Problem Sheet 10 for the Tutorial, December 1.

(Integrals and Vector Fields)

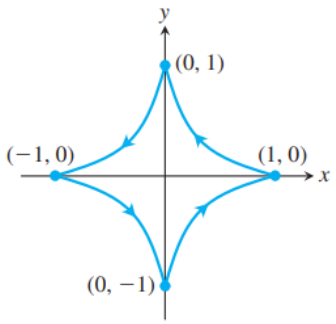
Problem 1. Evaluate the line integral $\int_C 2x\cos y\,dx - x^2\sin y\,dy$ along the following paths C in the xy-plane.

a) The parabola $y = (x-1)^2$ from $(1,0)$ to $(0,1)$.

b) The line segment from $(-1,\pi)$ to $(1,0)$.

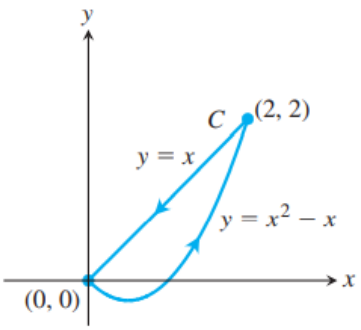
c) The x-axis from $(-1,0)$ to $(1,0)$.

d) The astroid $\mathbf{r}(t) = (\cos^3 t)\mathbf{i} + \sin^3 t)\mathbf{j}$, $0 \le t \le 2\pi$, counterclockwise from $(1,0)$ back to $(1,0)$.



Solution:

Problem 2. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field $\mathbf{F} = x^3y^2\,\mathbf{i} + \frac{1}{2}x^4y\,\mathbf{j}$ and curve C shown on the following Figure



Solution:

Problem 3. Evaluate the integral

$$\oint_C 4x^3y\,dx + x^4\,dy$$

for any closed path C.

Solution:

Problem 4. Use a parametrization to express the area of the surface as a double integral. Then evaluate the integral. (There are many correct ways to set up the integrals, so your integrals may not be the same. They should have the same values, however.)

Plane inside cylinder: The portion of the plane $z = -x$ inside the cylinder $x^2 + y^2 = 4$.

Solution: