Q1 i) let $n = n_0$ s.t. the width of the 95% conifene interval for μ is less than 3.06

MLE is $\hat{\mu} = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Hence. $\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$ and $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

let $P(a < \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} < b) = 1-\alpha = 0.95$, $\alpha = 0.05$

then $a = -1.96$, $b = 1.96$

$\Rightarrow L = \overline{X} - 1.65 \cdot \frac{\sigma}{\sqrt{n}}$, $U = \overline{X} + 1.65 \cdot \frac{\sigma}{\sqrt{n}}$

$3.06 \le U - L = 3.3 \times \frac{143}{\sqrt{n}}$

$238 \le n$

ii) (a) $P(\overline{Y} - 1.64 \cdot \sigma/\sqrt{n} < \mu < \overline{Y} + 2.33 \cdot \sigma/\sqrt{n})$

$= P\left(-2.33 < \frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < 1.6\right)$

$= P(-2.33 < T(\overline{Y}, \mu) < 1.6)$

$= 0.9353$

Hence confidence level is 93.53%

(b) $P(-\infty < \mu < \overline{Y} + 2.58\sigma/\sqrt{n})$

$= P\left(-2.58 < \frac{\overline{Y} - \mu}{\sigma/\sqrt{n}} < +\infty\right)$

$= P(-2.58 < T(\overline{Y}, \mu))$

$= 0.9951$

Hence confidence level is 99.51%

Q2: i) MLE is $\hat{p} = \frac{X}{n}$

Hence $\frac{X/n - p}{\sqrt{(X/n)(1-X/n)/n}} = T(\frac{X}{n}, p) \sim N(0,1)$

$P(a < T(\frac{X}{n}, p) < b) = 0.95$

hence $a = -1.96$, $b = 1.96$

$\Rightarrow P(-1.96 < \frac{120/400 - p}{\sqrt{(120/400)(1-120/400)/400}} < 1.96) = 0.95$

$\Rightarrow P(0.2551 < p < 0.3449) = 0.95$

ii) a) since $p_g \in (0.2, 0.4)$

$$\frac{z^2_{\alpha/2}}{E^2} p_g(1-p_g) \leq \frac{1.96^2}{0.01^2} 0.4(1-0.4) \approx 9220$$

Hence $n = 9220$

b) since $p_g \in (0, 0.2)$

$$\frac{z^2_{\alpha/2}}{E^2} p_g(1-p_g) \leq \frac{1.64^2}{0.02^2} 0.2(1-0.2) \approx 1076$$

Hence $n = 1076$