Q1:

i)

Range	Frequency	Relative Frequency
[0, 750)	5	0.147
[750, 800)	6	0.176
[800, 850)	8	0.235
[850, 900)	7	0.206
[900, 950)	5	0.147
[950, ∞)	3	0.088

It is symmetry

ii) $(725\times5+775\times6+825\times8+875\times7+925\times5+975\times3)\times\frac{1}{34}=839.71$

The group mean is 839.71

$\frac{1}{34}\times(5\times(725-839.71)^2+6\times(775-839.71)^2+7\times(875-839.71)^2+5\times(925-839.71)^2+3\times(975-839.71)^2+(825-839.71)^2\times8)$

$=\cancel{5666.09}\ 5837.79$

iii) • As we assume that the Random sample observed from normal distribution. Hence, with the confidence α. we can use the pivot of the population mean which is $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim N(1,0)$, which statisfied $P(-z_{\alpha/2}<\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}<z_{\frac{\alpha}{2}})=1-\alpha$. and get $P(\overline{X}-z_{\frac{\alpha}{2}}\sigma/\sqrt{n}<\mu<\overline{X}+z_{\frac{\alpha}{2}}\sigma/\sqrt{n})=1-\alpha$

Finally the C.I. is $(\overline{X}-z_{\frac{\alpha}{2}}\sigma/\sqrt{n},\ \overline{X}+z_{\frac{\alpha}{2}}\sigma/\sqrt{n})$

• As we talk before, the pivot R.V. is $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}\sim N(1,0)$

- The ~~ba~~ biased $\tilde{S}^2 = \frac{1}{n}\sum x_i^2 - (\frac{1}{n}\sum x_i)^2 = 5713.458$

The unbiased $S^2 = \frac{n}{n-1}\tilde{S}^2 = 5886.593$

~~Hence~~

- As we talk before the C.I. is $(\bar{X} - z_{\frac{\alpha}{2}}\sigma/\sqrt{n}, \bar{X} + z_{\frac{\alpha}{2}}\sigma/\sqrt{n})$

with $\alpha = 0.1$, we first compute the sample mean $\bar{X} = \frac{1}{n}\sum x_i = 839.795$

for the population mean

The C.I. is $(839.795 - 1.64 \times 76.724/\sqrt{34}, 839.795 + 1.64 \times 76.724/\sqrt{34})$

$\Rightarrow (818.216, 861.374)$

Q2: 1.

i) $E[\hat{p}] = E[\frac{X}{14}] = \frac{1}{14}E[X]$

Since $X \sim Bin(12, p)$, $E[X] = 12p$

We get $E[\hat{p}] = \frac{12}{14}p = \frac{6}{7}p$. the bias of this point estimator is

$E(\hat{p}) - p = -\frac{1}{7}p$.

ii) $Var[\hat{p}] = Var[\frac{X}{14}] = \frac{1}{14^2}Var[X] = \frac{1}{149} \times 12p(1-p) = \frac{3p-3p^2}{49}$

iii) The MSE

$E[(\hat{p}-p)^2] = E[\hat{p}^2 - 2\hat{p}p + p^2] = \cancel{E[\hat{p}^2]-2}$.

$E[\hat{p}^2] - 2E[\hat{p}]E[p] + E[p^2] = \frac{3p-2p^2}{49}$

2. $\cancel{P(X \mid \bar{p} = \bar{p}_i)} =$ $P(X_1 = x_1, \dots X_2 = x_2 \mid \theta) = \frac{P(X_1 = x_1, \dots X_2 = x_2 \cap \bar{p} = \bar{p}_i)}{P(\bar{p} = \bar{p}_i)} = P(X_1 = x_1, \dots X_2 = x_2)$

$\cancel{P(X \mid \theta)} =$ it is does not depend on θ

$\cancel{P(X \mid \bar{p} = \bar{p}_i) = \prod p^{k_i}(1-p)}$