Homework Sheet 5, available online November 29th, online delivery December 6th before 4:00 pm UK time.

(Integrals and Vector Fields)

Problem 1. (4 points) Evaluate $\int_C x\, ds$, where C is

a) the straight-line segment $x = t$, $y = t/2$, from $(0, 0)$ to $(4, 2)$.

b) the parabolic curve $x = t$, $y = t^2$, from $(0, 0)$ to $(2, 4)$.

Solution:

Problem 2. (4 points) Find the work done by the force $\mathbf{F} = y^2\mathbf{i} + x^3\mathbf{j}$, where force is measured in newtons, in moving an object over the curve $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j}$, $0 \leq t \leq 2$, where distance is measured in meters.

Solution:

Problem 3. (4 points) Is the vector field $\mathbf{F} = (y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$ conservative? Argue your answer.

Solution:

Problem 4. (4 points) Apply Green's Theorem to evaluate the integral

$$\oint_C (y^2\,dx + x^2\,dy)$$

C: The triangle bounded by $x = 0$, $x + y = 1$, $y = 0$.

Solution:

Problem 5. (4 points) Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 2$.

Solution: