
Exercise collection – Supervised Regression

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Lecture exercises

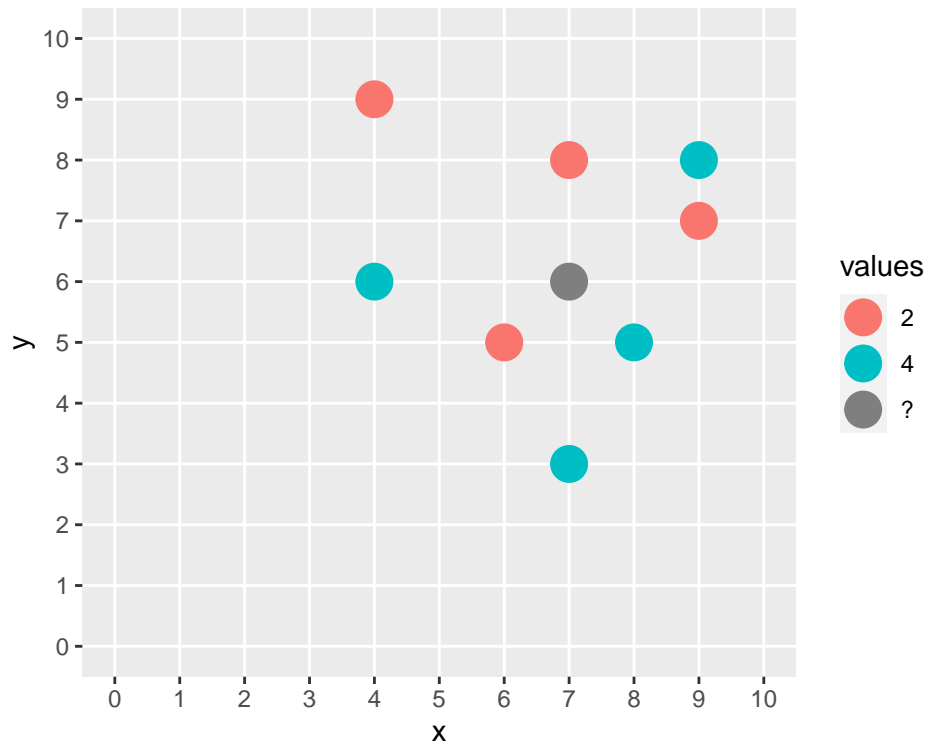
Exercise 1: k -NN regression

Let the 2D feature vectors in the following figure be with two different numeric target values (2 and 4). Predict the point (7,6) - represented by the grey point in the picture - with the k -nearest neighbor method. Distance function should be the L_1 norm (Manhattan distance):

$$d_{\text{manhattan}}(x, \tilde{x}) = \sum_{j=1}^p |x_j - \tilde{x}_j|$$

State as the prediction the unweighted and the weighted (according to the Manhattan distance) mean of the values of the k -nearest neighbors.

- a) $k = 3$
- b) $k = 5$
- c) $k = 7$

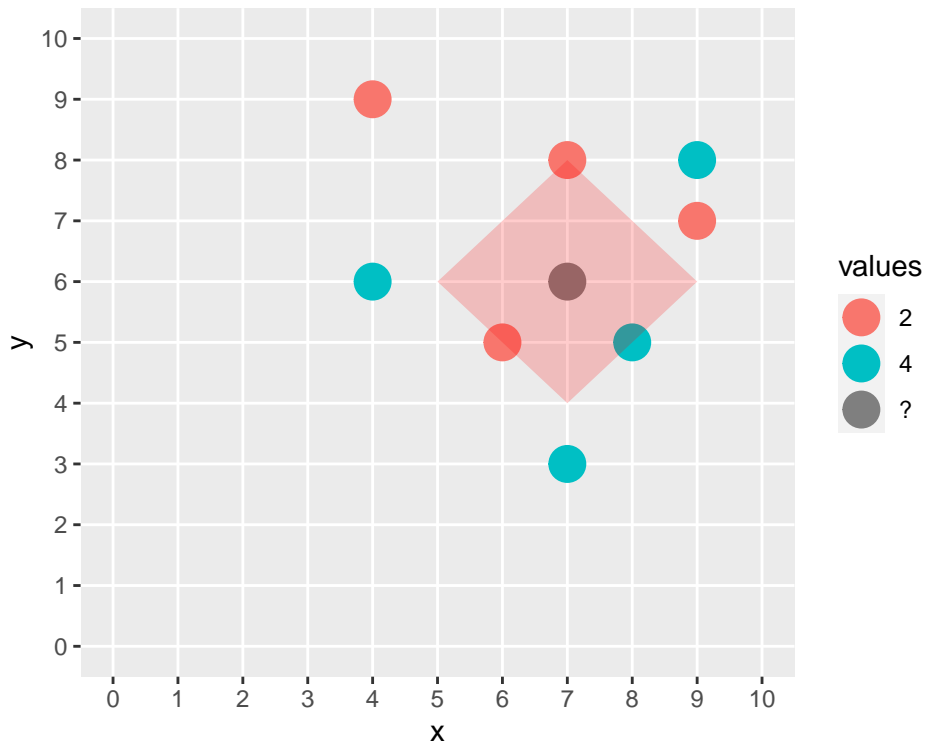


Solution 1:

a) $k = 3$

$$\hat{y} = \frac{2 + 2 + 4}{3} = \frac{8}{3} \approx 2.67$$

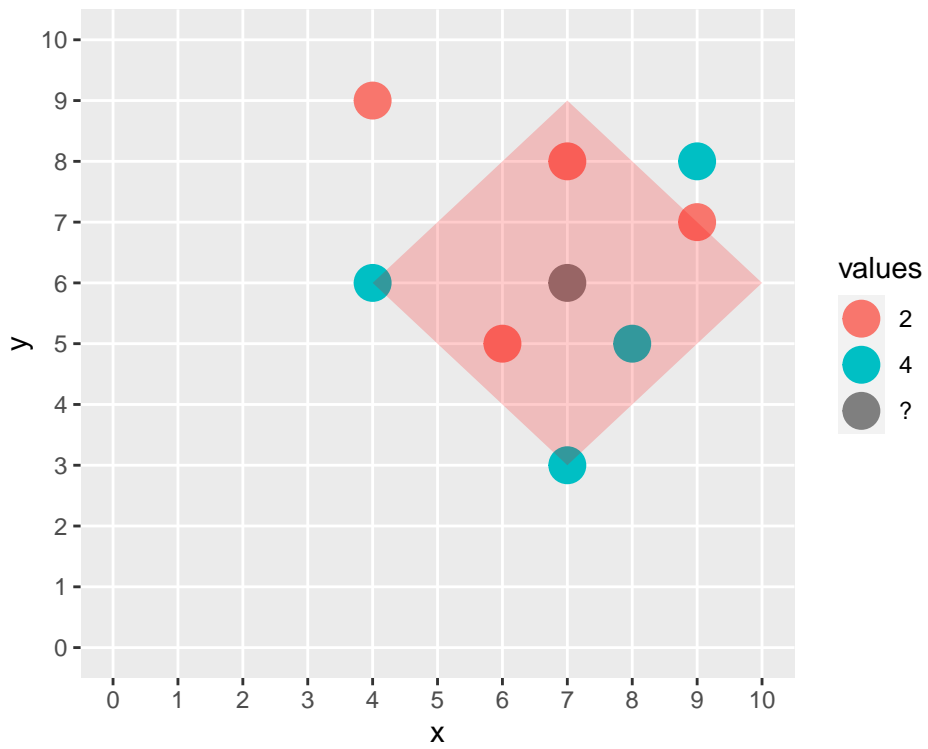
$$\hat{y}_{\text{weighted}} = \frac{\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 4}{\frac{3}{2}} = \frac{8}{3} \approx 2.67$$



