Assignment 3

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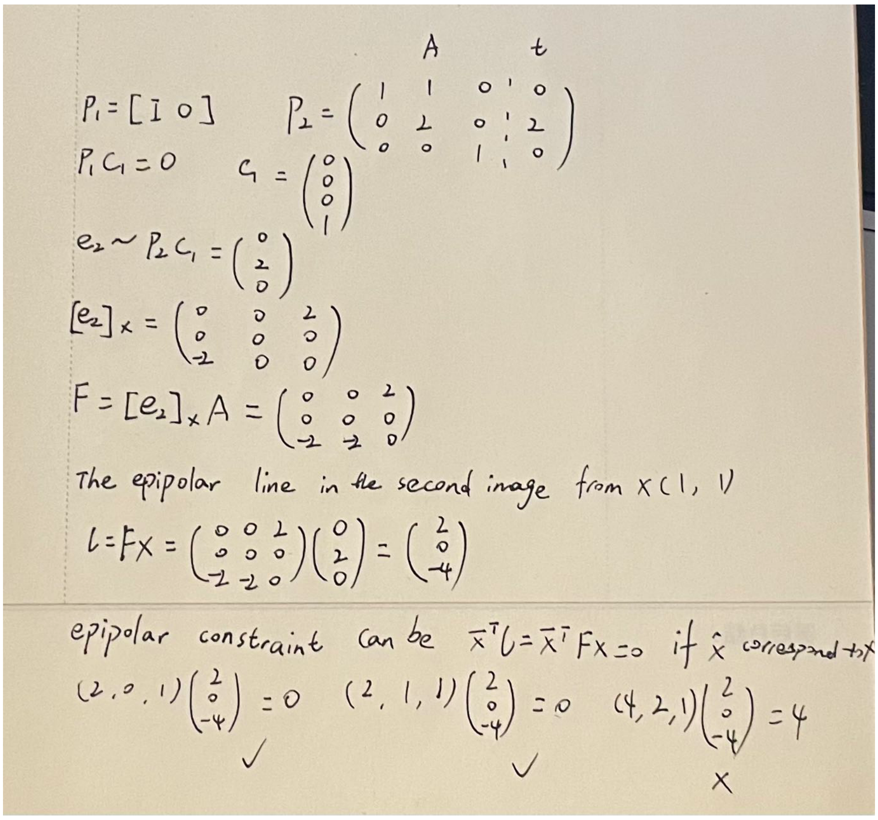
1 Fundamental Matrix

Ex1:



$$P_1 = [I \; 0] \qquad P_2 = \begin{array}{c} \quad A \qquad\quad t \\ \begin{pmatrix} 1 & 1 & 0 & \vert & 0 \\ 0 & 2 & 0 & \vert & 2 \\ 0 & 0 & 1 & \vert & 0 \end{pmatrix} \end{array}$$

$$P_1 C_1 = 0 \qquad C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$e_2 \sim P_2 C_1 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$[e_2]_\times = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$F = [e_2]_\times A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix}$$

The epipolar line in the second image from x(1, 1)

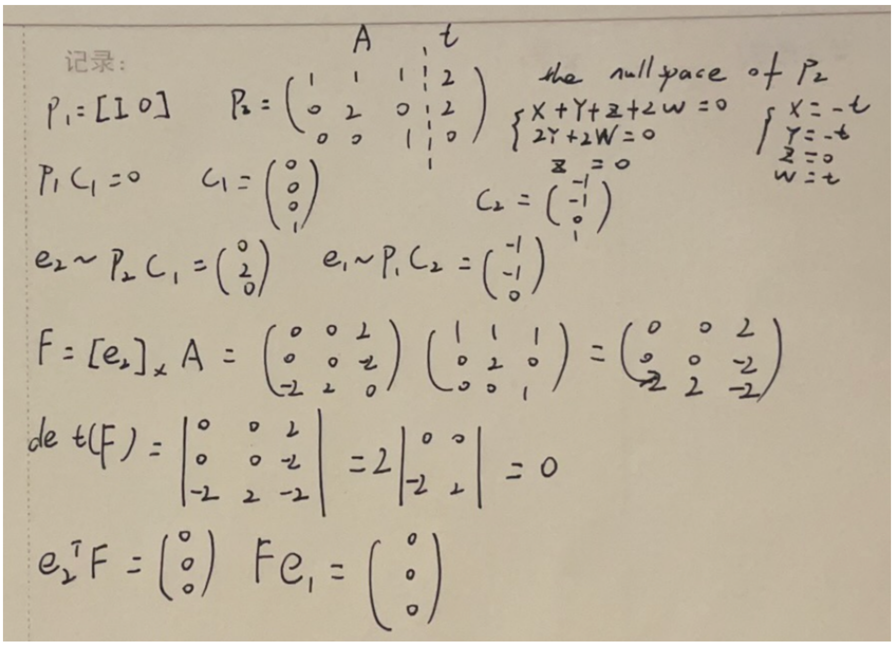
$$l = Fx = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ -2 & -2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

epipolar constraint can be $\bar{x}^T l = \bar{x}^T F x = 0$ if $\bar{x}$ correspond to x

$$(2, 0, 1)\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 0 \quad ✓ \qquad (2, 1, 1)\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 0 \quad ✓ \qquad (4, 2, 1)\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 4 \quad \times$$

Figure 1 Ex1

Ex2:



记录:

$$P_1 = [I \; 0] \qquad P_2 = \begin{array}{c} \quad A \qquad\quad t \\ \begin{pmatrix} 1 & 1 & 1 & \vert & 2 \\ 0 & 2 & 0 & \vert & 2 \\ 0 & 0 & 1 & \vert & 0 \end{pmatrix} \end{array}$$

the null space of P_2

$$\begin{cases} X + Y + Z + 2W = 0 \\ 2Y + 2W = 0 \\ Z = 0 \end{cases} \qquad \begin{cases} X = -t \\ Y = -t \\ Z = 0 \\ W = t \end{cases}$$

$$P_1 C_1 = 0 \qquad C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad C_2 = \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$e_2 \sim P_2 C_1 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \qquad e_1 \sim P_1 C_2 = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$F = [e_2]_\times A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{pmatrix}$$

$$\det(F) = \begin{vmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ -2 & 2 & -2 \end{vmatrix} = 2\begin{vmatrix} 0 & 0 \\ -2 & 2 \end{vmatrix} = 0$$

$$e_2^T F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad F e_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Figure 2 Ex2

OPTIONAL:

We know that the fundamental matrix relates corresponding points in the two views, and it is a rank 2 matrix since it is derived from the cross product of the camera matrices. Since the epipolar constraint e^T_2 F = 0 and Fe_1 = 0 must hold for all corresponding points, the fundamental matrix F must have a nullspace that contains the epipoles e1 and e_2. Since the dimension of the nullspace is at least 2, the determinant of F must be 0. This means that the fundamental matrix is singular and cannot be inverted.

Ex3:

$$F = N_2^T \tilde{F} N_1$$

CE1:

The fundamental matrix for the original (un-normalized) points (where F (3, 3) = 1, use F = F./F (3, 3))

3x3 double

	1	2	3
1	-3.3901e-08	-3.7201e-06	0.0058
2	4.6674e-06	2.8936e-07	-0.0267
3	-0.0072	0.0263	1

Figure 3 The fundamental matrix

The mean epipolar distances when F is computed both with normalization is 0.3612.

The mean epipolar distances when F is computed both without normalization is 0.4878.

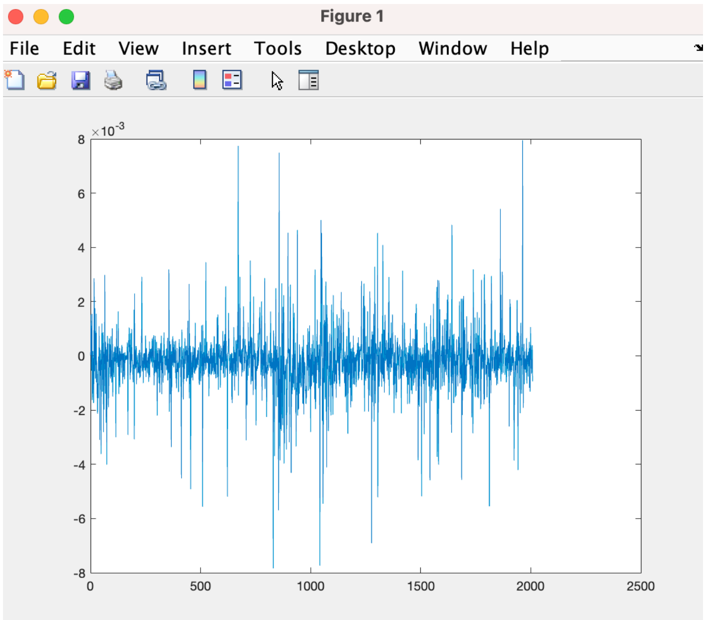


Figure 4 Epipolar constraints

The histogram and the plot of the epipolar lines are figure 5 and 6.

