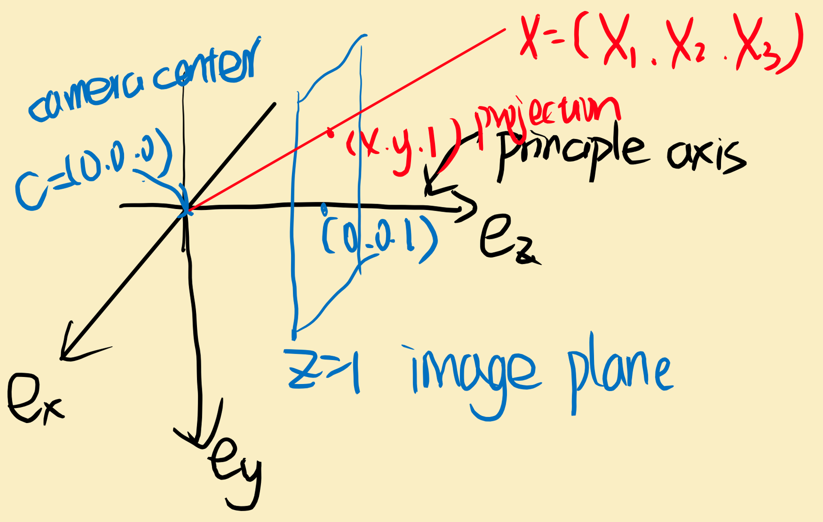
L1

Pinhole Camera Model



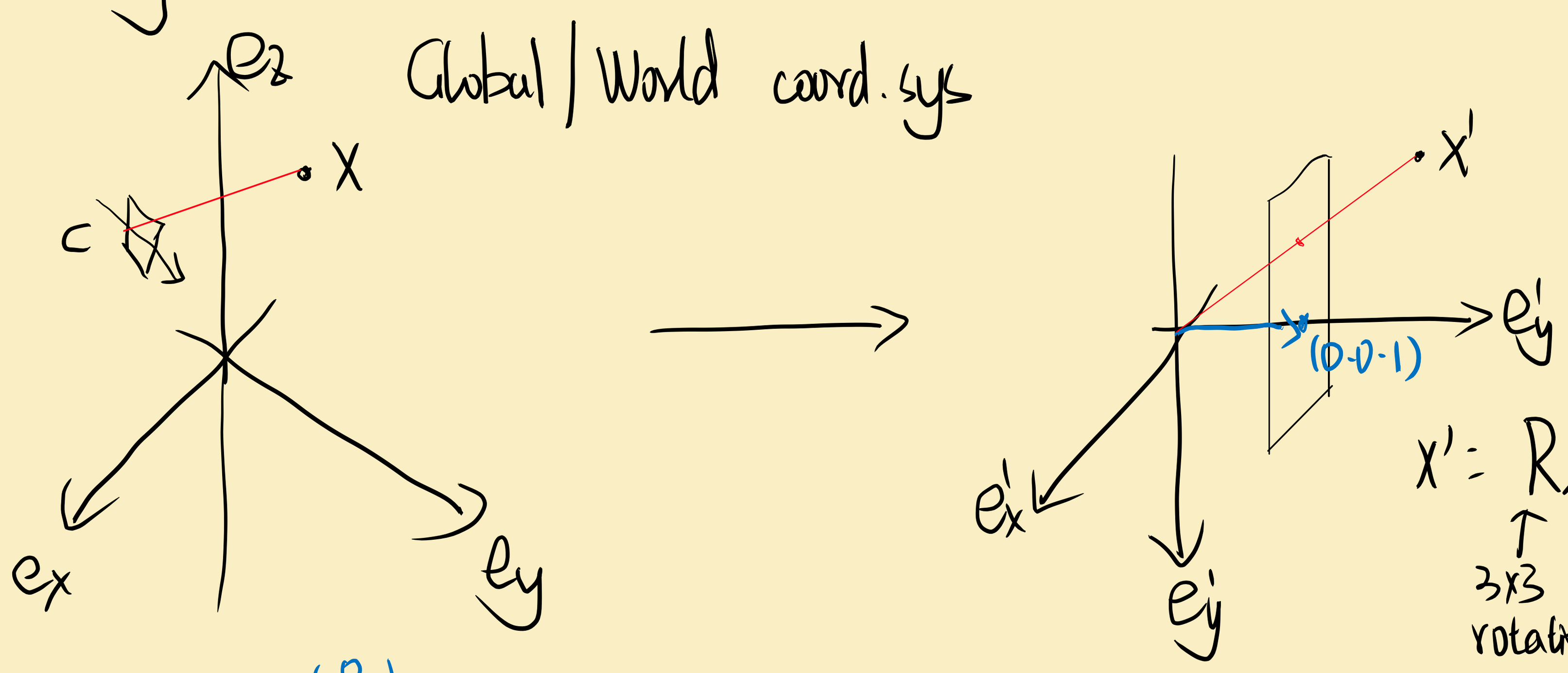
$$C+s(X-C)=\begin{pmatrix} sX_1 \\ sX_2 \\ sX_3 \end{pmatrix}$$

$sX_3=1$

$s=\frac{1}{X_3}$

$$(X_1,X_2,X_3)\longrightarrow\left(\frac{X_1}{X_3},\frac{X_2}{X_3}\right)$$

Moving Cameras



$X' = Rx + t$ ← translation vector

↑ 3x3 rotation matrix

Ex 2: $\cdot\ 0=\begin{pmatrix}0\\0\\0\end{pmatrix}=RC+t$

$\Leftrightarrow\ RC=-t\ \Rightarrow\ \underbrace{R^TR}_{I}C=C=-R^{-1}t=-R^Tt$

$\cdot\ \begin{pmatrix}0\\0\\1\end{pmatrix}=Rx+t\ \Rightarrow\ Rx=\begin{pmatrix}0\\0\\1\end{pmatrix}-t$

$\Rightarrow\ X=R^T\left[\begin{pmatrix}0\\0\\1\end{pmatrix}-t\right]$

$(V=X-C)$, $C=-R^Tt$

$V=R^T\left[\begin{pmatrix}0\\0\\1\end{pmatrix}-t+t\right]=R^T\begin{pmatrix}0\\0\\1\end{pmatrix}$

$R=\begin{bmatrix} x & x & x \\ x & x & x \\ \boxed{x \ \ x \ \ x} \end{bmatrix}$

$$X_3'\begin{pmatrix}\frac{X_1'}{X_3'}\\ \frac{X_2'}{X_3'}\\ 1\end{pmatrix}=\overbrace{\begin{pmatrix}X_1'\\X_2'\\X_3'\end{pmatrix}}^{\text{2D points}}=\underbrace{(R\ \ t)}_{3\times4\ P}\underbrace{\begin{pmatrix}X_1\\X_2\\X_3\\1\end{pmatrix}}_{\text{3D points}}$$

1) Computer $V=(R\ t)\begin{pmatrix}X\\1\end{pmatrix}$

2) Divide by last coord.

Ex 3: $P=\frac{1}{3}\begin{pmatrix}1&2&2&0\\2&1&-2&0\\-2&2&-1&1\end{pmatrix}$ $\quad X=\begin{pmatrix}0\\0\\-1\end{pmatrix}$

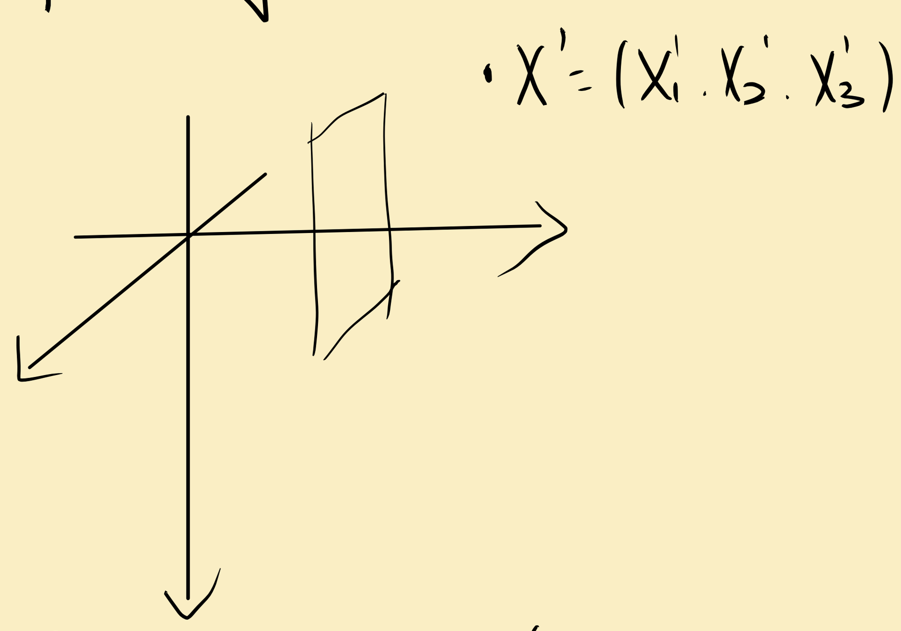
$V=P\begin{pmatrix}X\\1\end{pmatrix}=\frac{1}{3}\begin{pmatrix}-2\\2\\2\end{pmatrix}\rightarrow(-1,1)$

$\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}=(R\ t)\begin{pmatrix}X\\1\end{pmatrix}=RX+t$

$\Rightarrow\ \lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}=RX$

$\Rightarrow\ X=\begin{pmatrix}1&2&-2\\2&1&2\\2&-2&1\end{pmatrix}\left[\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}-\begin{pmatrix}0\\0\\1\end{pmatrix}\right]=\frac{\lambda}{3}\begin{pmatrix}-1\\1\\5\end{pmatrix}+\frac{1}{3}\begin{pmatrix}2\\-2\\1\end{pmatrix}$

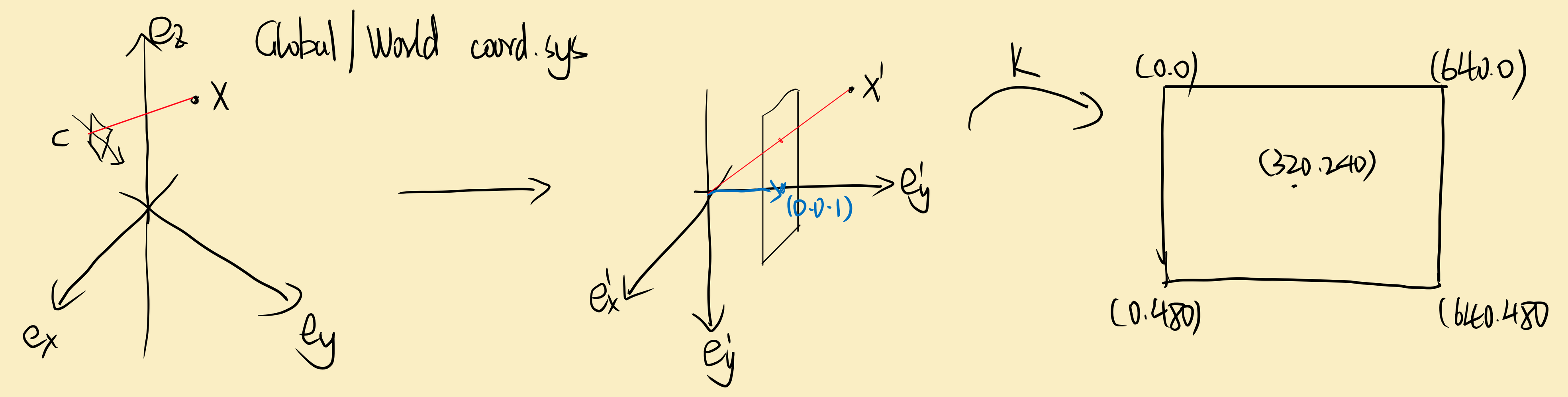
Depth of a Point



Camera Coordinate System

The depth of X is the z-coordinate

Ex 4:



K – 3x3 upper triangular matrix $K=[\,\triangledown\,]$
focal length, principal point

Calibration Matrix

$$K=\begin{bmatrix}\alpha f & sf & x_0\\ 0 & f & y_0\\ 0 & 0 & 1\end{bmatrix}$$

f – focal length
α – aspect ratio ($\alpha\approx 1$)
s – skew (≈ 0)
$\begin{pmatrix}x_0\\y_0\end{pmatrix}$ – principal point

$$\begin{bmatrix}f & & \\ & f & \\ & & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}=\begin{pmatrix}fx\\fy\\1\end{pmatrix}$$

$$\begin{bmatrix}f & & x_0\\ & f & y_0\\ & & 1\end{bmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}=\begin{pmatrix}fx+x_0\\fy+y_0\\1\end{pmatrix}$$

$$\begin{pmatrix}f & & x_0\\ & f & y_0\\ & & 1\end{pmatrix}\begin{pmatrix}X'_1\\X'_2\\X'_3\end{pmatrix}=\begin{pmatrix}fX'_1+x_0X'_3\\fX'_2+y_0X'_3\\X'_3\end{pmatrix}\sim\begin{pmatrix}f\frac{X'_1}{X'_3}+x_0\\f\frac{X'_2}{X'_3}+y_0\\1\end{pmatrix}$$

$$\left(\frac{X'_1}{X'_3},\ \frac{X'_2}{X'_3}\right)$$

$$\lambda\begin{pmatrix}x\\y\\1\end{pmatrix}=\overbrace{K[R\ t]}^{P}\begin{bmatrix}X\\1\end{bmatrix}$$ The camera equation

$$\Rightarrow\ \lambda x = PX$$

Camera Center

$P=K[R\ t]$

$0=RC+t \Leftrightarrow C=-R^Tt$

$\Leftrightarrow 0=\underbrace{K[R\ t]}_{P}\begin{pmatrix}C\\1\end{pmatrix}$ nullspace of P

nullspace $N(A)=\{x \mid Ax=0\}$

$v=K[R\ t]\begin{pmatrix}X\\1\end{pmatrix}$
$C: -R^Tt$ center (0,0) $\Rightarrow 0=P\begin{pmatrix}C\\1\end{pmatrix}$

