UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 11

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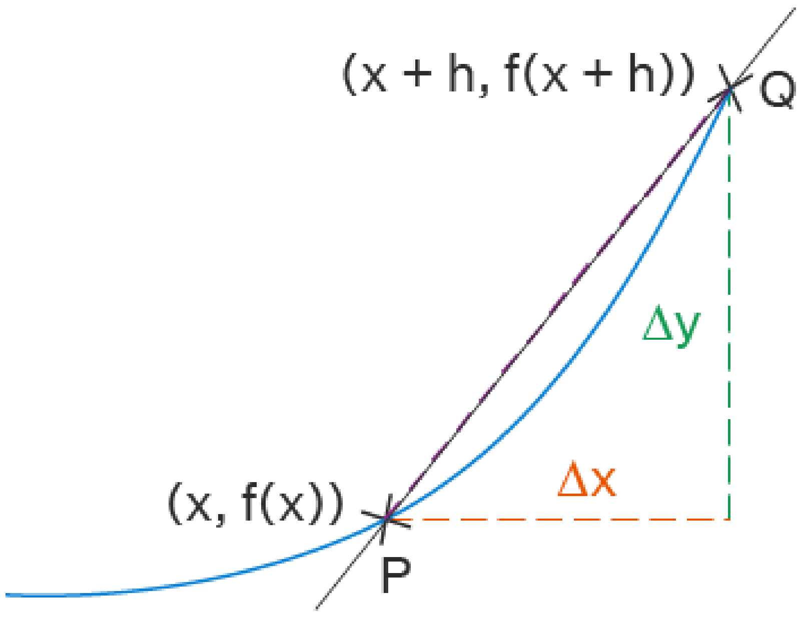
THE DERIVATIVE

- Rate of Change of a function w.r.t a variable

- Definition:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



EXERCISE

Consider $f(x) = \frac{1}{x}$ and $g(x) = x^3 + x^2$.
Determine $f'(x)$ and $g'(x)$ using first principles.

DIFFERENTIATION RULES

- Linearity: If $h(x) = \alpha f(x) + \beta g(x)$ where $f(x)$ and $g(x)$ are differentiable on $x \subseteq \mathbb{R}$ and $\alpha, \beta \in \mathbb{R}$, then

$$\boxed{h'(x) = \alpha f'(x) + \beta g'(x)}$$

- Product Rule:

$$\boxed{\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)}$$

- Quotient Rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

CHAIN RULE

If $F = f \circ g$ (i.e. $F(x) = f(g(x))$) where $g(x)$ is differentiable at x and $f(x)$ is differentiable at $g(x)$ then F is differentiable at x and,

$$\boxed{F'(x) = f\big(g(x)\big)' = f'\big(g(x)\big)g'(x)}$$

EXERCISE: DETERMINE THE DERIVATIVES OF THE FOLLOWING FUNCTIONS

a) $f(x) = x(2x+5)^3$

b) $g(x) = \sin(x)\cos(x)$

c) $h(x) = x\tan(x)$

d) $p(x) = \sin(x)\sqrt{x^2+7}$

e) $q(x) = \dfrac{e^{3x}}{\cos(4x)}$

f) $r(x) = \sin(\cos(1+x^3))$

g) $s(x) = \sqrt{\dfrac{x-3}{x^2+2}}$

h) $y(x) = (\sin(x)+1)^x$

INVERSE FUNCTIONS

Let $f(x)$ be one-to-one and differentiable on $I \subseteq \mathbb{R}$. Let $a \in I$ and $f(a) = b$, if $f'(a) \neq 0$ then f^{-1} is differentiable at b and

$$(f^{-1})'(b) = \frac{1}{f'(a)} = \frac{1}{f'\big(f^{-1}(b)\big)}$$

EXERCISE: DETERMINE THE DERIVATIVES OF THE FOLLOWING FUNCTIONS

a) $f(x) = \cot^{-1} x$

b) $g(x) = \sec^{-1} x$

c) $h(x) = \operatorname{cosec}^{-1} x$

Hint: Consider a function $y = f(x)$ and it's inverse $x = g(y) = f^{-1}(y)$, then $g'(y)f'(x) = 1$.

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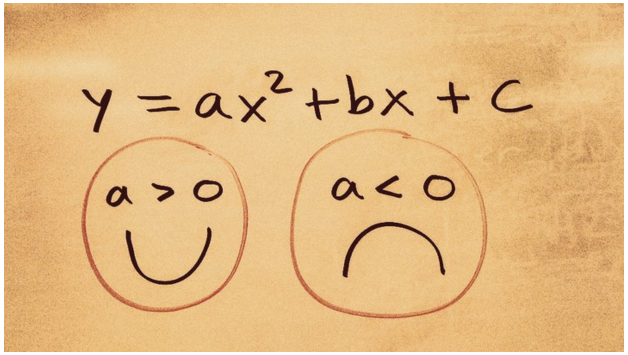

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$