Assignment 4

Shuai

March 12, 2023

1 Robust Homography Estimation and Stitching

E1:

记录：

Two cameras $P_1 = [A_1\ t_1]$ $P_2 = [A_2\ t_2]$ with same camera center. there is a homography H between two.

First, the expression for the camera center.

$$\begin{cases} P_1 \begin{bmatrix} C_1 \\ 1 \end{bmatrix} = A_1 C_1 + t_1 & C_1 = -A_1^{-1} t_1 \\ P_2 \begin{bmatrix} C_2 \\ 1 \end{bmatrix} = A_2 C_2 + t_2 & C_2 = -A_2^{-1} t_2 \end{cases}$$

$$C_1 = C_2 \Rightarrow -A_1^{-1} t_1 = -A_2^{-1} t_2$$

$$A_2 A_1^{-1} t_1 = A_2 A_2^{-1} t_2$$

$$A_2 A_1^{-1} t_1 = t_2$$

3D points $\mathbf{X}_i = \begin{bmatrix} X_i \\ 1 \end{bmatrix}$ are projected with two cameras

$$P_1 \begin{bmatrix} X_i \\ 1 \end{bmatrix} = A_1 X_i + t_1 \qquad P_2 \begin{bmatrix} X_i \\ 1 \end{bmatrix} = A_2 X_i + t_2$$

$$X_i = A_1^{-1} P X_i - A_1^{-1} t_1$$

$$P_2 X_i = A_2 (A_1^{-1} P X_i - A_1^{-1} t_1) + t_2$$

and $A_2 A_1^{-1} t_1 = t_2$

$$\Rightarrow P_2 X_i = \underline{A_2 A_1^{-1}} P X_i$$

so $H = A_2 A_1^{-1}$

Figure 1

E2:

Degrees of freedom is 8.

Minimal number of point correspondences is 4.

Iterations of RANSAC is $(1-0.9^4)^n < 1-0.98 \quad n \geq 3.66 \quad n=4$

CE1:

The number of SIFT features in a is 947.

The number of SIFT features in b is 865.

The number of matches is 204.

The number of inliers is 149.

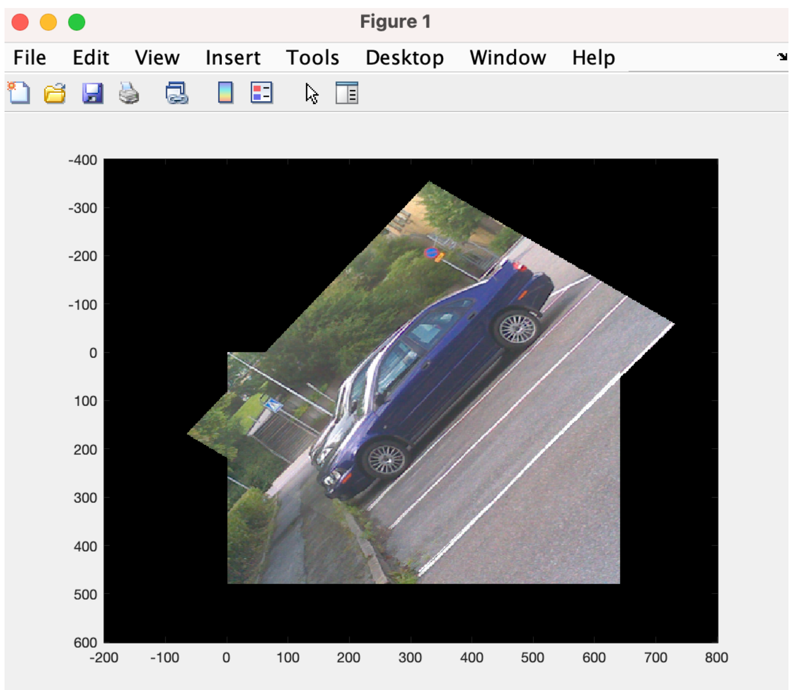

Figure 2

2 Robust Essential Matrix Estimation

E3:

Degrees of freedom is 5.

Minimal number of point correspondences is 5.

Iterations of RANSAC is $(1-0.9^5)^n < 1-0.98 \quad n \geq 4.38 \quad n=5$

CE2:

The number of inliers = 1465

RMS 0.4119

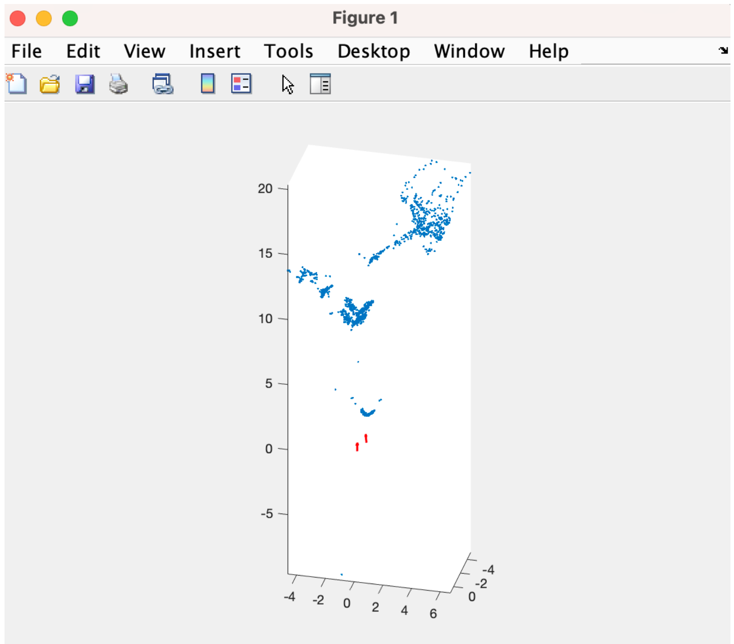

Figure 3

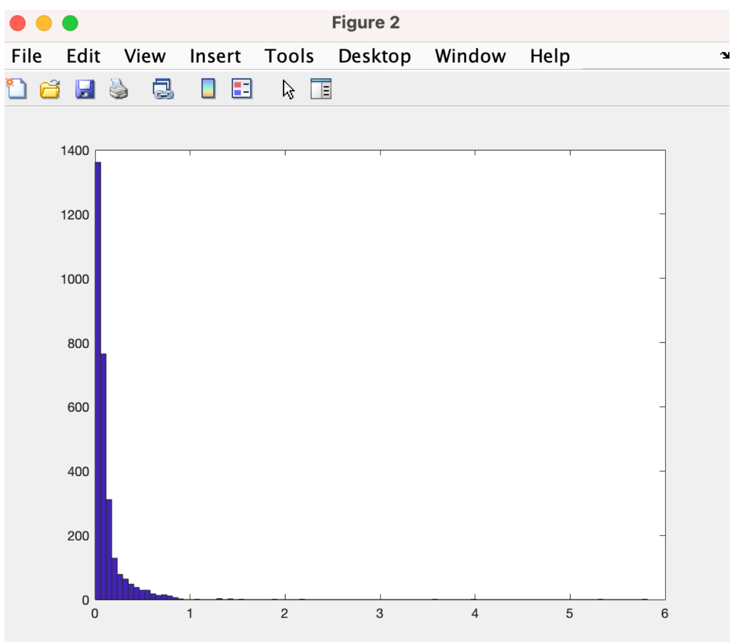

Figure 4

3 Calibrated Structure from Motion and Local Optimization

CE3:

The final RMS error is 0.3939.

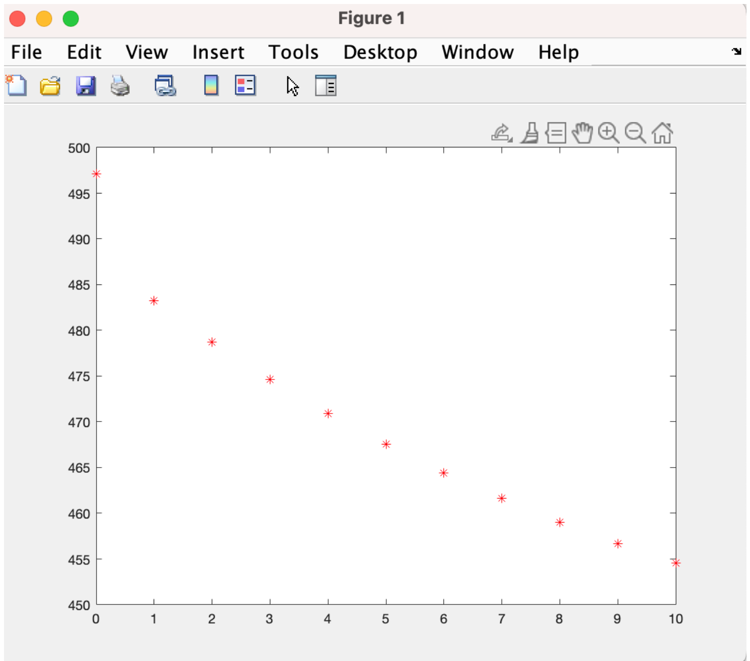

Figure 5

CE4:

The final RMS error is 0.2461.