Ex 1: P_1: $\begin{pmatrix}1/\sqrt2 & 0 & -1/\sqrt2 & 0\\ 0 & 1 & 0 & 0\\ 1/\sqrt2 & 0 & 1/\sqrt2 & 1\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\end{pmatrix}=\begin{pmatrix}-\frac{1}{\sqrt2}\\0\\\frac{1}{\sqrt2}+1\end{pmatrix}\to\left(\frac{-1}{1+\sqrt2},\ 0\right)$

P_2: $\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & -\sqrt2 & 0 & 0\\ -1 & 0 & -1 & -\sqrt2\end{pmatrix}\begin{pmatrix}0\\0\\1\\1\end{pmatrix}=\begin{pmatrix}1\\0\\-1-\sqrt2\end{pmatrix}\to\left(\frac{-1}{1+\sqrt2},\ 0\right)$

$P_2=-\sqrt2\,P_1$

$C=-R^Tt$ (compute nullspace) $\begin{pmatrix}-1 & 0 & 1 & 0\\ 0 & -\sqrt2 & 0 & 0\\ -1 & 0 & -1 & -\sqrt2\end{pmatrix}\to\begin{cases}x_1=t\\x_2=0\\x_3=t\\x_4=\frac{-2}{\sqrt2}t=-\sqrt2 t\end{cases}\to t\begin{pmatrix}1\\0\\1\\-\sqrt2\end{pmatrix}$

Homogeneous Coordinates

Ex: $(3,2,1)\to(3,2)$
$(6,4,2)\to(3,2)$

We say that $(\lambda x,\lambda y,\lambda)\to(x,y,1)$ $\lambda\neq0$
is equivalent to

The 2D projective space
$\mathbb{P}^2=\mathbb{R}^3\setminus\{0\}$ $\quad\mathbb{P}^n=\mathbb{R}^{n+1}\setminus\{0\}$

Ex: $(6,4,2)\sim(3,2,1)$ in $\mathbb{P}^2$

We associate the point $(x,y)\in\mathbb{R}^2$ with $(x,y,1)\in\mathbb{P}^2$
$(x,y,z)\in\mathbb{P}^2$ with $\left(\frac{x}{z},\frac{y}{z}\right)\in\mathbb{R}^2$, $z\neq0$

Ex: $v=[R\ t]\begin{bmatrix}X\\1\end{bmatrix}$
old interpretation: $v\in\mathbb{R}^3$ 3D point in camera coord. sys
new ——: $v\in\mathbb{P}^2$ homogeneous coordinates of projection in the image plane

$[R\ t]\begin{bmatrix}\lambda X\\\lambda\end{bmatrix}=\lambda v\sim v$

$P:\ \mathbb{P}^3\to\mathbb{P}^2$

Lines in $\mathbb{P}^2$

In $\mathbb{R}^2$: $ax+by+c=0$
line with parameter (a,b,c)
contains point $(x,y)\in\mathbb{R}^2$ if

In $\mathbb{P}^2$ $(x,y,z)\in\mathbb{P}^2$ belongs to $l(a,b,c)$
$a\frac{x}{z}+b\frac{y}{z}+c=0\Rightarrow ax+by+cz=0$

Point–Line duality

- Given two points there exist a line passing through them.
- Given two lines there exist a points in their intersection.

Ex2: $\begin{cases}l_1^Tx=0\\l_2^Tx=0\end{cases}$ $\begin{cases}-x+z=0\\-y+z=0\end{cases}\to\begin{cases}x=z\\y=z\\z\end{cases}\to t(1,1,1)$ $\quad(1,1)\in\mathbb{P}^2$

Ex3: $\begin{cases}-a+c=0\\-b+c=0\end{cases}$ $\cdots\to t(1,1,1)\to l(1,1,1)$

When parallel Ex: $l_1=(-1,0,1)$ $\quad l_2=(1,0,1)$
$\Rightarrow x=1$ $\quad x=-1$

in $\mathbb{P}^2$ $\begin{cases}-x+z=0\\x+z=0\\y=t\end{cases}$ $x=\begin{pmatrix}0\\1\\0\end{pmatrix}$ vanishing point: point that is infinitely far away in direction (x,y)

$l=(0,0,1)$ $\Rightarrow z=0\Rightarrow\begin{pmatrix}x\\y\\0\end{pmatrix}$ contains all the vanishing points
$x=(x,y,z)$

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Projective Transformations

$\mathbb{P}^n \to \mathbb{P}^n$ defined by $y \sim Hx \quad x, y \in \mathbb{P}^n$

H is an invertible $(n+1)\times(n+1)$ matrix

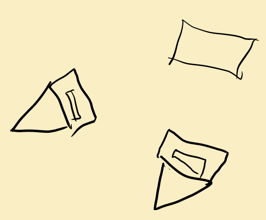
also called a homography (in CV only in $\mathbb{P}^2$)

$Hx \sim y \Rightarrow H(\lambda x) = \lambda Hx \sim \lambda y \sim y$

Examples of homographies

* $K \quad \underbrace{x}_{\in \mathbb{P}^2} \sim \underbrace{K[R\ t]X}_{\mathbb{P}^2}$

