Exercise 1: SVM - Kernel Trick

The polynomial kernel is defined as

$$k(x, \tilde{x}) = (x^T \tilde{x} + b)^d.$$

Furthermore, assume $x \in \mathbb{R}^2$ and d = 2.

(a) Derive the explicit feature map ϕ taking into account that the following equation holds:

$$k(x, \tilde{x}) = \langle \phi(x), \phi(\tilde{x}) \rangle$$

(b) Describe the main differences between the kernel method and the explicit feature map.