Lecture 3 4/10

$\mathbb{R}$ form an ordered field

3 axioms (ordered) 5 axioms (field)

Mult Identity

$\exists 1 \neq 0$ such that $a.1 = a \ \forall x$

Consequences of axioms:

$0 + 0 = 0$ $\quad (a + 0 = a \ \forall a)$

$(0+0)x = 0.x$ $\quad$ (Distrib.)

$0x + 0x = 0x$

$((0x+0x)) + (-0x) = 0x + (-0x)$ $\quad (+ \text{ inv})$

$0x + ((0x - 0x)) = 0x - 0x$ $\quad$ (Assoc)

$0x + 0 = 0 \Rightarrow \boxed{0x = 0}$

Let

$y = 0x$

$y + y = y$

$(y+y) + (-y) = y + (-y)$

$y + (y + (-y)) = y + (-y)$

1) $x > 0 \Rightarrow x^2 > 0$

$-x > 0 \Rightarrow x^2 > 0$

2) Notation $a < b \Leftrightarrow b > a$

$a \leq b \Leftrightarrow a < b$ or $a = b$

1) $0 < x$

$\Rightarrow 0x < x^2$

$\Rightarrow 0 < x^2$

2) $x < 0$

$0 < -x$

$0 \cdot (-x) < (-x)^2$

$0 < x^2$

$x \neq 0 \Rightarrow x^2 > 0$

More consequences

i) $x, y > 0 \Rightarrow xy > 0$

$[x > 0 \Rightarrow xy > 0y = 0]$

ii) $-x > 0 \Leftrightarrow x < 0$

iii) $x \neq 0 \Rightarrow x^2 > 0$ ✓

iv) $1 > 0$

$[1 \neq 0, 1 = 1^2 > 0]$

v) $0 < x < y \Leftrightarrow 0 < \frac{1}{y} < \frac{1}{x}$

$[xy > 0, (xy)^{-1} > 0$ also because $1 > 0$

so $\frac{x}{xy} < \frac{y}{xy}$ i.e. $\frac{1}{y} < \frac{1}{x}$

$$|x| = \begin{cases} 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

so $|x| \geq 0 \quad \forall x$

$|x| > 0 \iff x \neq 0$

absolute value of x.

Distance $d(a,b) = |a-b|$

is zero $\iff a = b$

Triangle Inequality

$\forall a,b \quad |a+b| \leq |a| + |b|$

Proof

	$a \geq 0$	$a < 0$
$b \geq 0$	$a+b = a+b$	$\pm(a+b) \leq -a+b$
$b < 0$	$a+b < a-b$ $-a-b \leq a-b$	$-(a+b) = -a-b$

$-a \leq a$

$b < -b$

$\pm(a+b)$