* projections of a scene plane in two views



* Pure rotation

$P_1 = [R_1\ 0] \qquad R_2 R_1^T$

$P_2 = [R_2\ 0]$

Ex: $H = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 0 & 1 & 1 \end{bmatrix}$ $(x, y) = (2, 1) \in \mathbb{R}^2$ find the homography

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 4 \\ 8 \\ 2 \end{bmatrix}}_{\in \mathbb{P}^2} \sim \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$

Prof: H preserves lines

$l, x \in \mathbb{P}^2 \quad l^T x = 0$

$y \sim Hx \quad 0 = l^T x = l^T \underbrace{H^{-1}H}_{I} x = \underbrace{(H^{-T} l)^T}_{\hat{l}} \underbrace{(Hx)}_{y} = \hat{l}^T y$

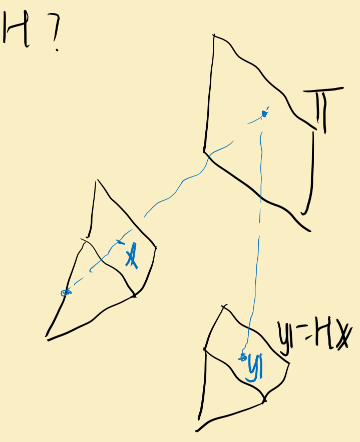
$\exists \hat{l}$ s.t. $\hat{l}^T y = 0$

$x_i \sim H y_i \quad i = 1, 2 \ldots n$ how to find H?

assume $\lambda x_i = H y_i$ $\quad H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$

λ, H unknown

$(h_{33} = 1)$



Def: $\overset{h}{8} + \overset{\lambda}{n} \qquad 3n \geq 8 + n$

Eq: $3n \qquad n \geq 4$

Remark: Vanishing points can be mapped to regular points:

$y \sim Hx = \begin{pmatrix} h & \cdots \\ & \\ h_{31} & h_{32}\ h_{33} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \text{---} \\ \text{---} \\ h_{31}x + h_{32}y + h_{33}z \end{pmatrix}$

y is an ideal point if x lies on $L = (h_{31}, h_{32}, h_{33})$

Classification of homographies

* General projections: perserves lines
* Affine tranform: perserves { prallel lines, finite/infinite lines }

$H = \begin{bmatrix} A_{2\times 2} & t_{2\times 1} \\ 0\ 0 & 1 \end{bmatrix}$

* Similarity transform

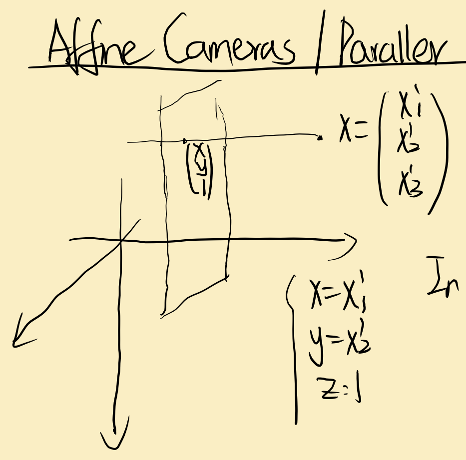
$H = \begin{bmatrix} sR_{2\times 2} & t_{2\times 1} \\ 0\ 0 & 1 \end{bmatrix}$ perseves angles

$R_{2\times 2}$ – rotation matrix

* Euclidean transform

$H = \begin{bmatrix} R_{2\times 2} & t_{2\times 1} \\ 0\ 0 & 1 \end{bmatrix}$ perseves distances

Affine Cameras / Paraller Projection



$\begin{cases} x = x_1' \\ y = x_2' \\ z = 1 \end{cases}$

In matrix form:

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ 1 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ 1 \end{pmatrix}$

From world to image plane:

first two rows of rotation matrix

$x = \begin{bmatrix} I_{2\times 3} & 0 \\ 0 & 1 \end{bmatrix}\underbrace{\begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}}_{\text{Euclidean } \mathbb{P}^3 \to \mathbb{P}^2} X = \begin{bmatrix} R_{2\times 3} & t_{2\times 1} \\ 0 & 1 \end{bmatrix} X$

$I_{2\times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

From world to image:

$x \sim K\begin{bmatrix} R_{2\times 3} & t_{2\times 1} \\ 0 & 1 \end{bmatrix} X$

if $K = I \quad P = \begin{bmatrix} R_{2\times 3} & t_{2\times 1} \\ 0 & 1 \end{bmatrix}$

Orthographic projection

if $K = \begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix} \quad P = \begin{bmatrix} sR_{2\times 3} & t_{2\times 1} \\ 0 & 1 \end{bmatrix}$ scaled ortho of weak projective

if $K = \begin{bmatrix} a & b & c \\ & d & e \\ & & 1 \end{bmatrix}$ Affine camera

$P = \begin{bmatrix} A_{2\times 3} & t_{2\times 1} \\ 0 & 1 \end{bmatrix}$

Ex $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0 \Rightarrow \phi = t\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

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Calibrated Cameras

$P = K[R\ t] \Rightarrow \tilde{x} = K^{-1}x \Rightarrow \tilde{x} \sim K^{-1}K[R\ t]X = [R\ t]X$

Finding K

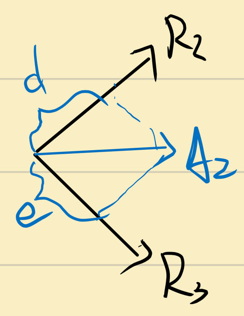
Given $P = [\underbrace{A}_{3\times3}\ \underbrace{a}_{3\times1}]$ we want RQ-fact

$A = KR$ — K upper triangular, R orthogonal: $R_1^T R_2 = 0 \cdots$, $\|R_1\| = 1$

if $K = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$ $R = \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix}$ $A = \begin{pmatrix} A_1^T \\ A_2^T \\ A_3^T \end{pmatrix}$

then $A = \begin{pmatrix} A_1^T \\ A_2^T \\ A_3^T \end{pmatrix} = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} R_1^T \\ R_2^T \\ R_3^T \end{pmatrix} = \begin{pmatrix} aR_1^T + bR_2^T + cR_3^T \\ dR_2^T + eR_3^T \\ fR_3^T \end{pmatrix}$

① $A_3 = fR_3$, $R_3 = \frac{1}{\|A_3\|} A_3$ $f = \|A_3\|$



② $A_2 = dR_2 + eR_3 \Rightarrow e = A_2^T R_3$. $dR_2 = A_2 - eR_3$

③ $b = A_1^T R_2$ $c = A_1^T R_3 \Rightarrow aR_1 = A_1 - bR_2 - cR_3$

Ex1: $P = \begin{pmatrix} 3000 & 0 & -1000 & 1 \\ 1000 & 2000\sqrt{2} & 1000 & 2 \\ 2 & 0 & 2 & 3 \end{pmatrix}$. find f, R_3

$A_3 = (2\ 0\ 2)^T$

$\|A_3\| = 2\sqrt{2} \Rightarrow f = 2\sqrt{2}$ $R_3 = \frac{1}{2\sqrt{2}}(2\ 0\ 2)$

Ex2: determine e. R_2 and d

$e = (1000\ 2000\sqrt{2}\ 1000) R_3 = 1000\sqrt{2}$

$dR_2 = \begin{pmatrix} 1000 \\ 2000\sqrt{2} \\ 1000 \end{pmatrix} - 1000\sqrt{2}\, R_3 = \begin{pmatrix} 0 \\ 2000\sqrt{2} \\ 0 \end{pmatrix} \Rightarrow d = 2000\sqrt{2}$ $R_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Ex3: determine $a.b.c$ and R_1

$b = (3000\ 0\ -1000) R_2 = 0$ $c = (3000\ 0\ -1000) R_3 = 1000\sqrt{2}$

$aR_1 = \begin{pmatrix} 3000 \\ 0 \\ -1000 \end{pmatrix} - 0 \cdot R_2 - 1000\sqrt{2}\, R_3 = \begin{pmatrix} 2000 \\ 0 \\ -2000 \end{pmatrix} \Rightarrow a = 2000\sqrt{2}$ $R_1 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

$\Rightarrow K = \begin{pmatrix} 2000\sqrt{2} & 0 & 1000\sqrt{2} \\ 0 & 2000\sqrt{2} & 1000\sqrt{2} \\ 0 & 0 & 2\sqrt{2} \end{pmatrix}$

$P = [A\ a] = [KR\ a] = K[R\ \underbrace{K^{-1}a}_{t = K^{-1}a}] = K[R\ t]$

$\xrightarrow{\text{scale}} \begin{pmatrix} 1000 & 0 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}$

Direct Linear Transform

$\lambda x = PX = \begin{bmatrix} P_1^T X \\ P_2^T X \\ P_3^T X \end{bmatrix} = \begin{bmatrix} X^T P_1 \\ X^T P_2 \\ X^T P_3 \end{bmatrix}$

Homogeneous Least Squares

We want to solve $Mv = 0$

To avoid $v = 0$ we add constraint $\|v\|^2 = 1$

$\boxed{\begin{array}{l} \min \|Mv\|^2 \\ \text{s.t. } \|v\|^2 = 1 \end{array}} \Rightarrow \begin{array}{l} \min \sum (\sigma_n w_n)^2 \\ \text{s.t. } \sum w_k^2 = 1 \end{array}$

$\xrightarrow{\text{SVD}} M = USV^T$ $\|Mv\|^2 = (Mv)^T(Mv) = v^T M^T M v$

$= v^T V S \underbrace{U^T U}_{I} S V^T v = \underbrace{v^T V}_{w^T} S^T S \underbrace{V^T v}_{w} = \|Sw\|^2$

