Lecture 1: Introduction. Vectors and the Geometry of Space.

MA2032 Vector Calculus

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September 21, 2022

Introduction

Lectures: Larissa Serdukova, ls563@leicester.ac.uk
Tutorials: Fang Chen, fc160@leicester.ac.uk

Some recommended **textbooks**:

1. Thomas, Calculus: Early Transcendentals, fourteenth edition.
2. E. Kreyszig, Advanced Engineering Mathematics.
3. L. Marder, Vector Analysis.
4. P. Moon and D.E. Spencer, Vectors.
5. J. Stewart, Multivariable Calculus (Seventh edition).
6. Schaum's Outlines (book series), Vector Analysis and Advanced Calculus.

Useful free Internet sources for **3D Graphics**:
https://www.geogebra.org/3d
https://www.geogebra.org/m/XUv5mXTmchapter/400289

Assessment:
Examination: 70%
Basic skills test: 15%
Continuous Assessment (Homework Problem Sheets): 15%

Problem sheets:

- Tutorial Problem Sheets for practice (not for evaluation) will be set up weekly on Tuesdays on Blackboard "Learning Material".
- Homework Problem Sheets for evaluation will be set up biweekly on Tuesdays on Blackboard "Assessment".
- Solutions must be submitted online as pdf-file: Please make sure to label your submission file by "lastname_firstname_sheetnr". In my case, a submission for the first sheet would be called "serdukova_larissa_sheet1.pdf"
- Submission deadline: Tuesday 16:00 (UK time) one week after being made available. Late submission of homework will not be accepted (unless there are clear mitigating circumstances).

Communication, questions feedback

- Three online live Lectures per week at 9:00 am-10:00 am UK time (Monday, Wednesday, Friday). Lectures Recordings will be published in "Learning Material".
- Online live Tutorial on Thursday at 9:00 am-10:00 am (questions, examples, feedback on problem sheets).
- Email communication is welcome but you can also post questions anonymously in the discussion board on blackboard.
- **Office hours** by appointment: please request a meeting by sending email (best times to meet: Monday 8:00 am-9:00 am UK time).

Introduction

We hope that you will enjoy Vector Calculus!

Please let us know when you have any questions or comments, we are here to support you in your learning.

Overview

- We begin the study of **multivariable calculus**.
- To apply calculus in many real-world situations, we introduce **three-dimensional coordinate systems** and **vectors**.
- We establish coordinates in space by adding a **third z-axis** that measures distance above and below the *xy*-plane.
- Then we define **vectors**, which provide simple ways to define equations for lines, planes, curves, and surfaces in space.

Three-Dimensional Coordinate Systems

To locate a point in space, we use three mutually perpendicular coordinate axes, arranged as in Figure 1.

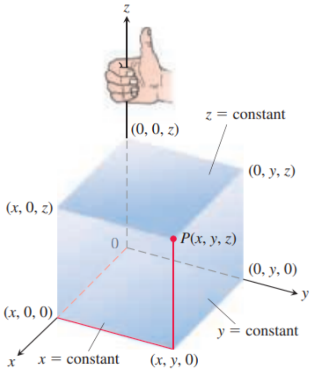


Figure 1: The Cartesian coordinate system is right-handed.

The coordinates (x, y, z) of a point P in space are the values at which the planes through P perpendicular to the axes cut the axes.

The xy-plane, $z = 0$.

The yz-plane, $x = 0$.

The xz-plane, $y = 0$.

They meet at the origin (0, 0, 0).

Three-Dimensional Coordinate Systems

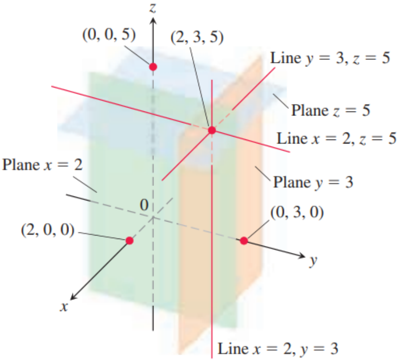


Figure 2: The planes $x = 2$, $y = 3$, and $z = 5$ determine three lines through the point $(2, 3, 5)$.

The plane $x = 2$ is the plane perpendicular to the x-axis at $x = 2$.

The plane $y = 3$ is the plane perpendicular to the y-axis at $y = 3$.

The plane $z = 5$ is the plane perpendicular to the z-axis at $z = 5$.

Example

We interpret these equations and inequalities geometrically.

(a) $z \geq 0$: The half-space consisting of the points on and above the xy-plane.

(b) x = -3: The plane perpendicular to the x-axis at x = -3.

(c) z = 0, $x \leq 0, y \geq 0$: The second quadrant of the xy-plane.

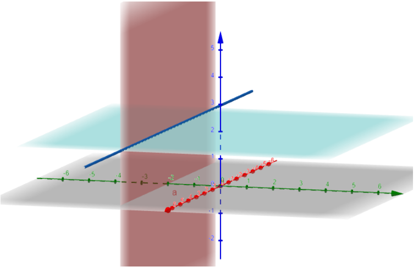


Figure 3: (f) The planes y = -2, z = 2 and their line of intersection.

(d) $x \geq 0, y \geq 0, z \geq 0$: The first octant.

(e) $-1 \leq y \leq 1$: The slab between the planes y = -1 and y = 1 (planes included).

(f) y = -2, z = 2 The line in which the planes y = -2 and z = 2 intersect.

Distance in Space

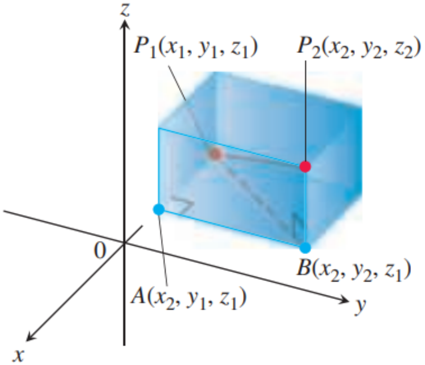
The Distance Between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \tag{1}$$

Proof: If $A(x_2, y_1, z_1)$ and $B(x_2, y_2, z_1)$ are the vertices of the box indicated in the figure, then the three box edges P_1A, AB, and BP_2 have lengths $|P_1A| = |x_2 - x_1|$, $|AB| = |y_2 - y_1|$, $|BP_2| = |z_2 - z_1|$.

Because triangles P_1BP_2 and P_1AB are both right-angled, two applications of the Pythagorean theorem give $|P_1P_2|^2 = |P_1B|^2 + |BP_2|^2$ and $|P_1B|^2 = |P_1A|^2 + |AB|^2$.

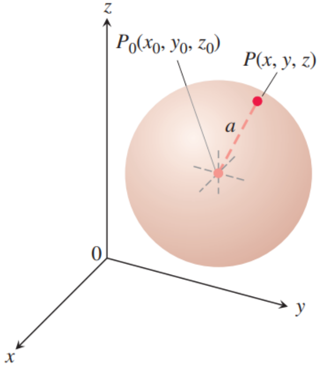
Substitution of $|P_1B|^2$ into $|P_1P_2|^2$ gives $|P_1P_2|^2 = |P_1A|^2 + |AB|^2 + |BP_2|^2 =$ $|x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2 =$ $(x_2 - x_1|)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$



Spheres in Space

We can use the distance formula to write equations for spheres in space (Right side Figure).

A point P(x, y, z) lies on the sphere of radius a centered at $P_0(x_0, y_0, z_0)$ precisely when $|P_0P| = a$



The Standard Equation for the Sphere of Radius a and Center (x_0, y_0, z_0):

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2 \tag{2}$$