UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 2

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ANNOUNCEMENTS

- I’m not here this week! ☹

- Make sure you keep up to date with all module content!

FUNCTIONS

A function takes an input, say x, does something to it ($f : x \to \mathbb{R}$), and gives an output $f(x)$.

More precisely, a function maps a **Domain** ($D = \mathrm{dom}(f) \subseteq \mathbb{R}$) to a **Range** ($\mathrm{range}(f) = \{f(x) : x \in D\}$)

Example: $f(x) = \sin(x)\,, \mathrm{dom}(f) = (-\infty, \infty)$ or $\mathbb{R}$ and $\mathrm{range}(f) = [-1,1]$

Further, it is possible to classify some functions as either **Odd** or **Even**:

- If $f(-x) = -f(x)$, then f is said to be **Odd**
- If $f(-x) = f(x)$, then f is said to be **Even**

Example: $g(x) = x^2, g(-x) = (-x)^2 = x^2 = g(x)$ i.e. even!

EXERCISE:
GIVE THE DOMAIN & RANGE OF EACH FUNCTION, AND ALSO CLASSIFY IT AS EITHER ODD, EVEN OR NEITHER

a) $f(x) = \cos(x)$

b) $g(x) = e^x$

c) $h(x) = x^3$

d) $k(x) = \sin^2(x)$

e) $l(x) = \ln(x)$

f) $m(x) = |x|$

g) $p(x) = \frac{x^2+1}{3x^3+x}$

h) $q(x) = x^5 + 4x^3 - 2x$

COMPOSITIONS OF FUNCTIONS

If $f: D_f \to \mathbb{R}, g: D_g \to \mathbb{R} :\ \mathrm{range}(g) \subseteq D_f$ then the Composition, $f \circ g$, on D_g can be defined as

$$\boxed{(f \circ g)(x) = f\big(g(x)\big), \qquad x \in D_g}$$

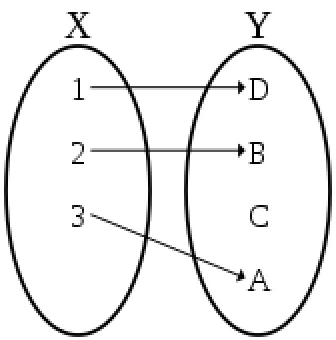
Note: Compositions are not generally commutative (i.e. $f \circ g \neq g \circ f$)

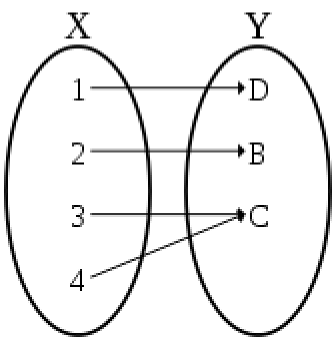
Exercise: If $f(x) = x^2 + 6$ and $g(x) = 2x - 1$, determine $(f \circ g)(x)$ and $(g \circ f)(x)$.

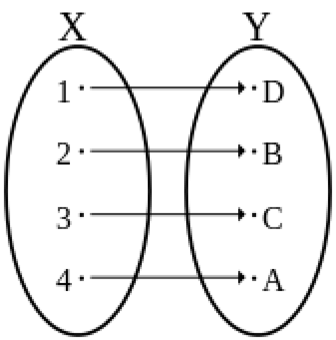
ONE-TO-ONE, ONTO & BIJECTIVITY

- A function, f, is **One-to-One** (Injective) if $\forall x_1, x_2 \in D_f, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$
- A function, f, is **Onto** (Surjective) if $\forall y \in \mathrm{range}(f), \exists x \in D_f : f(x) = y$
- A function is **Bijective** if it is **both** One-to-One and Onto

Example: Let $f: X \to Y$, classify the diagrams below.







INVERSE FUNCTIONS

If $f(x)$ is a one-to-one function, there exists a unique inverse function which we denote by

$$f^{-1} : \operatorname{ran}(f) \to \operatorname{dom}(f)$$

such that

$$f^{-1} \circ f(x) = x \; \forall x \in \operatorname{dom}(f)$$

and

$$f \circ f^{-1}(y) = y \; \forall y \in \operatorname{ran}(f)$$

Example: Find the inverse function of

$$f(x) = \frac{x+4}{2x-5}$$

EXERCISE:
DETERMINE IF THESE FUNCTIONS ARE ONE-TO-ONE. IF SO, FIND IT'S INVERSE.

a) $f(x) = 3x - 2$

b) $g(x) = \frac{x}{2} + 7$

c) $h(x) = x^2$

d) $j(x) = \sqrt[5]{2x + 11}$

e) $k(x) = \frac{1+2x}{7+x}$

f) $l(x) = |x|$

g) $m(x) = \cos(x)$

h) $n(x) = (x - 2)^3 + 1$

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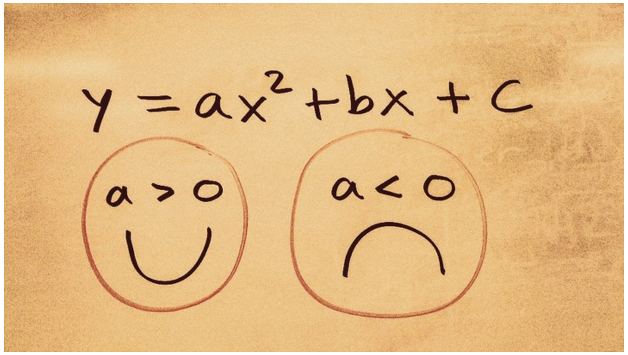

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$