MA1254 – Case Study 2
Tianyu Wang *Junbiao Li*
Zihao Ma *Jian Du*
Siyuan Fu
Group 8

Consulting for frozen food storage depot on cost reduction

Problem description:

The main problem of the case is how to minimize the gross electricity charge by using refrigeration units to automatically control the temperature within a specific range under different heating conditions. The main constraints are the minimum temperature, the maximum temperature, the efficiency of the cooling machine, the maximum number of refrigeration units, the difference between the temperature rise in summer and winter, the electric charge, and the power consumption of each refrigeration unit.
In task 1, the original constraints remain unchanged. The cost is minimized when the electricity charge at night differs from daytime. In other words, how to adjust the original plan to keep the process of high power consumption at night as much as possible to reduce the electric charge as much as possible.
In task 2, change the constrain in the different conditions, the number of cooling machines, and the minimum temperature after changing the constraint, whether the cost can be further reduced, and evaluate whether the reduced price is worth changing.

Analysis:

In the current scheme, the cost per day is different. To calculate the cost reasonably, the method of calculating the daily average cost is selected. The period of temperature change at the current activation temperature needs to be obtained to calculate the time each machine runs during a cycle and then allocate the running time to the day and night in proportion. With the above calculation, the cost of the current scheme is ￡5220 in summer and ￡3480 in winter.

The main reason for the high cost of the current scheme is that the temperature regulation at night is different every day. The current scheme cannot make full use of night time, so the optimal solution should be to make the machine operate the same at night and reach the minimum machine operating temperature at 6:30 a.m. (- 23 ℃), which implies the operation cycle is a factor of one day (24h). After mathematical calculations, we prove that if the temperature is the same every morning at 6:30, the cost of 24h is inversely proportional to the temperature at 23:30. Therefore, a better scheme can be obtained by dividing the year into winter and summer and controlling the operating cycle for one of the seasons to be the factor of 24h.
Take controlling the winter operation cycle as an example (i.e., ensuring the lowest cost in winter). As shown in Figure 1, the method to determine the optimal scheme is to make the temperature at 6:30 the lowest operating temperature (- 23 ℃), and the temperature at 23:30 the highest. Using "Solver" in Excel, we can get the time when three refrigeration units are not running (X) and the time when one, two, and three refrigeration units are running (Y, Z, W) at night. According to the night operation and cycle period, we can calculate the activation temperature of three machines and the cost.
Similarly, we can get the operation scheme with the lowest cost in summer.
By controlling the highest temperature at 23:30 and the lowest temperature at 6:30 to minimize the cost, we can also obtain the optimal scheme of four units or - 24 ℃.

		Best choice in winter				
	Machine number	Unit 1	Unit 2	Unit 3		
	Activation temperature	-20.2	-18.0	-18.0		
Winter		X	Y	Z	W	Total time
		0	2.833005584	0	4.167	7
	Temperature rise	Temperature at 23:30	Temperature at 6:30	Cycle period(h)	**Cost**	
	1.2	-19.1328	-23	12	**3093.28**	
Summer		Temperature rise		Cycle period(h)	Cycle period(day)	
		1.8		12.1	121	
	Running time				**Cost**	
	Total running time	Unit1	Unit 2	Unit 3	**4843**	
	2904	2528	2000	2000		

Figure 1

Recommendation:

In the analysis, there are two schemes with different costs. The set temperature of the first scheme is [-20.2, - 18.0, - 18.0], and then the price is £ 4843 per day in summer and £ 3093 per day in winter. The second scheme sets the temperature as [- 23, - 22, - 18.8], and the cost is £ 4800 per day in summer and £ 3480 per day in winter. If the same scheme is used in summer and winter, we recommend scheme 1, which can save £377 per day in summer and £387 per day in winter. However, we prefer to use different strategies in different seasons to minimize the cost consumption and save £ 807 per day. After changing the conditions, under the condition of using four sets, the cost is saved by £436 per day in summer and £387 per day in winter if different seasons use different schemes. When the minimum temperature reduces to - 24 ℃, the combination of the two schemes can reduce the cost by about £420 per day in summer and £487 per day if different seasons use different strategies. Therefore, we recommend upgrading the refrigeration unit to make the minimum temperature - 24 ℃ to reduce the cost further.

Technical formulas:

1. Calculate daily average cost

Let h_i be the running time of the ith unit($1 \leq i \leq 4$)The formula for calculating the daily cost is

$$(h_1 + h_2 + h_3 + h_4) \times \frac{17}{24} \times 120 + (h_1 + h_2 + h_3 + h_4) \times \frac{7}{24} \times 40$$

2. Prove that the cost is related to the temperature at 23:30

Let the machine reach the minimum operating temperature (a) at 6:30, b is the temperature rise without activating the machine, N is the temperature at 23:30, and h and $h^{'}$ are the sum of the operating hours of the three machines during the day and at night, respectively.

The total temperature rise range during the day is $17b$, and the total temperature drop range is $0.8h$, so the total temperature rise during the day is $17b - 0.8h$. So $17m - 0.8h = N - a$.

$$h = \frac{17b + a - N}{0.8}$$

The total temperature rise at night is $7b$, and the total temperature drop is $0.8h'$, then the temperature will reduce by $0.8h^{'} - 7b$ at night. So $0.8h^{'} - 7b = N - a$.

$$h' = \frac{N - a + 7b}{0.8}$$

Therefore, the calculation formula of the total cost is $M = 120h + 40h^{'} =- 100N + 2900b + 100a$. It can be seen that the total cost is inversely proportional to the temperature of 23:30.