Q1, i) let $L(x_i,\lambda)=\prod_{i=1}^{n} P(X_i=x_i)=\prod_{i=1}^{n}\frac{\lambda^{x_i}}{x_i!}e^{-\lambda}=e^{-n\lambda}\prod_{i=1}^{n}\frac{\lambda^{x!}}{x_i!}$

$$\ln L=-n\lambda+\sum_{i=1}^{n}x_i(\ln\lambda-\ln x_i!)$$

$$\frac{d\ln L}{d\lambda}=-n+\sum_{i=1}^{n}\frac{x_i}{\lambda}$$

let $\frac{d\ln L}{d\lambda}=0=-n+\sum_{i=1}^{n}\frac{x_i}{\lambda}$

$$\hat{\lambda}=\frac{1}{n}\sum_{i=1}^{n}x_i=\bar{x}$$

ii) Since $\bar{x}=\frac{1}{96}(0\times59+1\times27+2\times9+3\times1)=0.5$
we have $\hat{\lambda}=\bar{x}=0.5$

Q2 i) let $L(p) = P(x) = (1-p)^{x_1} p$

$$\ln L = x_1 \ln(1-p) + \ln p$$

$$\frac{d\ln L}{dp} = -\frac{x_1}{1-p} + \frac{1}{p} = 0$$

Hence $\hat{p} = \frac{1}{x_1+1}$

ii) let $L(p) = \prod_{i=1}^{n} P(X_i = x_i) = \prod_{i=1}^{n} (1-p)^{x_i} p = p^n \prod_{i=1}^{n} (1-p)^{x_i}$

$$\ln L = n \ln p + \sum_{i=1}^{n} x_i \ln(1-p)$$

$$\ln L = n \ln p + \ln(1-p) \sum_{i=1}^{n} x_i$$

let $\frac{d\ln L}{dp} = \frac{n}{p} - \frac{1}{1-p} \cdot \sum_{i=1}^{n} x_i = 0$

$$\frac{1}{p} - \frac{1}{1-p} \frac{1}{n} \sum_{i=1}^{n} x_i = 0$$

$$\frac{1}{p} = \frac{\bar{x}}{1-p}$$

$$\hat{p} = \frac{1}{\bar{x}+1}$$