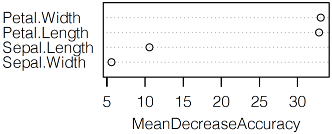
# Introduction to Machine Learning

# Random Forests: Feature Importance




## Learning goals

- Understand that the goal of defining variable importance is to enhance interpretability of the random forest
- Know definition of variable importance based on improvement in split criterion
- Know definition of variable importance based on permutations of OOB observations

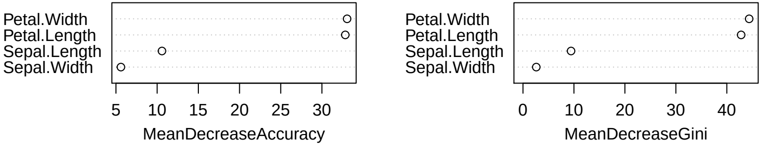
# VARIABLE IMPORTANCE

- Single trees are highly interpretable
- Random forests as ensembles of trees lose this feature
- Contributions of the different features to the model are difficult to evaluate
- Way out: variable importance measures
- Basic idea: by how much would the performance of the random forest decrease if a specific feature were removed or rendered useless?

# VARIABLE IMPORTANCE

Measure based on improvement in split criterion

**for** features $x_j$, $j = 1$ to $p$ **do**
  **for** tree base learners $\hat{b}^{[m]}(\mathbf{x})$, $m = 1$ to $M$ **do**
    Find all nodes $\mathcal{N}$ in $\hat{b}^{[m]}(\mathbf{x})$ that use $x_j$.
    Compute improvement in splitting criterion achieved by them.
    Add up these improvements.
  **end for**
  Add up improvements over all trees to get feature importance of $x_j$.
**end for**



# VARIABLE IMPORTANCE

## Measure based on permutations of OOB observations

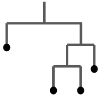
- While growing tree, pass down OOB observations and record predictive accuracy.
- Permute OOB observations of *j*-th feature. This destroys the association between the target and the permuted *j*-th feature.
- Pass down the permuted OOB observations and evaluate predictive accuracy again.
- The decrease of performance induced by permutation is averaged over all trees and is used as a measure for the importance of the *j*-th variable.

# VARIABLE IMPORTANCE BASED ON PERMUTATIONS OF OOB OBSERVATIONS

| | $x_1$ | ... | $x_p$ | y | $\hat{y}$ |
|---|---|---|---|---|---|
| 1 | 1.4 | | | 1 | 1 |
| 2 | 2 | | | 0 | 0 |
| 3 | 1.55 | | | 1 | 1 |
| 4 | 1.72 | | | 0 | 0 |
| 5 | 1.89 | | | 1 | 0 |
| ... | | | | | |
| n | 2.01 | | | 1 | 1 |

$$\mathrm{R}_{\mathrm{m}}^{\mathrm{OOB}} = \frac{1}{|\mathrm{OOB}_{\mathrm{m}}|} \sum_{\mathrm{i} \in \mathrm{OOB}_{\mathrm{m}}} \mathrm{L}\left(\mathrm{y}^{(\mathrm{i})}, \hat{\mathrm{y}}_{\mathrm{m}}^{(i)}\right)$$

Tree 1



Permuted OOB of tree 1

| | $x_1$ | ... | $x_p$ | y | $\hat{y}$ |
|---|---|---|---|---|---|
| 1 | 1.4 | | | 1 | |
| 2 | 2 | | | 0 | |
| 3 | 2.01 | | | 1 | 0 |
| 4 | 1.55 | | | 0 | 0 |
| 5 | 1.89 | | | 1 | |
| ... | | | | | |
| n | 1.72 | | | 1 | 0 |

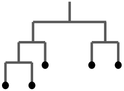
OOB obs. of $x_1$

| | $x_1$ | ... | $x_p$ | y | $\hat{y}$ |
|---|---|---|---|---|---|
| 1 | 1.4 | | | 1 | |
| 2 | 2 | | | 0 | |
| 3 | 1.55 | | | 1 | 1 |
| 4 | 1.72 | | | 0 | 0 |
| 5 | 1.89 | | | 1 | |
| ... | | | | | |
| n | 2.01 | | | 1 | 1 |

$$\underbrace{R_1^{OOB} \text{ with permutation} - R_1^{OOB} \text{ without permutation}}_{\Delta R_1^{OOB}}$$

....

Tree M



Permuted OOB of tree 1

| | $x_1$ | ... | $x_p$ | y | $\hat{y}$ |
|---|---|---|---|---|---|
| 1 | 1.89 | | | 1 | 0 |
| 2 | 2 | | | 0 | |
| 3 | 1.4 | | | 1 | 0 |
| 4 | 1.72 | | | 0 | |
| 5 | 1.55 | | | 1 | 1 |
| ... | | | | | |
| n | 2.01 | | | 1 | |

OOB obs. of $x_1$

| | $x_1$ | ... | $x_p$ | y | $\hat{y}$ |
|---|---|---|---|---|---|
| 1 | 1.4 | | | 1 | 1 |
| 2 | 2 | | | 0 | |
| 3 | 1.55 | | | 1 | 0 |
| 4 | 1.72 | | | 0 | |
| 5 | 1.89 | | | 1 | 1 |
| ... | | | | | |
| n | 2.01 | | | 1 | |

$$\underbrace{R_M^{OOB} \text{ with permutation - } R_M^{OOB} \text{ without permutation}}_{\Delta R_{\mathrm{m}}^{OOB}}$$

$$\text{variable importance for } x_1 = \frac{1}{M} \sum_{m=1}^{M} \Delta R_m^{OOB}$$