Alignment

The alignment is computed using the traceless symmetric part of the mean strain rate and the given by:

$$A = \frac{a_{ij} S'_{ji}}{\sqrt{a_{mn} a_{nm} S'_{pq} S'_{qp}}}$$

where

$$a_{ij} = \frac{\tau_{ij}}{2\rho k} - \frac{1}{3}\delta_{ij}$$
$$S'_{ij} = S_{ij} - \frac{1}{3}\delta_{ij} S_{kk}$$

For the spanwise homogeneous case, the traceless stress tensor is given by:

$$a = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$

$$S' = \begin{bmatrix} S'_{11} & S'_{12} & S'_{13} \\ S'_{21} & S'_{22} & S'_{23} \\ S'_{31} & S'_{32} & S'_{33} \end{bmatrix} = \begin{bmatrix} S_{11} - \frac{S_{11}+S_{22}+S_{33}}{3} & S_{12} & S_{13} \\ S_{21} & S_{22} - \frac{S_{11}+S_{22}+S_{33}}{3} & S_{23} \\ S_{31} & S_{32} & S_{33} - \frac{S_{11}+S_{22}+S_{33}}{3} \end{bmatrix}$$
$$= \begin{bmatrix} S_{11} - \frac{S_{11}+S_{22}}{3} & S_{12} & 0 \\ S_{21} & S_{22} - \frac{S_{11}+S_{22}}{3} & 0 \\ 0 & 0 & -\frac{S_{11}+S_{22}}{3} \end{bmatrix}$$

The numerator can be explicitly defined by:

$$\begin{aligned} N = a_{ij} S'_{ji} &= a_{11} S'_{11} + a_{12} S'_{21} + a_{22} S'_{22} + a_{21} S'_{12} + a_{33} S'_{33} \\ &= a_{11} S'_{11} + 2 \cdot a_{12} S'_{21} + a_{22} S'_{22} + a_{33} S'_{33} \end{aligned}$$

The denominator can be defined by:

$$\begin{aligned} D &= \sqrt{a_{mn} a_{nm} S'_{pq} S'_{qp}} \\ &= \sqrt{(a_{11}a_{11} + 2 \cdot a_{12}a_{12} + a_{22}a_{22} + a_{33}a_{33})(S'_{11}S'_{11} + S'_{22}S'_{22} + 2 \cdot S'_{12}S'_{21} + S'_{33}S'_{33})} \end{aligned}$$