Preparation Sheet for Final Exam

Problem 1. Vectors and the Geometry of Space. Vector-Valued Functions and Motion in Space.

a) [**5 marks**] Find the point in which the line through the origin perpendicular to the plane $2x - y - z = 4$ meets the plane $3x - 5y + 2z = 6$.

b) [**6 marks**] Find a vector of magnitude 2 parallel to the line of intersection of the planes $x + 2y + z - 1 = 0$ and $x - y + 2z + 7 = 0$.

c) [**6 marks**] Suppose $\mathbf{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j}$. Show that the angle between $\mathbf{r}$ and $\mathbf{a}$ never changes. What is the angle?

d) [**8 marks**] Find equations for the osculating, normal, and rectifying planes of the curve $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1, 1, 1)$.

Total: 25 mark

Problem 2. Partial Derivatives.

a) [**7 marks**] What is the largest value that the directional derivative of $f(x, y, z) = xyz$ can have at the point $(1, 1, 1)$?

b) [**10 marks**] Find the extreme values of $f(x, y, z) = x(y + z)$ on the curve of intersection of the right circular cylinder $x^2 + y^2 = 1$ and the hyperbolic cylinder $xz = 1$.

c) [**8 marks**] Find the points on the surface $(y + z)^2 + (z - x)^2 = 16$ where the normal line is parallel to the yz-plane.

Total: 25 mark

Problem 3. Multiple Integrals. Integrals and Vector Fields

a) **[7 marks]** Find the area of the "triangular" region in the xy-plane that is bounded on the right by the parabola $y = x^2$, on the left by the line $x + y = 2$, and above by the line $y = 4$.

b) **[10 marks]** Convert

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} 3 \;\, dz \, r \, dr \, d\theta, \qquad r \geq 0$$

to (a) rectangular coordinates with the order of integration $dz\ dx\ dy$ and

(b) spherical coordinates.

Then (c) evaluate one of the integrals.

c) **[8 marks]** Use Green's Theorem to find the outward flux of $\mathbf{F} = 2xy\mathbf{i} + 2yz\mathbf{j} + 2xz\mathbf{k}$ across the boundary of D: the entire surface of the upper cap cut from the solid sphere $x^2 + y^2 + z^2 \leq 25$ by the plane $z = 3$.

Total: 25 mark

Problem 4. Infinite Sequences and Series. Fourier series.

a) [**15 marks**] Given

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{n\, 3^n}$$

(i) find the series' radius and interval of convergence. Then identify the values of x for which the series converges

(ii) absolutely and

(iii) conditionally.

b) [**10 marks**] The series

$$\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \cdots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \cdots$$

is the value of the Maclaurin series at $x = 0$ of a function $f(x)$ at a particular point. What function and what point? What is the sum of the series?

Total: 25 mark