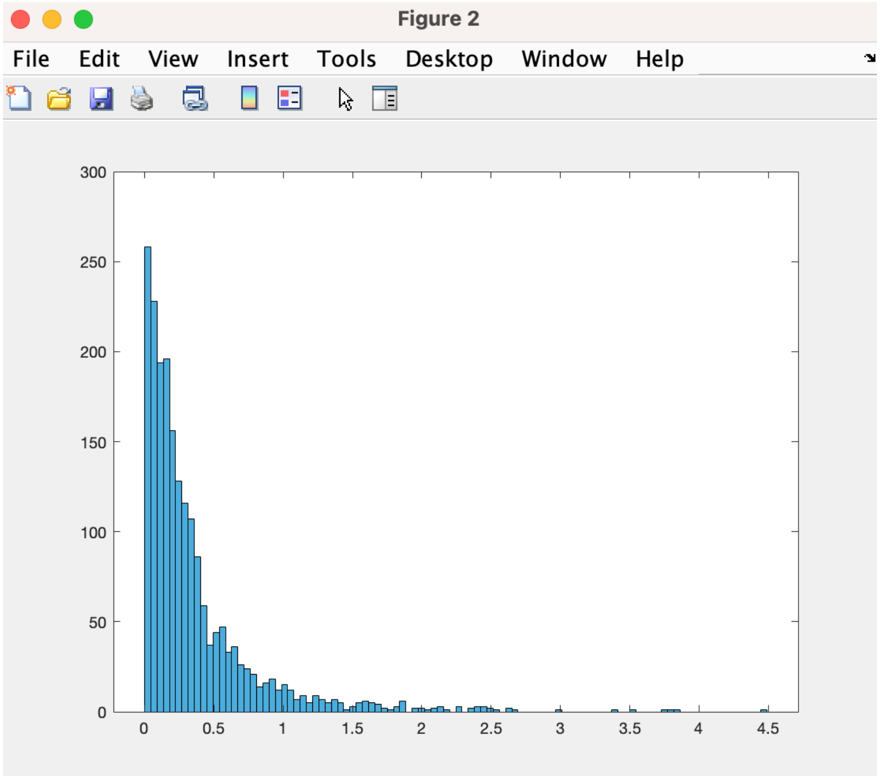


Figure 5 Histogram

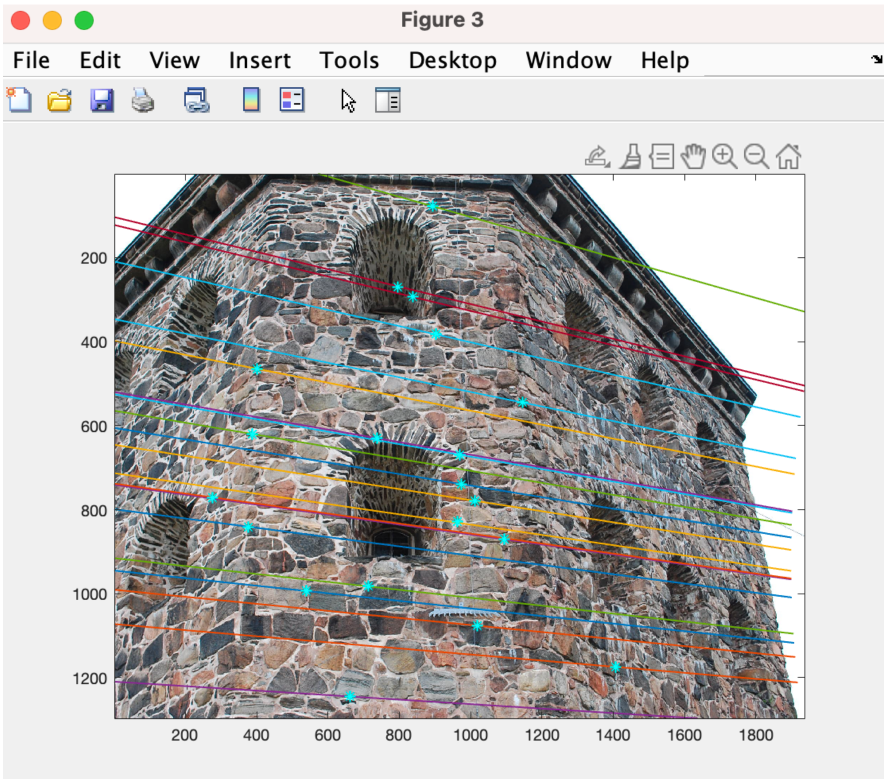


Figure 6 Random points and epipolar lines

Ex4:

记录：

$F = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $P_1 = [I \; 0]$ $x_1 = (1\ 2\ 3)$ $x_2 = (3\ 2\ 1)$

$P_2 = [[e_2]_\times F \; e_2]$

from before

$e_1 \sim P_2 C_1$ $e_2^T F = 0$ $e_2^T F = (x, y, z)\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = 0$

$e_2 \sim P_1 C_2$ $F e_1 = 0.$ $\begin{cases} x = -t \\ y = 0 \\ z = t \end{cases}$ $e_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$$[e_2]_\times F = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 2 \\ -1 & 0 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 2 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

x_1, x_2 are projected usig camera matrix

$x_{1,1} = P_1 x_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$ $x_{2,1} = P_1 x_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

$x_{2,2} = P_2 x_2 = \begin{pmatrix} -2 \\ 10 \\ 0 \end{pmatrix}$ $x_{1,2} = P_2 x_1 = \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

To verify that the epiplar constraint holds

$x_{1,2}^T F x_{1,1} = 0$ $x_{2,2}^T F x_{2,1} = 0.$

the nullspace of P_2 is the center.

$P_2 C_2 = 0$ $\begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 2 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0$ $\begin{cases} -x - w = 0 \\ 2y + 2z = 0 \\ -x + w = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = -t \\ z = t \\ w = 0 \end{cases}$

$C_2 \sim (0, -1, 1, 0).$

CE2:
The cameras matrices are P1 and P2

F | F_t | X | P1

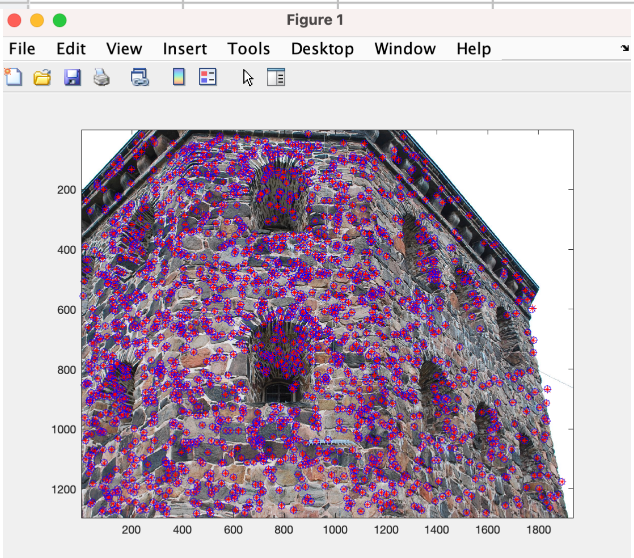
3x4 double

	1	2	3	4
1	420.2724	0	0	851.2594
2	0	333.0697	0	667.6759
3	0	0	0	1

F | F_t | X | P1 | P2

3x4 double

	1	2	3	4
1	29.9348	67.6594	-497.7678	-509.0913
2	90.7747	-228.7685	-110.2791	-458.8298
3	0.0588	0.0121	-0.0692	-0.6974



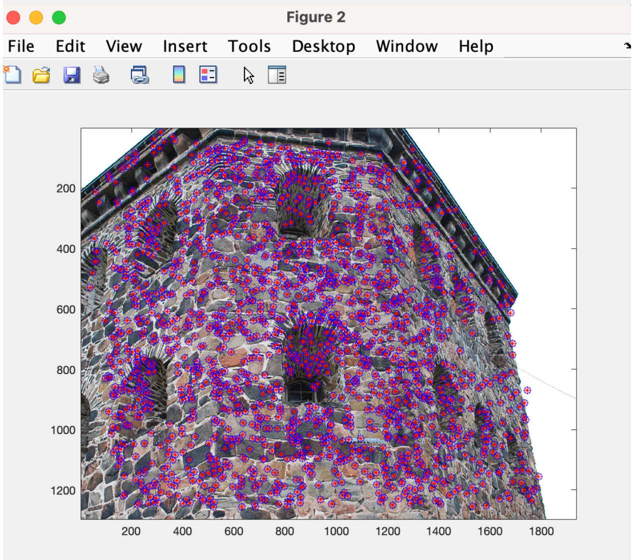


Figure 7 2D plot Red * are projection Blue are correct

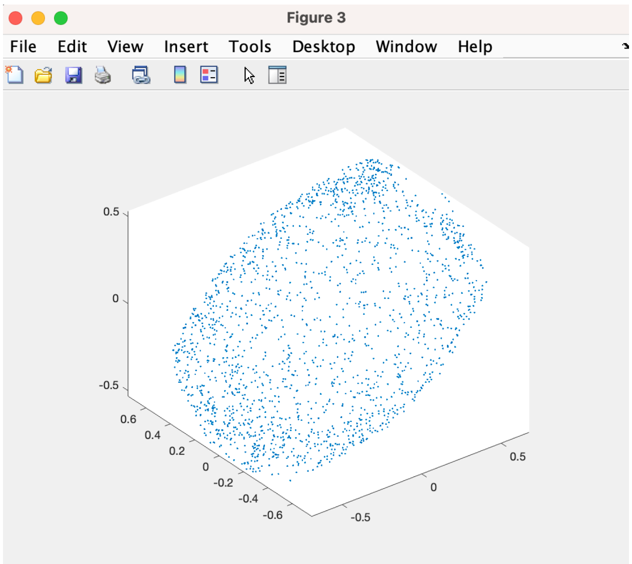


Figure 8

2 The Essential Matrix

Ex5:
To show that the eigenvalues of $[t]^T_\times[t]_\times$ are the squared singular values, we can use the fact that $[t]^T_\times[t]_\times = V^T S^T SU^T USV$, where $S^T S$ is diagonal with the singular values squared on the diagonal. Thus, the eigenvalues of $[t]^T_\times[t]_\times$ are the diagonal elements of $S^T S$, which are the singular values squared.

To verify equation (5), we first expand the left-hand side using the identity in equation (6):
$-t \times (t \times w) = (t \cdot w)t - (t \cdot t)w = ||t||^2w - (t \cdot t)w = (||t||^2 - ||t||^2)w = 0w$
Thus, we have shown that $w = t$ is an eigenvector of $[t]^T_\times[t]_\times$ with eigenvalue 0.

Next, we consider a vector w that is perpendicular to t. We have:
$[t]^T_\times[t]_\times w = (t \times (t \times w))_\times = (t \cdot w)t - ||t||^2w$
Using equation (5) again, we see that this expression is equal to λw, where $\lambda = ||t||^2$. Thus, any vector w perpendicular to t is an eigenvector of $[t]^T_\times[t]_\times$ with eigenvalue $||t||^2$.
Since $[t]^T_\times[t]_\times$ is a 3x3 matrix, we have found all three of its eigenvectors and corresponding eigenvalues.

To show that the singular values of $[t]_\times$ are 0, $||t||$ and $||t||$, we first note that $[t]_\times$ is skew-symmetric, which implies that its eigenvalues are purely imaginary or 0. We also know that the determinant of $[t]_\times$ is 0, which means that it has at least one eigenvalue of 0. Therefore, $[t]_\times$ has two purely imaginary eigenvalues, which are equal in magnitude and opposite in sign, and one eigenvalue of 0. The magnitude of the purely imaginary eigenvalues is equal to the norm of t, so the singular values of $[t]\times$ are 0, $||t||$ and $||t||$.
Finally, if $E = [t]_\times R$ and $[t]_\times$ has the SVD in (4), then we can write $E = U'SV'^T$, where $U' = [t]_\times U$ and $V' = RV$. The singular values of E are the same as the singular values of $[t]_\times$, which are 0, $||t||$ and $||t||$.

