Assignment 2

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1 Calibrated vs. Uncalibrated Reconstruction

1.1 Exercise 1

X is the estimated 3D-points and P is the camera matrix such that $\lambda_{ij}\mathbf{x}_{ij} = P_i\mathbf{X}_j$. And $\tilde{I} = TT^{-1}$, so $\lambda_{ij}\mathbf{x}_{ij} = P_i\mathbf{X}_j = P_iTT^{-1}\mathbf{X}_j = \tilde{P}_i\tilde{\mathbf{X}}_j$. $\tilde{P}_i = P_iT$ is the new cameras, $\tilde{X}_j = T^{-1}X_j$ is the new solution that solve the problem since $\lambda_{ij}\mathbf{x}_{ij} = P_i\mathbf{X}_j = P_iTT^{-1}\mathbf{X}_j = \tilde{P}_i\tilde{\mathbf{X}}_j$.

1.2 Computer Exercise 1

Figure 1 is the reconstruction of 3D points. It looks reasonable although there are some skew or distortion. We can find that the points in the upper part are relatively sparse, and the whole image or points stretch up a bit. The parallel lines are not preserved. The projections are close to the corresponding image points.

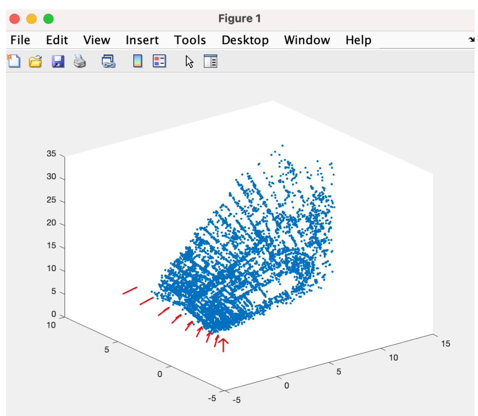


Figure 1 The reconstruction of 3D points

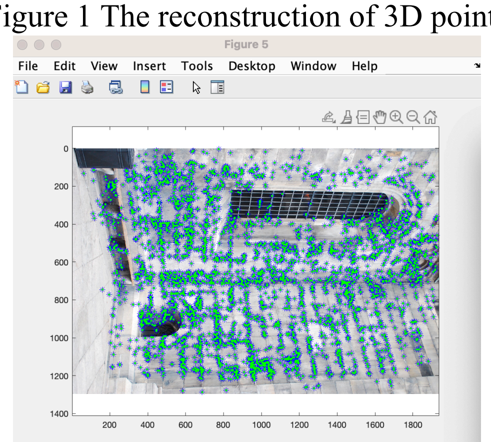


Figure 2 The points in the image

Figure 3 is the 3D points and cameras with projective transformations T_1.

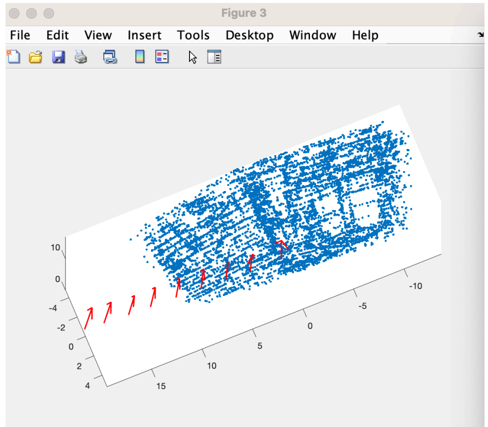


Figure 3 The 3D points and cameras with T1

Figure 4 is the 3D points and cameras with projective transformations T_2.

