Problem Sheet 2

MA1202, Introductory Statistics

Due date - 20/03/2022, 23:59 GMT

General information

Please upload your work to Blackboard as a single pdf document which is of good quality. Read the **Instructions on Scanning and Uploading handwritten work**. Please name your file *PS2*YourName.pdf.

Please submit to Blackboard only solutions to questions from Section 1.

Please prepare questions from Section 2 for Feedback Session - you are expected to participate in discussion of these questions, your input will contribute to the participation mark.

Section 1. [to be submitted to Blackboard by 20/03/22]

Question 1.

Let $X_1, ..., X_n$ be a random sample from a continuous distribution with pdf

$$f(x, \alpha) = \frac{1 + \alpha x}{2}, \quad -1 \leq x \leq 1, \quad -1 \leq \alpha \leq 1$$

Find the estimator for α using method of moments.

Question 2.

Let $X_1, ..., X_n$ be a random sample from a negative binomial distribution with pmf

$$p(x, r, p) = \binom{x + r - 1}{r - 1} p^x (1 - p)^x, \quad 0 \leq p \leq 1, \quad x = 0, 1, 2, ...$$

Find estimators for r and p using method of moments .
(Here $E[X] = r(1 - p)/p$ and $E[X^2] = r(1 - p)(r - rp + 1)/p^2$.)

Section 2. [to be discussed in Tutorial on 24/03/22]

Question 3.

In this question, X is a continuous random variable with density function

$$f(x) = \begin{cases} \alpha(\alpha+1)x^{\alpha-1}(1-x), & 0 \leq \ x \leq 1, \\ 0, & otherwise. \end{cases}$$

Here α is an unknown parameter which can be any strictly positive real number.

1) Write down the likelihood function $L(a)$ based on n independent observations $x_1, ..., x_n$ of X.

2) Show that the first derivative of the log likelihood function $l(\alpha)$ satisfies

$$l_\alpha = \frac{d\ l(\alpha)}{d\alpha} = \frac{n}{\alpha} + \frac{n}{\alpha+1} + \sum_{i=1}^{n} \log(x_i).$$

3) For which values of α is $l_\alpha = 0$? Which of these are positive?

4) Write down the maximum likelihood estimator for α.

Question 4.

Let $X_1, ..., X_n$ be iid $N(\mu, \sigma^2)$ random variables. Let $x_1, ..., x_n$ be the sample values.

Find the likelihood function. Using the method of maximum likelihood estimation show that the ML estimator of the parameter μ (assuming that σ^2 is constant) is

$$\hat{\mu} = \frac{\sum_{i=1}^{n} x_i}{n}.$$