UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 6

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ANNOUNCEMENTS

- Last session before Military Training

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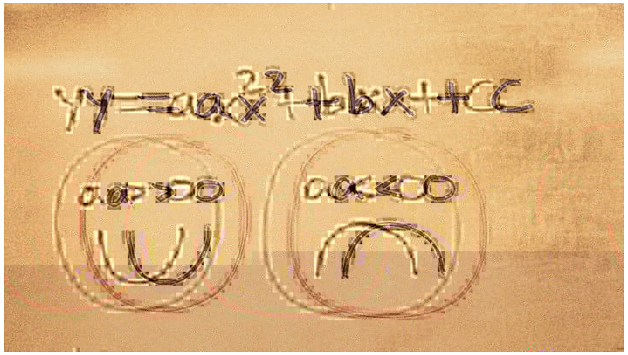


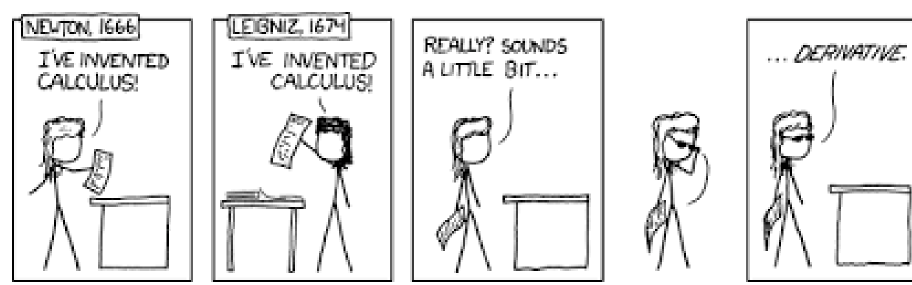
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$



LIMIT LAWS

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then

i. $\lim_{x \to c} [f(x) + g(x)] = L + M$

ii. $\lim_{x \to c} [\alpha f(x)] = \alpha L$ for $\alpha \in \mathbb{R}$

iii. $\lim_{x \to c} [f(x)g(x)] = LM$

EXERCISE:

Given that $\lim_{x\to 0}\frac{\sin(x)}{x}=1$, determine $\lim_{x\to 0}\frac{1-\cos(x)}{x}$. Hence, find

$$\lim_{x\to 0}\frac{\sqrt{1+\sin(x)}-\sqrt{1-\sin(x)}}{x}$$

CONTINUITY

Let $f(x)$ be defined on (a, b). Then $f(x)$ is **continuous** at $c \in (a, b)$ if

$$\boxed{\lim_{x \to c} f(x) = f(c).}$$

EXAMPLE

Let $f(x) = x^2 - 4x + 5$. Show that $\lim_{x \to 2} f(x) = 1$ by using an $\varepsilon - \delta$ argument. Hence, conclude that $f(x)$ is continuous at $x = 2$.

EXERCISE

Let

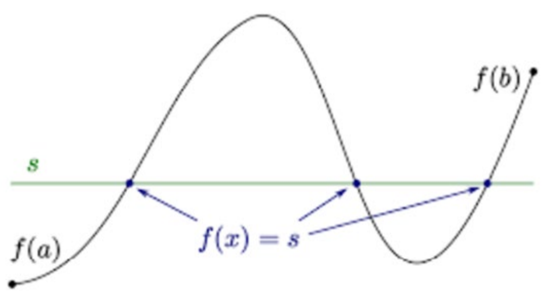
$$g(x) = \begin{cases} x^2 \quad , & x < 1 \\ 2 - x, & x \geq 1 \end{cases}$$

Show that $g(x)$ is continuous at $x = 1$ using an $\varepsilon - \delta$ argument.

INTERMEDIATE VALUE THEOREM

If a function, f, is continuous on $[a, b]$ and $f(a) < f(b)$. Then

$$\boxed{\forall\, f(a) < s < f(b), \exists c \in (a, b) : f(c) = s}$$

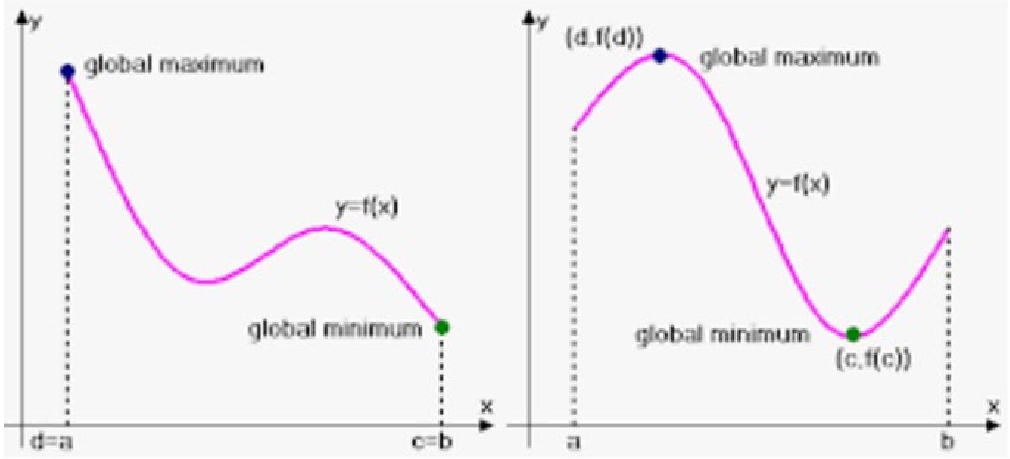


EXTREME VALUE THEOREM

Suppose $f(x)$ is continuous on the closed, bounded interval $[a, b]$. Then $f(x)$ is bounded on $[a, b]$ and achieves its bounds.

Or in other words,

$$\exists c, d \in [a, b] : f(c) \leq f(x) \leq f(d) \; \forall x \in (a, b)$$



EXAMPLE

Prove that $f(x) = x^3 - 3x^2 + 12x - 25$ has at least one real root.

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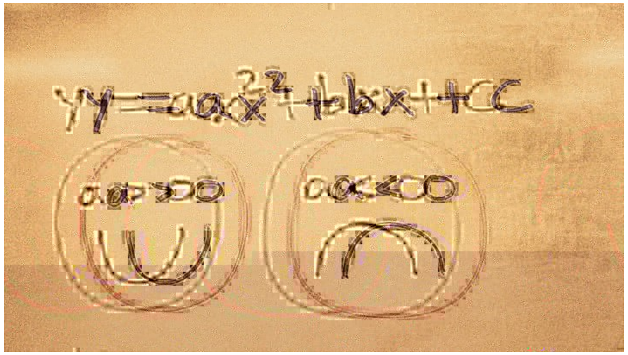

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