CE3:
The Essential matrix is

3x3 double

	1	2	3
1	-8.8885	-1.0058e+03	377.0783
2	1.2525e+03	78.3677	-2.4482e+03
3	-472.7888	2.5502e+03	1

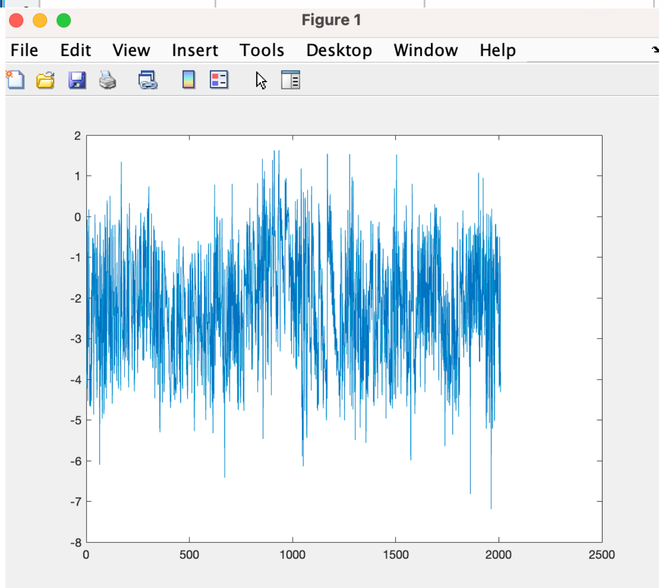


Figure 9 Errors of epiploar constraint

Figure 10 is the plot of the epipolar lines.

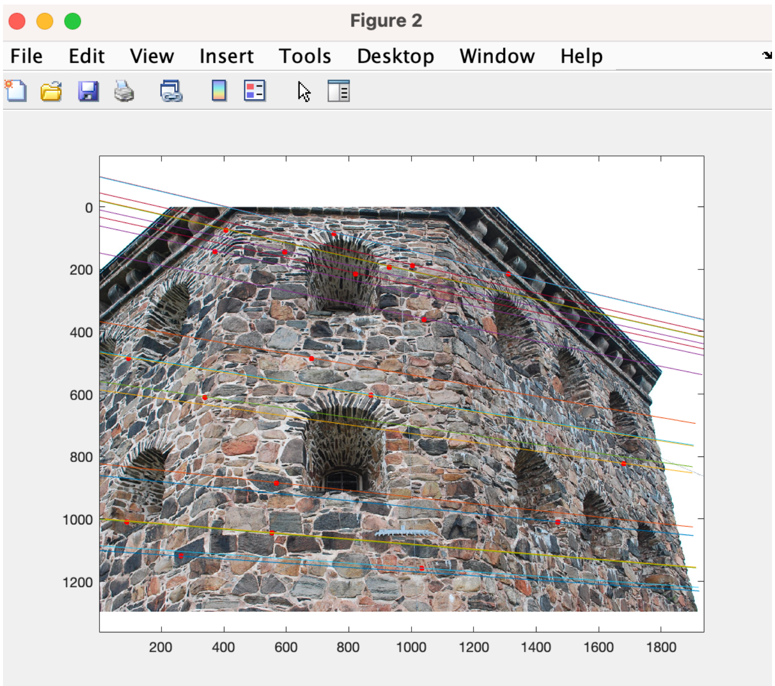


Figure 10 The epipolar lines

Figure 11 is the histogram of distances between points and lines.

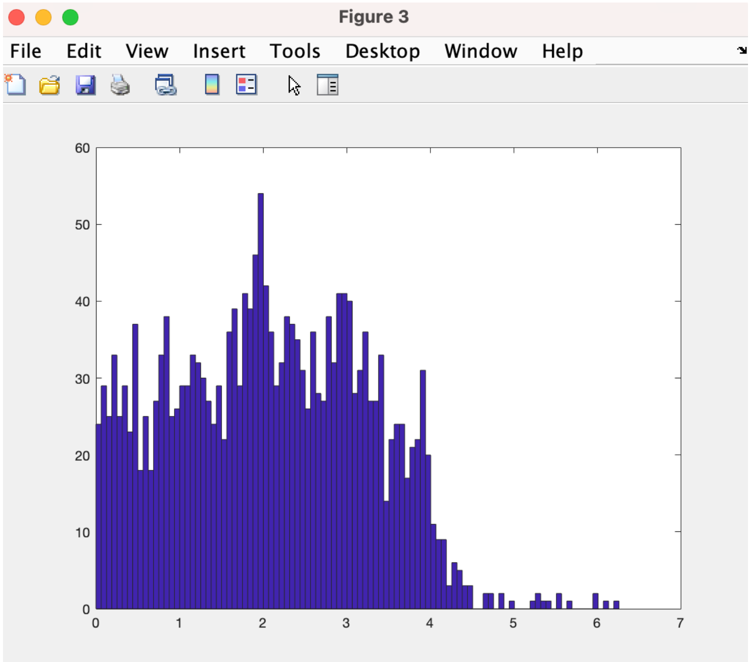


Figure 11 The histogram

Ex6:

$E = U\mathrm{diag}([1\ 1\ 0])V^T$

$$U = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$UV^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{pmatrix}$$

$$\det(UV^T) = \begin{vmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \end{vmatrix} = 0 + 0 + (+\tfrac{1}{2}) - 0 + \tfrac{1}{2} - 0$$
$$= 1$$

compute Essential matrix and verify $x_1(0,0)(P_1)$ $x_2(1,1)(P_2)$ is a plausible correspondence.

$$E = U\mathrm{diag}([1\ 1\ 0])V^T = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$
$$= \begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

记录：

To verify x_1, x_2. use the formula $(x')^T E x = 0$

$$x_2^T E x_1 = (1\ 1\ 1)\begin{pmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

x_1 x_2 are corresponding points.

$$X(s) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} \quad P_1[I\ 0]$$

$$P_1 X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} \Rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = 1 \end{cases}$$

we can not find W so $P_1 X$ is the projection $X(s)$.

$P_2 = [UWV^T\ u_3]$ or $[UWV^T\ -u_3]$ or $[UW^TV^T\ u_3]$ or $[UW^TV^T\ -u_3]$

$$W = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad u_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$[UWV^T u_3]X(s) = \begin{pmatrix} -1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ s \end{pmatrix} \quad s = -1/\sqrt{2}$$

$$[UWV^T - u_3]X(s) = \begin{pmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ -s \end{pmatrix} \quad s = 1/\sqrt{2}.$$

$$[UW^TV^T u_3]X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ s \end{pmatrix} \quad s = 1/\sqrt{2}$$

$$[UW^TV^T - u_3]X(s) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ -s \end{pmatrix} \quad s = -1/\sqrt{2}$$

if 3D point X in front of camera iff $r_3(X - C) > 0$.

$r_3 = v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. use depth.m we can find

$P_1[I\ 0]$, $P_2[UW^TV^T u_3]$ is infront of camera.

Both cameras and depths should be positive.

d1	-1.4142
d12	1
d2	1.4142
d22	-1
d3	1.4142
d32	1
d4	-1.4142
d42	-1

CE4:
I plot the image the points and the projected 3D-points in the same figure. The errors look small.

