UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 10

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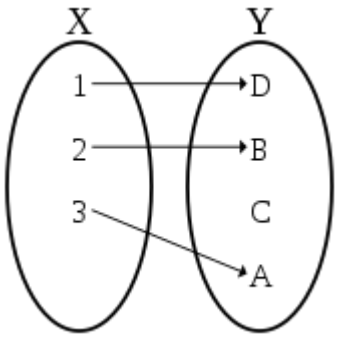
ANNOUNCEMENTS

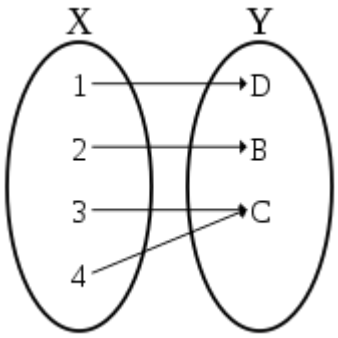
- Consolidation Week
- Wednesday Workshops with Andy, 5AM-6AM (UK time)
- Extra exercises?
  - Workbook
  - Text books
    - Larson/Edwards, "Calculus 9th edition"
    - Adams/Essex, "Calculus, a complete course, 7th edition"
  - Discuss solution(s)

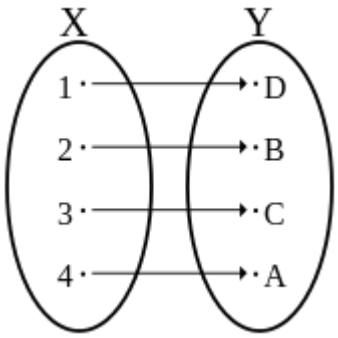
ONE-TO-ONE, ONTO & BIJECTIVITY

- A function, f, is **One-to-One** (Injective) if $\forall x_1, x_2 \in D_f, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- A function, f, is **Onto** (Surjective) if $\forall y \in \text{range}(f), \exists x \in D_f : f(x) = y$
- A function is **Bijective** if it is **both** One-to-One and Onto

Example: Let $f: X \to Y$, classify the diagrams below.







INVERSE FUNCTIONS

If $f(x)$ is a one-to-one function, there exists a unique inverse function which we denote by

$$f^{-1} : \text{ran}(f) \rightarrow \text{dom}(f)$$

such that

$$f^{-1} \circ f(x) = x \; \forall x \in \text{dom}(f)$$

and

$$f \circ f^{-1}(y) = y \; \forall y \in \text{ran}(f)$$

Example: Find the inverse function of

$$f(x) = \frac{x+4}{2x-5}$$

EXERCISE:
DETERMINE IF THESE FUNCTIONS ARE ONE-TO-ONE.
IF SO, FIND IT'S INVERSE.

a) $f(x) = 3x - 2$

b) $h(x) = x^2$

c) $k(x) = \frac{1+2x}{7+x}$

d) $m(x) = \cos(x)$

PROOF BY CONTRADICTION

- Want to prove a Proposition, P.
- If the negation, $\neg P$, leads to a contradiction

  i.e $\neg P \rightarrow C \wedge \neg C$
- Then P is true.

Example:

$P: a^2 - 4b \neq 2 : a, b \in \mathbb{Z}$

- $\neg P: a^2 - 4b = 2$
  $\Rightarrow a^2 = 2(1 + 2b)$
  $\Rightarrow a$ is even, let $a = 2c$
  $\Rightarrow 2(c^2 - b) = 1$
- C: 1 is odd and
  $\neg C$: 1 is even
- So P is true

EXERCISE:

Prove the Proposition:

$$P\text{: If } N^2 \text{ is even} \Rightarrow N \text{ is even}$$

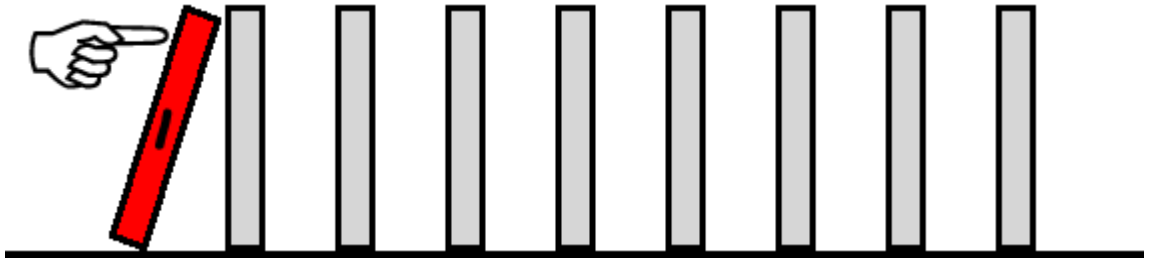
Hints:

- What is $\neg P$?
- If a number Q is even, then $Q = ?$

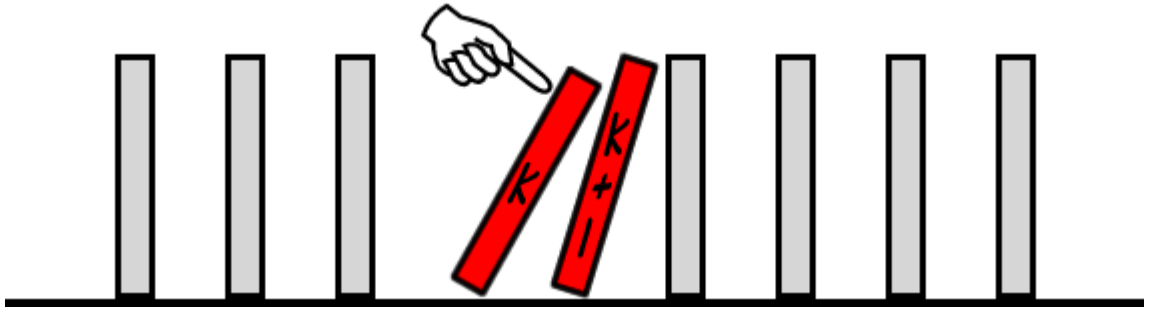
PROOF BY INDUCTION

Powerful way to prove iterative statements $P(n), n \in \mathbb{N}$.

1. First step: Prove Base Case (smallest possible n)



2. Assumptive step: Assume $n = k \in \mathbb{N}$ is true
3. Inductive step: Show $n = k$ is true $\Rightarrow n = k + 1$ is true



EXERCISE:

Prove by induction that $\forall n \geq 2, x_j \in \mathbb{R}, j \in \mathbb{N}$,

$$\left| \sum_{j=1}^{n} x_j \right| \leq \sum_{j=1}^{n} |x_j|$$

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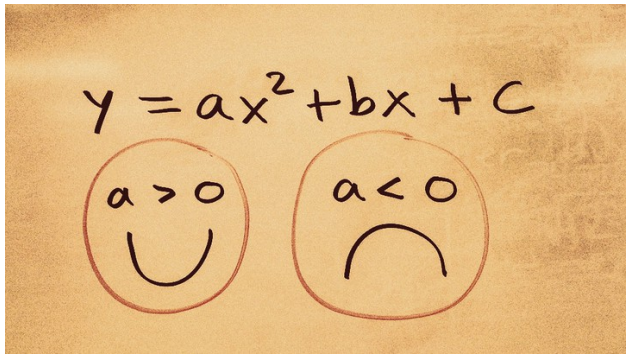

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

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ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$