$\Rightarrow Sw = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \ddots\ \sigma_n \end{pmatrix} w = \begin{pmatrix} \sigma_1 w_1 \\ \sigma_2 w_2 \\ \vdots \\ \sigma_n w_n \end{pmatrix} \Rightarrow \|Sw\|^2 = \sum_{n=1}^{n} (\sigma_n w_n)^2 \Rightarrow \|w\|^2 = v^T \underbrace{V V^T}_{I} v = v^T v = \|v\|^2$

Ex4: $M = USV^T$ $S = \begin{pmatrix} 3 & & \\ & 2 & \\ & & 1 \end{pmatrix}$ $V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ Find a vector $\min \|Mv\|^2$ and value

$\begin{cases} \min_w 3^2 w_1^2 + 2w_2^2 + 1 w_1^2 \\ w_1^2 + w_2^2 + w_3^2 = 1 \end{cases} \Rightarrow w = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$w = V^T v$

$v = Vw = \begin{pmatrix} 0 \\ -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix} \Rightarrow$ vector: Last column of V

value: Min value $\sigma_n^2 = 1$

L5

Structure from Motion

Given image points $\{x_{ij}\}$ find cam $\{P_1 \ldots P_n\}$
and 3D points $\{X_1 \ldots X_m\}$
s.t. $\lambda_{ij} x_{ij} = P_i X_j$

Projective Ambiguity

$$\lambda x_{ij} = P_i X_j = P_i \underbrace{H H^{-1}}_{I} X_j = \underbrace{(P_i H)}_{\tilde{P}_i} \underbrace{(H^{-1} X_j)}_{\tilde{X}_j}$$

Euclidean Reconstructions

when we have correct K

$P = K[R \; t]$

$PH = \tilde{P} = \tilde{K}[\tilde{R} \; \tilde{t}]$ $\quad H = \begin{bmatrix} sQ & v \\ 0 & 1 \end{bmatrix}$ (orthogonal)

Finding K

1) Find $P = [A \; a]$ camara matrix
2) Factorize $A = KR$

Resectioning

$x_i = \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix}$

Given x_i and X_i find P s.t. $\lambda_i x_i = P_i X_i \quad i = 1 \ldots n$

$$P = \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix} \quad \begin{cases} P_1^T X_i - \lambda_i x_i = 0 \\ P_2^T X_i - \lambda_i y_i = 0 \\ P_3^T X_i - \lambda_i = 0 \end{cases} \qquad P_3 X_i^T = X_i^T P_3$$

two points

$$\begin{bmatrix} X_1^T & & & -x_1 & 0 \\ & X_1^T & & -y_1 & 0 \\ & & X_1^T & -1 & 0 \\ X_2^T & & & 0 & -x_2 \\ & X_2^T & & 0 & -y_2 \\ & & X_2^T & 0 & -1 \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = 0$$

Homogeneous Least Squares

To avoid $v = 0$ and set $\|v\| = 1$

$\min \|Mv\|^2$ s.t. $\|v\| = 1$

SVD

1) $M = USV^T$ (U, V orthogonal; S diagonal)
2) v = last col of V

Triangulation

Find X s.t. $\lambda x_i = P_i X$

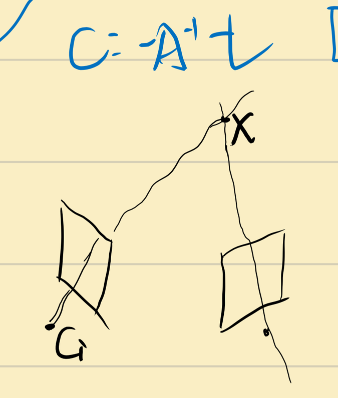
$X = \begin{bmatrix} X \\ 1 \end{bmatrix}$, $\quad x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad P_i = [A_i \; t_i]$

$$\lambda_i x_i = P_i X = [A_i \; t_i] \begin{bmatrix} X \\ 1 \end{bmatrix} = A_i X + t_i$$

$C = -R^T t$
$C = -A^{-1} t$ $\quad [A \; t] \begin{bmatrix} -A^{-1}t \\ 1 \end{bmatrix} = -\underbrace{AA^{-1}}_{I} t + t = 0$

$$\Leftrightarrow A_i X = \lambda_i x_i - t_i \Leftrightarrow X = \lambda_i A_i^{-1} x_i - \underbrace{A_i^{-1} t_i}_{C}$$

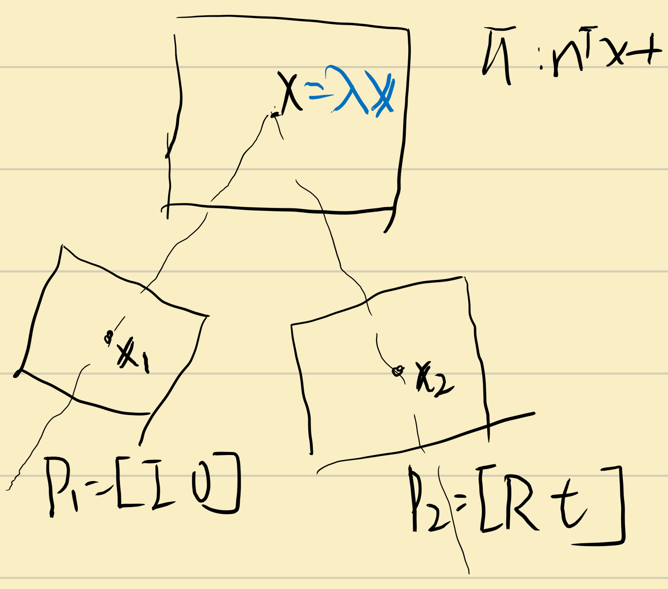
Line going through C_i with direction $A_i^{-1} x_i$



Ex 1 P_1 has camera center 0

P_2 has camera center $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{cases} X = \lambda_1 \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix} \\ X = \lambda_2 \begin{pmatrix} 0 \\ 1/2 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \end{cases} \Rightarrow \begin{cases} x = \frac{1}{2}\lambda_1 \\ y = \frac{1}{2}\lambda_1 \\ z = \lambda_1 \\ x = 1 \\ y = \frac{1}{2}\lambda_2 \\ z = \lambda_2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \\ z = 2 \\ \lambda_1 = 2 \\ \lambda_2 = 2 \end{cases} \quad X = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 1 \end{pmatrix}$$



$\Pi : n^T x + 1 = 0$

$X = \lambda x_1$

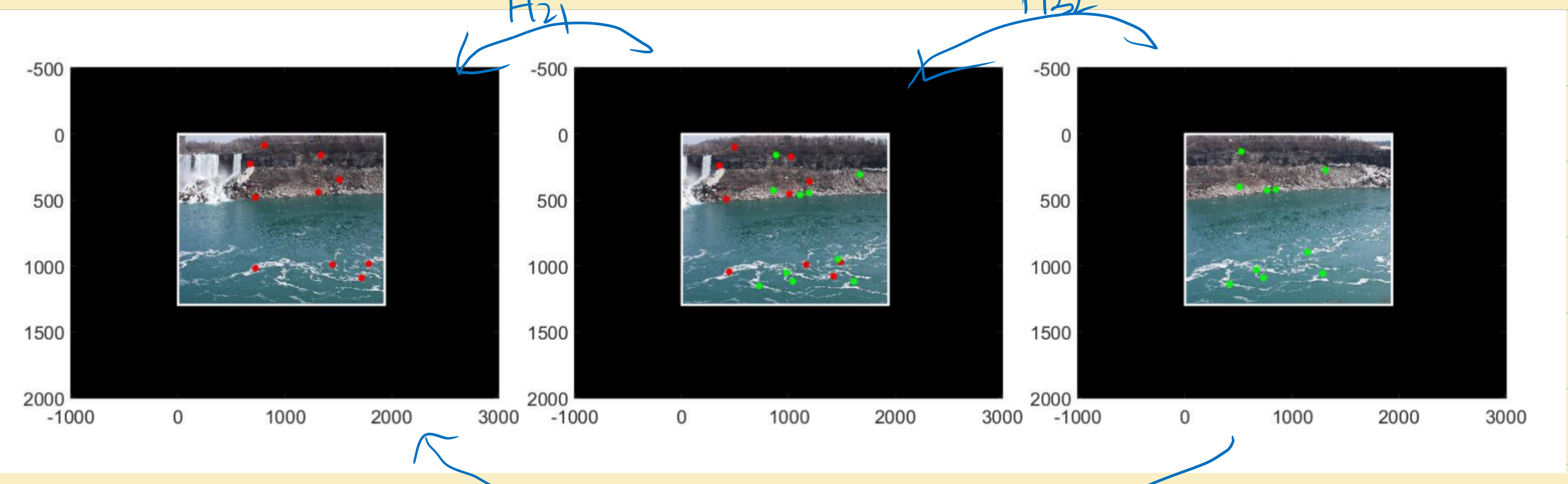
$n^T X + 1 = 0$

$\lambda n^T x_1 + 1 = 0 \qquad \lambda = \frac{-1}{n^T x_1}$

$$x_2 \sim [R \; t] \begin{bmatrix} X \\ 1 \end{bmatrix} = RX + t$$

$$= \frac{-1}{n^T x_1} R x_1 + t \sim R x_1 - t n^T x_1 = \underbrace{(R - t n^T)}_{H} x_1 = H x_1$$

H_{21} $\quad H_{32}$



$H_{31} = H_{32} H_{21}$