1 Instructions

2 Points in Homogeneous Coordinates

E1:

x_1=(4, -2, 2) gives(4/2, -2/2) = (2, -1)

x_2=(3, -2, -1) gives(3/-1, -2/-1) = (-3, 2)

x_3=(4λ, -2λ, 2λ) gives(4λ/2λ, -2λ/2λ) = (2, -1), λ≠0

x_4=(4, -2, 0) points that is infinitely far away in direction(4,-2).

CE1:

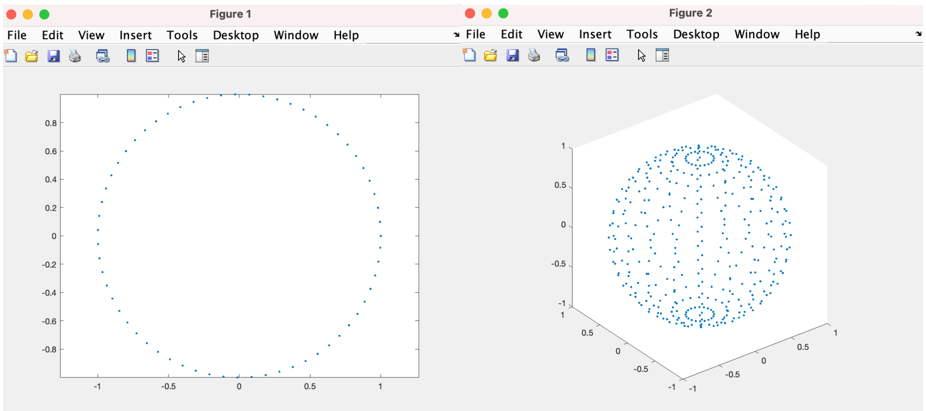


Figure 1 The result of x2D and x3D

3 Lines

E2:

记录：

$l_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$ $\quad l_2 = \begin{pmatrix}3\\2\\1\end{pmatrix}$

$\begin{cases} l_1^T x = 0 \\ l_2^T x = 0 \end{cases} \rightarrow \begin{cases} x+y+z=0 \\ 3x+2y+z=0 \end{cases} \rightarrow \begin{cases} x=-s \\ y=2s \\ z=-s \end{cases} \quad s \in R$

$x \sim (-1, 2, -1)$ which can be interpreted as $(1,-2)$ in R^2

记录：

$l_3 = \begin{pmatrix}1\\2\\3\end{pmatrix}$ $\quad l_4 = \begin{pmatrix}1\\2\\1\end{pmatrix}$

$\begin{cases} l_3^T x = 0 \\ l_4^T x = 0 \end{cases} \rightarrow \begin{cases} x+2y+3z=0 \\ x+2y+z=0 \end{cases} \rightarrow \begin{cases} x=-2t \\ y=t \\ z=0 \end{cases} \quad t \neq 0.$

The intersection in R^2 can be interpreted as a point infinitely far away in the direction $(-2, 1)$.

$x_1 = \begin{pmatrix}1\\1\end{pmatrix}$ $\quad x_2 = \begin{pmatrix}3\\2\end{pmatrix}$

$l \sim (a, b, c)$

$\begin{cases} l^T x_1 = 0 \\ l^T x_2 = 0 \end{cases} \rightarrow \begin{cases} a+b+c=0 \\ 3a+2b+c=0 \end{cases} \rightarrow \begin{cases} a=-s \\ b=2s \\ c=-s \end{cases} \quad s \in R$

$l \sim (-1, 2, -1)$

E3:

$$M = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad M = \begin{pmatrix} 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The nullspace of M is given by

$$\begin{cases} 3x + 2y + z = 0 \\ x + y + z = 0 \\ w = 0 \end{cases} \Rightarrow \begin{cases} x = -s \\ y = 2s \\ z = -s \\ w = 0 \end{cases} \quad s \in \mathbb{R}$$

$C \sim (-1, 2, -1, 0)$ which is a vanishing point we can interpret as a point infinitly far away in the direction $(-1, 2, -1)$ is the intersection point of l_1, l_2.

When I tried to find the null space of the matrix, only (-1, 2, -1) can be the solution. So, there is no other solution.

CE2

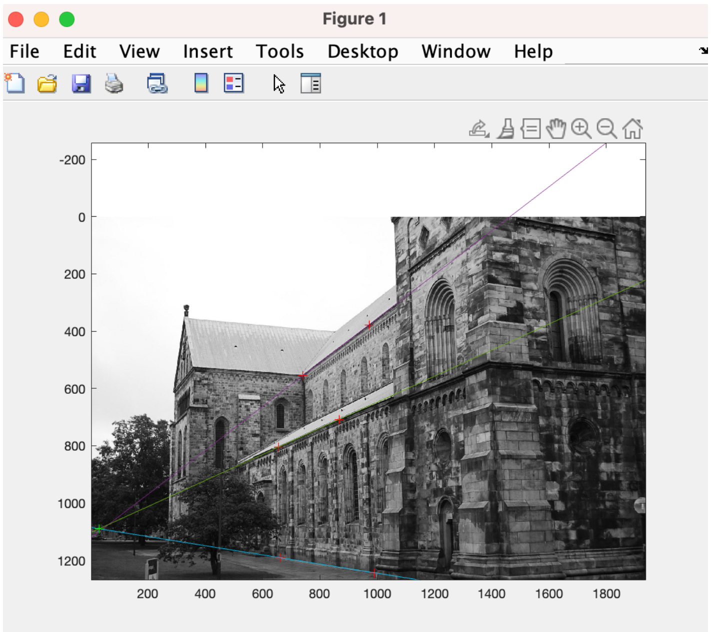


Figure 2 The result of three pairs of image points

The distance between the first line and the the intersection point is 8.1950. It is not close to zero. We can find that the three lines do not intersect at the same point. It means that three lines are not parallel in 3d. The reasons could be the image was altered or warped, or the positions of points were not exact.

4 Projective Transformations

E4

$$y_1 \sim Hx_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$y_2 \sim Hx_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$l_1 \quad x_1, x_2$

$$\begin{cases} x + z = 0 \\ y + z = 0 \end{cases} \rightarrow \begin{cases} x = -s \\ y = -s \\ z = s \end{cases} \quad s \in R \qquad \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$l_2 \quad y_1, y_2$

$$\begin{cases} x = 0 \\ x + y + z = 0 \end{cases} \rightarrow \begin{cases} x = 0 \\ y = -s \\ z = s \end{cases} \quad s \in R \qquad \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$(H^{-1})^T l_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} = l_2$$

The proof that projective transformations preserve lines are that we assume x is on the line L, so Lx=0, and y~Hx, it follows that y is on the line $\hat{l} = (H^{-1})^T l$, because

$0 = l^T x = l^T H^{-1}$ Hx~ $(H^{-T} l)^T$y=$\hat{l}^T y$

CE3

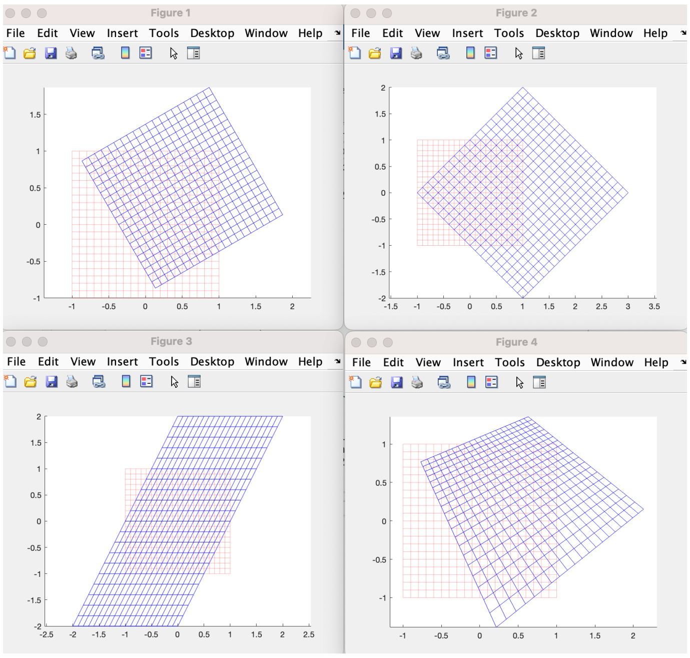


Figure 3 The results of the transformations

We can find that H1(Euclidean transformation) just rotated the position of lines and preserved lengths between points and preserved angles between lines. H2(similarity transformation) changed the position of lines, scaled the whole map and preserved angles between lines. H1, H2 and H3 (affine transformation) they were all parallel. H4(Projective transformation) only preserved the lines.

Projective transformation only preserves lines, affine transformation preserves parallel lines, similarity transformation preserves both parallel lines and angles, and Euclidean transformation preserves parallel lines, angles and the distances between points.

5 The Pinhole Camera

E5

$X_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$ $X_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $X_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$PX_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ $PX_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $PX_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

in cases division with the third coordinate

$\begin{pmatrix} \frac{1}{4} \\ \frac{1}{2} \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

The interpretation of the projection X_3 is the infinitely far away point in direction (1, 1).

nullspace of P

$x = 0$
$y = 0$
$z = -s$
$w = ts$

$C_1 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

P can be written $[R\ t]$

$R_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ view direction

CE4

The camera centers of two cameras were

Name ▲	Value
cc1	[0;0;0]
cc2	[6.6352;14.8460;-15.0691]

The principle axis were

```
pa1n =

    0.3129    0.9461    0.0837


pa2n =

    0.0319    0.3402    0.9398
```

Figure 4 is the projection plot. The points followed the objects.

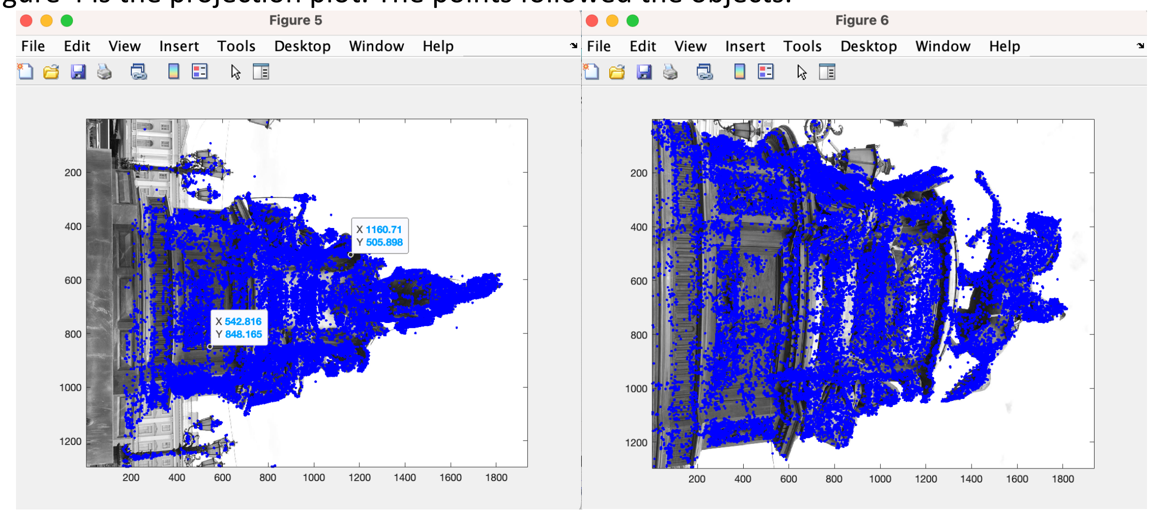


Figure 4 Projection of matrix

Figure 5 is the 3D image with camera center and principal axis.

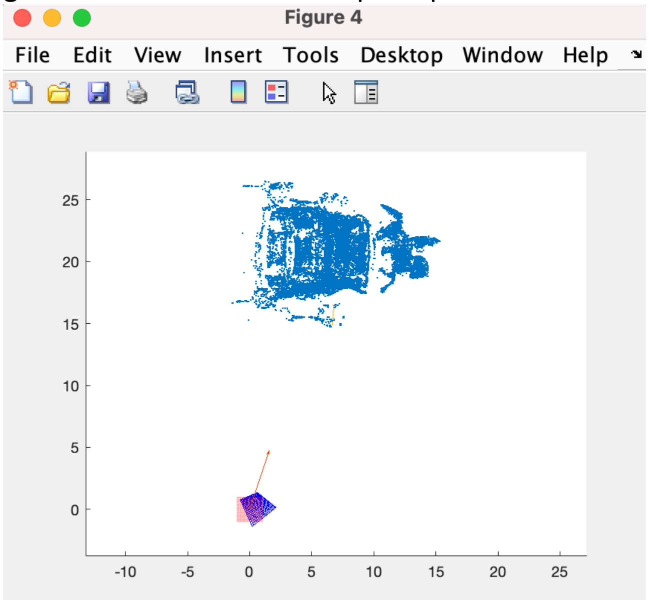


Figure 5 3D image with camera center and principal axis

E6:

x~P_1U=[I 0] $\begin{bmatrix} X \\ S \end{bmatrix} = \mathrm{IX}$

So the collection is the points in homogeneous coordination. And IX does not include s, so it is not possible to determine s using only information from P1.

$\pi^T U = 0$

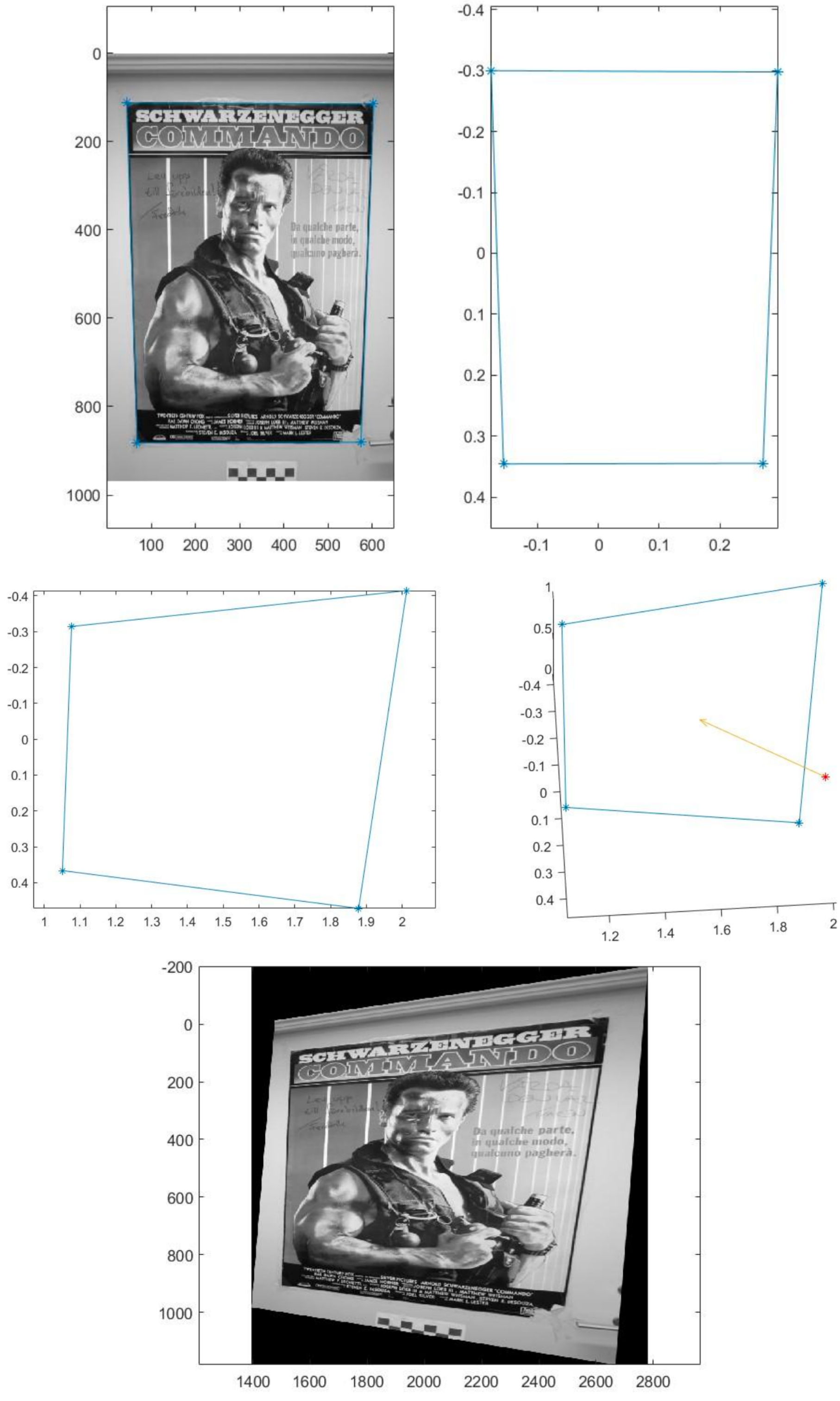
$(\pi^T \; 1)\begin{pmatrix} X \\ s \end{pmatrix} = \pi^T X + s = 0 \rightarrow s = -\pi^T X$

maps X to y means $y = HX$.

$y \sim P_2 U = (R \; t)\begin{pmatrix} X \\ -\pi^T X \end{pmatrix} = RX - t\pi^T X = (R - t\pi^T)X$

$\rightarrow (R - t\pi^T)X = HX \Rightarrow H = R - t\pi^T$

CE5:



The new camera matrix is:

$$P2 = 1.0e+03 * \begin{matrix} 0.9023 & 0.0000 & 0.8134 & 2.3762 \\ -0.2349 & 1.1913 & 0.4069 & 0 \\ -0.0005 & 0 & 0.0009 & 0 \end{matrix}$$