Solutions for Tutorial Problem Sheet 4, October 20.

(Partial Derivatives.)

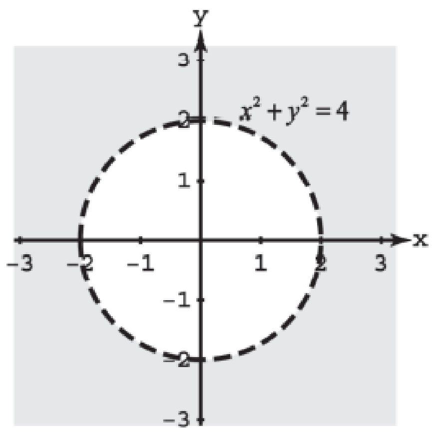
Problem 1. Find and sketch the domain for each function

a) $f(x, y) = \ln(x^2 + y^2 - 4)$.

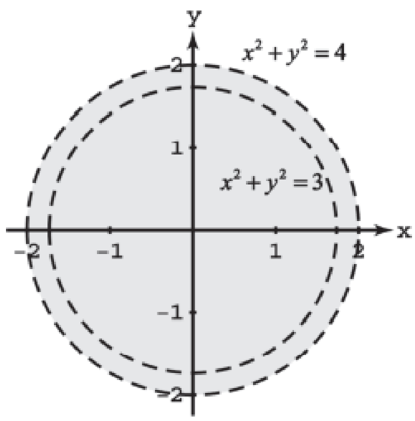
b) $f(x, y) = \frac{1}{\ln(4-x^2-y^2)}$

Solution:

Domain: all points (x, y) outside the circle $x^2 + y^2 = 4$



Domain: all points (x, y) inside the circle $x^2 + y^2 = 4$ such that $x^2 + y^2 \neq 3$



Problem 2. Given a function $f(x,y) = e^{-(x^2+y^2)}$,
(a) find the function's domain,
(b) find the function's range,
(c) describe the function's level curves,
(d) find the boundary of the function's domain,
(e) determine if the domain is an open region, a closed region, or neither, and
(f) decide if the domain is bounded or unbounded.

Solution:

(a) Domain: all points in the xy-plane
(b) Range: $0 < z \leq 1$
(c) level curves are the origin itself and the circles with center $(0, 0)$ and radii $r > 0$
(d) no boundary points
(e) both open and closed
(f) unbounded

Problem 3. Find an equation for and sketch the graph of the level curve of the function $f(x,y) = \frac{2y-x}{x+y+1}$ that passes through the point $(-1, 1)$.

Solution:

$$f(x, y) = \frac{2y-x}{x+y+1} \text{ and } (-1, 1) \Rightarrow z = \frac{2(1)-(-1)}{(-1)+1+1} = 3 \Rightarrow 3 = \frac{2y-x}{x+y+1} \Rightarrow y = -4x-3$$

Problem 4. Show that the limits do not exist

a)

$$\lim_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1}$$

b)

$$\lim_{(x,y)\to(0,1)} \frac{x\ln y}{x^2+(\ln y)^2}$$

Solution:

$$\lim_{\substack{(x,\,y)\to(1,\,1)\\ \text{along } x=1}} \frac{xy^2-1}{y-1} = \lim_{y\to1} \frac{y^2-1}{y-1} = \lim_{y\to1}(y+1) = 2; \quad \lim_{\substack{(x,\,y)\to(1,\,1)\\ \text{along } y=x}} \frac{xy^2-1}{y-1} = \lim_{y\to1} \frac{y^3-1}{y-1} = \lim_{y\to1}\left(y^2+y+1\right) = 3$$

$$\lim_{\substack{(x,\,y)\to(0,\,1)\\ \text{along } y=1}} \frac{x\ln y}{x^2+(\ln y)^2} = \lim_{x\to0} 0 = 0; \quad \lim_{\substack{(x,\,y)\to(0,\,1)\\ \text{along } y=e^x}} \frac{x\ln y}{x^2+(\ln y)^2} = \lim_{x\to0} \frac{x^2}{x^2+x^2} = \lim_{x\to0} \frac{1}{2} = \frac{1}{2}$$

Problem 5. At what points (x, y, z) in space is the functions continuous

a) $f(x, y, z) = \ln(xyz)$,

b) $f(x, y, z) = e^{x+y}\cos z$?

Solution:

(a) All (x, y, z) so that $xyz > 0$ (b) All (x, y, z)