Exercise Collection – Multiclass Classification

Contents

Lecture exercises

Exercise 1: Multiclass and Softmax Regression

(a) Read in the MNIST data set included in the data sets in the package `keras`, using either

```
library(keras)
mnist <- dataset_mnist()
```

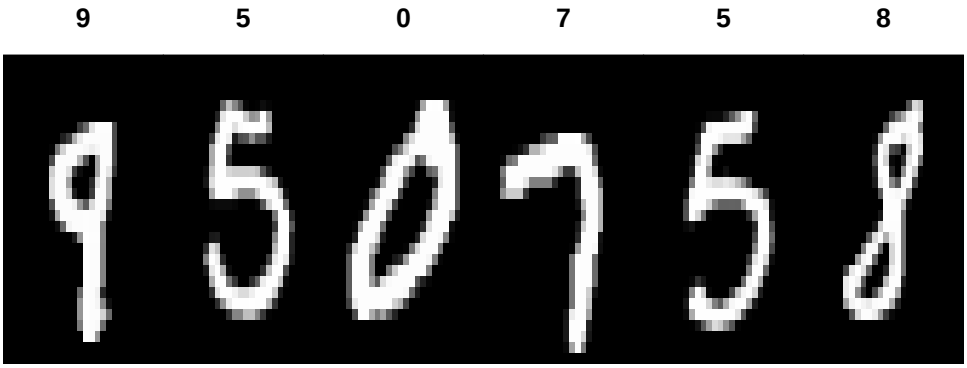
in R or

```
import tensorflow as tf
mnist = tf.keras.datasets.mnist.load_data(
    path='mnist.npz'
)
```

in Python. The data set should contain 4 (2x2) list elements for features (pictures) and outcomes (label) for training and testing (as separate list items).

(b) Optional: Check that the data is correctly loaded by visualizing it, e.g., like this:

```
## Loaded Tensorflow version 2.8.0
```



(c) Convert the features to a (`pandas`) data frame, by flattening the 28x28 images to a 784-entry-long vector, which represents one row in your data frame. Divide the intensity values of each pixel (each column) by 255 to get a value between 0 and 1.

(d) Fit a softmax regression to the given data and interpret the results. Use an existing package by searching for a package that fits *Multinomial Logistic Regression*, which is equal to a softmax regression.

(e) Use the deep learning API `keras` to fit a softmax regression by fitting a network like the following:

```
library(dplyr)
library(keras)

# convert outcome using one-hot encoding
y_train <- to_categorical(y_train)
y_test <- to_categorical(y_test)

neural_network <- keras_model_sequential()

neural_network %>%
  layer_dense(units = ..., # fill in the correct number of units
              activation = ... # fill in the correct activation function
              input_shape = list(784)) %>%
  compile(
    optimizer = "adam",
    loss      = "categorical_crossentropy",
    metric = "accuracy"
  )

history_minibatches <- fit(
  object           = neural_network,
  x                = as.matrix(x_train),
  y                = y_train,
  batch_size       = 24,
  epochs           = 80,
  validation_data = list(as.matrix(x_test), y_test)
)
```

and compare the learned `neural_network$weights` with the coefficients from the softmax regression.

(f) Compare the performances of the two algorithms using four different multiclass metrics (you are allowed to use existing implementations).

Exercise 2: Logistic Regression, Softmax, Cross-Entropy

(a) What is the relationship between the softmax

$$\pi_i(\mathbf{x}) = \frac{\exp\{\boldsymbol{\theta}_i^T \mathbf{x}\}}{\sum_k \exp\{\boldsymbol{\theta}_k^T \mathbf{x}\}}$$

and the logistic function

$$\pi(\mathbf{x}) = \frac{1}{1 + \exp\{\boldsymbol{\theta}^T \mathbf{x}\}}$$

?

(b) Derive the negative log-likelihood of a softmax regression assuming that the outcome follows a multinomial distribution with g classes.