Chapter 5

Advanced Encryption Standard (AES)

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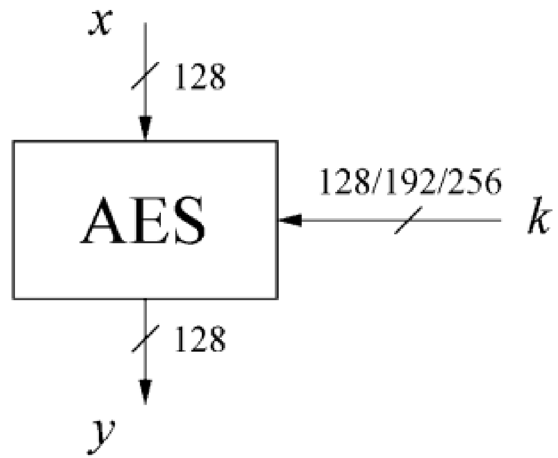
Origins

- Clear a replacement for DES was needed
  - have theoretical attacks that can break it
  - have demonstrated exhaustive key search attacks
- Triple-DES
  - but slow, has small blocks
- US NIST issued call for ciphers in 1997
  - 15 candidates accepted in Jun 98
  - 5 were shortlisted in Aug-99
  - Rijndael was selected as the AES in Oct-2000
  - issued as FIPS PUB 197 standard in Nov-2001

The AES Cipher - Rijndael

- Designed by Rijmen-Daemen in Belgium
- has 128/192/256 bit keys, 128 bit data
- an **iterative** rather than **feistel** cipher
  - processes data as block of 4 columns of 4 bytes
  - operates on entire data block in every round
- designed to be:
  - resistant against known attacks
  - speed and code compactness on many CPUs
  - design simplicity

AES: Overview

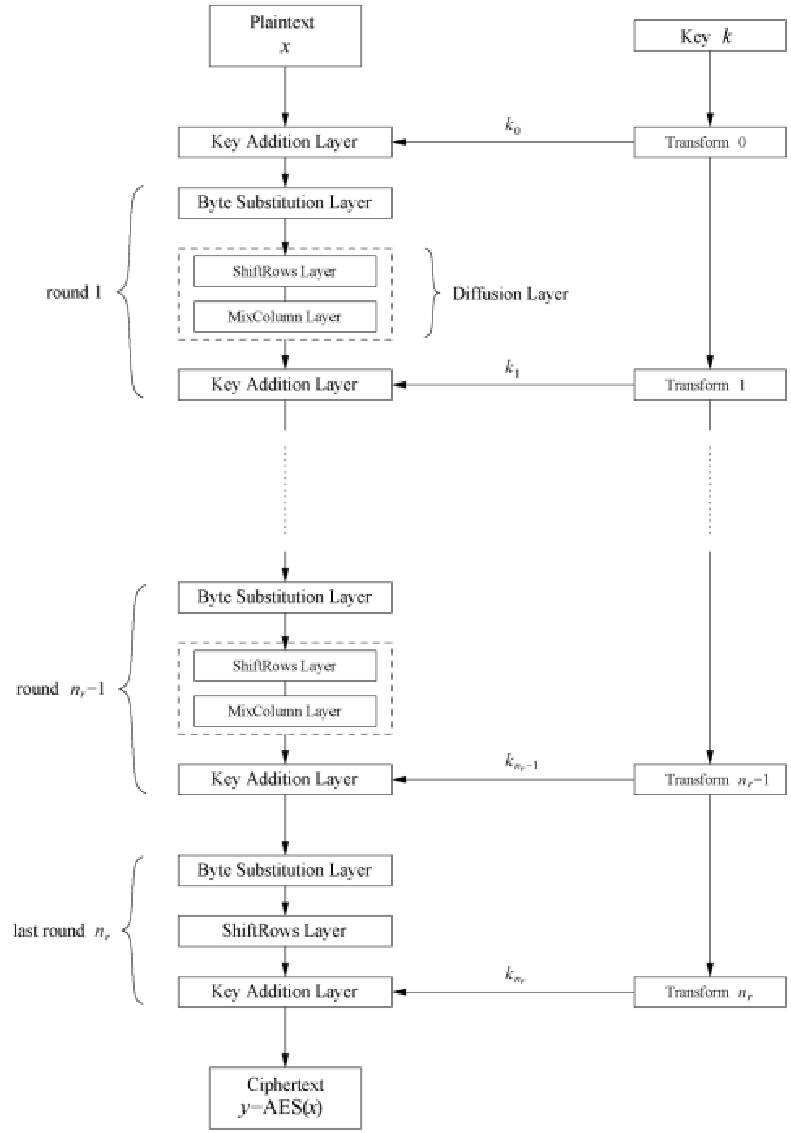


The number of rounds depends on the chosen key length:

Key length (bits)	Number of rounds
128	10
192	12
256	14

AES: Overview

- Iterated cipher with 10/12/14 rounds
- Each round consists of "Layers"



Content of this Chapter

- Overview of the AES algorithm
- **Internal structure of AES**
  - Byte Substitution layer
  - Diffusion layer
  - Key Addition layer
  - Key schedule
- Decryption
- Security

Internal Structure of AES

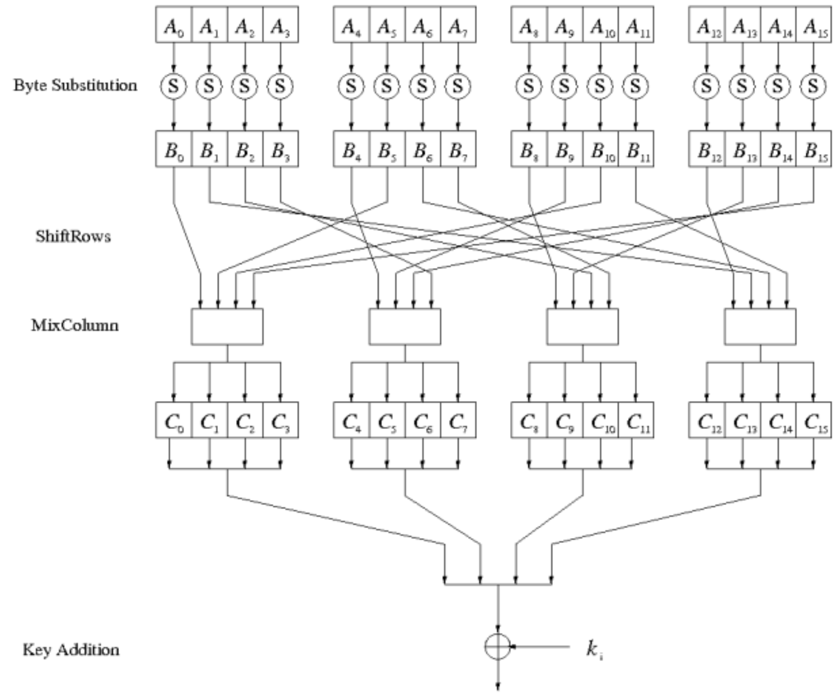
- AES is a byte-oriented cipher
- The state A (i.e., the 128-bit/16-byte data path) can be arranged in a 4×4 matrix:

A_0	A_4	A_8	A_{12}
A_1	A_5	A_9	A_{13}
A_2	A_6	A_{10}	A_{14}
A_3	A_7	A_{11}	A_{15}

  – with $A_0, \ldots, A_{15}$ denoting the 16-byte input of AES

Internal Structure of AES

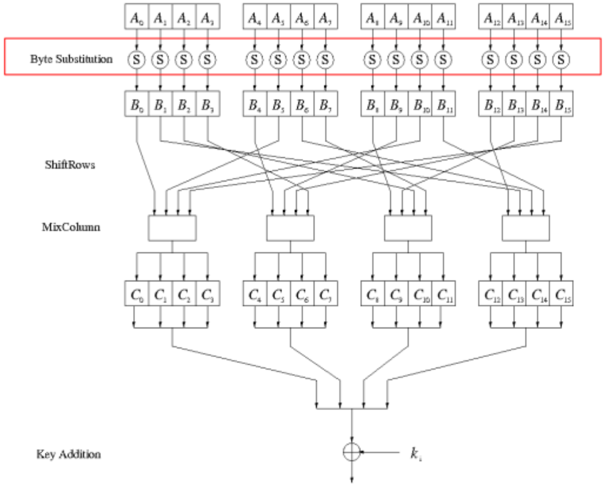
› Round function for rounds $1, 2, \ldots, nr_1$



– In the last round, the MixColumn transformation is omitted

Byte Substitution Layer

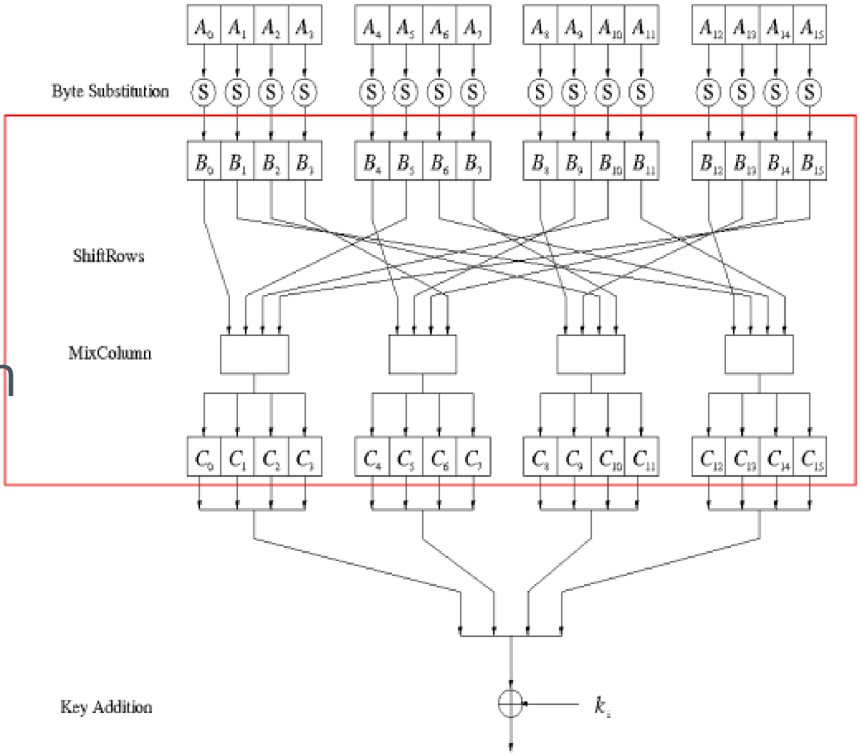
- The Byte Substitution layer consists of 16 S-Boxes with the following properties:
- The S-Boxes are
  - identical
  - the only **nonlinear** operation of AES,
  - $ByteSub(A_i) + ByteSub(A_j) \neq ByteSub(A_i + A_j)$, for $i, j = 0, \ldots, 15$



- Bijective
  - there exists a one-to-one mapping of input and output bytes
- S-Box can be uniquely reversed
  - In software implementations, the S-Box is usually realized as a lookup table

Diffusion Layer

- Provides diffusion over all input state bits
  - ShiftRows Sublayer: Permutation of the data on a byte level
  - MixColumn Sublayer: Matrix operation which combines ("mixes") blocks of four bytes
- Performs a linear operation on state matrices A, B
  - $DIFF(A) + DIFF(B) = DIFF(A + B)$

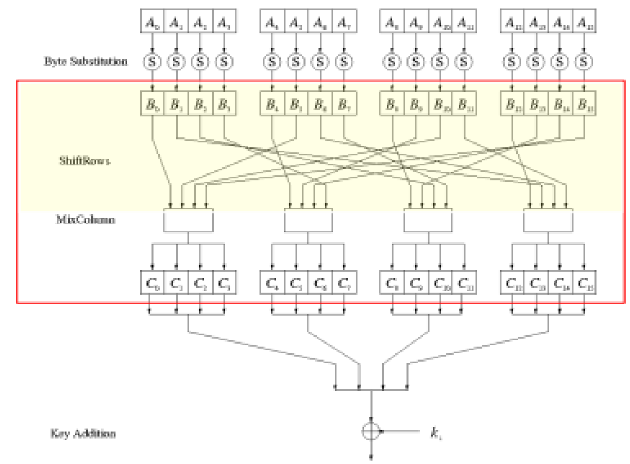


ShiftRows Sublayer

› Rows of the state matrix are shifted cyclically:

Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}



Output matrix

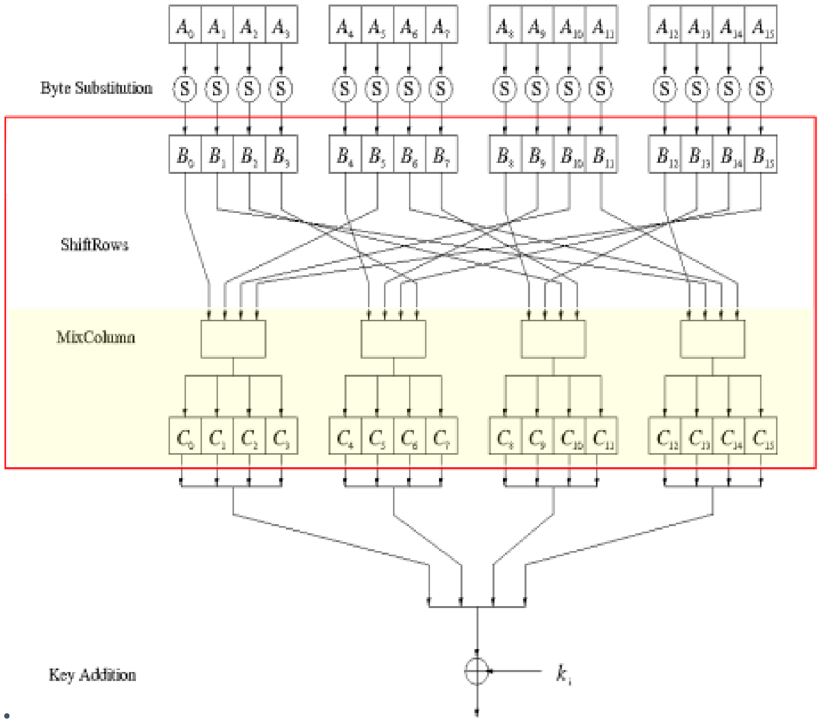
B_0	B_4	B_8	B_{12}	no shift
B_5	B_9	B_{13}	B_1	← one position left shift
B_{10}	B_{14}	B_2	B_6	← two positions left shift
B_{15}	B_3	B_7	B_{11}	← three positions left shift

MixColumn Sublayer

- Linear transformation which mixes each column of the state matrix
- Each 4-byte column is considered as a vector and multiplied by a fixed 4×4 matrix, e.g.,

