Problem 1.

$u = i + j - k$, $v = 2i + j + k$, $w = -i - 2j + 3k \Rightarrow u = (1, 1, -1)$, $v = (2, 1, 1)$, $w = (-1, -2, 3)$

The area of the parallelogram determined by u and v is $\|u \times v\| = \|2i - 3j - k\| = \sqrt{14}$.

The volume of the parallepiped determined by u, v, and w is $|(u \times v) \cdot w|$

$\Rightarrow \cancel{(-2+6-3)}$ $(2, -3, -1) \cdot (-1, -2, 3) = 1$

b) · We first make z to be the unit value. $z_0 = \frac{z}{|z|}$,

~~Next, we can find that from $|V| = |W|$, which~~

Since, $|V| = |W|$, $|V| \cdot \cos\beta = |W| \cdot \cos\beta$. $|V| \cdot \sin\beta = |W| \cdot \sin\beta$. ~~At~~

At the direction of L, v and w have the same direction

But, at the direction of z, v and w have the opposite direction.

Hence, we need to add 2 times $|v| \sin\beta$ at z direction.

to find w in terms of v and z, which is, $w = v + 2|v| \cdot \sin\beta \cdot \frac{z}{|z|}$

Problem 2.

a) $\frac{dy}{dx}\Big|_{x=3} = \frac{1}{3}x^2\Big|_{x=3} = 3$, which means the ~~directio~~ direction of.

v at the point (3, 3) is (3, 3), Hence. $v \cdot j = v \cdot i = 4$.

$\frac{d^2y}{dx^2}\Big|_{x=3} = \frac{2}{3}x\Big|_{x=3} = 2$, which means the direction of.

a at the point (3.3) is (3.2), Hence. ~~$3v \cdot j = 2$~~

$3a \cdot j = 2a \cdot i = -4$

$a \cdot j = -\frac{4}{3}$

b) velocity: $\frac{dr}{dt} = -5\sin t\, j + 3\cos t\, k = v(t)$

acceleration: $\frac{d^2r}{dt^2} = -5\cos t\, j - 3\sin t\, k. = a(t)$.

In order to make v and a orthogonal. $v(t) \cdot a(t) = 0$.

$\Rightarrow 25\sin t\cos t + (-9)\sin t\cos t = 0 \quad t \in [0, \pi]$.

$16\sin t\cos t = 0$.

when $t = 0, \frac{\pi}{2}, \pi$. the velocity and acceleration.

vectors orthogonal.

Problem 3. Partial Derivatives.

a) i) We can easy to know that the vector of direction toward (2,2). is (1,0), which means. $f_x(1,2)=2$. and the vector of direction toward (1,1) is (0.-1). Hence $\nabla f(1,2)\cdot(0,-1)=-f_y(2,1)=-2$.
$\Rightarrow f_y(1,2)=2$.

ii) Since $f_x(1,2)=2$. $f_y(1,2)=2$, $\nabla f(1,2)=(2,2)$.

We need to convert v to unit vector u.

$v=(4,6)-(1,2)=(3,4)$, $u=\frac{v}{|v|}=(\frac{3}{5},\frac{4}{5})$

The derivative of f at (1,2) in the direction of u is therefore $\nabla f|_{(1,2)}\cdot u=(2i+2j)\cdot(\frac{3}{5}i+\frac{4}{5}j)=\frac{14}{5}=2.8$.

b) Let $f(x,y,z)=xy+z-2=0$

$$\nabla f|_{P_0}=(xi+yj-k)|_{(1,1,1)}=i+j+k$$

The tangent plane is therefore the plane

$$(x-1)+(y-1)+(z-1)=0 \quad \text{or} \quad x+y+z=3.$$

Hence the line normal to the surface at P_0 is

$$x=1+t.\ y=1+t.\ z=1+t.$$

When $t=-1$, the point on the line is the origin.