

### Exercise 1: Kullback-Leibler Divergence

- (a) You want to approximate the binomial distribution with  $n$  number of trials and probability  $p$  with a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ . To find a suitable distribution you investigate the Kullback-Leibler divergence (KLD) in terms of the parameters  $\theta = (\mu, \sigma^2)^\top$ .
- (i) Write down the KLD for the given setup.
  - (ii) Derive the gradients with respect to  $\theta$ .
  - (iii) Is there an analytic solution for the optimal parameter setting? If yes, derive the corresponding solution. If no, give a short reasoning.
  - (iv) Independent of the previous exercise, state a numerical procedure to minimize the KLD.
- (b) Sample points according to the true distribution and visualize the KLD for different parameter settings of the Gaussian distribution (including the optimal one if available).
- (c) Create a surface plot with axes  $n$  and  $p$  and colour value equal to the KLD for the optimal normal distribution.
- (d) Based on the previous result,
- (i) how can the behaviour for varying  $p$  be explained?
  - (ii) how can the behaviour for varying  $n$  be explained?