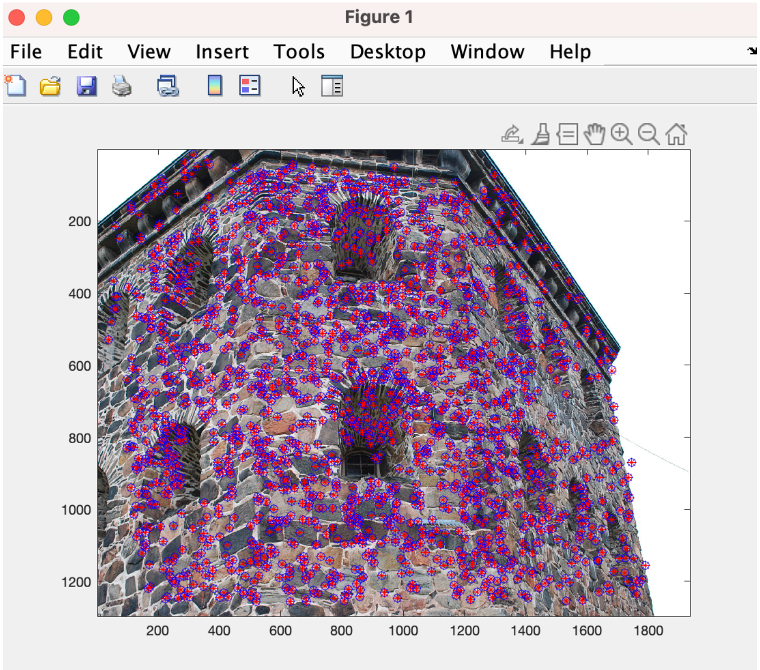


Figure 12

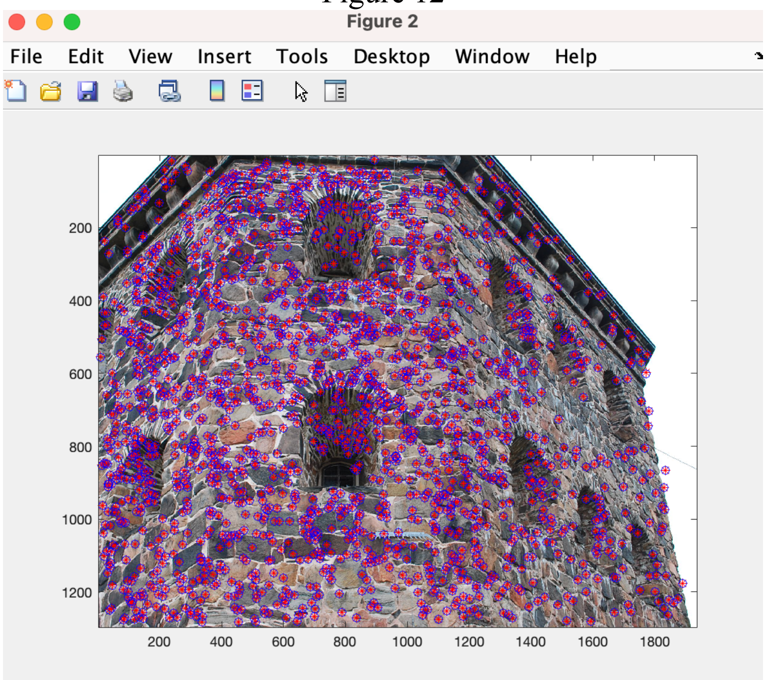


Figure 13

Figure 14 is 3D plot of the triangulated points. It looks like what I expected it to.

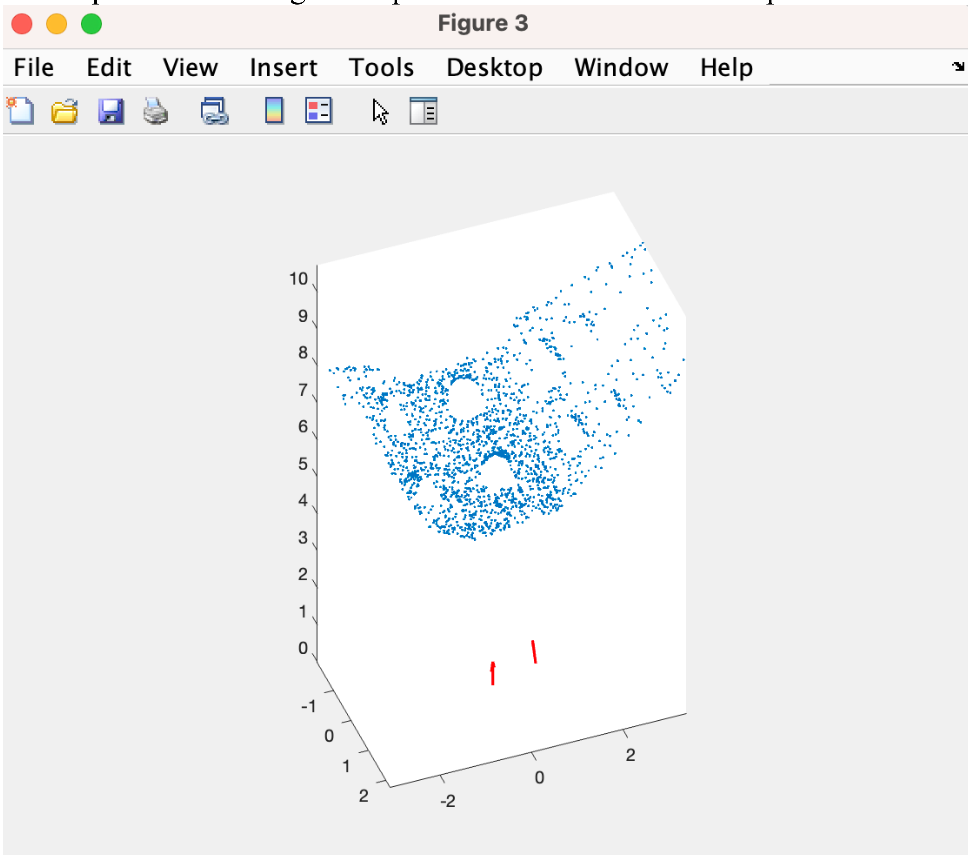


Figure 14 3D plot of the triangulated points