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MA3077 (DLI) Operational Research

Lecture 9 – Integer modelling

Dr Neslihan Suzen

Recap and lecture outline

Summary: we have learnt:

- how to model some mixed-integer linear programming problems,
- how to solve them in Matlab using `intlinprog`,
- what the branch and bound technique is,
- and that some mixed-integer problems have a so-called integer solution property.

Today: More modelling using integer variables, following closely chapter 9.1 of the Mosek Cookbook.

Mixed-integer linear modelling

A general *mixed-integer linear modelling problem* takes the form

where the set specifies which components of must be integers.

There are wo major modelling techniques:

1. *binary variables* (aka *indicator variables*) take values in and indicates the absence or presence of a particular event or choice. This can be model by
2. *big-* conditions: some relations can be modelled linearly only by assuming a fixed bound on the quantities involved.

Implication of positivity

If for some , we can model the conditional statement

by setting

Note: this only works if we know for sure that . Otherwise, the problem can become infeasible.

Implication of positivity - example

The cost to produce exemplars of a certain item is often affine, that is,

where is the material/energy cost to produce a unit of the item and is an initial investment (such as the purchase of specific equipment).

Example: Lemons are cheaper than oranges, but the lemon press is more expensive. Minimising the cost to produce liters of juice taken from lemons/oranges available takes the form

where is the juice extraction amount per lemon/orange.

Semi-continuous variable, indicator constraints

Semi-continuous variables: Let . The condition can be modeled by

Indicator constraints: Let . The conditions

can be modelled as

Note: if is bounded (say), then picking means we do not impose any extra constraint on if .

Disjunctive constraints

Let . If we want that at least one of the following constraints is satisfied,

we may choose large enough and use the linear model

Constraint satisfaction

If we can distinguish between the two options

with the linear model

Exact absolute value

Let In a previous lecture, we saw how to model

If , we can model the exact equality   as follows

Exact -norm

Let In a previous lecture, we saw how to model

We can model the exact equality as follows

Boolean operators

Let . Then, we can model Boolean operators as follows:

Bilinear equality

Let . The bilinear constraint , which models the alternative

can be modelled as

for a suitable constant .

Summary and self-study

Summary: today we have learnt

- how to model some nonlinear functions using mixed-integer linear programming.

Self-study: Consider the self-study exercise from OR Lecture 8_mixed_integer.pptx, but this time assume that I have collected 10 projects instead of 7. How should I modify the corresponding mixed-integer linear programming problem? Note that I cannot run all 10 projects because only I have only 32 students and each project should have at least 4 students.