Internet Appendix for
"Asset Prices and Portfolio Choice with Learning from Experience "

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This Internet Appendix serves as a companion to our paper "Asset Prices and Portfolio Choice with Learning from Experience." It provides additional theoretical results not reported in the main text due to space constraints. The Internet Appendix is organized as follows: Section 1 presents a simplified version of the model of the main text where the risky asset is in zero net supply. This is an old setting of the model that we had in previous versions of the paper. As we show below, the zero net supply security does not pay any dividends, therefore we impose that aggregate output is entirely distributed as endowments of earnings (i.e., we do not need the Lucas tree any longer). A possible interpretation of such an asset is that of a continuously resettled contract between agents to reallocate wealth. The assumption about the risky asset combined with logarithmic preferences and homogeneous endowments across cohorts gives more tractability and allows us to provide additional extensions and generalizations of this simplified model that we present in Section 2. Section 3 goes back to the setting of the main article with the risky asset is in positive net supply, but we now introduce a more realistic specification for the earnings profile of the agents. More precisely, we define dynamics of individual earnings to capture the hump-shaped profile of labor earnings that we observe in the data. Section 4 presents some results omitted from the main text. Finally, Section 5 briefly describes the methodology we follow to produce the numerical results of Section 3 in the main text. Throughout we refer to the paper "Asset Prices and Portfolio Choice with Learning from Experience" as the main text or main article.

1 A Model with Zero Net Supply Risky Security

This section presents a variant of our setting in which we assume the absence of the Lucas tree. In other words, the risky asset is still available to trade but it is in zero net supply. Although such a variant does not significantly affect the main insights of the model, it provides additional tractability allowing us to show further results in Section 2 below.

The setting is the same as in Section 2 of the main text, but now we let $\omega \to 1$. This assumption implies that the entire aggregate output, Y_t, is paid out as endowment of earnings and nothing is distributed in the form of dividends, i.e., $D_t = 0$. Consequently, the endowment at time t of an agent born at time s is equal to the aggregate output, i.e., $y_{s,t} = Y_t$. Lastly, the dynamics of aggregate endowment are as in Equation (1) in the main text.

In the absence of a representative firm, we introduce a security to span the set of states of the world and to complete the markets. For tractability, our choice is to employ an infinitely lived risky asset in zero net supply.[1] The dynamics of the risky asset evolve according to

$$\frac{dS_t}{S_t} = \left(\mu_t^S dt + \sigma^S dz_t\right) = \left(\mu_{s,t}^S dt + \sigma^S dz_{s,t}\right), \tag{1}$$

where the last part of the equation represents the dynamics perceived by agents born at time s. From Equation (1), notice that $\mu_{s,t}^S = \mu_t^S + \sigma^S \Delta_{s,t}$, as in the main article and that μ_t^S is determined in equilibrium. Additionally, in the presence of non-dividend paying risky asset in zero net supply the volatility can be chosen arbitrarily as the equilibrium does not separately determine both the risk premium and the diffusion coefficient, but only imposes a joint restrcition.

As in the main article, the set of available securities also includes an instantaneously risk-free asset, which is in zero net supply, and annuity contracts to hedge mortality risk.

1.1 Results

As we said in the introduction of this section, the absence of the Lucas tree does not significantly affect the results presented in the main article and all the insights hold in this setting as well. The only diference between the two settings is that the term coming from the overlapping generations structure, β, drops out in the absence of the Lucas tree. This happens because the wealth of a newborn agent corresponds exactly to the aggregate total

[1] The risky asset can be interpreted as continuously resettled contracts (e.g., futures contracts). The same asset structure is used, for example, in Basak (2000) and Karatzas, Lehoczky, and Shreve (1994).

wealth. To see this first notice that the individual optimization problems are the same as in the main text. Thus, we follow the same steps of Section 2 in the main paper to find the optimal consumption at time t of agents born at time s

$$c_{s,t} = c_{s,s} e^{-\rho(t-s)} \left(\frac{\eta_{s,t}}{\eta_{s,s}} \right) \left(\frac{\xi_s}{\xi_t} \right). \tag{2}$$

Similarly, the expressions for the consumption to wealth ratio, Equation (20), and for total wealth, Equation (22), follow from the main article. Differently in this setting, the individual human capital, defined as the discounted value of an agent's future earnings, coincides with aggregate human capital

$$H_{s,t} = \frac{1}{\xi_{s,t}} E_{s,t} \left[\int_t^\infty e^{-\nu(u-s)} \xi_{s,u} y_{s,u} du \right] = \frac{1}{\xi_{s,t}} E_{s,t} \left[\int_t^\infty e^{-\nu(u-s)} \xi_{s,u} Y_u du \right] = H_t, \tag{3}$$

where the second equality exploits the assumption that individual earnings equal aggregate output at each time. Additionally, aggregate financial wealth is now zero as the risky asset is in zero net supply. Therefore, total wealth corresponds exaclty to individual human capital, thus Equation (22) can be written as $H_{t,t} = \frac{Y_t}{\rho+\nu}$. Lastly, the assumption that each agent is born without financial wealth, i.e., $W_{s,s} = 0$, implies that the wealth of an agent born at time s is equal to his human capital. So, we can combine Equation (20) with the expression of individual human capital, $H_{t,t} = \frac{Y_t}{\rho+\nu}$, and obtain

$$c_{s,s} = Y_s. \tag{4}$$

Therefore, in this setting the fraction of aggregate output that is consumed by a newborn agent, defined as β in the main text, equals one.

Using the relation in Equation (4), we obtain the following proposition:

Proposition 1. *Optimal consumption at time t of agents born at time $s \leq t \leq \tau$, where τ*

denotes the stochastic time of death, is

$$c_{s,t} = Y_s e^{-\rho(t-s)} \left(\frac{\eta_{s,t}}{\eta_{s,s}}\right) \left(\frac{\xi_s}{\xi_t}\right). \tag{5}$$

Proof. *See main article.*

The next proposition characterizes the stochastic discount factor.

Proposition 2. *In equilibrium, the stochastic discount factor is*

$$\xi_t = \bar{\eta}_t \frac{e^{-\rho t}}{Y_t}. \tag{6}$$

where $\bar{\eta}_t$ and its dynamics are defined as in the main article.

Proof. *See main article.*

Comparing Equation (6) and Equation (23), the only difference is the absence of the overlapping generations term, $e^{-\nu(1-\beta)t}$. In this setting, this term drops out because β equals one. Nonetheless, the intuitions behind the other two terms are identical to the ones provided in the main article.

Next, we can characterize the real short rate and the market price of risk.

Proposition 3. *In equilibrium, the real short rate is*

$$r_t = \rho + \bar{\mu}_t - \sigma_Y^2, \tag{7}$$

and the market price of risk is

$$\theta_t = \sigma_Y - \frac{1}{\sigma_Y}\left(\bar{\mu}_t - \mu_Y\right). \tag{8}$$

Proof. *See main article.*

As expected comparing Proposition (3) to Proposition (4) in the main article, the market price of risk is identical, while the risk-free rate does not have the term, $\nu(1-\beta)$, representing the overlapping generations structure.

Finally, we have the optimal portfolio policy of an agent born at time s.

Proposition 4. *The optimal dollar amount invested in the risky asset for an agent born at time s is*

$$\pi_{s,t} = \frac{\hat{\mu}_{s,t} - \bar{\mu}_t}{\sigma^S}\hat{W}_{s,t} + \frac{\sigma_Y}{\sigma^S}W_{s,t}. \tag{9}$$

Proof. *See main article.*

The optimal portfolio above shows the impact of the assumption about the volatility of the risky asset. Whereas in the main article, the solution of the volatility of the risky asset, σ_t^S equals output volatility, σ_Y, in this setting they differ as the volatility of the risky asset is chosen arbitrarily.

2 Extensions

In this section, we consider several extensions to the model of the previous section. First, we start by considering the more general case of having heterogeneity within each cohort and not only across cohorts. In particular, we will study an economy in which agents of each cohort are normally distributed. Next, we will show that our results hold even after introducing some rational agents in our economy. Somehow surprisingly, the coexistence of rational agents with agents who learn from experience makes agents behave as extrapolators for a longer period of time than in the baseline case. Lastly, we will present some additional empirical features that the model is able to replicate when a second risky asset is added to our setting.

2.1 Within-Cohort Heterogeneity

In the model of Section 1 in this Internet Appendix, we assume that all agents within a cohort are identical. One question that naturally arises is how robust the results are to within cohort heterogeneity. Therefore, in this subsection, we extend the baseline mode with $\omega = 1$l to allow for heterogeneity in beliefs within cohorts.[2] We assume that in every period there is a distribution of initial beliefs. We index agent types by a and assume that agents within a cohort are distributed following a generic distribution $g(a)$ defined over the domain $[\underline{a}, \bar{a}]$, such that $\int_{\underline{a}}^{\bar{a}} g(a)da = 1$. An agent of type a has a prior mean and variance given by $\hat{\mu}_{a,s,s}$ and $V_{a,s,s}$, respectively. As in the main text, we do allow the prior mean and variance to potentially depend on the time when the agent is born. Given the prior mean and variance, we calculate the estimation error, $\Delta_{a,s,t}$, using standard filtering. The next proposition states the estimation error.

Proposition 5. *The estimation error at time t of an agent of type a born at time s is*

$$\Delta_{a,s,t} = \frac{\sigma_Y^2}{\sigma_Y^2 + V_{a,s,s}(t-s)} \Delta_{a,s,s} + \frac{V_{a,s,s}}{\sigma_Y^2 + V_{a,s,s}(t-s)} (z_t - z_s). \tag{10}$$

Moreover, we have that $\Delta_{a,s,s} = \frac{\hat{\mu}_{a,s,s} - \mu_Y}{\sigma_Y}$ *and* $\lim_{t-s\to\infty} \Delta_{s,t} = 0$.

The dynamics of the disagreement process is

$$d\eta_{a,s,t} = \Delta_{a,s,t}\eta_{a,s,t} dz_t. \tag{11}$$

An agent of type a born at time s maximizes

$$E_{a,s,s}\left[\int_s^\infty e^{-(\rho+\nu)(t-s)} log\left(c_{a,s,t}\right) dt\right], \tag{12}$$

where the expectation is taken with respect to the belief of an agent of type a born at time s. The equilibrium can be solved using the same approach as in Section 1. The next proposition characterizes the equilibrium real short rate and market price of risk.

[2] One can easily follow the same procedure with $\omega < 1$.

Proposition 6. *In equilibrium, the real short rate is*

$$r_t = \rho + \mu_Y - \sigma_Y^2 + \sigma_Y \bar{\Delta}_t^A, \tag{13}$$

and the market price of risk is

$$\theta_t = \sigma_Y - \bar{\Delta}_t^A, \tag{14}$$

where

$$\bar{\Delta}_t^A = \int_{-\infty}^{t} \int_{\underline{a}}^{\bar{a}} f_{a,s,t} \Delta_{a,s,t} da ds, \tag{15}$$

and

$$f_{a,s,t} = \nu e^{-\nu(t-s)} g(a) \left(\frac{X_s}{X_t} \right) \left(\frac{\eta_{a,s,t}}{\eta_{a,s,s}} \right), \tag{16}$$

with X_t *solving the integral equation*

$$X_t = \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} X_s \left(\int_{\underline{a}}^{\bar{a}} g(a) \frac{\eta_{a,s,t}}{\eta_{a,s,s}} da \right) ds. \tag{17}$$

From Proposition 6, we see that the real short rate and market price of risk take similar forms as in the model in Section 1. In the next two subsections, we discuss two specific examples of within-cohort heterogeneity. In the first example, we assume that the distribution of initial prior beliefs about the mean is Gaussian. In the second example, we let a fraction of the population learn from all historical data and, hence, these agents know the true mean instantaneously.

2.1.1 Gaussian Type Distribution

In this subsection, we follow Atmaz (2015) and specify the distribution function, $g(a)$, to be Gaussian, that is

$$g(a) = \frac{1}{\sqrt{2\pi\nu_0^2}} e^{-\frac{1}{2}\frac{a^2}{\Sigma}}, \tag{18}$$

with mean zero and variance Σ.[3] The belief of an agent of type a when born is $\hat{\mu}_{a,s,s} = \mu_Y + a$. Agents are homogeneous with respect to the prior variance, which is given by $\bar{v}$. As the bias parameter a has mean zero, the average agent is born with an unbiased prior for the mean. By standard filtering, the error process at time t of an agent born at time s with initial belief a is

$$\Delta_{a,s,t} = \frac{\sigma_Y}{\sigma_Y^2 + \bar{v}\,(t-s)} a + \frac{\bar{v}\,(z_t - z_s)}{\sigma_Y^2 + \bar{v}\,(t-s)}. \tag{19}$$

The dynamics of the disagreement process is

$$d\eta_{a,s,t} = \eta_{s,a,t}\Delta_{s,a,t}dz_t. \tag{20}$$

By aggregating total consumption within a cohort born at time s, we have

$$\begin{aligned} c_{s,t} = \int_{-\infty}^{\infty} c_{a,s,t} da &= \int_{-\infty}^{\infty} g(a) Y_s e^{-\rho(t-s)} \frac{\eta_{a,s,t}}{\eta_{a,s,s}} \frac{\xi_s}{\xi_t} da \\ &= Y_s e^{-\rho(t-s)} \frac{\xi_s}{\xi_t} \int_{-\infty}^{\infty} g(a) \frac{\eta_{a,s,t}}{\eta_{a,s,s}} da. \end{aligned} \tag{21}$$

From Equation (21), we see that the only term that differs between the agent types born at time s is the disagreement process. Define the aggregate disagreement process for the cohort born at time s as

$$\eta_{s,t} = \int_{-\infty}^{\infty} g(a) \frac{\eta_{a,s,t}}{\eta_{a,s,s}} da. \tag{22}$$

By following the same approach as in Atmaz (2015), one can show that

$$d\eta_{s,t} = \Delta_{s,t}\eta_{s,t}dz_t, \tag{23}$$

where

$$\Delta_{s,t} = \frac{(\bar{v} + \Sigma)\,(z_t - z_s)}{\sigma_Y^2 + (\bar{v} + \Sigma)\,(t-s)}. \tag{24}$$

[3] Atmaz (2015) derives equilibrium with a continuum of agents differing in their beliefs. His agents are infinitely lived however, so there are no generation specific beliefs.

Equation (24) has the same form as in the base case in Proposition 1 with $\bar{V} = \bar{v} + \Sigma$. Hence, we can interpret the base case model as a model with heterogeneous initial beliefs that are normally distributed with zero bias on average. Here, the dynamics of the cohort specific belief behave similar to the base case even without learning, i.e., when $\bar{v} = 0$. In this case, $\bar{V} = \Sigma$; thus, it only depends on the within cohort cross-sectional heterogeneity. Consequently, the convergence of the cohort specific belief does not occur because of learning, but due to market selection. Agents that start with a relatively more correct initial belief have a higher consumption growth and eventually dominate the cohort. However, when $\Sigma > 0$ and $\bar{v} = 0$, the individual agents do not change their beliefs and, therefore, a simple average of the beliefs in the economy is constant. This contrasts with the case when $\bar{v} > 0$ since the beliefs of individual agents do change over time in the baseline model in Section 1.

2.1.2 Introducing Rational Learners

Assuming that only a fraction $a \in [0, 1]$ of a cohort learn from realizations of the endowment during their own life yields another form of within-cohort heterogeneity. The remaining agents in the cohort, the fraction $1 - a$, use all past historical information and, therefore, know the true expected growth rate of the endowment.

The filtering problem for the agents who learn from experience is the same as in Section 1. Therefore, the estimation error at time t for an agent of this type born at time s is identical to that in Proposition 1 and denoted similarly by $\Delta_{s,t}$. Consequently, the dynamics of the disagreement process, $\eta_{a,s,t}$, for agents who learn from experience are also identical to the case in Section 1, namely $\frac{d\eta_{a,s,t}}{\eta_{a,s,t}} = \Delta_{s,t} dz_t$.

Rational learners do not make any estimation error, therefore their disagreement process, $\eta_{1-a,s,t}$, is equal to one for all states and times.

The equilibrium with this type of heterogeneity is a special case of the one presented in Proposition 6, where

$$\bar{\Delta}_t^A = a \int_{-\infty}^{t} \nu e^{-(\rho+\nu)(t-s)} \frac{X_s}{X_t} \frac{\eta_{a,s,t}}{\eta_{a,s,s}} ds. \tag{25}$$

Next, we investigate the implications of the coexistence of rational learners and agents who learn from experience on equilibrium asset prices.

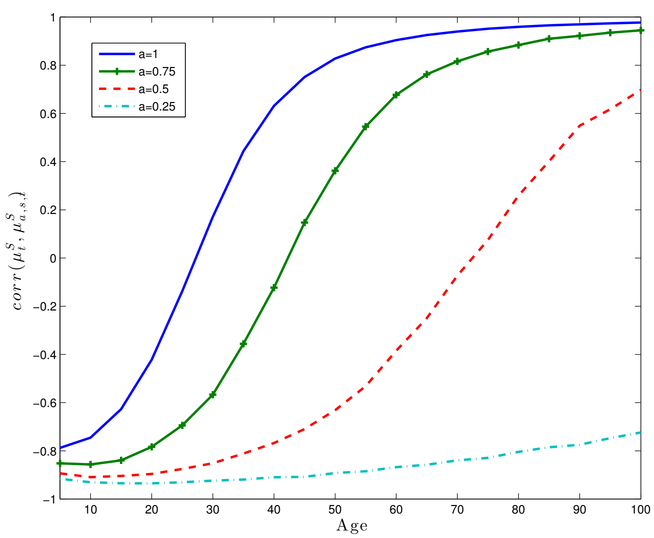
Figure 1 shows the correlation between the expected return as perceived by an agent of a particular age that is learning from experience and the true expected return for four economies with different fractions of agents who learn from experience. We see that as the fraction of the population that learns from experience is reduced, the older the agents who learn from experience have to be before the correlation between their perceived expected return and the true expected return is positive. The intuition for this result can be understood studying the expression for the time-t expected return as perceived by an agent born at time s

$$\mu^S_{a,s,t} = \underbrace{r_t + \sigma^S \sigma_Y - \sigma^S a \bar{\Delta}_t}_{\text{Expected return under the true measure}} + \sigma^S \Delta_{s,t}, \tag{26}$$

where we used the identity $\bar{\Delta}^A_t = a\bar{\Delta}_t$. We know that $\Delta_{s,t}$ and $\bar{\Delta}_t$ increase after positive shocks to aggregate endowment. Given that $\Delta_{s,t}$ and $\bar{\Delta}_t$ have opposite signs in the expression above, the true and perceived expected return can be negatively correlated. It is particularly so for young agents, for which $\Delta_{s,t}$ reacts strongly to shocks, thus the true and perceived expected return on the risky asset move in opposite directions. When agents accumulate experience, the variance of the estimation error, $\Delta_{s,t}$, declines and eventually the true and the perceived expected return correlate positively. The introduction of rational learners does not change this general intuition, but it postpones the time when the correlation between true and perceived expected return turns positive.

Suppose that a positive aggregate shock hits the economy, $dz_t > 0$. The response of the agents who learn from experience is exactly the same as in an economy without rational learners because the variance of their beliefs, $Var(\Delta_{s,t})$, is the same. The presence of rational learners changes instead the revision in the wealth weighted average belief, $\bar{\Delta}_t$. In fact, the response is dampened as the variance of the wealth weighted average belief is now multiplied by the squared fraction of learners from experience in the economy, $a^2 Var(\bar{\Delta}_t)$. Knowing that the true growth rate of endowment is constant, rational learners do not revise their beliefs when the economy is hit by a positive aggregate shock. This results in a weaker response of the wealth weighted average belief in the economy. The larger the fraction of rational learners in the economy, the smaller the response of the wealth weigthed average belief and the smaller $\Delta_{s,t}$ has to be for the correlation to turn positive. Alternatively, agents who learn from experience need to learn for a longer period of time the larger the fraction of rational learners in the economy.

Figure 1: *Objective and Perceived Returns.* The figure plots the correlation between the expected return under the objective measure and the perceived expected return of the agents that are learning from experience as a function of lifespan. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The fraction of each cohort that is learning from experience is $a = (1, 0.75, 0.5, 0.25)$. The figure is averaged from 10,000 simulations with 6000 periods or 500 years per simulation.



2.2 Multiple Risky Assets

In the model in Section 1, there is only one source of risk. Agents observe the endowment process and from that learn about the expected growth rate. Malmendier and Nagel (2011) show that individuals who experience high stock market returns invest more in the stock market, but do not alter their bond portfolios. Conversely, investors who experience high bond market returns invest more in bonds, but do not change their investment in stocks.

Hence, the experienced return is asset specific. In this subsection, we extend the baseline model to two risky assets to show that a model with learning from experience is consistent with an asset specific learning from experience. Rather than setting up the multi-asset model in full generality, we focus on two sources of risk: endowment risk and inflation risk. We do this for two reasons. First, Malmendier and Nagel (2016) show using survey data that consumers exhibit learning from experience behavior when forecasting inflation. Second, focusing on learning from inflation allows for the interpretation of the second risky asset as a nominal bond or some other asset for which inflation is an important driver of returns.

We follow Xiong and Yan (2010) and Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2016) and assume that inflation is exogenous and independent of aggregate endowment. Specifically, we keep the same structure as in Section 1, but in addition to learning about the expected growth rate of endowment agents also learn about the expected inflation. We model the price level as

$$d\Pi_t = \Pi_t \left(\mu_\Pi dt + \sigma_\Pi dz_t^\Pi \right). \tag{27}$$

Since agents do not know the expected inflation, μ_Π, they learn about it from observing the price level, Π_t, during their own life. As endowments and the price level are independent, the learning about the endowment growth can be separated from the learning about the expected inflation. To simplify, agents start with the correct value for the prior mean and the prior variance about inflation is $\bar{V}_\Pi$. The error process for the expected endowment growth, $\Delta_{s,t}$ is given in Proposition 1. The error process for expected inflation at time t for a cohort born at time s is

$$\Delta_{s,t}^\Pi = \frac{\bar{V}_\Pi \left(z_t - z_s \right)}{\sigma_\Pi^2 + \bar{V}_\Pi \left(t - s \right)}. \tag{28}$$

Similar to Proposition 1, we have that $\Delta_{s,s}^\Pi = 0$ and $\lim_{t \to \infty} \Delta_{s,t}^\Pi = 0$. The relation between the true Brownian motion and the perceived Brownian motion is $dz_{s,t}^\Pi = dz_t^\Pi - \Delta_{s,t}^\Pi dt$. The disagreement process, $\eta_{s,t}$, decomposes into two parts

$$\eta_{s,t} = \eta^Y_{s,t}\eta^\Pi_{s,t}, \tag{29}$$

where $d\eta^Y_{s,t} = \Delta_{s,t}\eta^Y_{s,t}dz_t$ and $d\eta^\Pi_{s,t} = \Delta^\Pi_{s,t}\eta^\Pi_{s,t}dz^\Pi_t$.

In addition to the asset specified in Section 1, the agents trade a risky asset in zero net supply with price P_t that is locally perfectly correlated with the shock to inflation. We refer to this asset as the long-term bond. The dynamics of it is

$$\frac{dP_t}{P_t} = \left(\mu^P_t dt + \sigma^P dz^\Pi_t\right) = \left(\mu^P_{s,t}dt + \sigma^P dz^\Pi_{s,t}\right). \tag{30}$$

The equilibrium is similar to the equilibrium in Section 1 and that the real short rate has the exact same form as in the case with only endowment risk. However, there are now two priced sources of risk in the economy. The market prices of endowment shocks, θ_t, and inflation shocks, θ^Π_t, are

$$\begin{aligned} \theta_t &= \sigma_Y - \bar{\Delta}_t, \\ \theta^\Pi_t &= -\bar{\Delta}^\Pi_t, \end{aligned} \tag{31}$$

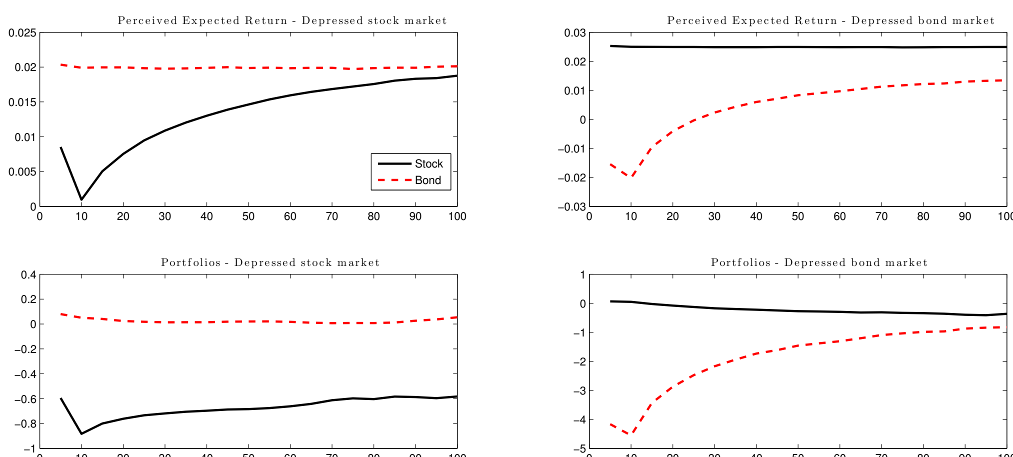
where $\bar{\Delta}_t$ has the same form as in the main text and $\bar{\Delta}^\Pi_t = \int_{-\infty}^t f_{s,t}\Delta^\Pi_{s,t}ds$. Let $\pi^S_{s,t}$ and $\pi^P_{s,t}$ be the dollar amount invested in the risky assets correlated with endowment shocks and inflation shocks, respectively. The next proposition characterizes the optimal portfolio.

Proposition 7. *The optimal dollar amount invested in the risky assets for an agent born at time s are*

$$\begin{aligned} \pi^S_{s,t} &= \frac{\Delta_{s,t} - \bar{\Delta}_t}{\sigma^S}\hat{W}_{s,t} + \frac{\sigma_Y}{\sigma^S}W_{s,t}, \\ \pi^P_{s,t} &= \frac{\Delta^\Pi_{s,t} - \bar{\Delta}^\Pi_t}{\sigma^P}\hat{W}_{s,t}. \end{aligned}$$

To examine the case with two risky assets, we reuse the parameters from the baseline model. For the risky bond, we assume that the volatility, σ_P, is 5%. We simulate two sets of paths. In the first set, we draw 200,000 five year paths for the endowment shocks, dz. Next, we pick the 10,000 worst realizations for the Brownian motion after five years. We call this set of paths depressed stock market. For the second set of paths, we draw the shocks to inflation rather than endowment. We refer to these paths as high inflation period or depressed bond market. Figure 2 shows the expectation of the stock market and bond market (top plots) and portfolio positions in the stock and the bond (bottom plots) for an agent born just before the bad realizations. The left plots show the depressed stock market and the right plots show the depressed bond market. We see from the plot that portfolio positions and expectations are asset specific, i.e., an agent who experiences a depressed stock market shorts the stock, but does not change the position in the bond relative to an agent who does not experience a depressed stock market.

Figure 2: *Expected Returns and Portfolio Policies in Depressed Markets.* The figure plots the perceived expected stock market return, $\mu^S_{s,t}$, and expected bond market return, $\mu^P_{s,t}$, (top plots) and the portfolio position in the stock, $\pi^S_{s,t}$ and the bond, $\pi^P_{s,t}$, (bottom plots) by cohort age in a depressed stock market (left-hand plots) and a depressed bond market (right-hand side). A depressed (stock and bond) market is defined as the bottom 5% realizations of the shocks during the first 5 years after the 6000 burn-in periods. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.



3 A Model with Life-cycle Earnings Profile

The aim of this section is to generalize our setup to additionally accomodate life-cycle earnings profile for each agent in the economy. This setting turns out to be equally tractable, leading to closed-form expressions similar to the ones of the main text. Importantly, all our previous conclusions and intuitions are robust to this generalization.

The model is as in Section 2, we therefore will focus on the effects that the introduction of more general earnings profile produce.

3.1 Life-cycle earnings profile

Section 2 assumes that the endowment that each agent receives does not depend on the age of the agent and it represents a fraction of aggregate output, i.e., $y_{s,t} = \omega Y_t$. In this section, we relax these assumptions and, following Gârleanu and Panageas (2015), we define individual earnings as

$$y_{s,t} = \omega Y_t \bar{h} G\left(t-s\right). \tag{32}$$

$G(t-s)$ is a deterministic function capturing the hump-shaped profile of life-cycle earnings. Aggregating over all the cohorts that are alive at time t gives

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} y_{s,t} ds = \omega Y_t \bar{h} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} G(t-s) ds = \omega Y_t. \tag{33}$$

where $\bar{h}$ is normalized such that aggregate output is as in the main article.
The remainder of the model remains exactly the same as in Section 2 and so they do the individual optimizations and the first order conditions.

3.2 Results

The introduction of life-cycle earnings does not change any of the dynamic properties of the model. This additional modelling ingredient does instead have an impact on the levels of asset prices through the consumption to wealth ratio. To see this, we rewrite for convenience our definition of β

$$\beta = \frac{c_{t,t}}{Y_t} = \frac{(\rho + \nu)\, H_{t,t}}{Y_t}. \tag{34}$$

β represents the consumption to wealth ratio of a newborn agent. As the last equality shows, it is only through the human capital that the hump-shaped profile of earnings influences β. Using the definition of earnings, Equation (32), we can write human capital as follows,

$$H_{t,t} = E_t \left[\int_t^{\infty} e^{-\nu(u-t)} \frac{\xi_u}{\xi_t} \omega \bar{h} Y_u G\left(u - t\right) du \right]. \tag{35}$$