b) $k = 5$

$$\hat{y} = \frac{2 + 2 + 2 + 4 + 4 + 4}{6} = 3$$

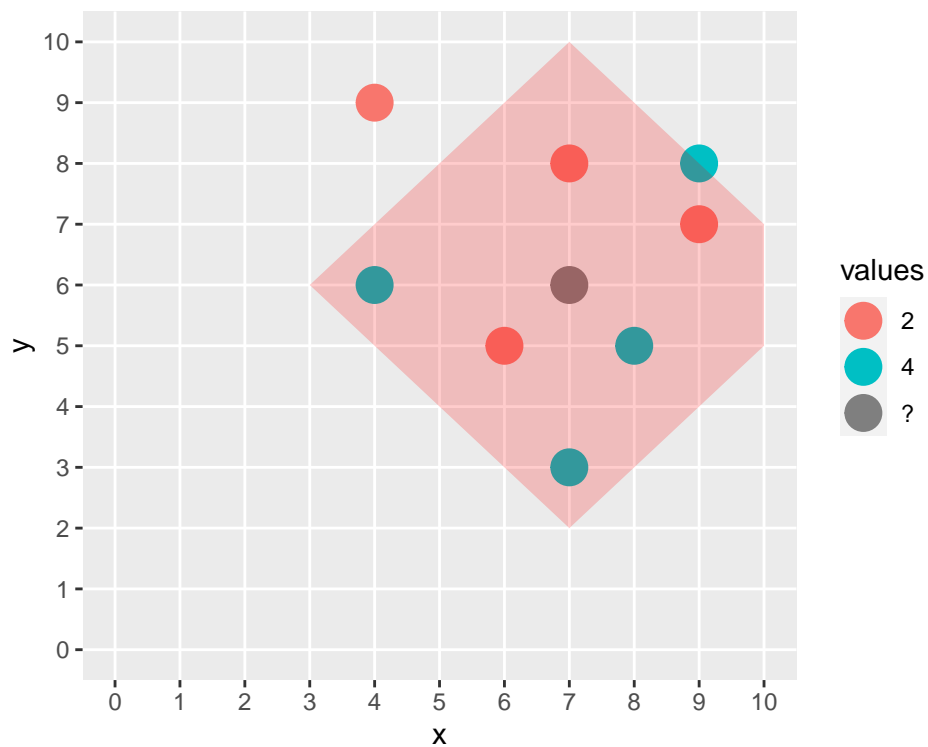
$$\hat{y}_{\text{weighted}} = \frac{\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 + \frac{1}{3} \cdot 2 + \frac{1}{2} \cdot 4 + \frac{1}{3} \cdot 4 + \frac{1}{3} \cdot 4}{\frac{5}{2}} = \frac{44}{15} \approx 2.93$$



c) $k = 7$

$$\hat{y} = \frac{2 + 2 + 2 + 4 + 4 + 4 + 4}{7} = \frac{22}{7} \approx 3.14$$

$$\hat{y}_{\text{weighted}} = \frac{\frac{1}{2} \cdot 2 + \frac{1}{2} \cdot 2 + \frac{1}{3} \cdot 2 + \frac{1}{2} \cdot 4 + \frac{1}{3} \cdot 4 + \frac{1}{3} \cdot 4 + \frac{1}{4} \cdot 4}{\frac{11}{4}} = \frac{100}{33} \approx 3.03$$



Exercise 2: mlr3 regression learners

How in mlr3 a learner can be constructed and what it represents can be found at <https://mlr3book.mlr-org.com/learners.html>.

- How does a learner in mlr3 compare to what you've learned in the videos?
- Pick an mlr3 learner of your choice. What are the different settings for this learner?
(Hint: Use `mlr_learners$keys()` to see all available learners)

Solution 2:

- Learning consists of *representation* (hypothesis space), *evaluation* (risk) and *optimization*.
A learner in mlr3 can be thought of as the implementation of these components, since

- a representation of the associated model learnt from the data by using the implemented optimization is stored in such a learner object,
- its performance measures can be accessed afterwards.

```
b) library(mlr3)
library(mlr3learners)

# show all available learners
mlr_learners$keys()

## [1] "classif.cv_glmnet" "classif.debug" "classif.featureless"
## [4] "classif.glmnet" "classif.kknn" "classif.lda"
## [7] "classif.log_reg" "classif.multinom" "classif.naive_bayes"
## [10] "classif.nnet" "classif.qda" "classif.ranger"
## [13] "classif.rpart" "classif.svm" "classif.xgboost"
## [16] "regr.cv_glmnet" "regr.featureless" "regr.glmnet"
## [19] "regr.kknn" "regr.km" "regr.lm"
## [22] "regr.ranger" "regr.rpart" "regr.svm"
## [25] "regr.xgboost" "surv.cv_glmnet" "surv.glmnet"
## [28] "surv.ranger" "surv.xgboost"

# see settings for a specific learner, e.g., for a regression tree
rpart_learner <- lrn("regr.rpart")
print(rpart_learner)

## <LearnerRegrRpart:regr.rpart>
## * Model: -
## * Parameters: xval=0
## * Packages: rpart
## * Predict Type: response
## * Feature types: logical, integer, numeric, factor, ordered
## * Properties: importance, missings, selected_features, weights
```

Exercise 3: regression for abalone data

We want to predict the age of an abalone using its longest shell measurement and its weight.

See: <http://archive.ics.uci.edu/ml/datasets/Abalone> for more details.

- a) Plot `LongestShell`, `WholeWeight` on the x - and y -axis and color points with `Rings`

Using the `mlr3`-package:

- b) Fit a linear model
c) Fit a k-nearest-neighbors model
d) Compare the fitted and observed targets for `lm` and `knn`, respectively (Hint: Use `autoplot()`)

Hint: See the official book manual of the `mlr3` package for usage:

<https://mlr3book.mlr-org.com/index.html>

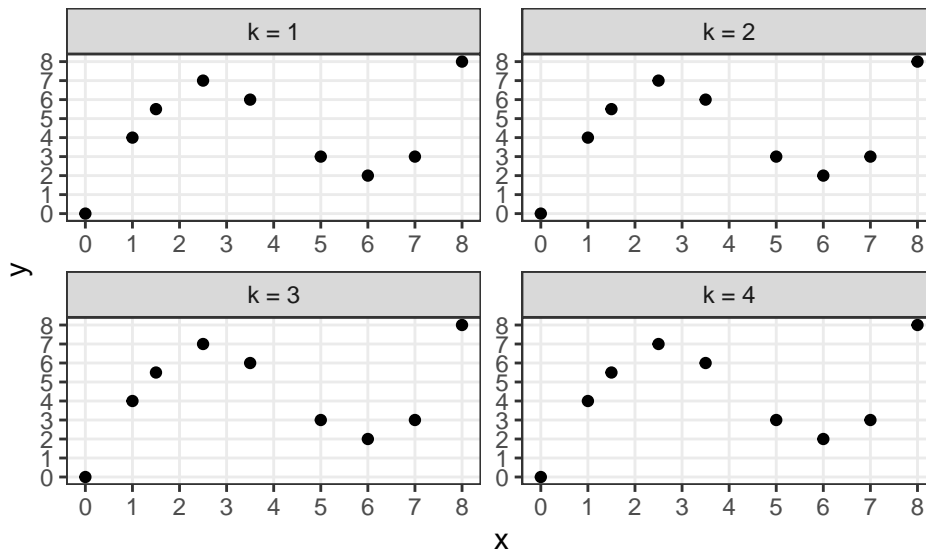
Solution 3:

See R code

Questions from past exams

Exercise 4: WS2020/21, retry exam, question 1

ID	\mathbf{x}	y
1	0.0	0.0
2	1.0	4.0
3	1.5	5.5
4	2.5	7.0
5	3.5	6.0
6	5.0	3.0
7	6.0	2.0
8	7.0	3.0
9	8.0	8.0



- (a) Assume we use k -nearest neighbours regression with the L1 norm (Manhattan distance) as a distance function. For every $k \in \{1, 2, 3, 4\}$, do the following:
- Mark the k -nearest neighbours of a new observation $\mathbf{x} = 4$ in the graphic below.
 - Calculate the predicted value \hat{y} for $\mathbf{x} = 4$ as the unweighted mean of the k -nearest neighbours and draw it in the graphic below.
- (b) Would using the euclidean distance as distance measure in a) have made a difference? Explain your answer.

Solution 4:

- (a) $\{x_5\}$, $\{x_5, x_6\}$, $\{x_4, x_5, x_6\}$, $\{x_4, x_5, x_6, x_7\}$
- (b) No. In the case of a single feature L1 and L2 are identical.

Ideas & exercises from other sources