Problem Sheet 5 for the Tutorial, October 27.

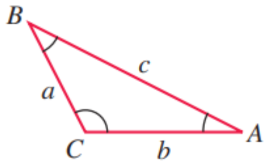
(Partial Derivatives.)

Problem 1. Find partial derivatives f_x, f_y, and f_z of the functions:

a) $f(x, y, z) = x - \sqrt{y^2 + z^2}$.
b) $f(x, y, z) = yz \ln(xy)$

Solution:

Problem 2. Given a triangle ABC as shown in Figure below



a) Express **A** implicitly as a function of **a, b**, and **c** and calculate $\partial A/\partial a$ and $\partial A/\partial b$.
b) Express **a** implicitly as a function of **A, b**, and **B** and calculate $\partial a/\partial A$ and $\partial a/\partial B$.

Solution:

Problem 3. Assuming that the following equations define y as a differentiable function of x, use Theorem 8 to find the value of dy/dx at the given point.

a) $xe^y + \sin xy + y - \ln 2 = 0, \quad (0, \ln 2),$

b) $(x^3 - y^4)^6 + \ln(x^2 + y) = 1, \quad (-1, 0).$

Solution:

Problem 4. Let $T = f(x, y)$ be the temperature at the point (x, y) on the circle $x = \cos t,\ y = \sin t$, $0 \leq t \leq 2\pi$ and suppose that

$$\frac{\partial T}{\partial x} = 8x - 4y, \qquad \frac{\partial T}{\partial y} = 8y - 4x.$$

a) Find where the maximum and minimum temperatures on the circle occur by examining the derivatives dT/dt and d^2T/dt^2.

b) Suppose that $T = 4x^2 - 4xy + 4y^2$. Find the maximum and minimum values of T on the circle.

Solution:

Problem 5. Find the directions in which the function

$$f(x, y, z) = \ln xy + \ln yz + \ln xz,$$

increase and decrease most rapidly at $P_0(1, 1, 1)$. Then find the derivatives of the function in these directions.

Solution: