MA3077 (DLI) Operational Research

Lecture 7 – Shadow prices and sensitivity analysis

Dr Neslihan Suzen

Recap and lecture outline

Summary: in previous lectures we learnt:

- that not all linear programming problems are feasible,
- how to derive the dual problem of a primal problem,
- weak duality: the optimal dual objective is a lower bound on the primal one,
- strong duality: if the primal or the dual is feasible and or is finite, then the bound is sharp.

Today: Shadow prices and sensitivity analysis, following closely chapter 2.4 of the Mosek Cookbook.

Shadow prices 1/2

Consider the primal-dual pair of linear programming problems

Question: if is subject to a small perturbation , that is, , how does this perturbation affect ?

Answer: Assuming the primal-dual pair is feasible, denote by the primal-dual optimal solution. A perturbation in affects , but not Assuming that was a unique vertex of , if is small enough, then remains also an optimal solution to the perturbed problem

Shadow prices 2/2

Question: if is subject to a small perturbation , that is, , how does this perturbation affect ?

Answer: The dual solution remains also an optimal solution to the perturbed problem

and its optimal dual objective value is

where denotes the optimal objective value of the perturbed primal problem.

Take home message: The dual variable quantifies how sensitive is with respect to perturbations on .

Shadow prices – example

(see OR07_shadow_prices.m**)**

Consider the linear programming problem

We can solve this problem in Matlab and read the value of the dual variables with

```
[x, fval, exitflag, output, lambda] = linprog(f,A,b,Aeq,beq,lb,ub);
```

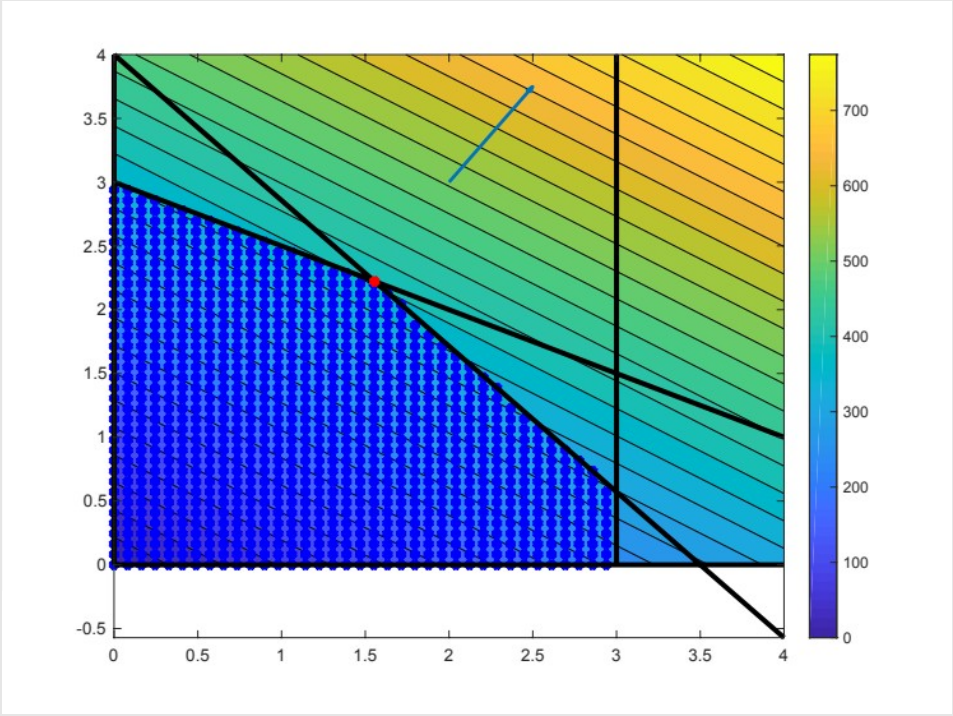
which returns . This implies that, for instance, if , then .

Sensitivity analysis – pertubations of

Maximise:

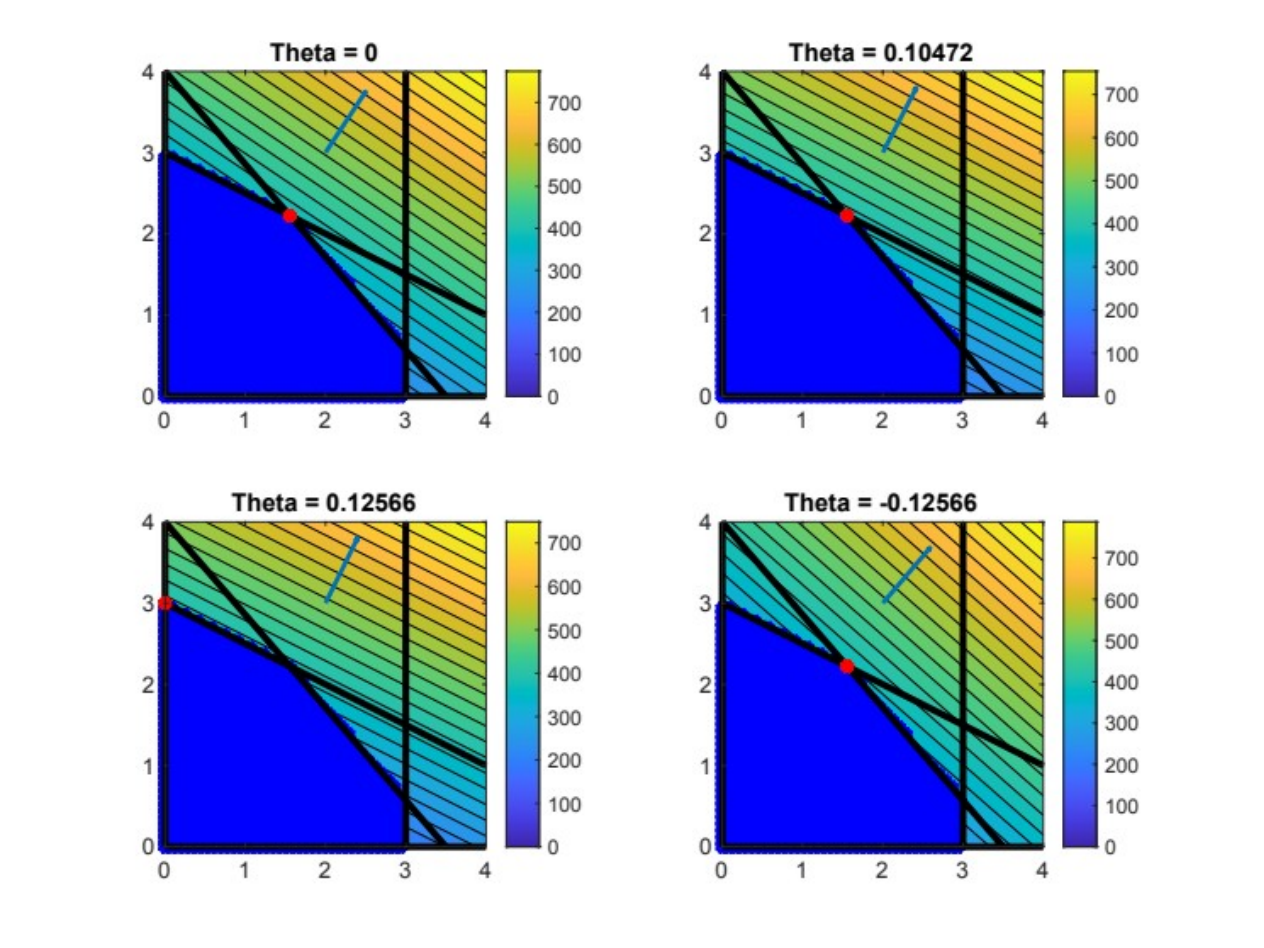
subject to:

Question: how does the solution change if we perturb ?



Perturbing the objective function

(see OR07_sensitivity_analysis.mlx an OR07_feasible_region.m)



Performing sensitivity analysis

Unfortunately, Matlab's `linprog` does not perform sensitivity analysis. There are alternative toolboxes e.g., Mosek, but setting them up can be technical and it's beyond the scope of this lecture.

Instead, sensitivity analysis in R or MS Excel is straightforward. On Blackboard you can find an R-script to perform sensitivity analysis (based on `lpsolve`). If you prefer MS Excel instead, here is a good tutorial. For this example, we get:

Objective's coefficients	Sensitivity analysis – lower bound	Sensitivity analysis – upper bound
80	60	137
120	70	160

Finally, for an interesting critique of sensitivity analysis, read S. W. Wallace.

Summary and self-study

Summary: today we have learnt

- the interpretation of the dual variables as shadow prices,
- how to retrieve the dual variables from linprog,
- how perturbations in the objective function can affect the location of the optimum.

Self-study: Analyze the shadow prices of your own linear programming problem. For example, you could use the problem for self-study section in OR Lecture 2_linear_programming.pptx .