Problem Sheet 3

MA1202, Introductory Statistics

Due date - 3/04/2022, 23:59 BST

General information

Please upload your work to Blackboard as a single pdf document which is of good quality. Read the **Instructions on Scanning and Uploading handwritten work**. Please name your file *PS3*YourName*Date*.pdf.

Please submit to Blackboard only solutions to questions from Section 1.

Please prepare questions from Section 2 for Feedback Session - you are expected to participate in discussion of these questions, your input will contribute to the participation mark.

Section 1. [to be submitted to Blackboard by 3/04/22, 23:59 BST]

Question 1.

The number of transistor failures in an electronic computer may be considered as a random variable X drawn from the Poisson distribution.
The numbers of transistor failures per hour for 96 hours are recorded in the table below.

Estimate the parameter(s) of the distribution for X based on these data by using the method of maximum likelihood:
i) Derive the maximum likelihood estimator for a random sample $X_1, ..., X_n$;
ii) Find the maximum likelihood estimate based on the observed values.

Hourly failures(No.)	Hours(No.)
0	59
1	27
2	9
3	1
> 3	0
	Total=96

Question 2.

Let X be a random variable with the geometric distribution describing the probability that the first occurrence of success requires n independent trials, each with success probability p. The pmf for this distribution is:

$$p_X(x) = (1-p)^x p, \ x = 0, 1, 2, 3, ...$$

i) Assume that a one value sample $X = x_1$ ($n = 1$) was collected. What is the likelihood function $L(p)$ in terms of p and x_1? Which value of p maximises $L(p)$?

ii) Consider a sample of size n: $X_1, X_2, ..., X_n$, find the maximum likelihood estimator for p.

Section 2. [to be discussed in Tutorial on 7/04/22]

Question 3.

If $X_1, X_2, ..., X_n$ constitute a random sample from a population with the mean μ, what condition must be imposed on the constants $a_1, a_2, ..., a_n$ so that

$$\hat{\mu} = a_1 X_1 + a_2 X_2 + ... + a_n X_n$$

is an unbiased estimator of μ.

Question 4.

If $\bar{X}_1$ and $\bar{X}_2$ are the means of independent random samples of sizes n_1 and n_2 from a normal population with the mean μ and the variance σ^2, show that the variance of the unbiased estimator

$$\hat{\theta} = \omega \bar{X}_1 + (1-\omega)\bar{X}_2$$

is a minimum when $\omega = \frac{n_1}{n_1+n_2}$