UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 3

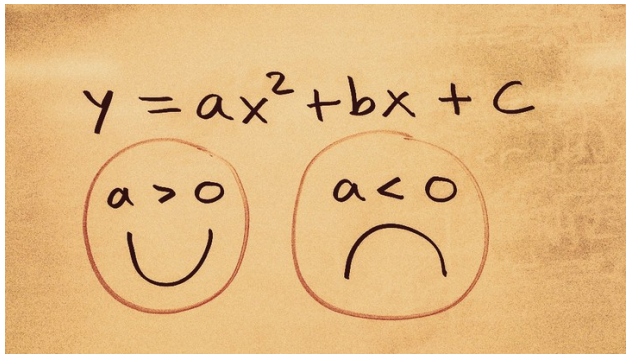
Dr. Andrew Tonks: apt12@le.ac.uk

Ben Smith: bjs30@le.ac.uk

ANNOUNCEMENTS

- I’mmmmmm baackkkkk!

UNIVERSITY OF LEICESTER



$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

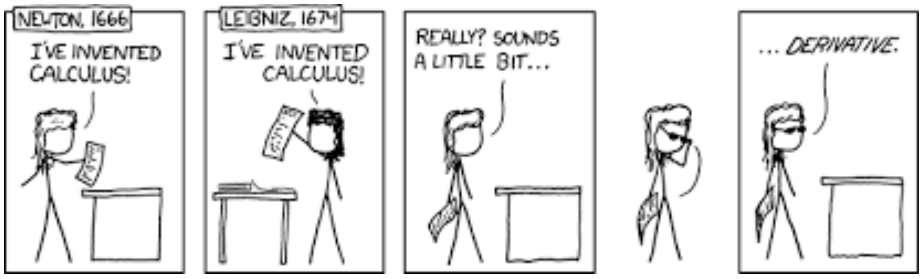
$$\int_a^b f(x)dx = F(b) - F(a)$$

L

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

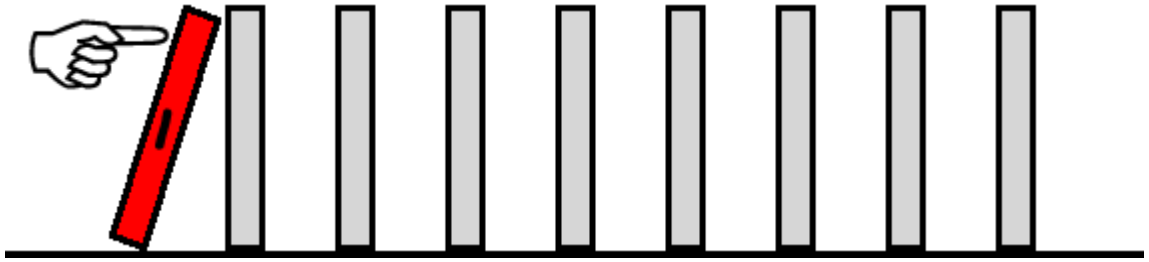
$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$



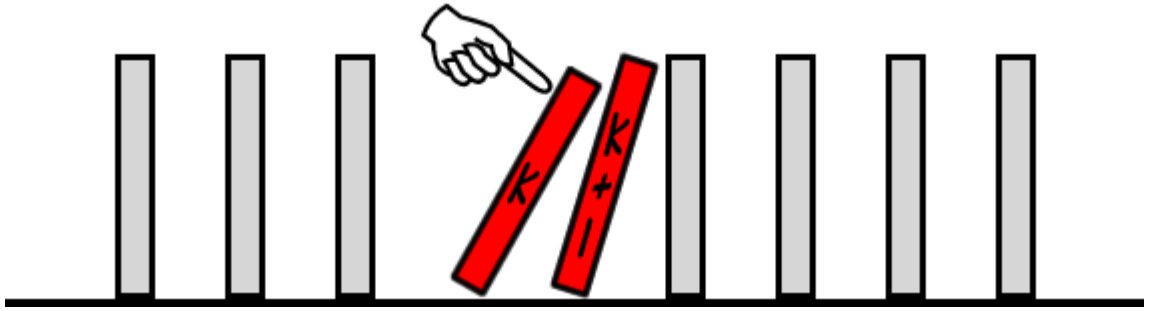
PROOF BY INDUCTION

Powerful way to prove iterative statements $P(n), n \in \mathbb{N}$.

1. First step: Prove Base Case (smallest possible n)



2. Assumptive step: Assume $n = k \in \mathbb{N}$ is true
3. Inductive step: Show $n = k$ is true $\Rightarrow n = k + 1$ is true



EXAMPLE:

Prove by induction that $\forall n \in \mathbb{N}$,

$$\sum_{i=1}^{n} (6i + 3) = 3n(n + 2)$$

EXERCISE:

Prove by induction that $\forall n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

EXERCISE:

Prove by induction that $\forall n \geq 2, x_j \in \mathbb{R}, j \in \mathbb{N}$,

$$\left| \sum_{j=1}^{n} x_j \right| \leq \sum_{j=1}^{n} |x_j|$$

Hint: $|x + y| \leq |x| + |y|$

EXERCISE:

Prove by induction that $\forall n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i^m = \frac{n^{m+1}}{m+1} + q(n)$$

where $q(n)$ is a polynomial of degree m.

Hints:

- Binomial Theorem: $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- Identity: $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

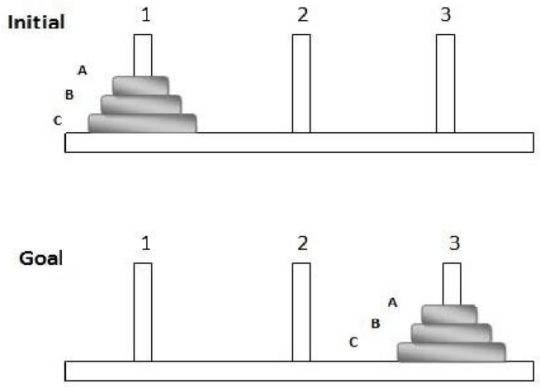
EXTRA TIME: HANOI TOWER

Prove: $N_n = 2^n - 1$

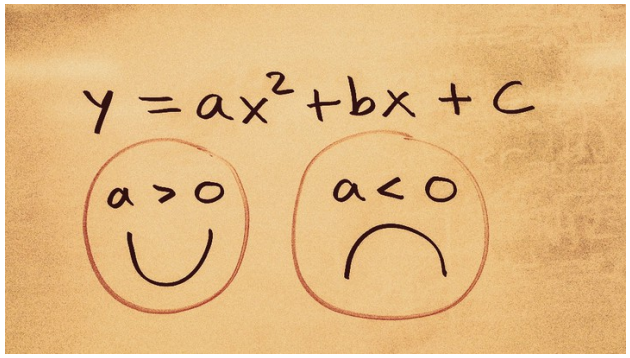
https://www.mathsisfun.com/games/towerofhanoi.html

Instructions:

- Solve puzzle for $n = 1,2,3,4$
- Convince yourself that $N_{n+1} = 2N_n + 1$
- Prove Statement by Induction



UNIVERSITY OF LEICESTER



$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

L

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$