In [ ]:
```python
import sympy as sp
from scipy import *
from sympy import *
init_printing()
from IPython.display import display, Latex, HTML, Math
import numpy as np
import pandas as pd
from sympy import Rational as R
```

Assignment 1

In [ ]:
```python
# a) FTFF
```

a. Determine whether each statement is true or false. Each correct answer is awarded with a point and each incorrect answer cancels out a correct answer.

If A is an $n \times n$ matrix such that $\lVert A\vec{v} \rVert = \lVert \vec{v} \rVert$ for every $\vec{v} \in \mathbb{R}^n$, then there exists some $\vec{b} \in \mathbb{R}^n$ such that the system of linear equations $A\vec{x} = \vec{b}$ has infinitely many solutions.	True	(•) False ✓
If A is an $m \times n$ matrix and $B = \mathrm{rref}(A)$, then $\mathrm{Nul}(A) = \mathrm{Nul}(B)$.	(•) True ✓	False
If A is an orthogonal matrix, there must exist an orthonormal basis of eigenvectors for A.	True	(•) False ✓
If A and B are matrices whose eigenvalues, counted with their algebraic multiplicities, are the same, then A and B are similar	True	(•) False ✓

In [ ]:
```python
# b) Must be 0
```

b. It is known that A singular. Based on this information, fill in the missing value in Matrix A.State your answer as an integer between 0 and 99

$$\begin{bmatrix} 3 & 7 & 1 & 2 \\ 0 & \boxed{0} & 6 & 8 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$ ✓

In [ ]:
```python
# c) Echelon form so eigenvalues are on diagonal. Determinant is product of diag
```

c. It is known that the determinant of the following matrix is 9. It is also know that one of its eigenvalues has algebraic multiplicity of 2. Based on this information, fill in the missing values. State your answers as integers between 0 and 99.

$$\begin{bmatrix} -\boxed{3} & 3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -\boxed{3} \end{bmatrix}$$ ✓

In [ ]:
```python
# d) Swap a and d and change signs on b and c
```

d. Let A be an invertible 2×2 matrix. Fill in the missing values in the below expressions. State your answers as integers between 0 and 99.

$$A^{-1} = \frac{1}{\boxed{10}} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}, \quad A = \begin{bmatrix} \boxed{4} & \boxed{7} \\ \boxed{2} & \boxed{6} \end{bmatrix}$$ ✓

Assignment 2

In [ ]:
```
# a)
A = Matrix([[1,1,0],[0,1,1],[0,0,1]])
B = Matrix([[2,0,0],[1,1,2],[2,0,1]])
C = Matrix([[1,1,0],[0,1,0],[0,1,2]])

X = C*(A - B)**-1
X.det()
```

Out[ ]: 1

Find the determinant of matrix X in the following matrix equation $XA = XB + C$. State your answer as an integer between 0 and 99.

$\det X = \boxed{1}$ ✔

In [ ]:
```
# b)
A = Matrix([[-1,2],[1,1]])
B = Matrix([[1,2],[0,1]])
C = Matrix([[2,2],[3,1]])
```

In [ ]:
```
BAC = B * A * C
CAB = C * A*B
```

In [ ]:
```
# Try to Isolate Y
# CY = BAC - AX and
# -BY = CAB - CX

# Get Y on its own:
# Y = C**-1*BAC - C**-1 * AX and
# - Y = B**-1*CAB -B**-1CX

# Now, calculate all parts not involving X and Y
```

In [ ]:
```
# Simplify a bit by making up new matrices that are known
Q = C**-1 * BAC
P = B**-1 * CAB

# Y = Q - C**-1 * AX og -Y = P - B**-1*CX
```

In [ ]:
```
# Add the two equations in order to elimaínate Y:

# Y + (-Y) = Q - C**-1 * AX + (P - B**-1*CX)

# The left side becomes 0. Now move all parts containing X to the left:
# C**-1 * AX + B**-1*CX = Q + P

# Take X out of the parenthesis:
# (C**-1 * A + B**-1*C)X = Q + P

# Since the matrices inside the parenthesis do not invlove X,
# you canb just calculate it, I assign it to R:
```

In [ ]:
```
R = C**-1 * A + B**-1 * C

# and you get

# Og du får så

# RX = Q+P
# X = R**-1(Q+P)
```

In [ ]:
```
X = R**-1*(Q+P)
X
```

Out[ ]: $\begin{bmatrix} -\frac{12}{13} & 0 \\ \frac{99}{26} & 3 \end{bmatrix}$

In [ ]:
```
det(X)
```

Out[ ]: $-\frac{36}{13}$

In [ ]:
```
# Use X to find Y in one of the expressions above
Y = Q-C**-1 *A*X
Y
# We see it is 0
```

Out[ ]: $\begin{bmatrix} -\frac{4}{13} & 0 \\ \frac{79}{26} & 0 \end{bmatrix}$

In [ ]:
```
det(Y)
```

Out[ ]: 0

b. Given the three matrices

$$A = \begin{bmatrix} -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$$

solve the following system of matrix equations (i.e. find matrices X and Y)

$$\begin{cases} AX + CY = BAC \\ CX - BY = CAB \end{cases}$$

and state the determinants of X and Y. State your answers as integers between 0 and 99.

$\det X = -\frac{36}{\boxed{13}}$, $\det Y = \boxed{0}$

Assignment 3

In [ ]:
```
# a)
a = symbols('a')
A = Matrix([[a,1,1],[1,a,1],[1,1,a]])
(A.cofactor(0,0)*A[0,0]+A.cofactor(0,1)*A[0,1]+A.cofactor(0,2)*A[0,2]).expand()
```

Out[ ]: $a^3 - 3a + 2$

$$= a\left(a^2 - \boxed{1}\right) - \left(\boxed{1}\right)(a-1) + \left(\boxed{1}\right)(1-a) = a^{\boxed{3}} - \boxed{3}\,a + \boxed{2}$$

In [ ]:
```
# b)
solve(A.det(), a)
```

Out[ ]: $[-2, 1]$

b. For what values of a is A not invertible? State your answer as integers between 0 and 99.

$$a = \left\{-\boxed{2}, \boxed{1}\right\}$$

In [ ]:
```
# c)
A = Matrix([[2,1,1],[1,2,1],[1,1,2]])
B = A.row_join(eye(3))
B[2,:] = B[2,:]-B[1,:] # r3 -r2
B[1,:] = B[1,:]-Rational(1,2) * B[0,:] # r2 - 1/2r1
B[1,:] = Rational(2,3)*B[1,:] # r2 -> 2/3 r2
B[2,:] = B[2,:] + B[1,:] # r3 + r2
B[1,:] = B[1,:] - Rational(1,4) * B[2,:] # r2 - 1/4 r3
B[2,:] = Rational(3,4)*B[2,:] # 3/4 r3
B[0,:] = B[0,:] - B[2,:] # r1 -r3
B[0,:] = B[0,:] - B[1,:] # r1 - r2
B[0,:] = Rational(1,2)*B[0,:] # 1/2 r1
B
```

Out[ ]:
$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

In [ ]:
```
# d)
B[:, 3:]*Matrix([2,4,6])
```

Out[ ]:
$$\begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

d. Use the result obtained in part (c) to solve the matrix equation below. State your answers as integers between 0 and 99.

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} -\boxed{1} \\ \boxed{1} \\ \boxed{3} \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

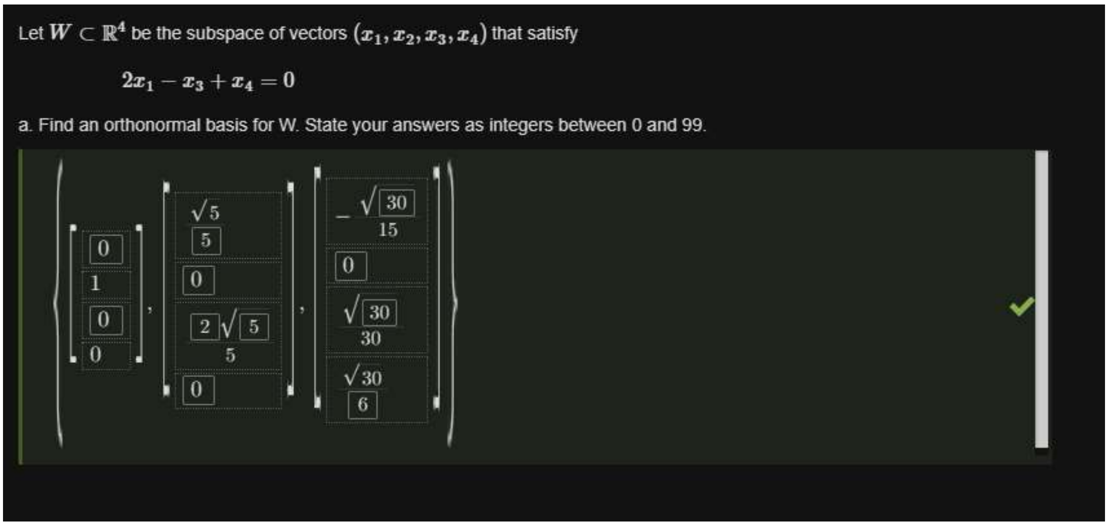
Assignment 4

In [ ]:
```python
# a)

u1 = GramSchmidt([Matrix([2, 0, -1, 1]).T.nullspace()[0],
                Matrix([2, 0, -1, 1]).T.nullspace()[1],
                Matrix([2, 0, -1, 1]).T.nullspace()[2]],
               True)[0]
u2 = GramSchmidt([Matrix([2, 0, -1, 1]).T.nullspace()[0],
                Matrix([2, 0, -1, 1]).T.nullspace()[1],
                Matrix([2, 0, -1, 1]).T.nullspace()[2]],
               True)[1]
u3 = GramSchmidt([Matrix([2, 0, -1, 1]).T.nullspace()[0],
                Matrix([2, 0, -1, 1]).T.nullspace()[1],
                Matrix([2, 0, -1, 1]).T.nullspace()[2]],
               True)[2]
u1, u2, u3
```

Out[ ]:

$$\left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{30}}{15} \\ 0 \\ \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{6} \end{bmatrix} \right)$$


Let $W \subset \mathbb{R}^4$ be the subspace of vectors (x_1, x_2, x_3, x_4) that satisfy

$$2x_1 - x_3 + x_4 = 0$$

a. Find an orthonormal basis for W. State your answers as integers between 0 and 99.

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{30}}{15} \\ 0 \\ \frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{6} \end{bmatrix} \right\}$$

In [ ]:
```python
# b)
u4 = u1.row_join(u2).row_join(u3).T.nullspace()[0].normalized()
u4
```

Out[ ]:

$$\begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \end{bmatrix}$$

b. Create a unit vector **u** that is orthogonal to the orthogonal basis for W, i.e. **u** will be the orthogonal complement to the base found in (a) and as a set all four vectors will make up an orthogonal basis for $\mathbb{R}^4$. Note there are two such unit vectors, whose entries are identical except with opposite signs. Please only enter the one whose signs match below.

$$u = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \end{bmatrix}$$

Assignment 5

In [ ]:
```
# a)
v1 = Matrix([1,1])
v2 = Matrix([2,3])
v = Matrix([3,4])

P = Matrix.hstack(v1,v2)
D = diag(2,1)
(P*D*P**-1)**3 * v
```

Out[ ]:

$$\begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

a. Let A be a 2×2 such that $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector for A with eigenvalue 2 and $\vec{v_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is an eigenvector for A with eigenvalue 1. If $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, compute $A^3\vec{v}$. State your answer as an integer between 0 and 99.

$$A^3 v = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

In [ ]:
```
# b)
p1 = Matrix([1,1,1,1])
p2 = Matrix([-1,0,0,1])
p3 = Matrix([0,-1,1,0])
p4 = Matrix([1,-2,0,1])
P = p1.row_join(p2).row_join(p3).row_join(p4)
D = diag(0,2,4,4)
P*D*P**-1
```

Out[ ]:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

b. Let A be a matrix. Given an eigenvalue 0 and corresponding eigenspace $\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\right\}$ of A, an eigenvalue of 2 and corresponding eigenspace $\left\{\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}\right\}$ of A, and an eigenvalue of 4 and corresponding eigenspace $\left\{\begin{bmatrix}0\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\-2\\0\\1\end{bmatrix}\right\}$ of A, find matrix A. State your answers as integers between 0 and 99.

$$A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

In [ ]:
```python
# c)
v1 = Matrix([Rational(2,3), Rational(-1,3), Rational(-2,3)])
v2 = Matrix([-sqrt(2)/2,  0, -sqrt(2)/2])
v1.row_join(v2).T.nullspace()[0].normalized()
```

Out[ ]:
$$\begin{bmatrix} -\frac{\sqrt{2}}{6} \\ -\frac{2\sqrt{2}}{3} \\ \frac{\sqrt{2}}{6} \end{bmatrix}$$

c. Let $\vec{v}_1$ and $\vec{v}_2$ denote the following vectors in $\mathbb{R}^3$:

$$\vec{v}_1 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} -\sqrt{2}/2 \\ 0 \\ -\sqrt{2}/2 \end{bmatrix}$$

Find a vector $\vec{v}_3$ so that $\vec{v}_1, \vec{v}_2, \vec{v}_3$ form an orthonormal basis for $\mathbb{R}^3$. Note there are two such vectors, whose entries are identical except with opposite signs. Please enter both. State your answers as integers between 0 and 99.

$$v_3 = \begin{bmatrix} -\frac{\sqrt{2}}{6} \\ -\frac{2\sqrt{2}}{3} \\ \frac{\sqrt{2}}{6} \end{bmatrix}, or \quad v_3 = \begin{bmatrix} \frac{\sqrt{2}}{6} \\ \frac{2\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{6} \end{bmatrix}$$

Assignment 6

In [ ]:
```python
# a)

x = pd.DataFrame([
    35,
    28,
    32,
    27,
    37,
    38,
    34,
    35,
    34,
    31

])
y = pd.DataFrame([
    21,
```

```
    15,
    16,
    13,
    25,
    27,
    21,
    20,
    21,
    15
])

X1 = Matrix([ones(len(x), 1)]).row_join(Matrix(x**3))
X2 = Matrix([ones(len(x), 1)]).row_join(Matrix(x)).row_join(Matrix(x**3))
X3 = Matrix([ones(len(x), 1)]).row_join(Matrix(x))

display(Math(r'X_1 = ' + latex(X1) + r'\quad X_2 = ' + latex(X2) + r'\quad X_3 =
            + latex(Matrix(y))))
```

$$X_1 = \begin{bmatrix} 1 & 42875 \\ 1 & 21952 \\ 1 & 32768 \\ 1 & 19683 \\ 1 & 50653 \\ 1 & 54872 \\ 1 & 39304 \\ 1 & 42875 \\ 1 & 39304 \\ 1 & 29791 \end{bmatrix} \quad X_2 = \begin{bmatrix} 1 & 35 & 42875 \\ 1 & 28 & 21952 \\ 1 & 32 & 32768 \\ 1 & 27 & 19683 \\ 1 & 37 & 50653 \\ 1 & 38 & 54872 \\ 1 & 34 & 39304 \\ 1 & 35 & 42875 \\ 1 & 34 & 39304 \\ 1 & 31 & 29791 \end{bmatrix} \quad X_3 = \begin{bmatrix} 1 & 35 \\ 1 & 28 \\ 1 & 32 \\ 1 & 27 \\ 1 & 37 \\ 1 & 38 \\ 1 & 34 \\ 1 & 35 \\ 1 & 34 \\ 1 & 31 \end{bmatrix} \quad y = \begin{bmatrix} 21 \\ 15 \\ 16 \\ 13 \\ 25 \\ 27 \\ 21 \\ 20 \\ 21 \\ 15 \end{bmatrix}$$

In [ ]:
```
X1tX1 = X1.T*X1
X1ty = X1.T*Matrix(y)
Mat, _ = X1tX1.row_join(X1ty).rref()
B1 = Mat[:,-1]


X2tX2 = X2.T*X2
X2ty = X2.T*Matrix(y)
Mat, _ = X2tX2.row_join(X2ty).rref()
B2 = Mat[:,-1]


X3tX3 = X3.T*X3
X3ty = X3.T*Matrix(y)
Mat, _ = X3tX3.row_join(X3ty).rref()
B3 = Mat[:,-1]
```

In [ ]:
```
display(Math(r'X_1^TX_1 = ' + latex(X1tX1)))
display(Math(r'X_1^Ty = ' + latex(X1ty)))
display(Math(r'X_2^TX_2 = ' + latex(X2tX2)))
display(Math(r'X_2^Ty = ' + latex(X2ty)))
display(Math(r'X_3^TX_3 = ' + latex(X3tX3)))
display(Math(r'X_3^Ty = ' + latex(X3ty)))
```

$$X_1^TX_1 = \begin{bmatrix} 10 & 374077 \\ 374077 & 15173359173 \end{bmatrix}$$

$$X_1^Ty = \begin{bmatrix} 194 \\ 7712824 \end{bmatrix}$$

$$X_2^T X_2 = \begin{bmatrix} 10 & 331 & 374077 \\ 331 & 11073 & 12751413 \\ 374077 & 12751413 & 15173359173 \end{bmatrix}$$

$$X_2^T y = \begin{bmatrix} 194 \\ 6562 \\ 7712824 \end{bmatrix}$$

$$X_3^T X_3 = \begin{bmatrix} 10 & 331 \\ 331 & 11073 \end{bmatrix}$$

$$X_3^T y = \begin{bmatrix} 194 \\ 6562 \end{bmatrix}$$

In [ ]:
```
# b)
display(Latex("$$y_1(t) = {}+{}t^3$$".format(round(B1[0],2), round(B1[1], 4))))
display(Latex("$$y_2(t) = {}{}t+{}t^3$$".format(round(B2[0],2), round(B2[1], 4),
display(Latex("$$y_3(t) = {}+{}t$$".format(round(B3[0],2), round(B3[1], 4))))
```

$$y_1(t) = 4.95 + 0.0004t^3$$

$$y_2(t) = 41.66 - 1.7164t + 0.0009t^3$$

$$y_3(t) = -20.41 + 1.2027t$$

In [ ]:
```
# c)
display(Latex("$$e_1 = {}$$".format(round((Matrix(y)-X1*B1).norm(), 2))))
display(Latex("$$e_2 = {}$$".format(round((Matrix(y)-X2*B2).norm(), 2))))
display(Latex("$$e_3 = {}$$".format(round((Matrix(y)-X3*B3).norm(), 2))))
```

$$e_1 = 3.52$$

$$e_2 = 2.97$$

$$e_3 = 4.39$$

In [ ]:
```
# d)
display(Latex("$$f_2(40) ={}$$".format(round(float(B2[0]+B2[1]*40 + B2[2]*40**3)
```

$$f_2(40) = 32$$

Assignment 7

In [ ]:
```
A = Matrix([[2,0,0],[2,1,0],[0,-2,0]])
AtA = A.T*A
vecs1 = AtA.eigenvects()
```

```
display(A)
display(vecs1)
```

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\left[\left(0, \ 1, \ \left[\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right] \right), \ \left(4, \ 1, \ \left[\begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} \right] \right), \ \left(9, \ 1, \ \left[\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right] \right) \right]$$

In [ ]:

```
s1 = sqrt(vecs1[2][0])
s2 = sqrt(vecs1[1][0])
s3 = sqrt(vecs1[0][0])

v1 = vecs1[2][2][0].normalized()
v2 = vecs1[1][2][0].normalized()
v3 = vecs1[0][2][0].normalized()

u1 = ((s1**-1)*A*v1)
u2 = ((s2**-1)*A*v2)

# We need one more eigenvectors for U

u3 = Matrix([u1.T, u2.T]).nullspace()[0].normalized()

U1 = u1.row_join(u2).row_join(u3)
V = v1.row_join(v2).row_join(v3)
Vt1 = V.T
S1 = diag(s1,s2,s3)

display(Math('U \Sigma V^T = {}{}{}'.format(latex(U1), latex(S1), latex(Vt1))))
display(Latex("Test:"))
display(U1*S1*Vt1)
display(A)
display(U1*S1*Vt1==A)
```

$$U\Sigma V^T = \begin{bmatrix} \frac{4\sqrt{5}}{15} & -\frac{\sqrt{5}}{5} & -\frac{2}{3} \\ \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ -\frac{2\sqrt{5}}{15} & -\frac{2\sqrt{5}}{5} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Test:

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

True

In [ ]:

```
# b: U and V wont be the same, but Sigma will.
```

Consider the following matrix:

$$\begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

a. Do a full singular value decomposition of the form $A = U\Sigma V^T$ departing from A^TA. State your answers as integers between 0 and 99.

$$A = \begin{bmatrix} \frac{4\sqrt{5}}{15} & -\frac{\sqrt{5}}{5} & -\frac{2}{3} \\ \frac{\sqrt{5}}{3} & \boxed{0} & \frac{\boxed{2}}{3} \\ -\frac{2\sqrt{5}}{\boxed{15}} & -\frac{2\sqrt{5}}{5} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \boxed{3} & 0 & 0 \\ 0 & \boxed{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{2\sqrt{5}}{\boxed{5}} & \frac{\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{5}}{5} & \frac{\boxed{2}\sqrt{5}}{5} & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix}$$

Check answer

b. Now, try to redo the SVD but this time depart from AA^T. The question is whether any of the matrices are identical to the matrices found in question (a). Please note that each incorrect answer is awarded with a negative score and no answer is awarded a score of 0.

The U matrix of AA^T is not identical to the U matrix of A^TA.

The Σ matrix of AA^T is identical to the Σ matrix of A^TA.

The V^T matrix of AA^T is not identical to the V^T matrix of A^TA.

Check answer