MA1112
All candidates

UNIVERSITY OF LEICESTER

JANUARY EXAMINATIONS 2017

DO NOT OPEN THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE CHIEF INVIGILATOR	
Department	Mathematics
Module Code	MA1112
Module Title	Linear Algebra I
Exam Duration	Two hours
CHECK YOU HAVE THE CORRECT QUESTION PAPER	
Number of Pages	3
Number of Questions	4
Instructions to Candidates	Answer all questions. All marks gained will be counted. All questions carry equal weight.
FOR THIS EXAM YOU ARE ALLOWED TO USE THE FOLLOWING:	
Calculators	Permitted calculators are the Casio FX83 and FX85 models
Books/Statutes provided by the University	No
Are students permitted to bring their own Books/Statutes/Notes?	No
Additional Stationery	No

1. Find all the solutions of the following linear systems.

   (a) **[5 marks]**

$$\begin{aligned} 4x_1 + x_2 - 3x_3 &= 11 \\ -3x_2 + 2x_3 &= 9 \\ x_1 + x_2 + x_3 &= -3 \end{aligned}$$

   (b) **[5 marks]**

$$\begin{aligned} x - 2y &= 5 \\ 3x + y &= 1 \end{aligned}$$

   (c) **[7 marks]**

$$\begin{aligned} x_2 - x_3 &= 2 \\ x_1 + 2x_3 - x_4 &= 0 \\ x_1 + 2x_2 - x_4 &= 4 \end{aligned}$$

   (d) **[8 marks]**

$$\begin{aligned} x_1 + 2x_2 + x_3 - 2x_4 &= 0 \\ 2x_3 - 4x_4 &= 0 \\ -2x_1 - 4x_2 + x_3 - 2x_4 &= 0 \end{aligned}$$

Total: 25 marks

2. (a) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}.$$

      i. **[5 marks]** Compute $A^T A$.

      ii. **[5 marks]** Compute AA^T.

   (b) Find the inverse A^{-1} for the given matrix A, if it exists, in each case below.

      i. **[5 marks]**

$$A = \begin{bmatrix} -2 & 3 \\ 4 & 6 \end{bmatrix}$$

      ii. **[5 marks]**

$$A = \begin{bmatrix} -2 & 3 \\ 3 & 6 \end{bmatrix}$$

      iii. **[5 marks]**

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 6 & 5 \\ 0 & 1 & 0 \end{bmatrix}$$

Total: 25 marks

3. (a) **[4 marks]** Let V be a vector space, $S = \{\vec{u}_1, \dots \vec{u}_r\} \subseteq V$. Under what conditions do we say that S is linearly independent?

   (b) **[4 marks]** Give the definition of a basis of a vector space.

   (c) **[5 marks]** Is $S = \{1-t, \quad 2t+3t^2, \quad t^2-2t^3, \quad 2+t^3\}$ a basis of $\mathbb{P}_3$, the space of all polynomials of degree up to three? Briefly justify your answer.

   (d) **[5 marks]** Define the rank and the nullity of a $m \times n$ matrix. Find the rank and the nullity of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -3 & 6 & 5 \\ 0 & 1 & 0 \end{bmatrix}$$

   (e) **[7 marks]** Write $\vec{v}$ as a linear combination of $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$ where

$$\vec{v} = (1,2,3,5), \quad \vec{u}_1 = (1,2,3,4), \quad \vec{u}_2 = (-1,-2,-3,-4), \quad \vec{u}_3 = (0,0,1,1),$$

   or explain why it is impossible to do so.

Total: 25 marks

4. (a) For each of the following sets $S \subseteq V$, state if they are linearly independent or not, briefly justifying your answer.

      i. **[4 marks]** $S = \{(2,-1,3),(5,0,4)\}$, V is the real vector space $\mathbb{R}^3$.
      ii. **[4 marks]** $S = \{(2,0,7),(2,4,5),(2,-12,13)\}$, V is the real vector space $\mathbb{R}^3$.
      iii. **[4 marks]** $S = \{(6,2,1),(-1,3,2),(1,0,0),(0,1,0)\}$, V is the real vector space $\mathbb{R}^3$.
      iv. **[4 marks]** $S = \{(i,1),(-1,i)\}$, V is the complex vector space $\mathbb{C}^2$.

   (b) For each of the following sets $W \subseteq V$, state whether W is a subspace of V, briefly justifying your answer. If W is indeed a subspace also state what its dimension is.

      i. **[3 marks]** $W = \{(x,y)|x = 2y\} \subseteq V = \mathbb{R}^2$.
      ii. **[3 marks]** $W = \{(x,y)|x - y = 1\} \subseteq V = \mathbb{R}^2$.
      iii. **[3 marks]** $W = \{(x,y,z)|x \geqslant 0\} \subseteq V = \mathbb{R}^3$.

Total: 25 marks