MA2261 - DLI, Linear Statistical Models, Year 2022-2023

Solutions of exercises for feedback class 7
(Note: the exercise number refers to the workbook)

EXERCISE 4.3

The sum of squares of the model is

$$SS = \sum_{i=1}^{10} (y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)^2 \, .$$

The equation $\dfrac{\partial SS}{\partial \beta_1} = 0$ is solved by $\hat{\beta}_0 = 1, \; \hat{\beta}_2 = 0.1, \; \hat{\beta}_3 = -3.$

Since

$$\frac{\partial SS}{\partial \beta_1} = 2 \sum_{i=1}^{10} (y_i - \beta_0 - \beta_1 x_i - \beta_2 x_i^2 - \beta_3 x_i^3)(-x_i)$$

The equation $\dfrac{\partial SS}{\partial \beta_1} = 0$ is equivalent to

$$\sum_{i=1}^{10} y_i x_i - \beta_0 \sum_{i=1}^{10} x_i - \beta_1 \sum_{i=1}^{10} x_i^2 - \beta_2 \sum_{i=1}^{10} x_i^3 - \beta_3 \sum_{i=1}^{10} x_i^4 = 0 \, .$$

that is $7 - 1.2\beta_0 - 2\beta_1 - 7\beta_2 - 10\beta_3 = 0$

It follows that

$$\hat{\beta}_1 = \frac{7 - 1.2\hat{\beta}_0 - 7\hat{\beta}_2 - 10\hat{\beta}_3}{2} = \frac{7 - 1.2 - 7 \times 0.1 - 10 \times (-3)}{2} = 17.55 \, .$$

EXERCISE 4.4 i)

i) To test the significance of the individual variable x_1, we need to test the hypothesis $\beta_1 = 0$. We calculate $t = \frac{\hat{\beta}_1}{se(\beta_1)} = \frac{-5.905}{1.610} = -3.667 \sim t_{22}$. The critical region is $(-\infty, -2.074) \cup (2.074, +\infty)$. Hence the hypothesis is rejected: the individual variable x_1 is statistically significant.

   For the variable x_2, we need to test the hypothesis $\beta_2 = 0$. We calculate $t = \frac{\hat{\beta}_2}{se(\beta_2)} = \frac{-6.261}{2.099} = -2.983 \sim t_{22}$. The critical region is $(-\infty, -2.074) \cup (2.074, +\infty)$. Hence the hypothesis is rejected: the individual variable x_2 is also statistically significant.