Problem Sheet 6 for the Tutorial, November 3.

(Partial Derivatives.)

Problem 1. Find equations for the

(a) tangent plane and

(b) normal line at the point $P_0(0,0,1)$ on the given surface defined by $ye^x - ze^{y^2} = z$.

Solution:

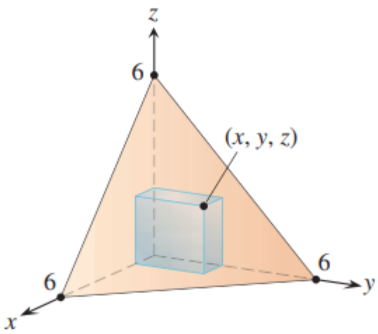
Problem 2. Find the linearization $L(x, y)$ of the function $f(x, y) = x^2 - 3xy + 5$ at $P_0(2, 1)$. Then find an upper bound for the magnitude $|E|$ of the error in the approximation $f(x, y) \approxeq L(x, y)$ over the rectangle $R : |x - 2| \leq 0.1, \quad |y - 1| \leq 0.1$.

Solution:

Problem 3. Find the absolute maxima and minima of the function $T(x, y) = x^2 + xy + y^2 - 6x + 2$ on the rectangular plate $0 \leq x \leq 5, \quad -3 \leq y \leq 0$.

Solution:

Problem 4. A rectangular box is inscribed in the region in the first octant bounded above by the plane with x-intercept 6, y-intercept 6, and z-intercept 6.



a) Find an equation for the plane.

b) Find the dimensions of the box of maximum volume.

Solution:

Problem 5. Use the method of Lagrange multipliers to find

a) Minimum on a hyperbola: The minimum value of $x+y$, subject to the constraints $xy = 16,\ x > 0,\ y > 0$.

b) Maximum on a line: The maximum value of xy, subject to the constraint $x + y = 16$.

Comment on the geometry of each solution.

Solution: