Data representation and Boolean algebra

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In this workshop, you will solve data representation and Boolean algebra problems. The fluency in these two mathematical topics forms the foundation of a good understanding of the subject of computer architecture. You will use a pen and paper or any functionally equivalent devices to work on these problems. The discussion of these problems is an essential part of the learning process in this lab. So, please discuss your ideas and answers with your colleagues and show your work to a teaching assistant.

Credits: Problem (4) and Problem (5) are authored by Prof. Kerstin Eder as problems for the Mathematical Methods for Computer Scientists unit.

Goals of this lab

This workshop aims to help you to understand the essence of number representation, improve your fluency in number conversion, solidify your understanding of negative and real number representations, and give you a chance to practice truth tables and logic inference rules. In addition, through practical examples, this worksheet is designed to help you to generalise your understanding from pure mathematical concepts to more specific computer architecture applications.

It is recommended that you work in small informal lab groups, e.g. with two to three students located close to you in the lab. Please note, however, that each of you needs to develop a good understanding of the material so that you can perform the tasks on your own once you have worked through the lab sheets.

Questions

If you have any questions, please ask. Make use of the TAs, but do not expect them to give you complete designs, they are there to guide you, not do the work for you. Please be patient. You may need several attempts to get something working.

When requesting help, TAs will expect you to show them what you have done so far (no matter how sketchy) and they will ask you to clearly explain your reasoning. This makes it easier for the TAs to help you. For this reason, priority will be given to students who can show and explain how far they have got.

1 Number Representation

As you know, each numbering system needs a base, and numbers are represented with symbols. So, with this in mind, how many symbols do we need for a numbering system with base five, base 85 and base n? Invent your own numbering system without using the standard numbering systems' symbols. You might want to use objects as symbols; be creative and have fun. Think about a number, then format it in your numbering system. Next, invite a classmate to decode your number. What is the least amount of information they need to know to interpret your number correctly?

base 7

2 Number Conversion

Attempt the following conversion problems. Be careful; some of the questions are invalid. That means it is impossible to represent the given number in the requested numbering system with the given number of digits.

1. What is the base-2 value 0b1100 when converted to base-10? Use 2 digits and unsigned representation. 12
2. What is the base-10 value 7 when converted to base-2? Use 4 digits and unsigned representation. 0111
3. What is the base-16 value 0x2 when converted to base-2? Use 4 digits and unsigned representation. 0010
4. What is the base-16 value 0xE when converted to base-2? Use 4 digits and 2's complement signed representation. 1110
5. What is the base-2 value 0b1111 when converted to base-10? Use 2 digits and signed (negative symbol where needed) representation. -1 or -7
6. What is the base-16 value 0xFC when converted to base-2? Use 6 digits and 2's complement signed representation. 111100
7. What is the base-16 value 0xFC when converted to base-2? Use 4 digits and 2's complement signed representation. 1100
8. What is the base-16 value 0x1F when converted to base-2? Use 6 digits and 2's complement signed representation. 011111
9. What is the base-16 value 0x1F when converted to base-2? Use 5 digits and 2's complement signed representation. cannot be converted

Checkpoint: show your answers to a TA or staff member for feedback.

Well done for completing the number conversion questions in this worksheet. Now try a few questions from the online number conversion quiz available at https://anas-shrinah.github.io.

3 Usage of Numbers

3.1 representation of quantities - Battery charge

A processor that controls a robot uses a register with eight bits to store its battery charge level. In addition, this robot has a solar plane that charges its battery when it is idle. After completing a task, the robot becomes and starts charging; the battery charge level was at 0xCC. After half an hour, the robot processor estimated the charge of the battery had increased by 0x3E. Calculate the new charge level of the battery. Remember, you can only store eight bits. This robot is programmed to operate in normal mode if the battery charge is above 5% and to go into hibernation mode if the battery charge is less than 5%. What mode would the robot be in after completing the half an hour of charge?

11001100
00111110
——————
100001010

204 + 62(increased by) = 266 in base-10
100001010 in base-2
10a in base-16

Checkpoint: show your answers to a TA or staff member for feedback.

3.2 Representation of characters

i 0x69
l 0x6c
o 0x6f
v 0x76
e 0x65
c 0x63
o 0x6f
m 0x6d
p 0x70
u 0x75
t 0x74
e 0x65
r 0x72
a 0x61
r 0x72
c 0x63
h 0x68
i 0x69
t 0x74
e 0x65
c 0x63
t 0x74
u 0x75
r 0x72
e 0x65

Characters are represented with binary numbers like everything else in computers. But how can we use binary numbers to represent characters? The answer to this question is in the American Standard Code for Information Interchange (ASCII). This standard encodes characters with eight bits each. First, do your own research to find out more about ASCII. Then, based on your research, what are the hexadecimal ASCII values of the charters in the string "i love computer architecture", ignore the spaces? Write down these values, then use them to do the two following tasks:

0x69, 0x6c, 0x6f, 0x76, 0x65, 0x63, 0x6f, 0x6d, 0x70, 0x75, 0x74, 0x65, 0x72, 0x61, 0x72, 0x63, 0x68, 0x69, 0x74, 0x65, 0x63, 0x74, 0x75, 0x72, 0x65

1. Add 0xE0 to the codes of the characters of the string "i love computer architecture" and convert the new values back to characters.
2. Subtract 0x20 from the codes of the characters of the string "i love computer architecture" and convert the new values back to characters.

Compare the two strings produced from the previous addition and subtraction operations. What did you notice?

Checkpoint: show your answers to a TA or staff member for feedback.

4 Logical equivalence

Which of the following statements are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$?

i) $(\neg a \wedge \neg c) \vee (a \wedge \neg b) \vee (a \wedge \neg c) \vee (b \wedge \neg c)$

ii) $c \vee \neg(a \Rightarrow b)$

iii) $c \Rightarrow (a \wedge \neg b)$

A. All are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$.

B. Both (i) and (ii) are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$.

C. Both (i) and (iii) are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$.

D. Both (ii) and (iii) are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$.

E. None of these statements are logically equivalent to $\neg((a \Rightarrow b) \wedge c)$

Checkpoint: show your answers to a TA or staff member for feedback.

5 Logical formula minimisation

Which of the following formulas represents the minimized Disjunctive Normal Form of the formula (1)?

$$\neg(((p \iff q) \wedge (p \vee r)) \implies \neg q) \tag{1}$$

A. $(p \wedge q \wedge \neg r) \vee (p \wedge q \wedge r)$

B. $(p \wedge q) \vee (r \wedge \neg r)$

C. $p \wedge q \wedge \neg r$

D. $(p \wedge q) \vee r$

E. $p \wedge q$

Checkpoint: show your answers to a TA or staff member for feedback.

6 Design a Boolean logic formula

An Emergency Shutdown system (ESD) supervises the operation of a nucleonic power plant. The ESD system shuts down the reactor when the core temperature exceeds a specific limit. The system has one temperature transmitter connected to the ESD system's processor. There were a couple of incidents when the ESD system shut down the reactor when the temperature had not exceeded the limit. The engineers at the plant suspect that the temperature transmitter might give wrong readings sometimes. Therefore, they decided to add two more temperature transmitters to the system. For the ESD to work with three transmitters, they have suggested using a voting system. The philosophy is the processor of the ESD system must only perform the emergency shutdown procedure if two out of the three transmitters indicate a high-temperature alarm. The plant engineers asked you, as the chief computer architecture designer, to design a Boolean algebra formula that takes as input three propositions:

1. p: High temperature alarm of the first temperature transmitter.
2. q: High temperature alarm of the second temperature transmitter.
3. r: High temperature alarm of the third temperature transmitter.

The logic formula should evaluate to true only and only if two out of the three high-temperature alarms are true. Write the truth table of your formula to check its behaviour accurately captures the requirement of the proposed control philosophy.

((P∧Q) ∨ (P∧R) ∨

Checkpoint: show your answers to a TA or staff member for feedback.

7 Real numbers representation

Yes, with first digit as sign

1. Can we represent negative numbers with the fixed point representation introduced in the lecture?
2. What are the largest and smallest (non-zero) positive numbers that can be represented by the fixed point representation introduced in the lecture? in 8 bit system, for 4 digit precision, largest number is 15.9375, smallest number 0.0625.
   If consider negative, -8~7.9375
3. Pick a floating point decimal number (a positive or negative whole number with a decimal point) from the range defined in Item 2 and convert it to binary with the fixed point representation introduced in the lecture.
   3.34 => 0011.0101 (3.3125)
   => 0011.1000 (3.5)
   => 0011.01011 (3.34375)
4. What are the decimal values of the largest and smallest (non-zero) positive numbers that can be represented by the floating point representation introduced in the lecture? 1/16 ~ 120
5. Pick a floating point decimal number from the range defined in Item 4 and convert it to binary with the floating point representation introduced in the lecture.
   3.34 => 01110111 (3.5)
   3.34 => 01110111 (3.25)
6. What are the decimal values of the largest and smallest (non-zero) negative numbers that can be represented by the floating point representation introduced in the lecture? -1/16 ~ -120
7. Pick a floating point decimal number from the range defined in Item 6 and convert it to binary with the floating point representation introduced in the lecture.
   -3.34 => 11110111 (-3.5)
   -3.34 => 11110111 (-3.25)

Checkpoint: show your answers to a TA or staff member for feedback.

The following tasks require you to read and research beyond the content delivered in the lectures. Please only attempt these questions when you have done all other sections.

8. What is the decimal value of the binary encoding 0 10000111 01101101001111100000000 when interpreted as IEEE 754 single-precision number?
9. What is the IEEE 754 single-precision binary encoding of the decimal value 10.59375?
10. What is the IEEE 754 single-precision binary encoding of the decimal value 10.61?
11. What are the binary encodings of the largest and smallest (non-zero) positive numbers that can be represented by IEEE 754 single-precision representation?
12. What are the binary encodings of the largest and smallest (non-zero) negative numbers that can be represented by IEEE 754 single-precision representation?

Checkpoint: show your answers to a TA or staff member for feedback.

Well done for completing the first workshop of the Overview of Computer Architecture unit.