UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 16

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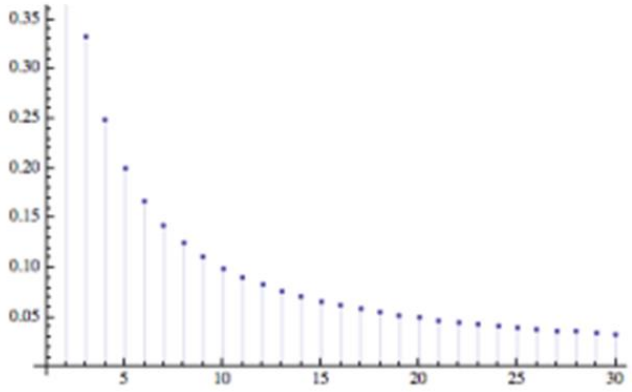
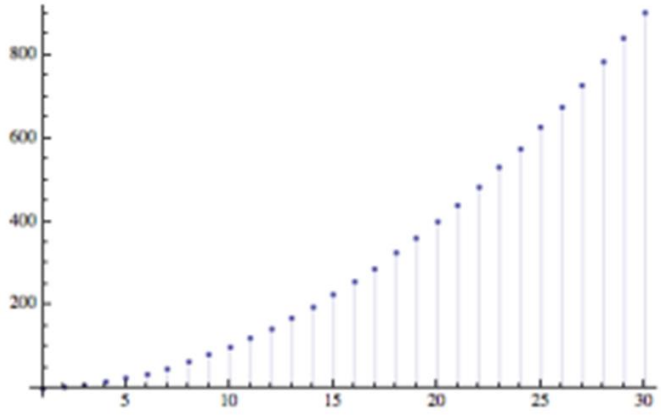
SEQUENCES

- A sequence is a function, $a : \mathbb{N} \to \mathbb{R}$ with $a_n = a(n),\ n \in \mathbb{N}$
  e.g. $a_n = n^2 \Rightarrow a_1 = 1, a_2 = 4, a_3 = 9, \ldots$

- Monotonic: If $\forall n \in \mathbb{N}, a_n \leq a_{n+1}$ (increasing) or $a_n \geq a_{n+1}$ (decreasing)

- Bounded: If $\exists m, M \in \mathbb{R} : m \leq a_n \leq M\ \forall n \in \mathbb{N}$





LIMITS OF SEQUENCES

If

$$\forall \varepsilon > 0, \exists K \in \mathbb{N} : |a_n - L| < \varepsilon \; \forall n \geq K$$

then $\lim_{n \to \infty} a_n = L$ (convergent)

EXAMPLE

Prove that

$$\lim_{n\to\infty} \frac{2n+4}{n} = 2$$

EXERCISE

Let

$$a_n = \frac{3n - 8}{4n + 1}$$

a) Determine $K \in \mathbb{N}$ such that $\left|a_n - \frac{3}{4}\right| < 0.01$ for all $n \geq K$

b) Given $\varepsilon > 0$, determine $K \in \mathbb{N}$ such that $\left|a_n - \frac{3}{4}\right| < \varepsilon$ for all $n \geq K$

c) Prove that $\lim_{n \to \infty} a_n = \frac{3}{4}$

EXERCISE

If

$$a_n = \frac{4n+1}{n}$$

Calculate a_{10}, a_{100} and a_{1000} and make a guess for the limit, L, as $n \to \infty$.

Prove a_n tends to this limit.

EXERCISE

Prove the following limits:

a) $a_n = \frac{\cos(an)}{n} \to 0$ as $n \to \infty$ where $a \in \mathbb{R}$

b) $b_n = \frac{n^2+1}{n^2-1} \to 1$ as $n \to \infty$

c) $c_n = \left(\sqrt{n+1} - \sqrt{n}\right) \to 0$ as $n \to \infty$

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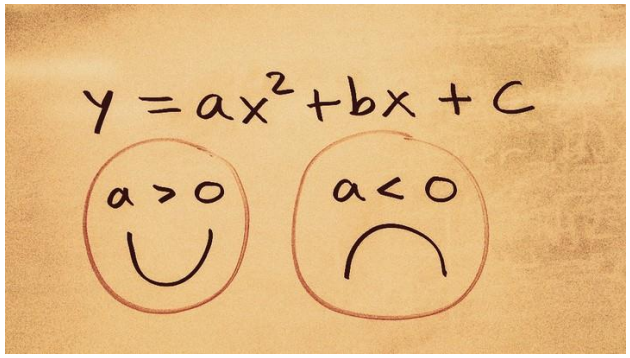

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

L

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$