MA1014
CALCULUS AND ANALYSIS
TUTORIAL 20

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ANNOUNCEMENTS

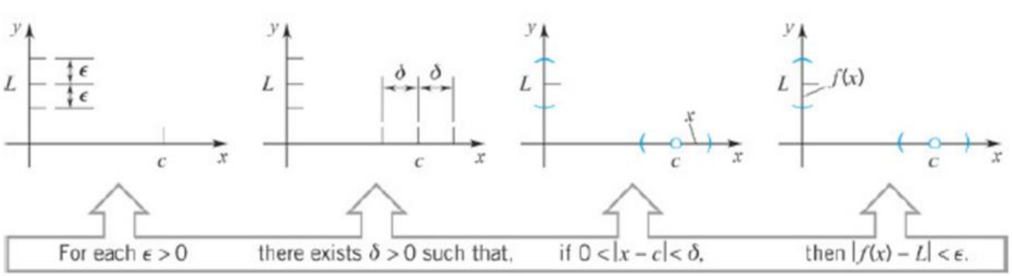
- Chapter 2 revision

FORMAL DEFINITION OF THE LIMIT

A function, f, has a limit, L, at $x = c \in (a, b) \subset \mathbb{R}$, if

$$\boxed{\forall \varepsilon > 0, \exists \delta : |f(x) - L| < \varepsilon, \forall |x - c| < \delta}$$

and $f(x)$ is well defined on $x \in (a, b) \backslash \{c\}$



CONTINUITY

Let $f(x)$ be defined on (a, b). Then $f(x)$ is **continuous** at $c \in (a, b)$ if

$$\boxed{\lim_{x \to c} f(x) = f(c).}$$

EXERCISE:
USING THE ε-δ DEFINITION OF THE LIMIT, SHOW THE FOLLOWING

a) $f(x) = \sqrt{x-1}$ is continuous at $x = 5$

b) $f(x) = |x|$ is continuous on all of $\mathbb{R}$

c) $f(x) = \sqrt{x}$ is continuous at $x = c > 0$

PINCHING THEOREM

Suppose that

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$$

and $f(x) \leq g(x) \leq h(x)\ \forall x \neq c$. Then,

$$\lim_{x \to c} g(x) = L.$$

LIMIT LAWS

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then

i. $\lim_{x \to c}[f(x) + g(x)] = L + M$

ii. $\lim_{x \to c}[\alpha f(x)] = \alpha L$ for $\alpha \in \mathbb{R}$

iii. $\lim_{x \to c}[f(x)g(x)] = LM$

EXERCISE:
FIND THE FOLLOWING LIMITS
(ε-δ PROOF NOT REQUIRED)

a) $$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$$

b) $$\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$$

c) $$\lim_{x\to 9} \frac{\frac{1}{\sqrt{x}}-\frac{1}{3}}{x-9}$$

d) $$\lim_{x\to c} \frac{\sqrt{x}-\sqrt{c}}{x-c}$$

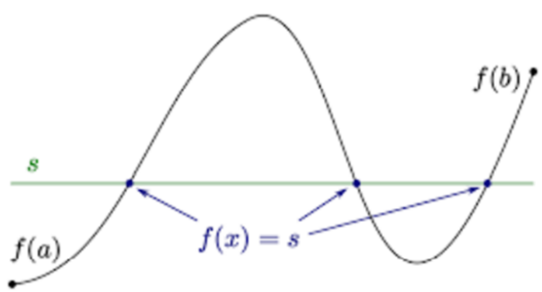
e) $$\lim_{x\to c} \frac{x^{-\frac{1}{2}}-c^{-\frac{1}{2}}}{x-c}$$

f) $$\lim_{h\to 0} \frac{\sin(c+h)-\sin(c)}{h}$$

INTERMEDIATE VALUE THEOREM

If a function, f, is continuous on $[a, b]$ and $f(a) < f(b)$. Then

$$\boxed{\forall\, f(a) < s < f(b), \exists c \in (a, b) : f(c) = s}$$

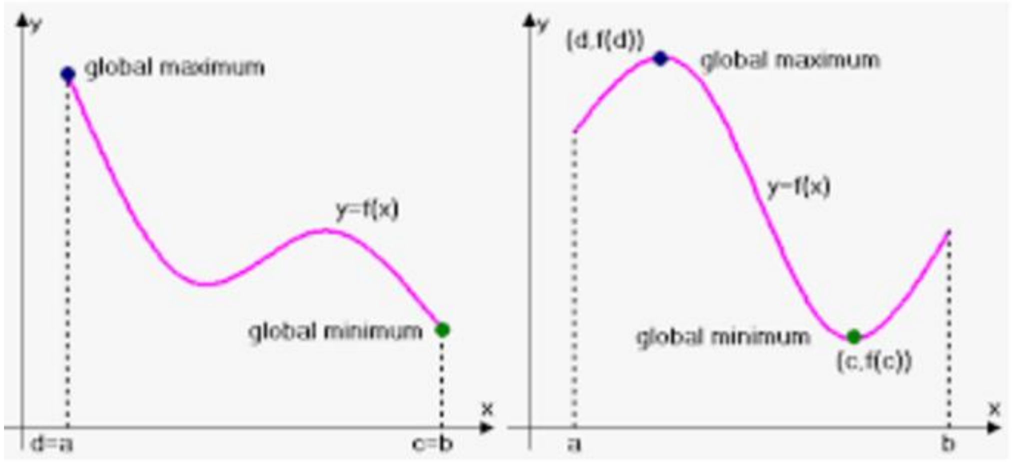


EXTREME VALUE THEOREM

Suppose $f(x)$ is continuous on the closed, bounded interval $[a, b]$. Then $f(x)$ is bounded on $[a, b]$ and achieves its bounds.

Or in other words,

$$\exists c, d \in [a, b] : f(c) \leq f(x) \leq f(d)\ \forall x \in (a, b)$$



EXERCISE:

Prove that

$$\sin(x) = 1 - x$$

has a solution between $x = 0$ and $x = \frac{\pi}{2}$.

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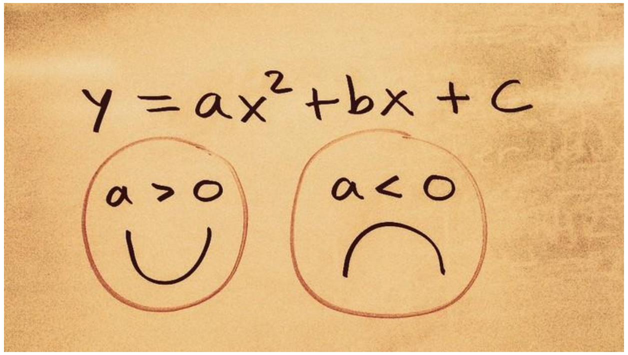

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$