Next, we introduce the following functional form for $G(t-s)$ which is able to approximate the hump-shaped profile of life-cycle earnings

$$G(t-s) = B_1 e^{-\delta_1 (t-s)} + B_2 e^{-\delta_2 (t-s)}, \tag{36}$$

where $B_1 > B_2$ and $\delta_1 < \delta_2$. Following the same steps that lead to Proposition 3 in the main article, we get an expression for the stochastic discount factor similar to the one in Equation (23), with the only difference being β. Particularly, β now solves the following equation

$$\beta = \omega \bar{h} \left[\frac{B_1}{\rho + \nu + \nu(1-\beta) + \delta_1} + \frac{B_2}{\rho + \nu + \nu(1-\beta) + \delta_2} \right]. \tag{37}$$

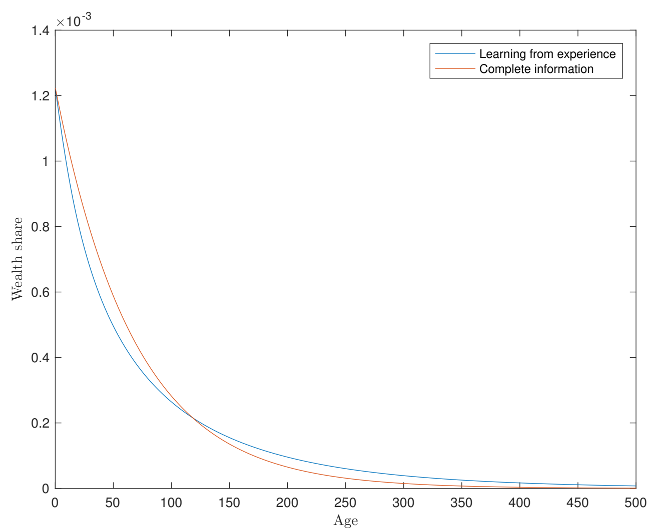
Notice that if $B_1 = 1, B_2 = 0$ and $\delta_1 = \delta_2 = 0$, we are back to the case where $G(t-s) = 1$. Therefore, life-cycle earnings profile impacts the level of the stochastic discount factor, Equation (23), and of the risk-free rate, Equation (28), through the consumption to wealth ratio of a newborn agent, i.e., β.

4 Additional Results Omitted from the Main Paper

4.1 Wealth Shares

This subsection presents a figure which compares the wealth shares of a cohort over time under two scenarios: our baseline specification and an overlapping generations model with complete information. The figure shows the cost of learning from experienced based bias. Until the age of about 120, the wealth share of an agent in our model is lower than the wealth share of an agent in a model with complete information. Therefore, the mistakes young agents make due to inexperience are large and have an impact that persists a ripe old age. The longer the time series of events they experience, the more precise the estimate about expected output growth of old agents is, but it still takes a long time for them to catch up with the loss of wealth they undergo when younger.

Figure 3: *Cohort Wealth Shares.* The figure shows the wealth share of different cohorts in two different economies: 1) the main specification in the paper 2) an OLG economy with complete information.



4.2 Particularly Bad or Good Experiences Early in Life

If mistakes made by young inexperienced agents are short lived, then the cumulative impact might be low relative to a situation in which mistakes are very long lasting. One way to examine how persistent mistakes are is to examine how particularly good or bad consumption path trajectories experienced early on in life impacts the belief about the risk premium later in life.

To do so, we sort on average consumption growth during the first 5 years after the agent starts trading. We define the top 5% and bottom 5% average consumption growth sample paths as good and bad times, respectively. Figure 4 shows the objective and the perceived risk premium in good and bad times. We see that in good (bad) times the true risk premium declines (increases) over the consumption boom (depressed) period but gradually increases (declines) back to the unconditional risk premium. In contrast, in good (bad) times the perceived expected return remains significantly above (below) the unconditional expected return even after 100 years and after 40 years less than half the effect has faded away.[4]Given persistent mistakes shown in Figure 4, it is not surprising that this also influences the optimal portfolio choice. Indeed, Figure 5 shows the portfolio policies in good and bad times and one can see that large negative or positive realizations early on in life have a long lasting impact on the optimal portfolio choice of agents.

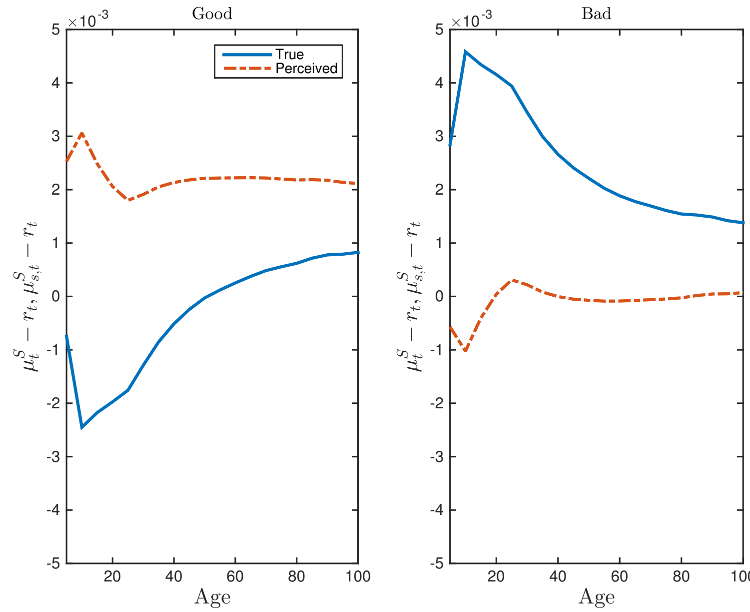
Therefore, the model is consistent with the empirical evidence on how significant experiences can be a source of persistent disagreement.

4.3 Constant Gain Learning

In the main paper, agents the young agents are updating more aggressively than the old agents due to a decreasing gain. It is useful to compare the main specification to a model with "constant gain." To do so, consider the dynamics of the belief at time t of someone

[4]The perceived risk premium is not monotonically decreasing after the initial five years. The reason for this due to the pre-learning period of 20 years. Hence, agents born after the cohort we are tracking are also significantly biased when entering the market.

Figure 4: *True and Perceived Risk premium in Good and Bad Times.* The figure plots the true risk premium, $\mu_t^S - r_t$, and the perceived risk premium, $\mu_{s,t}^S - r_t$, by cohort age in good and bad times. Good (bad) times are defined as the top (bottom) 5% average consumption growth during the first 5 years after the 6000 burn-in periods. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.



born at time s:

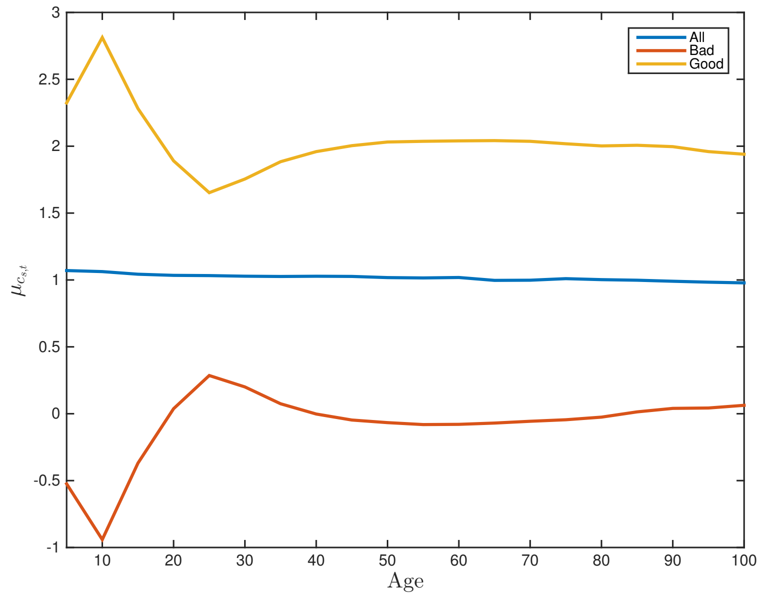
$$d\hat{\mu}_{s,t} = -K\Delta_{s,t}dt + Kdz_t,$$

where K is the constant gain and $\Delta_{s,t} = \frac{\hat{\mu}_{s,t} - \mu_Y}{\sigma_Y}$ is the estimation error. Following the same calculations as in the paper, but with the constant gain learning instead, one can show that the market view dynamics (similar to Proposition 7 in the paper) is

$$d\bar{\mu}_t = \beta\nu\left(\hat{\mu}_{t,t} - \bar{\mu}_t\right)dt - \frac{\bar{V}_t}{\sigma_Y}\bar{\Delta}_t dt + \frac{\bar{V}_t}{\sigma_Y}dz_t,$$

where $\bar{V}_t = K + \mathcal{V}\left(\hat{\mu}_t\right) > K$. The diffusion of the average belief (not wealth weighted average as for the market view) is K, as all agents have the same constant gain. Hence, we have that in such an economy the market view is updating more aggressively than any of the agents due to the wealth reallocations after a shock. This is important, as in this constant gain economy the correlation between the average belief about the risk premium and the true

Figure 5: *Portfolio Policies in Good and Bad Times.* The figure plots the portfolio policy, $\pi_{s,t}$, by cohort age in good and bad times. Good (bad) times are defined as the top (bottom) 5% average consumption growth during the first 5 years after the 6000 burn-in periods. Each observation is calculated using a window of 60 non-overlapping observations (5 years). The figure is averaged from 10,000 simulations with 1200 periods or 100 years per simulation.



risk premium is likely to be positive and, therefore, it cannot explain the negative relation found in Greenwood and Shleifer (2014). Put differently, as the market view updates more aggressively than any of the agents in the economy, the variance of the market view is higher than the covariance between the market view and the average beliefs, which according to Equation (37) in the paper implies a positive correlation between the average belief about the risk premium and the true risk premium.

5 Numerical Illustrations

To produce Figure 1, Figure 2, Figure 3, and Table 1 in the main text, we simulate an economy populated by a large number of cohorts where one cohort is born every period. One period in the simulation represents one month. After 6000 burn-in periods, we obtain an economy with 6000 cohorts. We use the final values from the burn-in simulation as starting values for simulating this economy forward for another 1200 periods. We generate data from $10,000$ simulations, each with 1200 periods or 100 years.

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