UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 9

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ANNOUNCEMENTS

- Consolidation Week
- Coursework coming soon!

LIMITS

Let $f(x) = e^x$, what would happen if I let x get very large?

Try different values for x. What happens?

Now, what would happen if you let x get large but imposed that $x < 0$?

It should be clear that the following seem to happen,

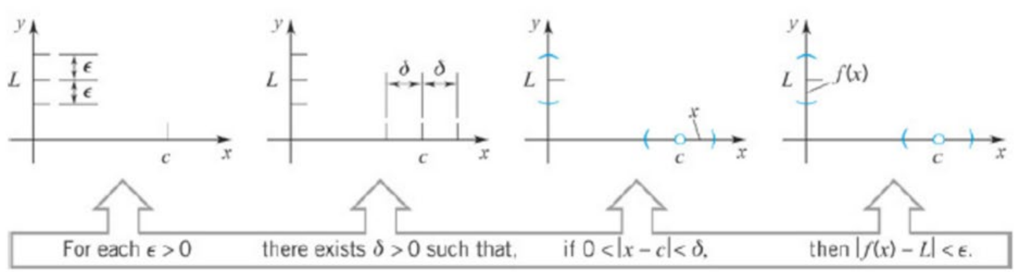
$$\lim_{x\to\infty} f(x) = \infty \ \& \ \lim_{x\to-\infty} f(x) = 0$$

FORMAL DEFINITION OF THE LIMIT

A function, f, has a limit, L, at $x = c \in (a, b) \subset \mathbb{R}$, if

$$\boxed{\forall \varepsilon > 0, \exists \delta : |f(x) - L| < \varepsilon, \forall |x - c| < \delta}$$

and $f(x)$ is well defined on $x \in (a, b) \setminus \{c\}$



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EXERCISE

Let $f(x) = -2x - 5$.

a) Find $A > 0$ such that if $0 < |x+2| < A$ then $|f(x) + 1| < \frac{1}{200}$

b) Given $\varepsilon > 0$ find $\delta > 0$ so that if $0 < |x+2| < \delta$ then $|f(x) + 1| < \varepsilon$.

c) Hence, prove $\lim_{x \to -2} f(x) = -1$.

EXERCISE

Let $f(x) = x^2 - 4x + 5$. Show that $\lim_{x \to 2} f(x) = 1$ by using an $\varepsilon - \delta$ argument.

LIMIT LAWS

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$, then

i. $\lim_{x \to c} [f(x) + g(x)] = L + M$

ii. $\lim_{x \to c} [\alpha f(x)] = \alpha L$ for $\alpha \in \mathbb{R}$

iii. $\lim_{x \to c} [f(x) g(x)] = LM$

PINCHING THEOREM

Suppose that

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$$

and $f(x) \leq g(x) \leq h(x)\ \forall x \neq c$. Then,

$$\lim_{x \to c} g(x) = L.$$

EXERCISE:

Use the Pinching Theorem to determine:

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$$

CONTINUITY

Let $f(x)$ be defined on (a, b). Then $f(x)$ is **continuous** at $c \in (a, b)$ if

$$\boxed{\lim_{x \to c} f(x) = f(c).}$$

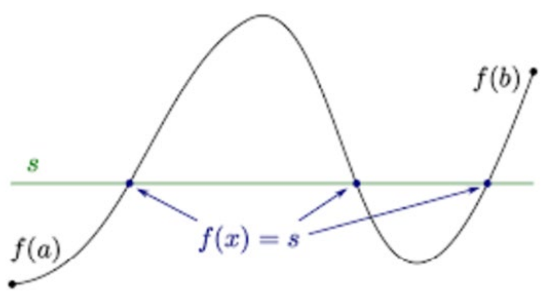
EXERCISE

Let $f(x) = x^2 - 4x + 5$. Show that $\lim_{x \to 2} f(x) = 1$ by using an $\varepsilon - \delta$ argument. Hence, conclude that $f(x)$ is continuous at $x = 2$.

INTERMEDIATE VALUE THEOREM

If a function, f, is continuous on $[a, b]$ and $f(a) < f(b)$. Then

$$\boxed{\forall\, f(a) < s < f(b), \exists c \in (a, b) : f(c) = s}$$



EXAMPLE

Prove that $f(x) = x^3 - 3x^2 + 12x - 25$ has at least one real root.

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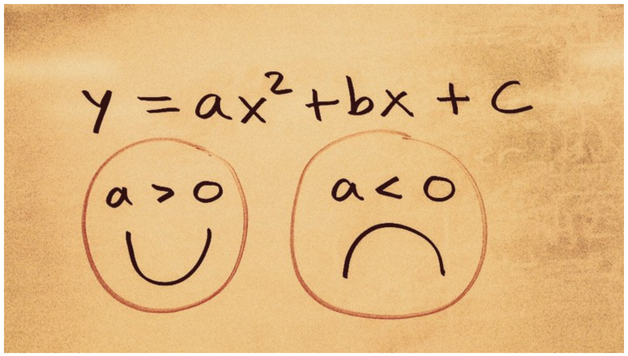

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$