UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 18

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SUBSEQUENCES

If $(a_n)_{n\geq 1}$ is a sequence, then a subsequence of a_n is a sequence

$$\boxed{(b_k)_{k\geq 1} : \forall k \in \mathbb{N}, \exists n_k \in \mathbb{N} : n_1 < n_2 < \cdots < n_k \text{ with } b_k = a_{n_k}}$$

e.g. $a_n = n^2 : n_k = 2k, b_k = a_{n_k} = a_{2k} = (2k)^2 = 4k^2$

<u>Bolzano-Weierstrass Theorem:</u>

If $(a_n)_{n\geq 1}$ is a bounded sequence, then there exists a convergent subsequence $\left(a_{n_k}\right)_{k\geq 1}$

CAUCHY SEQUENCES

- A sequence $(a_n)_{n\geq 1}$ is Cauchy if

$$\boxed{\forall \varepsilon > 0, \exists N \in \mathbb{N} : \ \forall n, m \geq N \Rightarrow |a_n - a_m| < \varepsilon}$$

- Every Cauchy sequence is bounded
- A sequence is convergent iff it is Cauchy

EXERCISE

Consider the sequence $(a_n)_{n\geq 1} = (-1)^n$

a) Is a_n bounded? Is it monotonic? Explain your answers.

b) Show that a_n diverges.

c) Does there exist a convergent subsequence $(b_k)_{k\geq 1} = a_{n_k}$? If so, explain why and give an example of such a subsequence.

d) Show that your chosen b_k is a Cauchy Sequence. Why is a_n not a Cauchy sequence?

EXERCISE

1) Consider $x^3 = 3$.
   a) Write down Newton's method to solve this equation.
   b) By choosing $x_0 = 3$, prove by induction that the sequence is monotonic decreasing and bounded below.
   c) Hence, why does the sequence converge, and find the limit.

2) Let $(a_n)_{n\geq 1}$ be a sequence, and $\left(a_{n_k}\right)_{k\geq 1}$ be a subsequence. Prove by induction that $n_k \geq k,\ \forall k \geq 1$.

3) Show that all bounded divergent sequences possess at least two convergent subsequences.

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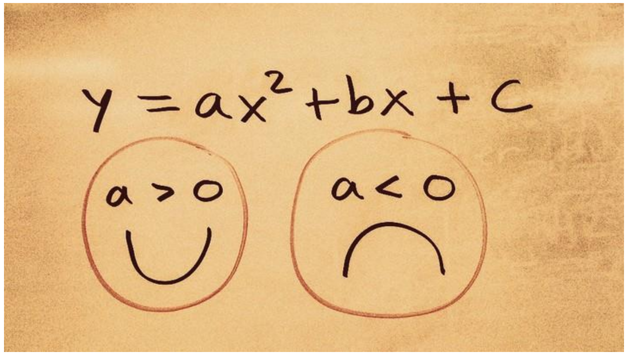

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$