Problem Sheet 12 for the Tutorial, December 15.

(Infinite Sequences and Series)

Problem 1.

a) Use the Ratio Test to determine if the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^2(n+2)!}{n!\,3^{2n}}$ converges absolutely or diverges.

b) Use the Root Test to determine if the series $\sum_{n=1}^{\infty}\left(\frac{4n+3}{3n-5}\right)^n$ converges absolutely or diverges.

c) Use any method to determine if the series $\sum_{n=1}^{\infty}\frac{n^{10}}{10^n}$ converges or diverges. Give reasons for your answer.

Solution:

Problem 2. Determine if the series converges absolutely, converges, or diverges? Give reasons for your answers.

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\ \frac{3+n}{5+n}$,

b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ (n!)^2}{(2n)!}$.

Solution:

Problem 3. Use any method to determine whether the series converges or diverges. Give reasons for your answer.

a) $\sum_{n=0}^{\infty}(-1)^n \, \frac{(n+2)!}{(2n)!}$,

b) $\sum_{n=2}^{\infty} \frac{3}{10+n^{4/3}}$.

Solution:

Problem 4. Given a series $\sum_{n=0}^{\infty} \frac{(-1)^n \, x^n}{n!}$

a) find the series' radius and interval of convergence.

For what values of x does the series converge b) absolutely, c) conditionally?

Solution:

Problem 5.

a) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{8}{4^{n+2}} x^n$.

b) Represent the power series in part a) as a power series about $x = 3$ and identify the interval of convergence of the new series.

Solution: