

TURBO

CENTRE OF MASS & TORQUE

Detailed Technical Summary • Class 11 Physics

I. Fundamentals of Centre of Mass (CM)

Definition: The Centre of Mass is a hypothetical point where the entire mass of a system is assumed to be concentrated for the analysis of external forces.

- **Existence:** It is a unique point that always exists.
- **Location:** It may lie inside or outside the physical body.
- **Significance:** If all external forces are applied at the CM, the body undergoes pure translational motion.

II. Mathematical Formulas

System Type	Vector / Coordinate Form	Other Information
Discrete Particle System (n-particles)	$\vec{r}_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{\text{Total Linear Moment}}{\text{Total Mass}}$	$x_{CM} = \frac{\sum m_i x_i}{M}, y_{CM} = \frac{\sum m_i y_i}{M}$
Rigid Body System (Continuous)	$\vec{r}_{CM} = \frac{\int \vec{r} dm}{\int dm}$	$x_{CM} = \frac{\int x dm}{M}, y_{CM} = \frac{\int y dm}{M}$
Two-Particle System	$m_1 r_1 = m_2 r_2$ (Relative to CM)	CM lies on the line joining the masses and is closer to the heavier mass.

III. Motion Dynamics of CM

- **Total Momentum:** $M\vec{v}_{CM} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = \vec{P}_{total}$.
- **Acceleration:** $M\vec{a}_{CM} = \vec{F}_{ext}$. Internal forces do not affect the motion of the CM.
- **Zero External Force:** If $\vec{F}_{ext} = 0$, then $\vec{a}_{CM} = 0$ and \vec{v}_{CM} remains constant. If initially at rest, the CM position remains fixed.
- **CM vs. Centre of Gravity:** They coincide only in a uniform gravitational field. CG is where the total gravitational force acts.

IV. Moment of Force (Torque - $\vec{\tau}$)

- **Definition:** A measure of the rotating tendency of a force about a point. $\vec{\tau} = \vec{r} \times \vec{F}$.
- **Magnitude:** $|\tau| = (r \sin \theta)F$, where $r \sin \theta$ is the "moment arm."
- **Cartesian Components:**
 - $\tau_x = yF_z - zF_y$
 - $\tau_y = zF_x - xF_z$
 - $\tau_z = xF_y - yF_x$
- **Properties:**
 - **Direction:** Found via Right-Hand Screw Rule.
 - **Sign Convention:** Anticlockwise is Positive (+); Clockwise is Negative (-).
 - **Units/Dimensions:** SI Unit is **N m**. Dimensions: $[ML^2T^{-2}]$.