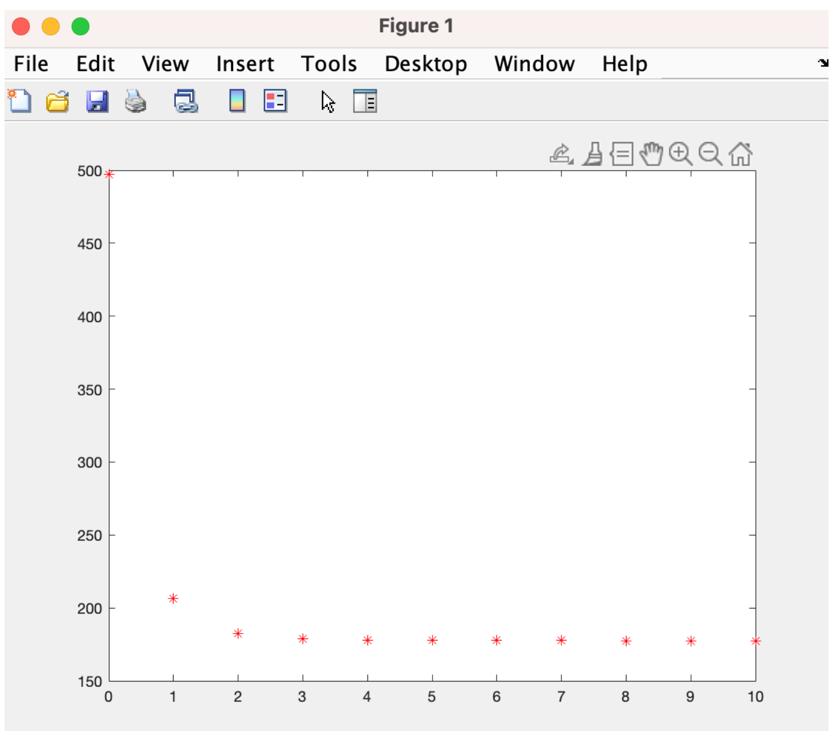

Figure 6

4 Dynamic Objects and Factorization

E4:

The matrix X can be rewritten as $X = C_kB + t$, where C = [c1 c2 ... cm] is a matrix with the vectors cik as columns, B = [B1 B2 ... Bn] is a matrix with the vectors Bj as rows, and t1 is a vector of all ones. Since the row vectors B1,...,Bn and 1 are linearly independent, the rank of X is n+1.

To minimize $\|A - X\|^2_F$, we can set the gradient of the objective function with respect to X to zero:

$\nabla X(\|A - X\|^2_F) = -2(A - X) = 0$

This yields X = A. Substituting X in the equation for X, we get:

$A = C_KB + t$

Multiplying both sides by 1t1, we get:

$tA = tC_KB + tt$

Since tB = [sum(B1) sum(B2) ... sum(Bn)] = $n\bar{B}$, where $\bar{B}$ is the mean row vector of B, and (1t1)t1 = n, we have:

$tA = nC_K\bar{B} + n$

Dividing both sides by n, we get:

$\bar{A} = C_K\bar{B} + 1$,

where $\bar{A}$ is the row mean vector of A. Solving for ti, we get:

$t_i = \bar{A}i - c_{ik}\bar{B}_k$,

where $\bar{A}_i$ is the row mean vector of $[a_{i1}\ a_{i2} \ldots a_{in}]$ and $\bar{B}_k$ is the mean row vector of B_k.

To find B_k and c_{ik} when t_i has been eliminated:

Compute the mean row vector $\bar{B}$ of B.

Subtract $\bar{B}$ from each row vector B_j to get the centered row vectors $B_j\tilde{} = B_j - \bar{B}$.

Compute the centered matrix $C\tilde{} = [c_1\tilde{}\ c_2\tilde{} \ldots c_m\tilde{}]$, where $c_{ik}\tilde{} = c_{ik} - t_i\bar{B}_k$.

Solve for the centered matrix $C\tilde{}$ by minimizing the objective function $\|A\tilde{} - \|C\tilde{}B\tilde{}\|^2_F$, where $A\tilde{}$ is the centered matrix of A, and $B\tilde{} = [B_1\tilde{}\ B_2\tilde{} \ldots B_n\tilde{}]$ is the matrix with the centered row vectors $Bj\tilde{}$ as rows.

Once $C\tilde{}$ and $B\tilde{}$ have been found, we can compute B_k as $B_k = B\tilde{}_k + \bar{B}$, and c_{ik} as $c_{ik} = c_{ik}\tilde{} + t_i$.

Note that this method assumes that the noise in the measurements a_{ij} is uncorrelated and has zero mean.