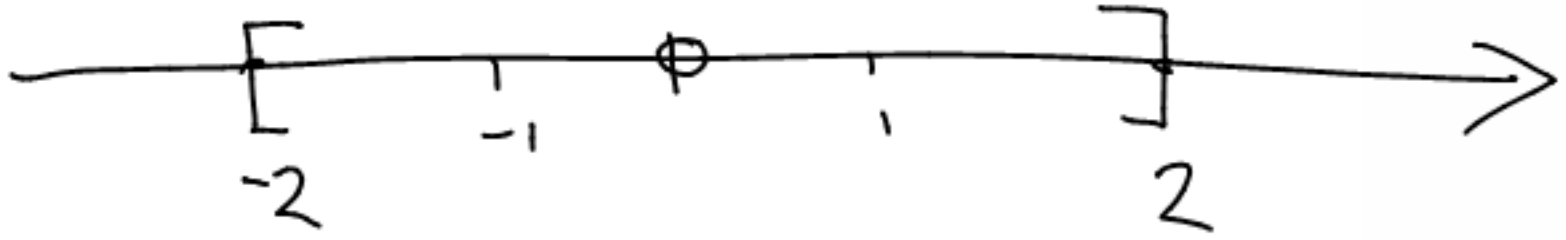
Lecture 2 Wed 3/10

W+ Sec. 1.2] A set is a collection of elements

$x \in S$ $y \notin S$

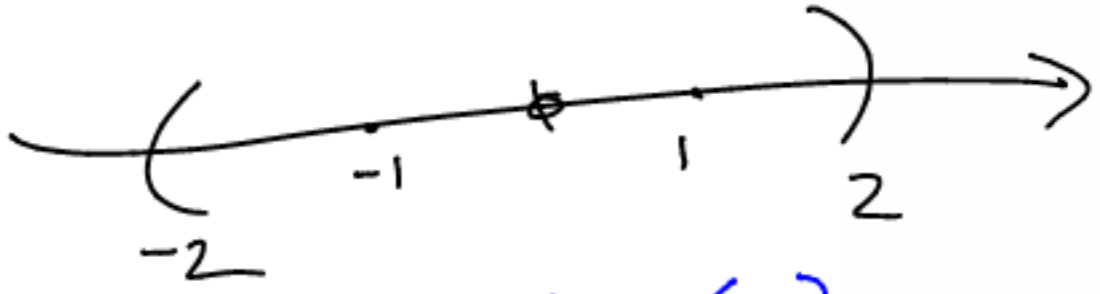
$S = \{ x \in \mathbb{R} : x^2 \leq 4 \}$

$= \{ x \in \mathbb{R} : -2 \leq x \leq 2 \}$



$[-2,2]$ closed interval

open interval $(-2,2) = \{ x : x^2 < 4 \}$

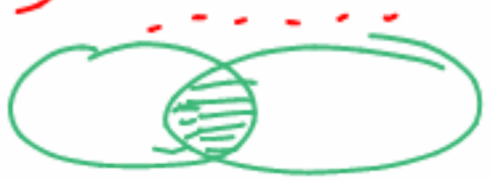


"half open" $(-2, 2]$, $[-2,2)$

$\emptyset = \{ \}$

Operations: $A \cup B$ union $x \in A \cup B \iff x \in A$ or $x \in B$

$A \cap B$ intersection $x \in A \cap B \iff$ ...... and ......



__Subsets__ $A \subseteq B$ $x \in A \Rightarrow x \in B$

A is a subset of B

B contains A

__Examples__

$$\underset{\text{natural}}{\mathbb{N}} \subseteq \underset{\substack{\text{whole} \\ \text{INTEGERS}}}{\mathbb{Z}} \subseteq \underset{\text{rational}}{\mathbb{Q}} \subseteq \underset{\text{real}}{\mathbb{R}} \subseteq \underset{\text{complex}}{\mathbb{C}}$$

$\mathbb{N} = \{0, 1, 2, \ldots\}$

$\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z},\ q \geq 1 \right\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$e, \pi, \sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$

__Def__ A __FIELD__ is a set with operations $+$, $\times$ satisfying

- Associativity $a+(b+c)=(a+b)+c$ $\qquad a(bc)=(ab)c \quad \forall a,b,c\in\mathbb{R}$
- Commutativity $a+b=b+a$ $\qquad ab=ba \quad \forall a,b\in\mathbb{R}$
- Identity $\exists 0 \quad a+0=a$ $\qquad \exists 1 \quad a.1=a \quad \forall a$
- Inverse $\exists -a \quad a+(-a)=0$ $\qquad \exists a^{-1} \quad a.(a^{-1})=1$ if $a\neq 0$
- Distributivity $a(b+c)=ab+ac \qquad \forall a,b,c\in\mathbb{R}$

Additive $\qquad$ Multiplicative

Proof that $\boxed{x.0=0 \quad \forall x}$

$$x.0 \underset{\text{Add. Id.}}{=} x.(0+0) \underset{\text{Dist.}}{=} x.0+x.0$$

Add. Inv. $-(x.0)+x.0 = -(x.0)+(x.0+x.0) \underset{\text{Assoc}}{=} \left(-(x.0)+x.0\right)+x.0$

That is: $0 = 0+x.0$

$= x.0+0$ (comm.)

$= x.0$ (add. id.)

$\mathbb{R}$ form a field, & so does $\mathbb{C}$, & so does $\mathbb{Q}$

& so does $\mathbb{Z}/_3 = \{\bar{0}, \bar{1}, \bar{2}\}$ $\quad \bar{2}.\bar{2} = \bar{4} = \bar{1}$ modulo 3

$\bar{2}+\bar{2} = \bar{4} = \bar{1}$

$-\bar{1} = \bar{2}$ because $\bar{1}+\bar{2}=\bar{0}$

$\bar{2}^{-1} = \bar{2}$ as $\bar{2}.\bar{2} = \bar{1}$

$\mathbb{R}$ forms an ordered field

Order: $a<b$ satisfying

- $\forall a,b,c$, either $a=b$ or $a<b$ or $b<a$
- $\forall a,b,c$ $\quad a<b \Rightarrow a+c<b+c$ & if $c>0$, $a<b$, $ac<bc$
- $\forall a,b,c$ $\quad a<b$ & $b<c \Rightarrow a<c$