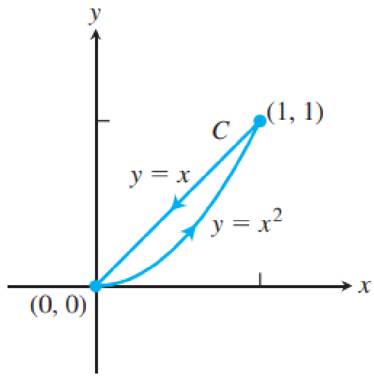
Problem Sheet 9 for the Tutorial, November 24.

(Integrals and Vector Fields)

Problem 1. Evaluate $\int_C (x + \sqrt{y})\, ds$ where C is given in the accompanying figure.



Solution:

Problem 2. Integrate $f(x, y) = x^2 - y$ over the curve $C:\ x^2 + y^2 = 4$ in the first quadrant from $(0, 2)$ to $(\sqrt{2}, \sqrt{2})$.

Solution:

Problem 3. Along the curve $\mathbf{r}(t) = t\mathbf{i} - \mathbf{j} + t^2\mathbf{k}$, $0 \le t \le 1$, evaluate each of the following integrals:
a) $\int_C (x + y - z)\, dx$; *b)* $\int_C (x + y - z)\, dy$; *c)* $\int_C (x + y - z)\, dz$.

Solution:

Problem 4. Find the flow of the velocity field $\mathbf{F} = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ along each of the following paths from $(1, 0)$ to $(-1, 0)$ in the xy-plane.

a) The upper half of the circle $x^2 + y^2 = 1$.

b) The line segment from $(1, 0)$ to $(-1, 0)$.

c) The line segment from $(1, 0)$ to $(0, -1)$ followed by the line segment from $(0, -1)$ to $(-1, 0)$.

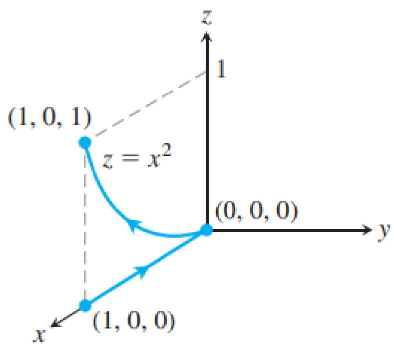
Solution:

Problem 5. Find the work done by $\mathbf{F} = (x^2+y)\mathbf{i} + (y^2+x)\mathbf{j} + ze^z\mathbf{k}$ over the following paths from $(1,0,0)$ to $(1,0,1)$.

a) The line segment $x = 1, \quad y = 0, \quad 0 \le z \le 1$.

b) The helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (t/2\pi)\mathbf{k}, \quad 0 \le t \le 2\pi$.

c) The x-axis from $(1,0,0)$ to $(0,0,0)$ followed by the parabola $z = x^2, \quad y = 0$ from $(0,0,0)$ to $(1,0,1)$.



Solution: