UNIVERSITY OF LEICESTER

MA2261 LINEAR STATISTICAL MODELS
FORMULA SHEET

I) Simple linear regression

(1) $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2 = \sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}$

(2) $S_{yy} = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{(\sum_{i=1}^{n} y_i)^2}{n}$

(3) $S_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}$

(4) $\hat{\rho}\sqrt{\frac{n-2}{1-\hat{\rho}^2}} \sim t_{n-2}$

(5) $l_1 = \dfrac{e^{-\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)}\frac{1+\hat{\rho}}{1-\hat{\rho}} - 1}{e^{-\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)}\frac{1+\hat{\rho}}{1-\hat{\rho}} + 1}, \qquad l_2 = \dfrac{e^{\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)}\frac{1+\hat{\rho}}{1-\hat{\rho}} - 1}{e^{\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)}\frac{1+\hat{\rho}}{1-\hat{\rho}} + 1}$

(6) $RSS = S_{yy} - \frac{S_{xy}^2}{S_{xx}}$

(7) $\frac{RSS}{\sigma^2} \sim \chi^2_{n-2}$

(8) $\hat{b} \sim N\left(b, \frac{\sigma^2}{S_{xx}}\right), \qquad \hat{a} \sim N\left(a, \left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)\sigma^2\right)$

(9) $r_i = y_i - \hat{a} - \hat{b}x_i, \qquad E(r_i) = 0, \qquad \mathrm{var}(r_i) = \left(1 - \frac{1}{n} - \frac{(x_i - \bar{x})^2}{S_{xx}}\right)\sigma^2$

(10) $\dfrac{\hat{a} - a}{\hat{\sigma}\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}}} \sim t_{n-2}$

(11) $\dfrac{\hat{b} - b}{\hat{\sigma}/\sqrt{S_{xx}}} \sim t_{n-2}$

(12) $\left(\hat{\sigma}\sqrt{\frac{n-2}{\chi^2_{0.025,n-2}}}, \hat{\sigma}\sqrt{\frac{n-2}{\chi^2_{0.975,n-2}}}\right)$

(13) $\left(\hat{a} + \hat{b}x_0 - t_{0.025,n-2}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}\ ,\ \hat{a} + \hat{b}x_0 + t_{0.025,n-2}\hat{\sigma}\sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}\right)$

(14) $\left(\hat{a} + \hat{b}x_0 - t_{0.025,n-2}\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}\ ,\ \hat{a} + \hat{b}x_0 + t_{0.025,n-2}\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}\right)$

(15) $SST = S_{yy}, \qquad RSS = SSE, \qquad SSM = SST - SSE$

(16) $\frac{SST}{\sigma^2} \sim \chi^2_{n-1}, \qquad \frac{SSE}{\sigma^2} \sim \chi^2_{n-2}, \qquad \frac{SSM}{\sigma^2} \sim \chi^2_1$

(17) $SST = SSW + SSB$

(18) $SSE = SSW + SSL$

(19) $\frac{SST}{\sigma^2} \sim \chi^2_{N-1}, \qquad \frac{SSW}{\sigma^2} \sim \chi^2_{N-k}, \qquad \frac{SSB}{\sigma^2} \sim \chi^2_{k-1}, \qquad \frac{SSE}{\sigma^2} \sim \chi^2_{N-2}, \qquad \frac{SSL}{\sigma^2} \sim \chi^2_{k-2}$

II) Multiple linear regression

(1) $\mathbf{Y} = \mathbf{X}\beta + \varepsilon, \qquad \hat{\beta} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}, \qquad \hat{\beta} \sim N(\beta, \sigma^2(\mathbf{X}^T\mathbf{X})^{-1})$

(2) $\hat{\beta}_k \sim N\left(\beta_k, K^2\sigma^2\right), \qquad K^2 = [(\mathbf{X}^T\mathbf{X})^{-1}]_{k+1,k+1}$

(3) $\frac{\hat{\beta}_k - \beta_k}{K\hat{\sigma}} \sim t_{n-p}$

(4) $\mathbf{a}^T\hat{\beta} \sim N(\mathbf{a}^T\beta, \sigma^2\mathbf{a}^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{a})$ where $\mathbf{a}^T = (a_0, \ldots, a_{p-1})$

(5) $\dfrac{\mathbf{a}^T\hat{\beta} - \mathbf{a}^T\beta}{\hat{\sigma}\sqrt{\mathbf{a}^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{a}}} \sim t_{n-p}$

(6) $\left(\mathbf{x}_0^T\hat{\beta} - t_{0.025,n-p}\hat{\sigma}\sqrt{\mathbf{x}_0^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x}_0}\ ,\ \mathbf{x}_0^T\hat{\beta} + t_{0.025,n-p}\hat{\sigma}\sqrt{\mathbf{x}_0^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x}_0}\right)$

(7) $\left(\mathbf{x}_0^T\hat{\beta} - t_{0.025,n-p}\hat{\sigma}\sqrt{1+\mathbf{x}_0^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x}_0}\ ,\ \mathbf{x}_0^T\hat{\beta} + t_{0.025,n-p}\hat{\sigma}\sqrt{1+\mathbf{x}_0^T(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{x}_0}\right)$

(8) $\frac{SST}{\sigma^2} \sim \chi^2_{n-1}, \qquad \frac{SSE}{\sigma^2} \sim \chi^2_{n-p}, \qquad \frac{SSM}{\sigma^2} \sim \chi^2_{p-1}$

(9) $\frac{SSE_R}{\sigma^2} \sim \chi^2_{n-q} \qquad \frac{SSE_F}{\sigma^2} \sim \chi^2_{n-p} \qquad \frac{SS_{extra}}{\sigma^2} \sim \chi^2_{p-q}$

III) One way analysis of variance

(1) $CF = \frac{\bar{y}^2N^2}{N} = N\bar{y}^2$

(2) $SSB = \sum_{j=1}^{k} n_j\bar{y}_j^2 - CF$

(3) $SST = \sum_{j=1}^{k}\sum_{i=1}^{n_j} y_{ij}^2 - CF$

(4) $\hat{C} = \sum_{j=1}^{k} c_j\bar{y}_j$

(5) $s^2(\hat{C}) = MSE\sum_{j=1}^{k}\frac{c_j^2}{n_j}$

(6) $\frac{\hat{C}-C}{s(\hat{C})} \sim t_{N-k}$