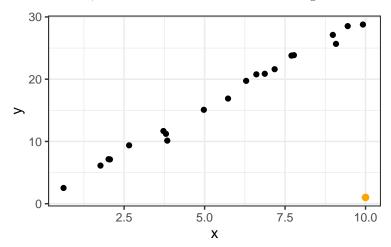
Exercise 1: HRO in mlr3

Throughout the lecture, we will frequently use the R package mlr3 and its descendants, providing an integrated ecosystem for all common machine learning tasks. Let's recap the HRO principle and see how it is reflected in mlr3. An overview of the most important objects and their usage, illustrated with numerous examples, can be found at https://mlr3book.mlr-org.com/basics.html.

- a) How are the key concepts (i.e., hypothesis space, risk and optimization) you learned about in the lecture videos implemented in mlr3?
- b) Have a look at mlr3::tsk("iris"). What attributes does this task object store?
- c) Pick an mlr3 learner of your choice. What are the different settings for this learner? (Hint: use mlr3::mlr_learners\$keys() to see all available learners.)

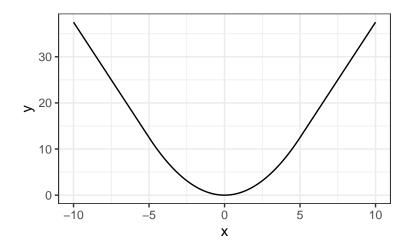
Exercise 2: Loss Functions for Regression Tasks

In this exercise, we will examine loss functions for regression tasks somewhat more in depth.



- a) Consider the above linear regression task. How will the model parameters be affected by adding the new outlier point (orange) if you use
 - i) L1 loss
 - ii) L2 loss

in the empirical risk? (You do not need to actually compute the parameter values.)



- b) The second plot visualizes another loss function popular in regression tasks, the so-called *Huber loss* (depending on $\epsilon > 0$; here: $\epsilon = 5$). Describe how the Huber loss deals with residuals as compared to L1 and L2 loss. Can you guess its definition?
- c) Derive the least-squares estimator, i.e., the solution to the linear model when using L2 loss, analytically via

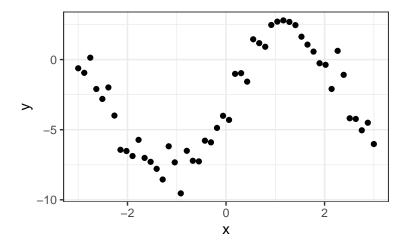
$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta} \in \Theta} \|\mathbf{y} - \mathbf{X}\boldsymbol{\theta}\|_2^2.$$

Exercise 3: Polynomial Regression

Assume the following (noisy) data-generating process from which we have observed 50 realizations:

$$y = -3 + 5 \cdot \sin(0.4\pi x) + \epsilon$$

with $\epsilon \sim \mathcal{N}(0,1)$.



- a) We decide to model the data with a cubic polynomial (including intercept term). State the corresponding hypothesis space.
- b) Demonstrate that this hypothesis space is simply a parameterized family of curves by plotting in R curves for 3 different models belonging to the considered model class.
- c) State the empirical risk w.r.t. θ for a member of the hypothesis space. Use L2 loss and be as explicit as possible.
- d) We can minimize this risk using gradient descent. In order to make this somewhat easier, we will denote the transformed feature matrix, containing x to the power from 0 to 3, by $\tilde{\mathbf{X}}$, such that we can express our model by $\tilde{\mathbf{X}}\boldsymbol{\theta}$ (note that the model is still linear in its parameters, even if \mathbf{X} has been transformed in a non-linear manner!). Derive the gradient of the empirical risk w.r.t $\boldsymbol{\theta}$.

- e) Using the result from d), state the calculation to update the current parameter $\theta^{[t]}$.
- f) You will not be able to fit the data perfectly with a cubic polynomial. Describe the advantages and disadvantages that a more flexible model class would have. Would you opt for a more flexible learner?

Exercise 4: Predicting abalone

We want to predict the age of an abalone using its longest shell measurement and its weight.

See https://archive.ics.uci.edu/ml/machine-learning-databases/abalone/ for more details.

```
url <- "https://archive.ics.uci.edu/ml/machine-learning-databases/abalone/abalone.data"
abalone <- read.table(url, sep = ",", row.names = NULL)
colnames(abalone) <- c(
    "sex", "longest_shell", "diameter", "height", "whole_weight",
    "shucked_weight", "visceral_weight", "shell_weight", "rings")
abalone <- abalone[, c("longest_shell", "whole_weight", "rings")]</pre>
```

a) Plot LongestShell and WholeWeight on the x- and y-axis, respectively, and color points according to Rings.

Using mlr3:

- b) Create an mlr3 task for the abalone data.
- c) Define a linear regression learner (for this you will need to load the mlr3learners extension package first) and use it to train a linear model on the abalone data.
- d) Compare the fitted and observed targets visually. (Hint: use autoplot().)
- e) Assess the model's training loss in terms of MAE.

 (Hint: losses are retrieved by calling \$score(), which accepts different mlr_measures, on the prediction object.)



https://en.wikipedia.org/wiki/Abalone#/media/File:LivingAbalone.JPG