Tutorial 3 on 28 Sep

Pak-Hin Lee
phl8@le.ac.uk

Topic: integer programming

Lecture 8 self-study

Summary and self-study

Summary: today we have learnt

- how to model some mixed-integer linear programming problems,
- how to solve them in Matlab using `intlinprog`,
- and what the branch and bound technique is.

Self-study: Formulate the following problem as mixed-integer linear programming problem.

Scenario: A colleague of mine convenes the module MA3513 Industrial Mathematics Project. In this module, students work in groups for one year on a project set by an industrial partner.
This summer my colleague collected 7 potential projects and now he must allocate students to projects. This year there are 32 students and he would like to let them choose what project they'd like to work on. To this end, he asked them to rank their top-three preferred projects.

Question: How should he allocate students to projects to maximise their happiness while at the same time ensuring that no project has fewer than 4 or more than 5 students?

Lecture 9 self-study

Summary and self-study

Summary: today we have learnt

- how to model some nonlinear functions using mixed-integer linear programming.

Self-study: Consider the self-study exercise from `OR Lecture 8_mixed_integer.pptx`, but this time assume that I have collected 10 projects instead of 7. How should I modify the corresponding mixed-integer linear programming problem? Note that I cannot run all 10 projects because only I have only 32 students and each project should have at least 4 students.

8 Self-study

7 projects, 32 students

each student: submit top 3 pref

each project: has 4-5 students

Goal: maximise happiness

Let $x_{ij} \in \{0, 1\}$ denote whether

i: student, $1 \le i \le 32$

j: project, $1 \le j \le 7$

student i is allocated to project j

$p_{ij} \in \{0, 1, 2, 100\}$ denote

3rd choice

1st choice, 2nd choice, other projects

unhappiness of student i if he/she is allocated to project j.

Model · objective function

(minimise total unhappiness for all students)

student i → project j unhappiness p_{ij}

unhappiness of student $i = \sum_{j=1}^{7} p_{ij} x_{ij}$

Total unhappiness $= \sum_{i=1}^{32} \sum_{j=1}^{7} p_{ij} x_{ij}$

$$\min \quad \sum_{i=1}^{32} \sum_{j=1}^{7} p_{ij} x_{ij}$$

s.t.

- each student has 1 project

$$\sum_{j=1}^{7} x_{ij} = 1 \quad \text{for each } i = 1, \dots, 32$$

- each project has 4 or 5 students

$$4 \le \sum_{i=1}^{32} x_{ij} \le 5 \quad \text{for each } j = 1, \dots, 7$$

- each $x_{ij} \in \{0, 1\}$ (in terms of vectors)

$$x \in \{0,1\}^{224} \quad , 7 \times 32$$

Note: $x \in \{0,1\}^{224}$ can be expressed as $x \in \mathbb{Z}^{224}$, $0 \le x_j \le 1$

---

9 self-study Now: 10 projects

Replace 7 by 10 in previous scenario?

$$\min \quad \sum_{i=1}^{32} \sum_{j=1}^{10} p_{ij} x_{ij}$$

s.t.

$$\sum_{j=1}^{10} x_{ij} = 1 \quad \text{for each } i = 1, \dots, 32$$

$$4 \le \sum_{i=1}^{32} x_{ij} \le 5 \quad \text{for each } j = 1, \dots, 10$$

$$x \in \{0,1\}^{224}$$

Warning This is infeasible because 10 projects
4-5 students each
↝ need at least 40 students

Fix Issue: for each project, # students should be from $\{0, 4, 5\}$

Semi-continuous variables

Recall: Condition $x \in \{0\} \cup [a,b]$ can be modeled as $az \le x \le bz$ $\quad z \in \{0,1\}$

$$4 z_j \le \sum_{i=1}^{32} x_{ij} \le 5 z_j \quad \text{for each } j=1,\dots,10$$

$$z \in \{0,1\}^{10}$$

Model

$$\min \sum_{i=1}^{32} \sum_{j=1}^{10} p_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{10} x_{ij} = 1 \quad , \quad i=1,\dots,32$$

$$4 z_j \le \sum_{i=1}^{32} x_{ij} \le 5 z_j \quad j=1,\dots,10$$

$$x \in \{0,1\}^{224} \quad , \quad z \in \{0,1\}^{10}$$

$x \in \mathbb{Z}^{224}$, $0 \le x \le 1$

$z \in \mathbb{Z}^{10}$, $0 \le z \le 1$