Homework Sheet 6, available online December 14th, online delivery December 21 before 5:00 pm UK time.

(Infinite Sequences and Series)

Problem 1. (1.5+1.5+1 = 4 points)

a) Use the Ratio Test to determine if the series $\sum_{n=2}^{\infty}(-1)^n \frac{3^{n+2}}{\ln n}$ converges absolutely or diverges.

b) Use the Root Test to determine if the series $\sum_{n=1}^{\infty}\left(-\ln\left(e^2 + \frac{1}{n}\right)\right)^{n+1}$ converges absolutely or diverges.

c) Use any method to determine if the series $\sum_{n=1}^{\infty}\frac{n!}{10^n}$ converges or diverges. Give reasons for your answer.

Solution:

Problem 2. (2+2 = 4 points) Determine if the series converges absolutely, converges, or diverges? Give reasons for your answers.

a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$,

b) $\sum_{n=1}^{\infty}(-1)^n \frac{(n!)^2 \, 3^n}{(2n+1)!}$.

Solution:

Problem 3. (2+2 = 4 points) Use any method to determine whether the series converges or diverges. Give reasons for your answer.

a) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$,

b) $\sum_{n=3}^{\infty} \sin\left(\frac{1}{\sqrt{n}}\right)$.

Solution:

Problem 4. (2+1+1 = 4 points) Given a series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{\sqrt{n+3}}$

a) find the series' radius and interval of convergence.

For what values of x does the series converge b) absolutely, c) conditionally?

Solution:

Problem 5. (4 points) For what values of x does the series

$$1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 + \ldots + \left(-\frac{1}{2}\right)^n (x-3)^n + \ldots$$

converge? What is its sum? What series do you get if you differentiate the given series term by term? For what values of x does the new series converge? What is its sum?

Solution: