

Homework Sheet 3, available online October 26th, **online delivery November 2nd before 4:00 pm UK time.**

(Partial Derivatives)

Problem 1. (4 points) Find an equation for the level surface of the function $f(x, y, z) = \ln(x^2 + y + z^2)$, through the given point $(-1, 2, 1)$.

Solution:

Problem 2. (4 points) By considering different paths of approach, show that the function

$$f(x, y) = -\frac{x}{\sqrt{x^2 + y^2}}$$

have no limit as $(x, y) \rightarrow (0, 0)$.

Solution:

Problem 3. (4 points) Find all the second-order partial derivatives of the function

$$w = x \sin(x^2 y).$$

Solution:

Problem 4. (4 points) Assume that $z = \ln(f(w))$, $w = g(x, y)$, $x = \sqrt{r - s}$, and $y = r^2 s$. If $g_x(2, -9) = -1$, $g_y(2, -9) = 3$, $f'(-2) = 2$, $f(-2) = 5$, and $g(2, -9) = -2$, find $\frac{dz}{dr} \big|_{r=3, s=-1}$ and $\frac{dz}{ds} \big|_{r=3, s=-1}$.

Solution:

Problem 5. (4 points) Is there a direction \mathbf{u} in which the rate of change of the temperature function $T(x, y, z) = 2xy - yz$ (temperature in degrees Celsius, distance in feet) at $P(1, -1, 1)$ is -3°C/ft ? Give reasons for your answer.

Solution: