UNIVERSITY OF LEICESTER

More Examples for Lecture 1.

MA2032 Vector Calculus

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Geometric Interpretations of Equations

Example 1

Give a geometric description of the set of points in space whose coordinates satisfy the given pairs of equations:

a) $x = 2,\ y = 3,$

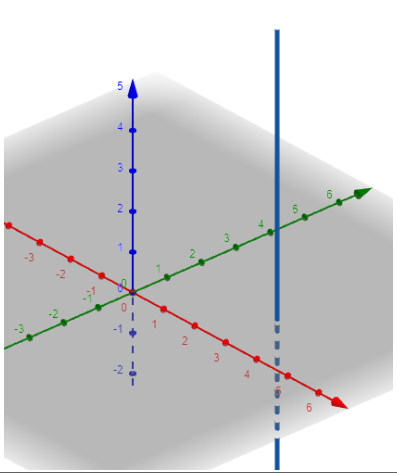
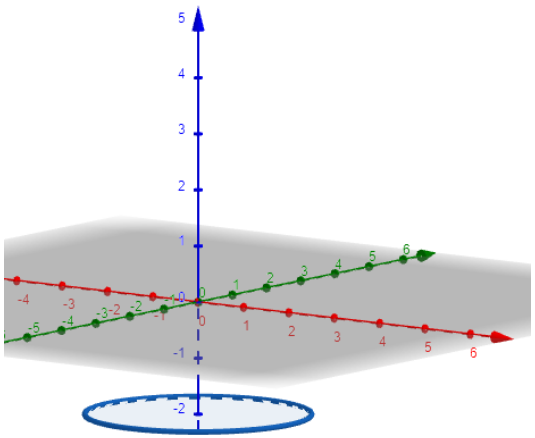
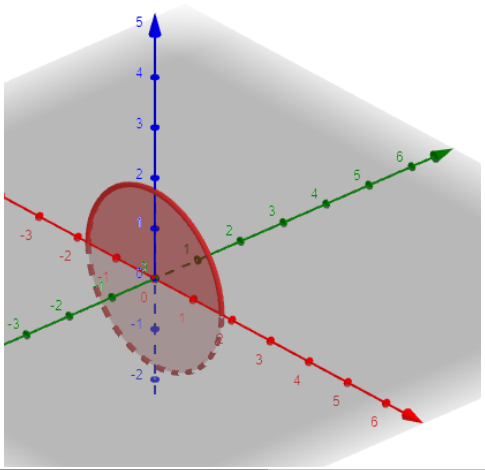
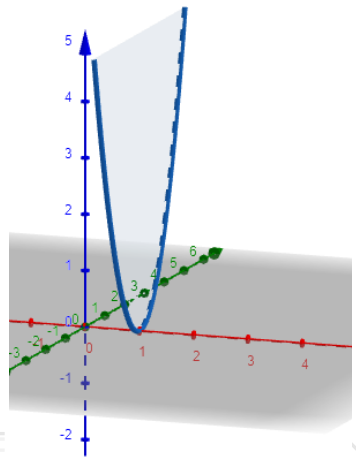
b) $x^2 + y^2 = 4,\ z = -2,$

c) $x^2 + (y-1)^2 + z^2 = 4,\ y = 0,$

d) $z = y^2,\ x = 1.$

Solution:

a) The line through the point $(2, 3, 0)$ parallel to the z-axis.
b) The circle $x^2 + y^2 = 4$ in the plane $z = -2$.
c) The circle $x^2 + z^2 = 3$ in the xz-plane.
d) The parabola $z = y^2$ in the plane $x = 1$.

Geometric Interpretations of Inequalities and Equations

Example 2

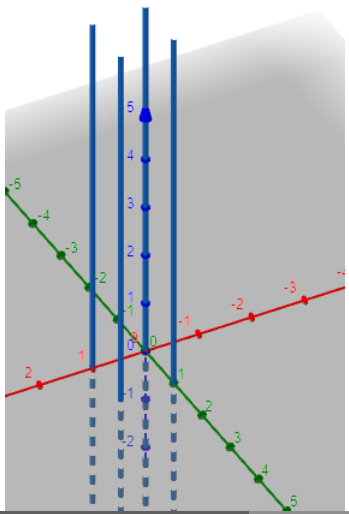
Describe the sets of points in space whose coordinates satisfy the given inequalities or combinations of equations and inequalities:

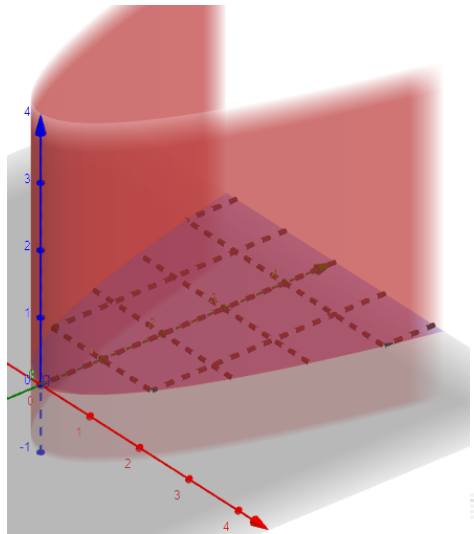
a) $0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 1,$ b) $y \geqslant x^2,\ z \geqslant 0.$

Solution:

a) The square column bounded by the planes $x = 0, x = 1, y = 0, y = 1$.

b) The region on or inside the parabola $y = x^2$ in the xy-plane and all points above this region.





Distance

Example 3

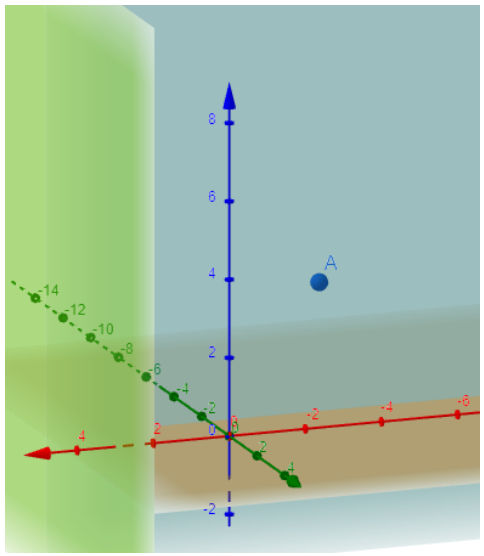
Find the distance from the point $(-2, 1, 4)$ to the:
a) plane $x = 3$, b) plane $y = -5$, c) plane $z = -1$.

Solution:

(a) the distance between $(-2, 1, 4)$ and the plane $x = 3$ is $3 - (-2) = 5$

(b) the distance between $(-2, 1, 4)$ and the plane $y = -5$ is $1 - (-5) = 6$

(c) the distance between $(-2, 1, 4)$ and the plane $z = -1$ is $4 - (-1) = 5$



Spheres

Example 4

Find the center C and the radius a for the sphere
$(x-1)^2+(y-2)^2+(z+1)^2=103+2x+4y-2z$.

Solution:

$$(x-1)^2+(y-2)^2+(z+1)^2=103+2x+4y-2z \Rightarrow\ x^2-2x+1+y^2-4y+4+z^2+2z+1=103+2x+4y-2z$$
$$\Rightarrow (x^2-4x+3+1)+(y^2-8y+12+4)+(z^2+4z+3+1)=103+3+12+3\Rightarrow$$
$$(x-2)^2+(y-4)^2+(z+2)^2=11^2 \Rightarrow \text{ the center is at } (2,4,-2) \text{ and the radius is } 11$$

Example 5

Find a formula for the distance from the point $P(x, y, z)$ to the
a) x-axis, b) y-axis, c) z-axis.

Solution:

(a) the distance between (x, y, z) and $(x, 0, 0)$ is $\sqrt{y^2 + z^2}$

(b) the distance between (x, y, z) and $(0, y, 0)$ is $\sqrt{x^2 + z^2}$

(c) the distance between (x, y, z) and $(0, 0, z)$ is $\sqrt{x^2 + y^2}$