MA2252 Introduction to Computing

Lecture 14
Interpolation

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Learning outcomes

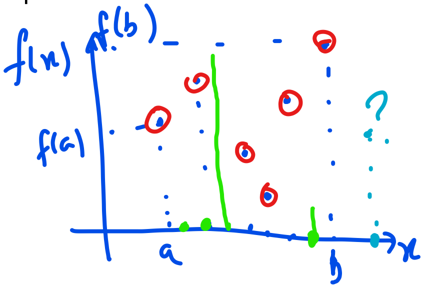
At the end of lecture, students will be able to

- understand interpolation problem
- understand theory of interpolation methods
- implement interpolation methods in MATLAB

Introduction

In mathematics, **interpolation** means to estimate the value of a function $f(x)$ in a given interval $x \in [a, b]$ based on some known values of the function inside and at end points of this interval.

extrapolation



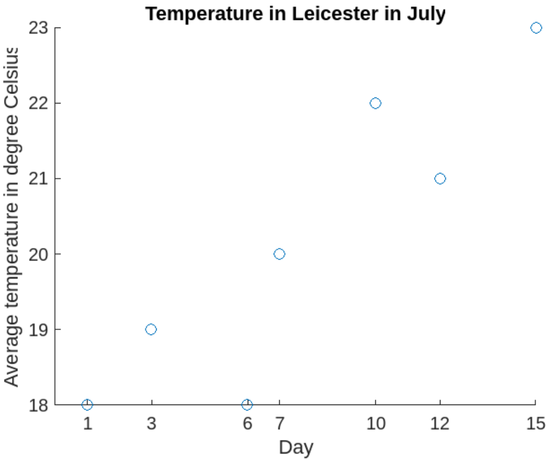
Example: The table below shows the average temperatures in Leicester in July. Can we predict temperature on other days between 1st and 15th July?

Day	Average Temperature ℃
1	18
3	19
6	18
7	20
10	22
12	21
15	23

2^{nd} July $\rightarrow$?

Days = 2, 4, 5, 8, 9, 11, 13, 14

Introduction (contd.)



Interpolation vs Regression

- Both techniques are used to describe the given data set as good as possible.
- Unlike regression, interpolation requires the estimation function to pass through all data points.

Interpolation Problem Statement

Suppose we have a data set containing n data points $(x_i, y_i), i = 1, 2, \cdots, n$.

Goal: To find an estimation function $\hat{y}(x)$ with domain $x \in [x_1, x_n]$ such that $\hat{y}(x_i) = y_i$. → ~~or~~ $\hat{y}$ should pass through all data points.

The function $\hat{y}(x)$ is called **interpolation function**.

Note: The choice of interpolation function depends on other factors such as accuracy, underlying physics etc. Depending on this choice, there are different interpolation methods.

Some Interpolation methods

- Linear Interpolation
- Cubic Spline Interpolation
- Lagrange Polynomial Interpolation

Here, the interpolation function $\hat{y}(x)$ is defined piecewise by linear polynomials (or straight lines). So,

$mx+c$

$$\hat{y}_i(x) = y_i + \frac{(y_{i+1} - y_i)(x - x_i)}{x_{i+1} - x_i}, \quad x_i \le x \le x_{i+1} \quad (i = 1, 2, \cdots, n-1) \tag{1}$$

$\hat{y}(x)$

$\hat{y}_1$ $\hat{y}_2$ (x_3, y_3) $\hat{y}_{n-1}$

$n-1$ segments

(x_1, y_1) (x_2, y_2) (x_{n-1}, y_{n-1}) (x_n, y_n)

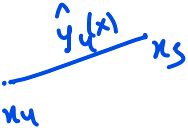
MATLAB's interp1() function can be used to make life easier.

$$\hat{y}_4(x) = y_4 + \frac{(y_5 - y_4)}{x_5 - x_4}(\underset{\substack{\parallel \\ 8}}{x} - x_4) = 20.66\cdots$$

Linear interpolation (contd.)

Example: Use linear interpolation to find the average temperature in Leicester on 8th July.

	Day	Average Temperature ℃	
x_1	1	18	y_1
x_2	3	19	y_2
x_3	6	18	y_3
x_4	7	20	y_4
x_5	10	22	y_5
	12	21	
	15	23	



Linear interpolation (contd.)

Demo

Cubic spline interpolation

A cubic spline is a function defined piecewise by cubic polynomials.

In cubic spline interpolation, the interpolating function is a cubic spline defined as

$$S_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i, \quad x_i \leq x \leq x_{i+1} \quad (i = 1, 2, \cdots, n) \quad (2)$$

n-1 cubics

Again, MATLAB's interp1 function can be used by giving 'cubic' as argument.

Cubic spline interpolation (contd.)

Example: (from book)

Demo

Cubic spline interpolation (contd.)

The unknown parameters a_i, b_i, c_i and d_i are found using these conditions:

cubic should pass through nearest point on x_i the left

$$S_i(x_i) = y_i \quad (i = 1, 2, \cdots, n-1) \tag{3a}$$

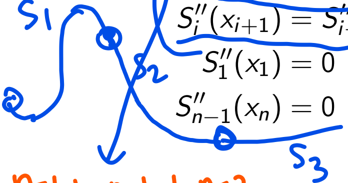
$$S_i(x_{i+1}) = y_{i+1} \quad (i = 1, 2, \cdots, n-1) \tag{3b}$$

$$S_i'(x_{i+1}) = S_{i+1}'(x_{i+1}) \quad (i = 1, 2, \cdots, n-2) \tag{3c}$$

$$S_i''(x_{i+1}) = S_{i+1}''(x_{i+1}) \quad (i = 1, 2, \cdots, n-2) \tag{3d}$$

$$S_1''(x_1) = 0 \tag{3e}$$

$$S_{n-1}''(x_n) = 0 \tag{3f}$$



curvature should be equal

n data points → $n-1$ cubics → $4(n-1)$ parameters → $4(n-1)$ equations → $4n-4$ eq's

$n-1 + n-1 + n-2 + n-2 = 4n-6$

2 eqⁿs

Cubic spline interpolation (contd.)

Derivation of parameters: Please refer book and lecture recording.

Lagrange Polynomial Interpolation

Here, the interpolation function is a **Lagrange polynomial** defined as

$$L(x) = \sum_{i=1}^{n} y_i P_i(x) \tag{4}$$

where

$$P_i(x) = \prod_{j=i, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}. \tag{5}$$

The Lagrange polynomial $L(x)$ of degree $n-1$ passes through n data points i.e. it satisfies $L(x_i) = y_i \quad (i = 1, 2, \cdots, n)$.

Example: (from book)

End of Lecture 14

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