0 \\ \tilde{l}_2 X = 0 \end{cases} \Rightarrow \begin{cases} 3z = 0 \\ 2x - y + 3z = 0 \end{cases} \qquad \begin{cases} x = t \\ y = 2t \\ z = 0 \end{cases}$$

So $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ is the representative of intersection point.

c)

The new intersection point means that the new lines $\widetilde{l_1}$, $\widetilde{l_2}$ are parallel in R^2 and the point is infinity far away in direction (1, 2).

3.

a)

$$P_1 = [I \; 0] \qquad P_2 = \begin{pmatrix} 1 & 0 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} = [A_2 \;\; t_2]$$

$$[t_2]_\times = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$F_{12} = [t_2]_\times A_2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} -1 & 1 & -2 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix} = [A_3 \;\; t_3] \qquad [t_3]_\times = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$F_{13} = [t_3]_\times A_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & 1 & -2 \\ -1 & 0 & -1 \\ 0 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

b)

the epipolar constraint can be written

$$x'^T l = x'^T F x = 0 \qquad F_{21} = F_{12}^T$$

x_1 correspond x_2, so

$$x_2'^T F_{12} x_1 = 0 \text{, so}$$

$$x_1'^T F_{12}^T x_2 = 0$$

$$l_1 = F_{12}^T x_2 \qquad l_3 = F_{13}^T x_3$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ -2 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \qquad = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$$

$$x_2 : x + 3z = 0 \qquad x_3 : -x + 2y - z = 0$$

$$(l_1(1) \;\; l_1(2))\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -l_1(3) \qquad (l_2(1) \;\; l_2(2))\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -l_2(3)$$

$$(1 \;\; 0)\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = -3 \qquad (-1 \;\; 2)\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = 1$$

$$x_1 = -3 \qquad y_1 = -1$$

$$x_1 = (-3, -1)$$

4.

a)

$P = [_{3} \quad ^{4}]$

An uncalibrated pinhole camera has 11 degrees of freedom

3 eq + 1 unknown

$3n \geq 11 + n$

$n \geq 5.5$

$n = 6$

We need at least 6 point matches to complute the camera matrix

b)

prob of inlier sample 0.65^6

prob of failing to do $1 - 0.65^6$

prob of failing n times $(1-0.65^6)^n$

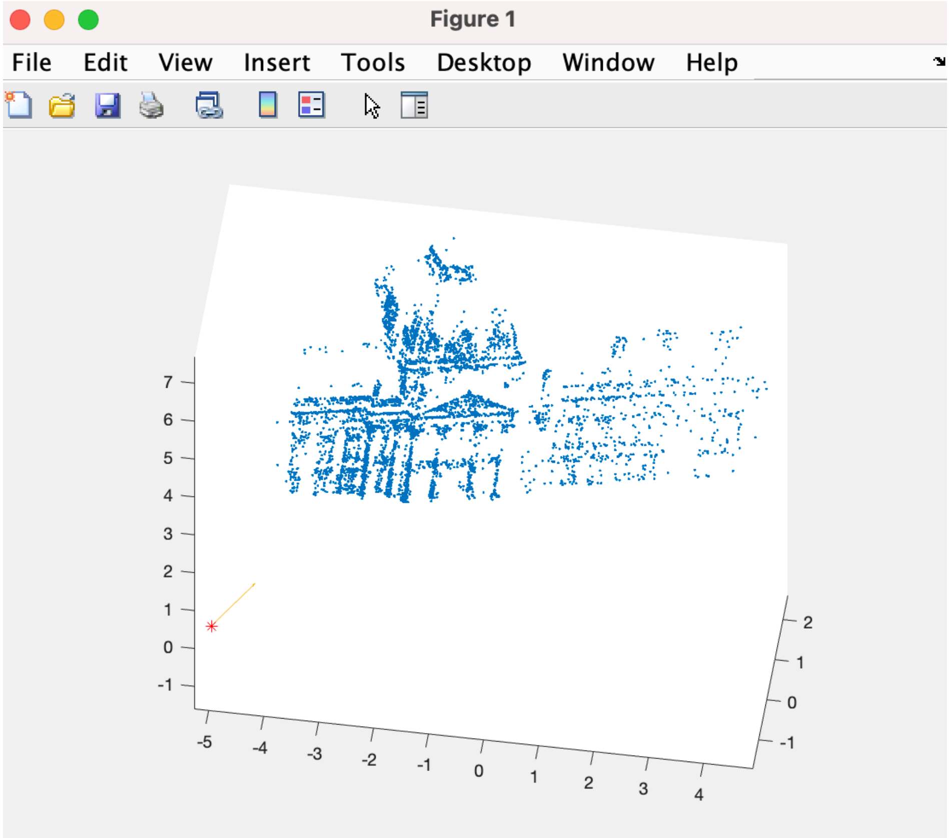
$(1-0.65^6)^n < 1 - 99.99\%$

$n \log(1-0.65^6) \leq \log(0.0001)$

$n \geq 117.5$

$n = 118$ We need 118 RANSAC iterations.

c)



P =

3x4 double

	1	2	3	4
1	-14.1741	-18.4157	77.5002	-387.2859
2	-7.4642	-6.5460	31.6026	-157.8700
3	-3.3386e-05	-1.7328e-05	8.6191e-05	-4.3674e-04
4				

best_E =

3x4 double

	1	2	3	4
1	-13.8153	-83.5759	16.7025	-407.4312
2	54.7713	-16.4362	-8.5006	-119.4317
3	-2.4243e-06	-3.9484e-05	-5.9909e-05	-3.0169e-04
4				

5.

a)

P_1 $\quad$ P_2 $\quad$ x_1 x_2 are corresponding points in two views

$E = k_2^T R k_1$

For P_1, $k_1 = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$ $k_2 = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$

So. $E = \begin{pmatrix} 0 & -z & y_0 \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$ $\qquad z = x_2 - x_1$

$F = k_2^{-T} E k_1^{-1} = \begin{pmatrix} 0 & -1 & y_0 \\ 1 & 0 & -x_0 \\ -y_0 & x_0 & 0 \end{pmatrix}$ it doesn't depend on f and α they cancel out during computation

$$F = \begin{pmatrix} 0 & -1 & y \\ 1 & 0 & -x \\ -y & x & 0 \end{pmatrix} \begin{pmatrix} 1/f^2 & 0 & -x/f^2 \\ 0 & 1/f^2 & -y/f^2 \\ x/f^2 & y/f^2 & 1/f^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1/f & y/f - x/f^2 \\ 1/f & 0 & -x/f - y/f^2 \\ -y/f & x/f & 0 \end{pmatrix} \alpha^2$$

$$= \begin{pmatrix} 0 & -1 & y_0 \\ 1 & 0 & -x_0 \\ -y_0 & x_0 & 0 \end{pmatrix}$$ it doesn't depend on f and α

b)

记录:

corresponding point (x, y) $(\bar{x}, \bar{y})$.

the epipolar constraint is given by $\bar{x}^T F x = 0$

$\bar{x}^T F x = 0 \Rightarrow \bar{x}^T [e]_\times P_2 P_1^{-1} x = 0$. e is epipolar.

$[e]_\times$ skew-symmetric

$\bar{x}^T [e]_\times P_2 P_1^{-1} x = (\bar{x}^T [e]_\times P_2)(P_1^{-1} x)$.

$y = P_1^{-1} x \quad \bar{y} = P_2^{-1} \bar{x}$

$\bar{y}^T [e]_\times y = 0$

$\det([\bar{y}]_\times [e]_{\times y}) = 0$ $\quad [\bar{y}]_\times$ is skew-symmetrix of $\bar{y}$

$[a]_{\times b} = a \times b^T - b \times a^T$

$[\bar{y}]_\times [e]_{\times y} = \bar{y}(e_\times y)^T - y(e_\times \bar{y})^T$

$[\bar{y}]_\times [e]_{\times y} = (\bar{x} - x)(e_\times (P_1^{-1} x)^T - e_\times (P_2^{-1} \bar{x})^T)$

$\det([\bar{y}]_\times [e]_{\times y}) = \det(\bar{x} - x) \det([e_\times][P_1^{-1} x - P_2^{-1} x])$

$\det([e_\times]) = 0$.

So $\det([\bar{y}]_\times [e]_{\times y}) = 0$

the corresponding points follow epipolar constraint and the expandation of $\det\left[\begin{pmatrix} x - x_0 & \bar{x} - x_0 \\ y - y_0 & \bar{y} - y_0 \end{pmatrix}\right]$ is epipolar constraint so ~~it is equal to 0~~

c)

The projections of (x, y) and $(\bar{x}, \bar{y})$ onto the line joining the epipolars e_1 and e_2 are collinear. The determinant $\det\begin{pmatrix} x - x_0 & \bar{x} - x_0 \\ y - y_0 & \bar{y} - y_0 \end{pmatrix} = 0$ means that the two corresponding points (x, y) and $(\bar{x}, \bar{y})$ lie on a common epipolar line in the two views.

6.

I estimate the normal vector of a plane from corresponding points (I used x1a x2a and x1b x2b) using the given camera matrices P1 and P2.

$x1a \quad x2a \qquad x1b \quad x2b$

so $x2a = H\,x1a, \quad x2b = k(R\,x1b + t)$

$H = k(R - tn^T)k^{-1} \Rightarrow x2a = k(R - tn^T)k^{-1}$

$x2b = kR\,x1b + kt$

$\Rightarrow k\,x2a = kRk^{-1}x1a - k\,t\,n^T k^{-1} x1a$

$R^T x2b = R^T kR\,x1b + R^T kt.$

$n_1 = k^{-T} n \qquad n_2 = R n_1$

$[x2a\,;\,0] = [k(R - tn^T)k^{-1}, 0]\,[x1a\,;1;\,0]\,n^T k^{-1} x1a$

$[x2b\,;\,0] = [0, Rk, Rt]\,[x1b\,;1;0].$

$x = A \setminus b$

$n = k^{-T} x(1{:}3)$

n

3x1 double

	1	2
1	0.4991	
2	0.5375	
3	-0.6797	
4		