UNIVERSITY OF LEICESTER

MA1014 CALCULUS AND ANALYSIS TUTORIAL 1

Dr. Andrew Tonks: apt12@le.ac.uk

Ben Smith: bjs30@le.ac.uk

ANNOUNCEMENTS

- I’m not here this week! ☹
- Make sure you keep up to date with all module content!

PROOF BY CONTRADICTION

- Want to prove a Proposition, P.
- If the negation, $\neg P$, leads to a contradiction

  i.e $\neg P \to C \land \neg C$
- Then P is true.

Example:

$P: a^2 - 4b \neq 2 : a, b \in \mathbb{Z}$

- $\neg P: a^2 - 4b = 2$
  $\Rightarrow a^2 = 2(1 + 2b)$
  $\Rightarrow a$ is even, let $a = 2c$
  $\Rightarrow 2(c^2 - b) = 1$
- C: 1 is odd and
  $\neg C$: 1 is even
- So P is true

WORKED EXAMPLE:

Prove the Proposition:

P: If N^2 is even $\Rightarrow N$ is even

Hints:

- What is $\neg P$?
- If a number Q is even, then $Q = ?$

EXERCISE:

Prove by Contradiction that $\sqrt{2}$ is irrational.

Hints:

- Can you write down a proposition P?
- What is $\neg P$?
- If a number Q is even, then $Q = $?

Extra Time: Try a similar approach to prove that $\sqrt{3}$ is irrational. (Hint: There is a useful theorem from Number Theory you can use)

ORDER & INEQUALITIES

The real numbers $\mathbb{R}$ are an ordered field, i.e. $\exists$ a relation $<$, that $\forall a, b, c \in \mathbb{R}$:

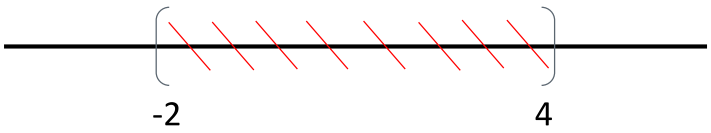
- Total Order: Either $a < b, b < a$ or $b = a$
- Transitivity: If $a < b$ and $b < c \ \Rightarrow a < c$
- Compatibility: If $a < b \ \Rightarrow a + c < b + c$. If $a < b$ and $c > 0$ then $ac < bc$

Example:

If $x, y > 0$, then Compatibility $\Rightarrow x + y > y$ and $xy > 0$ (i.e. $a = 0, b = x, c = y$)

Also we can denote intervals with these inequalities.

Example: $-2 < x < 4$



EXERCISE:
SOLVE THE FOLLOWING INEQUALITIES

a) $x^2 - 5x + 4 < 0$

b) $3x - 4 \geq 0$

c) $4(x + 2) - 1 > 5 - 7(4 - x)$

d) $2x^2 + 3x - 9 \leq 0$

WORKED EXAMPLE: TRIANGLE INEQUALITY

The Triangle Inequality will be extremely useful during this course!

$$|x + y| \leq |x| + |y|$$

It sets a bound on the addition of two real numbers.

Prove the Triangle Inequality. Provide an example where there is equality and another where there is inequality.

BOUNDS

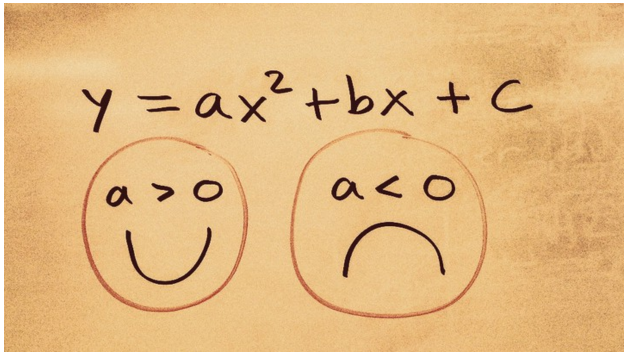
Let $S \subset F$, then

- An **Upper Bound** of S is $M \in F : \forall s \in S,\ s \leq M$.
- A **Lower Bound** of S is $m \in F : \forall s \in S,\ m \leq s$.
- The **Supremum** of S or $\sup(S)$ is the smallest/least upper bound
- The **Infimum** of S or $\mathrm{Inf}(S)$ is the largest/greatest lower bound

EXERCISE: FILL THIS IN AS MUCH AS YOU CAN

Set	Upper bound	Lower bound	Max	Min	Sup	Inf	Is the sup in the set?	Is the inf in the set?
$\{x \in \mathbb{R} : 0 \leq x < 1\}$	1	0	N/A	0	1	0	No	Yes
$\{x \in \mathbb{R} : 0 \leq x \leq 1\}$								
$\{x \in \mathbb{R} : 0 < x < 1\}$								
$\{1/n : n \in \mathbb{Z} - \{0\}\}$								
$\{1/n : n \in \mathbb{N}\}$								
$\{x \in \mathbb{R} : x < \sqrt{2}\}$								
$\{1, 4, 7, 97\}$								
$\{(-1)^n(2 - \frac{1}{n}) : n \in \mathbb{N}\}$								
$\{\ln(x) : x \in \mathbb{R}, x > 0\}$								
$\{n^{1/n} : n \in \mathbb{N}\}$								
$\{\arctan(x) : x \in \mathbb{R}\}$								
$\{(-1)^n : n \in \mathbb{N}\}\}$								
$\{e^x : x \in \mathbb{R}\}\}$								

UNIVERSITY OF LEICESTER



$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$