Problem Sheet 7 for the Tutorial, November 10.

(Partial Derivatives. Multiple Integrals)

Problem 1. Use Taylor's formula to find a quadratic approximation of $f(x, y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$.

Solution:

Problem 2. Find the volume of the region bounded above by the surface $z = 4 - y^2$ and below by the rectangle R : $0 \leq x \leq 1, \quad 0 \leq y \leq 2$.

Solution:

Problem 3. Sketch the region of integration, reverse the order of integration, and evaluate the integral

$$\text{a)} \int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy\, dx,$$

$$\text{b)} \int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dy\, dx.$$

Solution:

Problem 4. Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

Solution:

Problem 5. Change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral

$$\text{a)} \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dy\, dx,$$

$$\text{b)} \int_{1}^{2} \int_{0}^{\sqrt{2x-x^2}} \frac{1}{(x^2+y^2)^2} dy\, dx.$$

Solution: