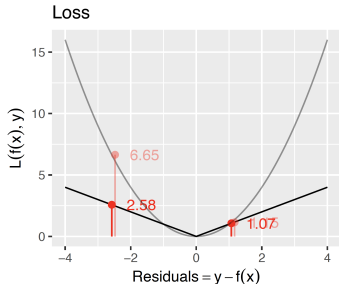


# Introduction to Machine Learning

## Loss Functions for Regression



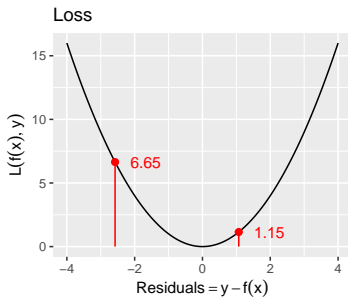
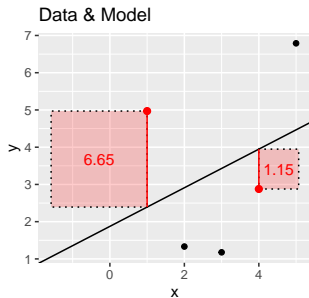
### Learning goals

- Know definitions of L1 and L2 loss
- Understand difference between L1 and L2 loss
- Understand why optimization for L1 loss is harder than for L2 loss

# REGRESSION LOSSES - L2 / SQUARED ERROR

- $L(y, f(\mathbf{x})) = (y - f(\mathbf{x}))^2$  or  $L(y, f(\mathbf{x})) = 0.5(y - f(\mathbf{x}))^2$
- Convex
- Differentiable, gradient no problem in loss minimization
- For later:  $\frac{\partial 0.5(y - f(\mathbf{x}))^2}{\partial f(\mathbf{x})} = f(\mathbf{x}) - y = -\epsilon$ , derivative is negative residual
- Tries to reduce large residuals (if residual is twice as large, loss is 4 times as large), hence outliers in  $y$  can become problematic
- Connection to Gaussian distribution (see later)

# REGRESSION LOSSES - L2 / SQUARED ERROR



# REGRESSION LOSSES - L2 / SQUARED ERROR

What's the optimal constant prediction  $c$  (i.e. the same  $\hat{y}$  for all  $\mathbf{x}$ )?

$$L(y, f(\mathbf{x})) = (y - f(\mathbf{x}))^2 = (y - c)^2$$

We search for the  $c$  that minimizes the empirical risk.

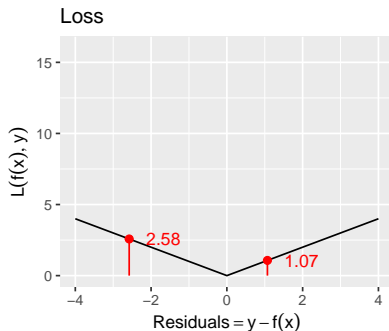
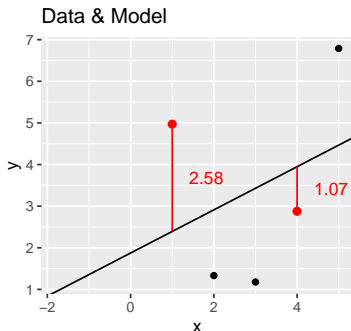
$$\hat{c} = \arg \min_{c \in \mathbb{R}} \mathcal{R}_{\text{emp}}(c) = \arg \min_{c \in \mathbb{R}} \frac{1}{n} \sum_{i=1}^n (y^{(i)} - c)^2$$

We set the derivative of the empirical risk to zero and solve for  $c$ :

$$\begin{aligned} -\frac{1}{n} \sum_{i=1}^n 2(y^{(i)} - c) &= 0 \\ \hat{c} &= \frac{1}{n} \sum_{i=1}^n y^{(i)} \end{aligned}$$

# REGRESSION LOSSES - L1 / ABSOLUTE ERROR

- $L(y, f(\mathbf{x})) = |y - f(\mathbf{x})|$
- Convex
- No derivatives for  $L(y, f(\mathbf{x})) = 0$ , optimization becomes harder
- $\hat{f}(\mathbf{x}) = \text{median of } y|\mathbf{x}$



# REGRESSION LOSSES - L1 / ABSOLUTE ERROR

- $L(y, f(\mathbf{x})) = |y - f(\mathbf{x})|$
- Convex
- No derivatives for  $\epsilon = 0$ ,  $y = f(\mathbf{x})$ , optimization becomes harder
- $\hat{f}(\mathbf{x}) = \text{median of } y|\mathbf{x}$
- More robust, outliers in  $y$  are less influential than for L2