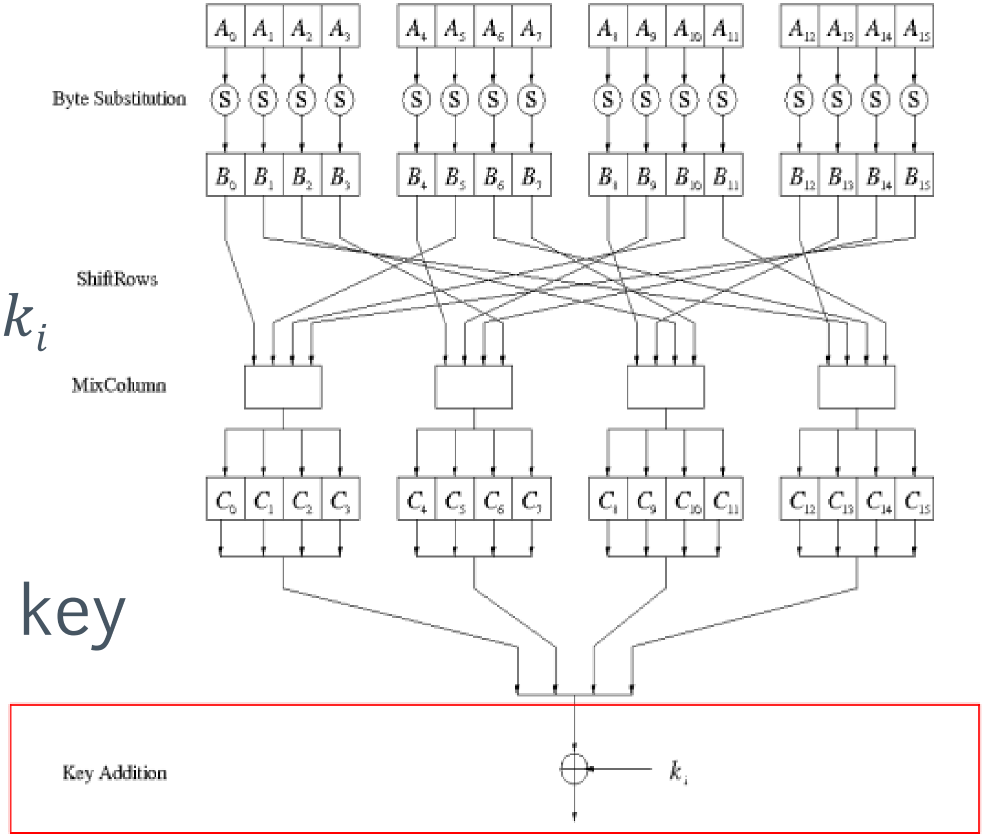
$$\begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \cdot \begin{pmatrix} B_0 \\ B_5 \\ B_{10} \\ B_{15} \end{pmatrix}$$

where 01, 02 and 03 are given in hexadecimal notation



Key Addition Layer

- Inputs:
  - 16-byte state matrix C
  - 16-byte subkey k_i
- Output: $C \oplus k_i$
- The subkeys are generated in the key schedule



Key Schedule

› Subkeys are derived recursively from the original 128/192/256-bit input key

› Each round has 1 subkey, plus 1 subkey at the beginning of AES

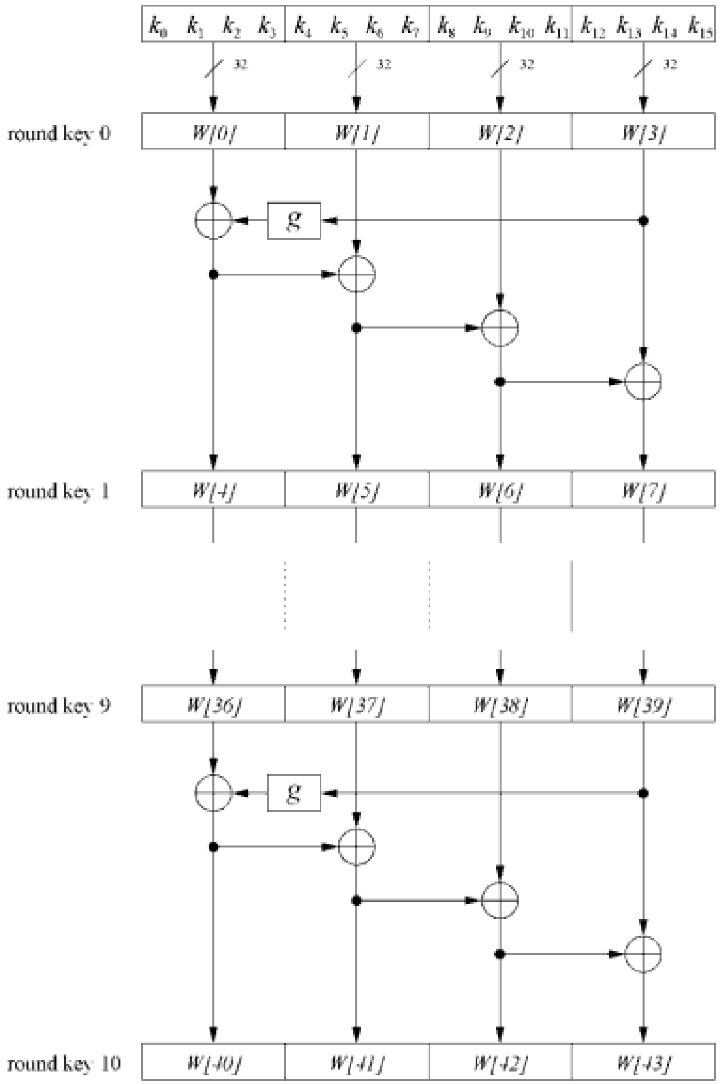
Key length (bits)	Number of subkeys
128	11
192	13
256	15

› Key whitening: Subkey is used both at the input and output of AES

$\Rightarrow \#\, subkeys = \#\, rounds + 1$

› There are different key schedules for the different key sizes

Key Schedule



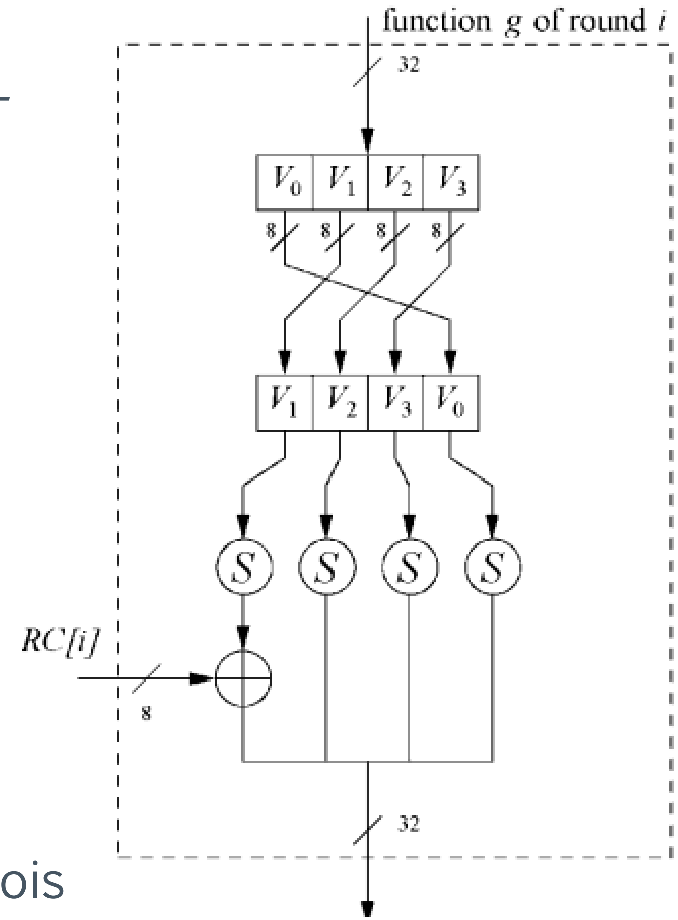
- Word-oriented:
  - $1\, word = 32\, bits$
- 11 subkeys are stored in
  - $W[0], \ldots, W[3], W[4], \ldots, W[7], \ldots, W[40], \ldots, W[43]$
- First subkey $W[0] \ldots W[3]$ is the original AES key

Key Schedule

- Function g rotates its four input bytes and performs a bytewise S-Box substitution
  - Nonlinearity
- The round coefficient RC is only added to the leftmost byte and varies from round to round:

$$RC[1] = x^0 = (00000001)_2$$
$$RC[2] = x^1 = (00000010)_2$$
$$RC[3] = x^2 = (00000100)_2$$
$$\ldots$$
$$RC[10] = x^9 = (00110110)_2$$

  - x^i represents an element in a Galois field



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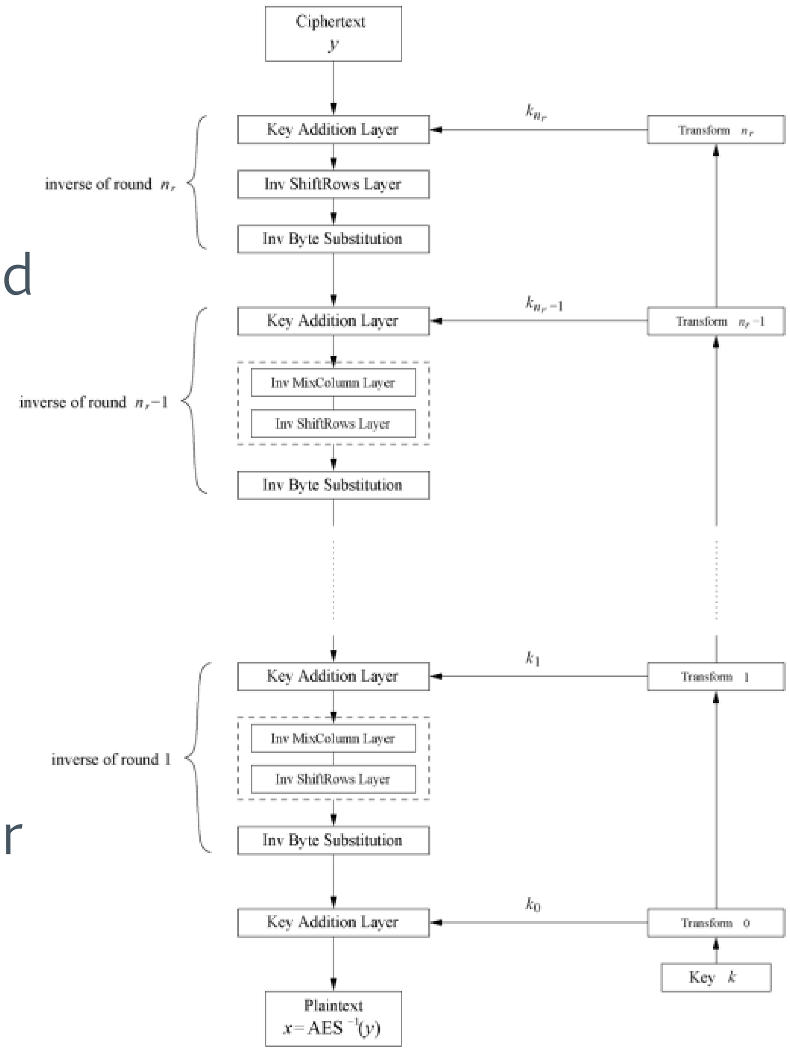
- Security

Decryption

- AES is not based on a Feistel network
  - All layers must be inverted for decryption:
- MixColumn layer
  - Inv MixColumn layer
- ShiftRows layer
  - Inv ShiftRows layer
- Byte Substitution layer
  - Inv Byte Substitution layer
- Key Addition layer is its own inverse



Decryption

- Inv MixColumn layer:
  - To reverse the MixColumn operation, each column of the state matrix C must be multiplied with the inverse of the 4×4 matrix, e.g.,

$$\begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \cdot \begin{pmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{pmatrix}$$

  - where 09, 0B, 0D and 0E are given in hexadecimal notation

- Again, all arithmetic is done in the Galois field $GF(2^8)$

Decryption

› Inv ShiftRows layer:
  - All rows of the state matrix B are shifted to the opposite direction:

Input matrix

B_0	B_4	B_8	B_{12}
B_1	B_5	B_9	B_{13}
B_2	B_6	B_{10}	B_{14}
B_3	B_7	B_{11}	B_{15}

Output matrix

B_0	B_4	B_8	B_{12}	no shift
B_{13}	B_1	B_5	B_9	→ one position right shift
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B_7	B_{11}	B_{15}	B_3	→ three positions right shift

Decryption

› Inv Byte Substitution layer:
  – Since the S-Box is bijective, it is possible to construct an inverse, such that

  $A_i = S^{-1}(B_i) = S^{-1}(S(Ai))$

  ⇒The inverse S-Box is used for decryption. It is usually realized as a lookup table

› Decryption key schedule:
  – Subkeys are needed in reversed order (compared to encryption)
  – In practice, for encryption and decryption, the same key schedule is used. This requires that all subkeys must be computed before the encryption of the first block can begin

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Security

- Brute-force attack:
  - Due to the key length of 128, 192 or 256 bits, a brute-force attack is not possible
- Analytical attacks:
  - There is no analytical attack known that is better than brute-force
- Side-channel attacks:
  - Several side-channel attacks have been published
  - Note that side-channel attacks do not attack the underlying algorithm but the implementation of it