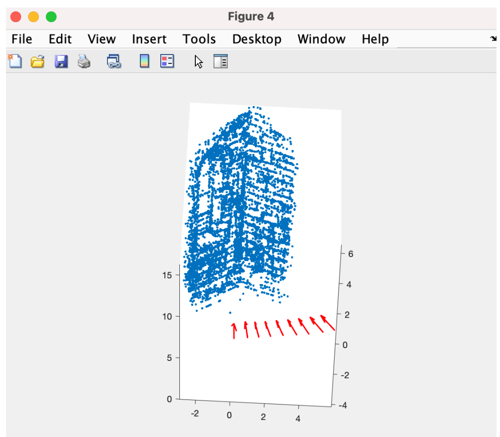


Figure 4 The 3D points and cameras with T2

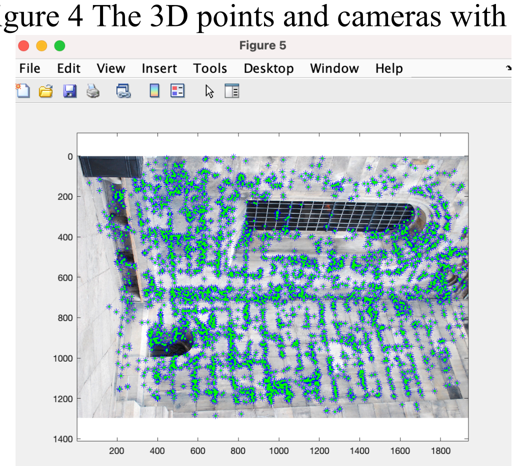


Figure 5 The image points

The two projective transformations look more reasonable. We still can find two results are different a bit due to the different projective transformations. They provide the same projection.

1.3 Exercise 2

When we use calibrated cameras, the parameter K is known, so x˜ = k−1x and then structure form motion problem to find normalized cameras (calibrated cameras to that of Exercise 1) and X and λx˜= [R t] X. Then we can try to do the same trick. However, when multiplying [R t] with H, the result does not have a R in first 3x3 block. We need H to be similarity transformation to achieve a valid solution. Hence, we do not have the same distortion.

2 Camera Calibration

2.1 Exercise 3

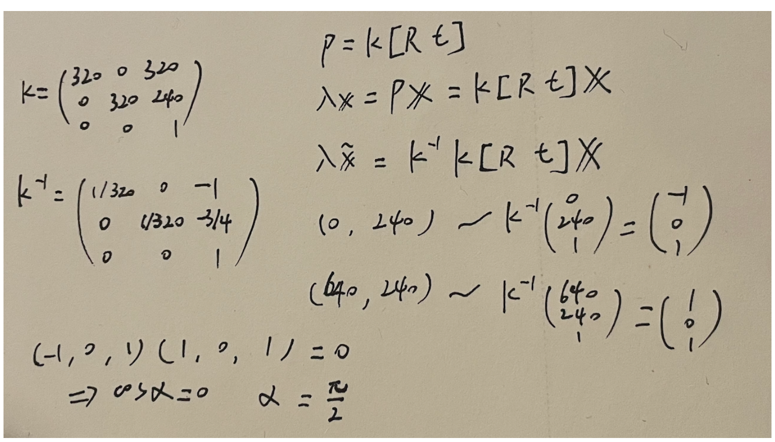
focal length f, principal point (x0, y0). The coordinates are scaled by the focal length and translated by the principal point.

A is scaling by a factor one over the focal length. The points are transformed from pixel to meters. B is the negative translation of the principal point which centers the point around (0,0).

When multiplying the image with transformation K^{-1}, it would transform points on the projective plane in the camera coordinate system.

The principal point (x_0, y_0) ends up at (0,0,1).

A point with distance f to the principal point ends up with distance 1 to the principal point after normalization.



P_1=K[R t] P_2=K[R t] The camera centers are the null space of P_1 and P_2.

So, $P_1\begin{bmatrix}C_1\\1\end{bmatrix} = K[R\ t]\begin{bmatrix}C_1\\1\end{bmatrix} = 0$, $P_2\begin{bmatrix}C_2\\1\end{bmatrix} = [R\ t]\begin{bmatrix}C_2\\1\end{bmatrix} = 0$. And $K^{-1}K[R\ t]\begin{bmatrix}C_1\\1\end{bmatrix} = 0$, so $C_1 = C_2$. Hence, they have the same center. The principal axis is the third row of matrix R.

The third row of K that is (0,0,1) cannot change the third row of R. It results that P_1, P_2 are the same.

2.2 Exercise 4

记录:

focal length 1000 $f = 1000$ $(500, 500)$.

$$K = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{pmatrix} \quad K^{-1} = \begin{pmatrix} 1/1000 & 0 & -1/2 \\ 0 & 1/1000 & -1/2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(0,0) = K^{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix}$$

$$(500,500) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad (0, 1000) = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

$$(1000, 0) = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ 1 \end{pmatrix} \qquad (1000, 1000) = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix}$$

3 RQ Factorization and Computation of K

3.1 Exercise 5

$$KR = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}\begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix}$$

the last row of P $A_3 = (-1/\sqrt{2}, 0, 1/\sqrt{2})$

$$\|A_3\| = \sqrt{\tfrac{1}{2}+\tfrac{1}{2}} = 1 = f \quad R_3 = (-1/\sqrt{2}, 0, 1/\sqrt{2}).$$

$$e = \left(-\frac{700}{\sqrt{2}}, 1400, \frac{700}{\sqrt{2}}\right)\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = 700$$

$$dR_2 = \begin{pmatrix} -\frac{700}{\sqrt{2}} \\ 1400 \\ \frac{700}{\sqrt{2}} \end{pmatrix} - 700\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1400 \\ 0 \end{pmatrix} \quad d = 1400 \quad R_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

$$b = \left(\frac{800}{\sqrt{2}}, 0, \frac{2400}{\sqrt{2}}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0 \quad c = \left(\frac{800}{\sqrt{2}}, 0, \frac{2400}{\sqrt{2}}\right)\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = 800$$

$$aR_1 = \begin{pmatrix} \frac{800}{\sqrt{2}} \\ 0 \\ \frac{2400}{\sqrt{2}} \end{pmatrix} - 0\,R_2 - 800\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \frac{1600}{\sqrt{2}} \\ 0 \\ \frac{1600}{\sqrt{2}} \end{pmatrix} \Rightarrow a = 1600 \quad R_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

$$\Rightarrow K = \begin{pmatrix} 1600 & 0 & 800 \\ 0 & 1400 & 700 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} rf & sf & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

$f = 1400$ $r = \frac{8}{7}$ $s = 0$ $(x_0, y_0) = (800, 700)$.

3.2 Computer Exercise 2

I used the first camera; the results are following.

```
K1 =

   1.0e+03 *

    2.3940         0    0.9324
         0    0.5995    0.6283
         0         0    0.0010


K2 =

   1.0e+03 *

    2.3940    0.0000    0.9324
         0    2.3981    0.6283
         0         0    0.0010
```

4 Direct Linear Transformation DLT

4.1 Exercise 6

4.2 Exercise 7

The formulate to compute P from $\tilde{P}$ would be $P = N^{-1}\tilde{P}$

4.3 Computer Exercise 3

Figure 6 is the normalization of the points X. The points are are centered around (0, 0) with mean distance 1 to (0, 0).

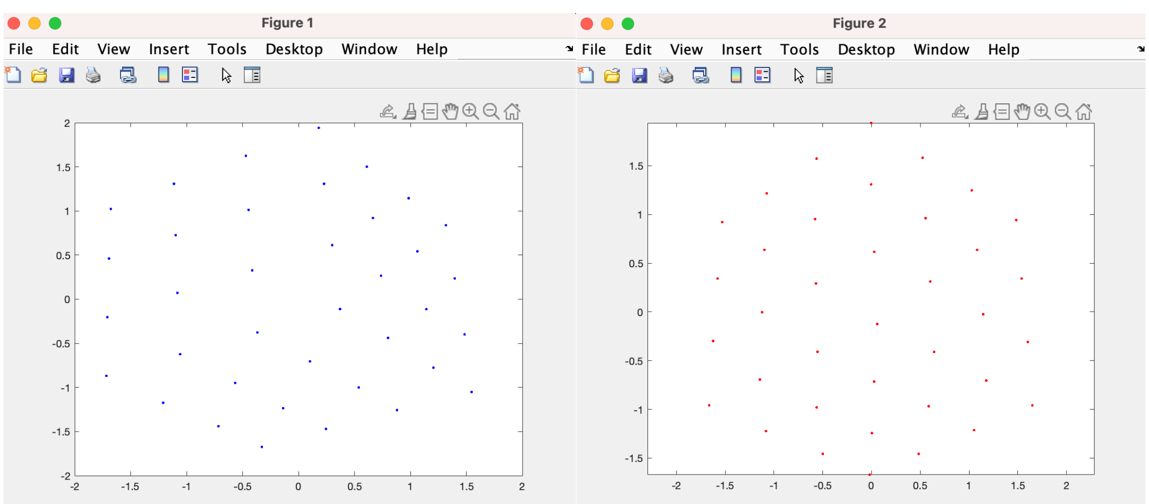


Figure 6 The normalized point

After finding the M metricize for two cameras, the singular value decomposition is computed to find the v and the smallest eigenvalues. The smallest eigenvalues are close to zero (0.0151 and 0.0122). ||Mv|| is equal to the smallest eigenvalue.

```
miniMize1        0.0151
miniMize2        0.0122
K1 =

   1.0e+03 *

    2.4486   -0.0181    0.9598
         0    2.4468    0.6759
         0         0    0.0010


K2 =

   1.0e+03 *

    2.3885   -0.0245    0.8142
         0    2.4009    0.7905
         0         0    0.0010
```

Figure 7 is the model points that were projected into images. The model points and the projected ones are close to each other. They look reasonable although there are still some differences. The cameras both points to the cube but from slightly different centers. From the two results K1 and K2, they are true parameters. The two f are close, two s are close to 0, γ are close to 1. And the principal points are (959.8, 675.9) and (814.2, 790.5). I thought the points of image are clearer than the exercise 1 and when only two directions so there is no ambiguity.

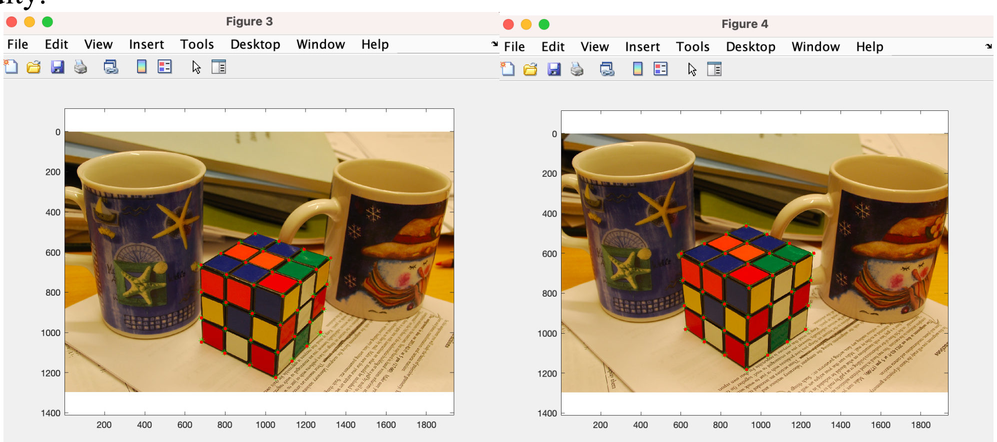


Figure 7 Model points on the images

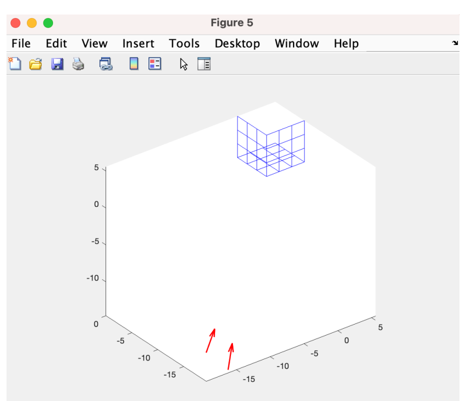
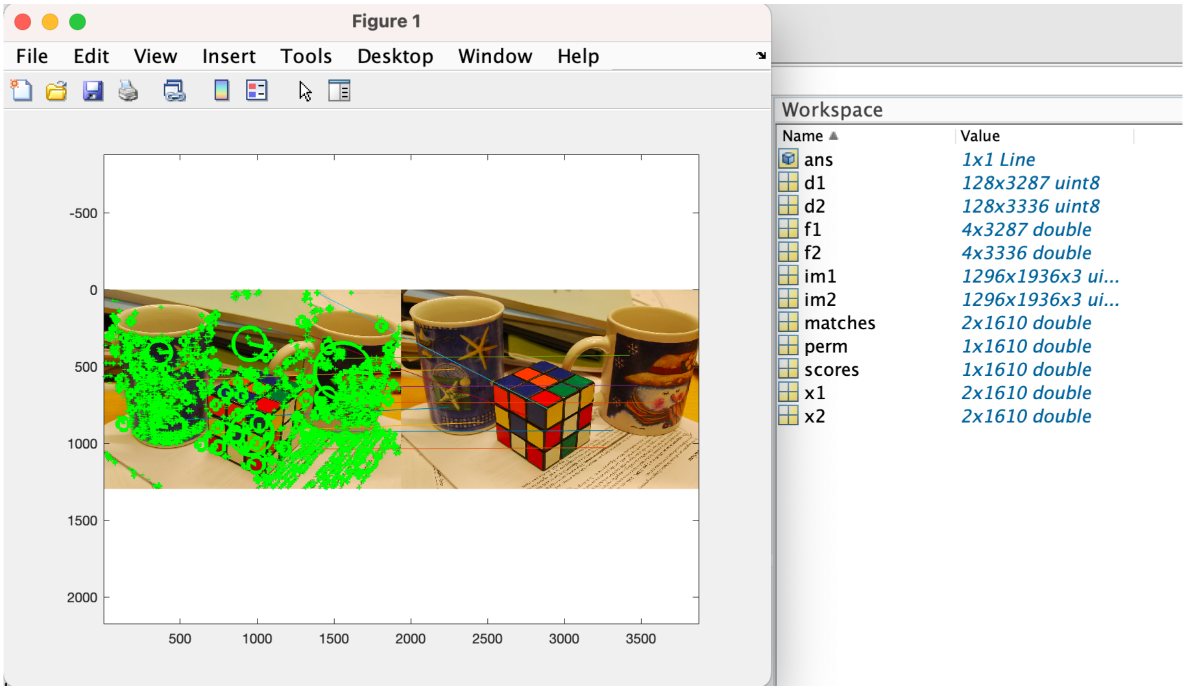


Figure 8 Camera centers and model

5 Feature Extraction and Matching using SIFT

5.1 Computer Exercise 4



6 Triangulation using DLT

6.1 Computer Exercise 5

Figure 9 plots the remaining 3D points, the cameras and the cube model in the same 3D plot.

We can find there are some improvements like the text, and points on cups image. And I distinguish the dominant objects in Figure 10.

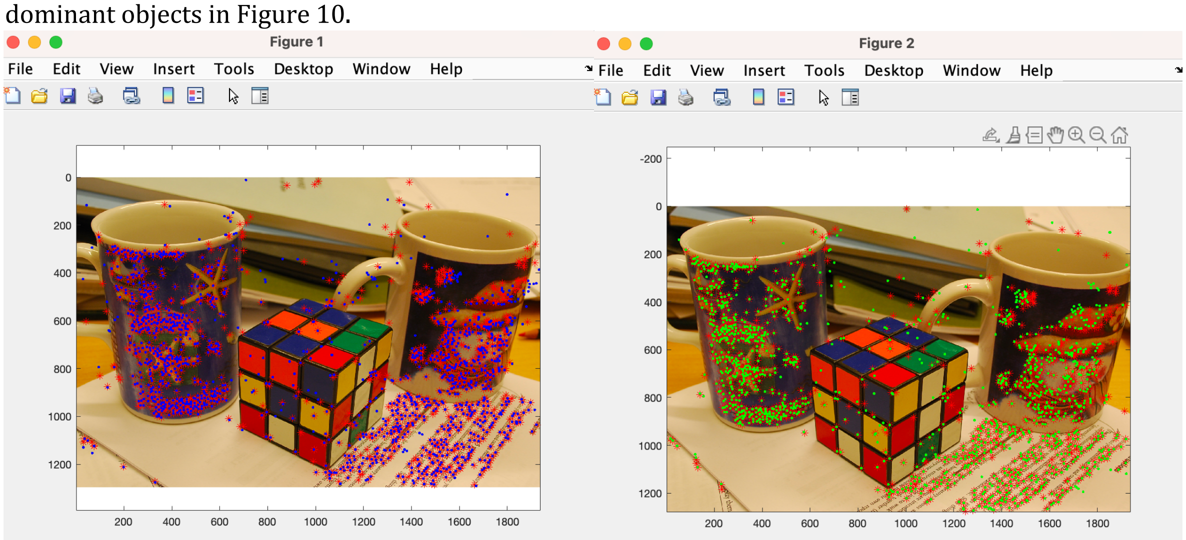


Figure 9 3D points and the corresponding SIFT points

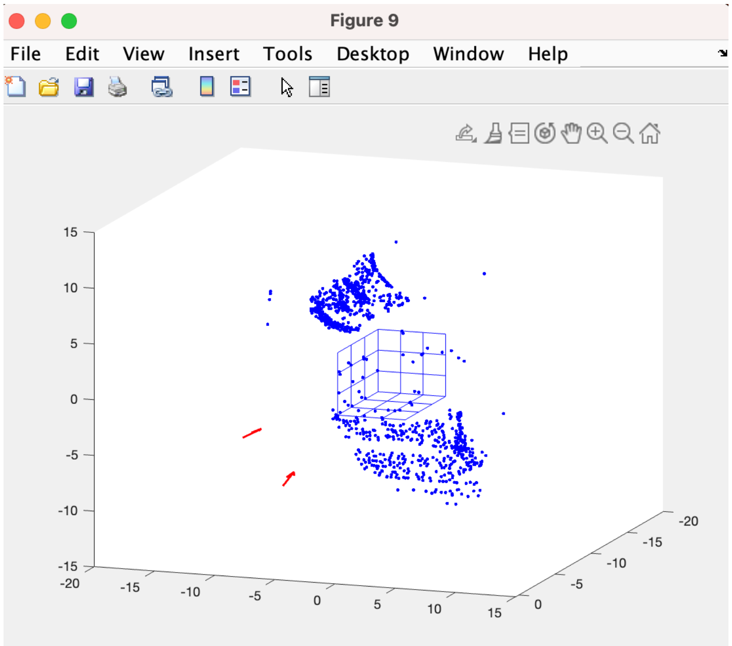


Figure 10 Cameras and cube model