Q1

(i) use $P(a \le x \le b) = \int_a^b f_X(x)dx$

(ii) $P(X<1) = \int_{-\infty}^{1} f_X(x)dx = \int_0^1 e^{-x}dx = -e^{-x}\Big|_0^1 = 1-\frac{1}{e}$

$P(1 \le X < 2) = \int_1^2 e^{-x}dx = \frac{1}{e} - \frac{1}{e^2}$

$P(2 \le X) = \int_2^{\infty} e^{-x}dx = \frac{1}{e^2}$

(iii) let X_0 X_1 X_2 represent the observation lying in each of intervals $[0,1)$, $[1,2)$ and $[2,+\infty)$

$E[X_0] = 1000 \times P(X<1) \approx 632$
$E[X_1] = 1000 \times P(1 \le X < 2) \approx 232$
$E[X_2] = 1000 \times P(2 \le X) \approx 136$

Q2

(i) $E[X] = \int_{-\infty}^{+\infty} x \cdot f(x)\,dx = \int_0^4 \frac{3}{64} x^3 (4-x)\,dx$

$= \frac{3}{64}\left(\int_0^4 4x^3\,dx - \int_0^4 x^4\,dx\right)$

$= \frac{3}{64}\left(256 - \frac{1024}{5}\right) = \frac{12}{5}$

$E[X^2] = \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \int_0^4 \frac{3}{64} x^4 (4-x)\,dx$

$= \frac{3}{64}\left[\frac{4}{5}x^5 - \frac{1}{6}x^6\right]\Big|_0^4 = \frac{96}{15}$

Thus we get $V[X] = E[X^2] - (E[X])^2 = \frac{16}{25}$

(ii) $E[Y] = E[300X + 50] = 300E[X] + 50 = 770$

$V[Y] = V[300X + 50] = 300^2 V[X] = 57600$

(iii) $P(Y>750) = P(300X+50>750) = P\left(X>\frac{7}{3}\right)$

$$= \int_{\frac{7}{3}}^{4} f(x)\,dx = \int_{\frac{7}{3}}^{4} \frac{3}{64}x^2(4-x)$$

$$= \frac{425}{768} \approx 0.553$$