UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 15

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ANNOUNCEMENTS

- Coursework Deadline: **TODAY** (13:00 GMT)

TAYLOR'S THEOREM

Suppose $f(x)$ is n-times differentiable over $[a, x]$. Then,

$$\boxed{f(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{k!}(x-a)^k + \frac{(x-a)^n}{n!} f^{(n)}(c)}$$

for some $c \in (a, x)$.

TAYLOR SERIES

Suppose f is infinitely continuously differentiable on $[a, x]$. Then,

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x - a)^n}$$

EXAMPLE

Find the Taylor series of $\cos(x)$ at $x = 0$

EXERCISE

Determine the Taylor series of the following functions around $x = 0$,

a) $f(x) = \cos(ax)$

b) $g(x) = \sin(bx)$

c) $h(x) = e^{cx}$

Thus show that $e^{inx} = \cos(nx) + i\sin(nx) : i = \sqrt{-1}$

EXERCISE: EINSTEIN'S ENIGMA

Albert is very clever and has figured out that the Energy of a particle is

$$E = \sqrt{m^2c^4 + p^2c^2}$$

Where p is the particle's momentum, v is the particle's velocity, m is the particle's mass and c is the speed of light ($\sim 3 \times 10^5 \text{ km}/s$). What is E if

$$p = \frac{mv}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Find the first two terms of the Taylor Series of $\frac{1}{\sqrt{1-x^2}}$ around $x = 0$. Hence, assume that $v \ll c$ and help Albert find a nice expression for E. What is E if $v = 0$?

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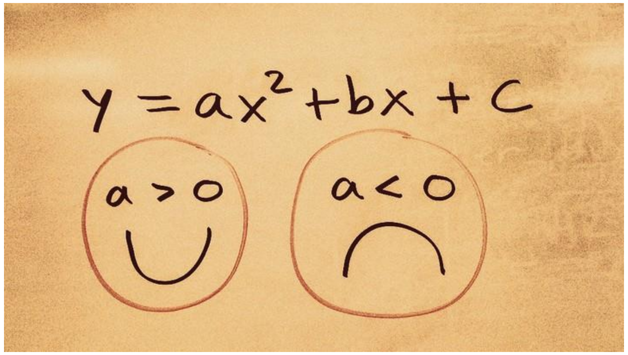

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$