

# TURBO

## ROTATIONAL MOTION

Detailed Technical Summary • Class 11 Physics

### I. Kinematics of Rotation

Equation	Rotational Form	Linear Analogy
First Equation	$\omega = \omega_0 + \alpha t$	$v = u + at$
Second Equation	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at^2$
Third Equation	$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = u^2 + 2as$

Note:  $\omega_0$  = initial angular velocity,  $\omega$  = final angular velocity,  $\alpha$  = angular acceleration,  $\theta$  = angular displacement.

### II. Moment of Inertia (I)

- **Physical Meaning:** A measure of rotational inertia. Higher  $I$  makes it harder to change the rotational state.
- **Nature:** It is a **Tensor** (neither purely scalar nor vector).
- **Formulas:**  $I = \sum m_i r_i^2$  (discrete) and  $I = \int r^2 dm$  (rigid).
- **Radius of Gyration (K):**  $I = MK^2$ . It is the effective perpendicular distance where the mass is considered concentrated for rotation.
- **Units/Dimensions:** Unit: **kg m<sup>2</sup>**. Dimensions:  $[ML^2T^0]$ .
- **Dependencies:** Depends on mass distribution, shape, size, and axis orientation.

### III. Work, Energy & Momentum

- **Work Done:**  $dW = \tau d\theta \Rightarrow W = \int \tau d\theta$ .
- **Kinetic Energy:**  $K.E_{rot} = \frac{1}{2}I\omega^2$ .
- **Angular Momentum ( $\vec{L}$ ):**  $\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$ .
- **Conservation of Angular Momentum:** If  $\vec{\tau}_{ext} = 0$ , then  $\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$  ( $I_1\omega_1 = I_2\omega_2$ ).

### IV. Dynamics of Rolling on an Incline

- **Acceleration (a):**  $a = \frac{g \sin \theta}{1 + (K^2/R^2)}$ .
- **Rolling Time:** As  $K/R$  increases, acceleration decreases, and time taken to reach the bottom increases.
- **Condition for Pure Rolling:**  $\mu_s \geq \frac{\tan \theta}{1 + (R^2/K^2)}$ .
  - For a Disc ( $R^2/K^2 = 2$ ):  $\mu_s \geq \frac{1}{3} \tan \theta$ .
- **Friction Note:** In pure rolling, static friction does no work, thus mechanical energy is conserved. Friction always acts **up** the plane (whether rolling up or down).