Q1:

Since $\sigma_{\bar{X}} = \sqrt{\frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right)}$

when $n=4$ $\sigma_{\bar{X}_1} = \sqrt{\frac{2^2}{4}\left(\frac{60-4}{60-1}\right)} = \sqrt{\frac{56}{59}}$

when $n=16$ $\sigma_{\bar{X}_2} = \sqrt{\frac{2^2}{16}\left(\frac{60-16}{60-1}\right)} = \sqrt{\frac{11}{59}}$

Hence, $\sigma_{\bar{X}_1} > \sigma_{\bar{X}_2}$ the $\bar{X}$ has a more precise estimator of μ when sample size was increase from $n=4$ to $n=16$

Q2:

i) Since we know μ and we can easily write $\sigma^2 = E[(X_i-\mu)^2]$ $\forall i = 1 \dots n$ we don't need to estimated μ. then

$$E[\hat{\sigma}^2] = E\left[\frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2\right] = \sigma^2$$

ii) With R script, we can estimate each sample varience $\hat{\sigma}_i$ with $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2$

$\hat{\sigma}_1 = 4.4779$
$\hat{\sigma}_2 = 3.4185$
$\hat{\sigma}_3 = 4.5980$
$\hat{\sigma}_4 = 2.1545$
$\hat{\sigma}_5 = 5.9574$

Hence the MSE of estimator is

$$\frac{1}{n}\sum_{i=1}^{n}(\hat{\sigma}_i-\sigma)^2 = 1.6305$$