UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 5

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ANNOUNCEMENTS

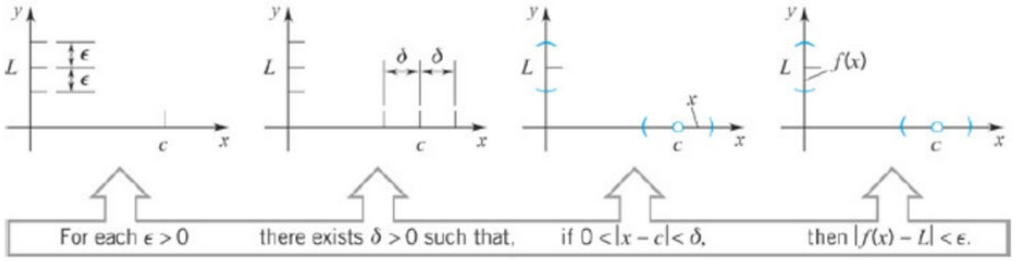
- No sessions on Friday (01/10) or Tuesday (07/10)

REVISION: FORMAL DEFINITION OF THE LIMIT

A function, f, has a limit, L, at $x = c \in (a, b) \subset \mathbb{R}$, if

$$\forall \varepsilon > 0, \exists \delta : |f(x) - L| < \varepsilon, \forall |x - c| < \delta$$

and $f(x)$ is well defined on $x \in (a, b) \backslash \{c\}$



EXERCISE:

Prove that

$$\lim_{x \to 0} x^2 = 0$$

PINCHING THEOREM

Suppose that

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$$

and $f(x) \leq g(x) \leq h(x)\ \forall x \neq c$. Then,

$$\lim_{x \to c} g(x) = L.$$

EXAMPLE:

Use the Pinching Theorem to determine:

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

EXERCISE:

Prove that

$$\lim_{x \to 1} x - 1 = 0$$

Thus, determine

$$\lim_{x \to 1} (x - 1) \cos\left(\frac{\pi}{x - 1}\right)$$

EXERCISE:

Determine the following limit

$$\lim_{x \to 0^-} \frac{x + |x|(1+x)}{x} \sin\left(\frac{1}{x}\right)$$

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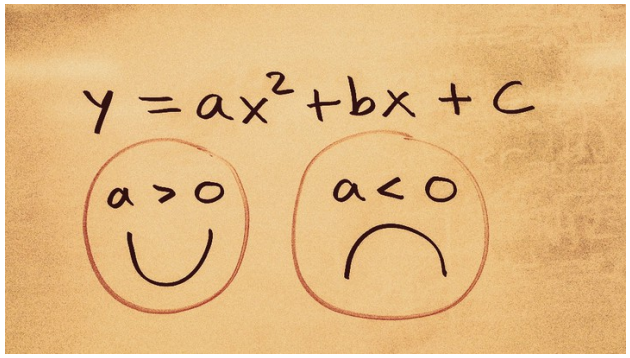

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

L

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$