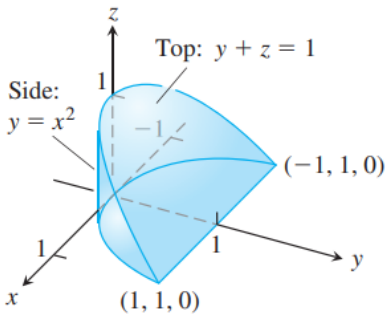
Problem Sheet 8 for the Tutorial, November 17.

(Multiple Integrals)

Problem 1. Here is the region of integration of the integral

$$\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\, dy\, dx.$$



Rewrite the integral as an equivalent iterated integral in the order
a) $dy\, dz\, dx$; b) $dy\, dx\, dz$; c) $dx\, dy\, dz$; d) $dx\, dz\, dy$; e) $dz\, dx\, dy$.

Solution:

Problem 2. Find the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 4$.

Solution:

Problem 3. Let D be the region in xyz-space defined by the inequalities $1 \leq x \leq 2$, $0 \leq xy \leq 2$, $0 \leq z \leq 1$. Evaluate

$$\iiint_D (x^2y + 3xyz)dx\ dy\ dz$$

by applying the transformation $u = x$, $y = xy$, $w = 3z$ and integrating over an appropriate region G in uyw-space.

Solution: