UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 12

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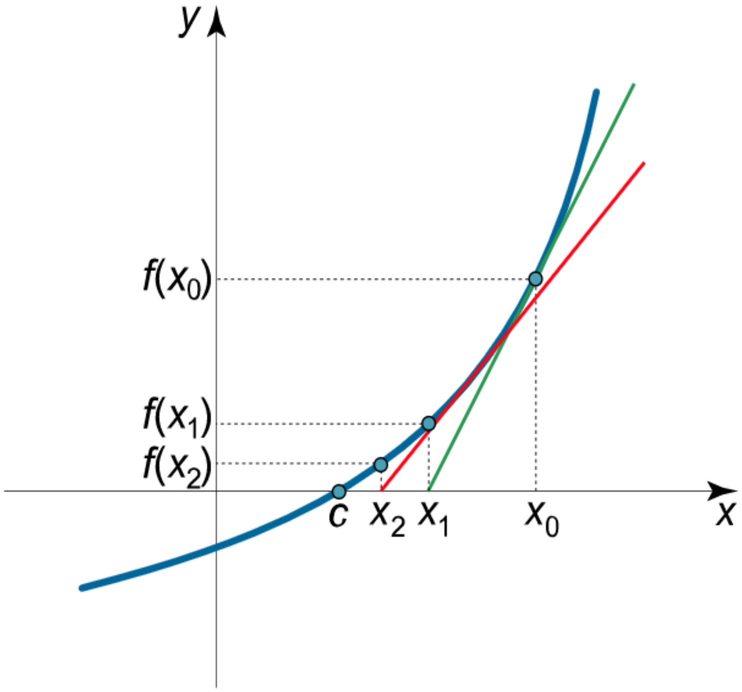
NEWTON (RAPHSON) METHOD

Finds the root of a function, so it can solve non-linear equations.

1. Choose an initial guess x_0
2. Iterate

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



EXAMPLE

Find an approximation of the solution to $x^5 = 3$ with $x_0 = 4$.

EXERCISE

Find an approximation to the solution of

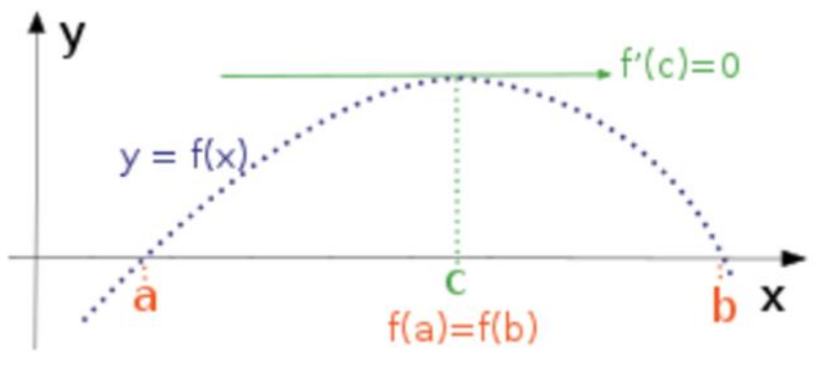
$f(x) = x^3 - \cos(x) = 0$ to 4 decimal places

EXERCISE

Use the Newton method on the function $y(x) = 2x^3 - 6x^2 + 6x - 1$ with $x_0 = 1$. What happens? Why? Try again using $x_0 = 0.5$.

ROLLE'S THEOREM

Suppose $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b). Suppose further that $f(a) = f(b)$. Then, for some $c \in (a, b), f'(c) = 0$.



MEAN VALUE THEOREM

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). Then for some $c \in (a, b)$,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

EXERCISE

Prove that $f(x) = x^3 - 3x^2 + 12x - 25$ has exactly one real root.

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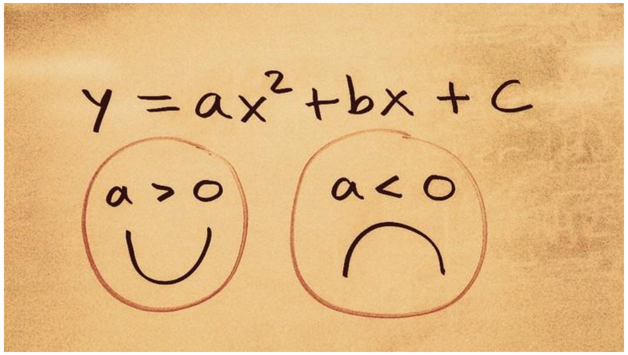


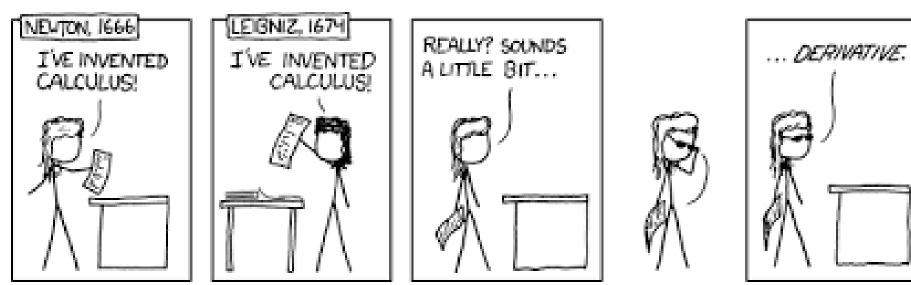
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$



CHALLENGE: NEWTON'S METHOD

Recall Newton's Method for solving Non-Linear Equations of the form $g(x) = 0$,

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

Implement/program this method in order to solve equations of the form $f(x) = 0$.

Hence, find the root of the function

$$f(x) = x^5 + 5x^4 + 4x^3 + 3x^2 + 2x + 1$$