Q1.

We know that

$$\mu_1 = \int_{-1}^{1} x f(x) dx = \int_{-1}^{1} \frac{x+\alpha x^2}{2} dx = \int_{-1}^{1} \left(\frac{1}{2}x + \frac{\alpha}{2}x^2\right) dx$$

$$\Rightarrow \mu_1 = \frac{1}{4}x^2 + \frac{\alpha}{6}x^3 \Big|_{-1}^{1} = \frac{\alpha}{3}$$

$$m_1 = \frac{1}{n}\sum_{i=1}^{n} X_i \Rightarrow m_1 = \mu_1 = \frac{\alpha}{3}$$

Hence, $\hat{\alpha} = 3\overline{X}$

Q2. Since $\mu_1 = \frac{r(1-p)}{p}$ and $\mu_2 = \frac{r(1-p)(r-rp+1)}{p^2}$

$$m_1 = \frac{1}{n}\sum_{i=1}^{n} X_i, \quad m_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 \Rightarrow \begin{cases} m_1 = \mu_1 = \frac{r(1-p)}{p} \\ m_2 = \mu_2 = \frac{r(1-p)(r-rp+1)}{p^2} \end{cases}$$

$$\Leftrightarrow \begin{cases} \overline{X} = \frac{r(1-p)}{p} \\ m_2 - m_1 = S^2 = \frac{r(1-p)}{p^2} \end{cases}$$

Solving these equations for p and r gives.

$$\hat{p} = \frac{\overline{X}}{S^2} \qquad \hat{r} = \frac{\overline{X}}{S^2 - \overline{X}}$$