Problem Sheet 4

MA1202, Introductory Statistics

Due date - 10/04/2022, 23:59 BST

General information

Please upload your work to Blackboard as a single pdf document which is of good quality. Read the **Instructions on Scanning and Uploading handwritten work**. Please name your file *PS4*YourName*Date*.pdf.

Please submit to Blackboard only solutions to questions from Section 1.

Please prepare questions from Section 2 for Feedback Session - you are expected to participate in discussion of these questions, your input will contribute to the participation mark.

Section 1. [to be submitted to Blackboard by 10/04/22]

Question 1.

Consider a random sample of size n taken without replacement from a finite population of $N = 60$ with $\sigma = 2.0$. Show that $\bar{X}$ would be a more precise estimator of μ if if the sample size was increased from $n = 4$ to $n = 16$. (Hint: consider the standard error of the mean).

Question 2.

i)If $X_1, X_2, ..., X_n$ constitute a random sample from a normal distribution, suppose μ is known and does not have to be estimated. Show that $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$ is unbiased for σ^2.

ii) the following 5 samples of size 20 were obtained from a population described by normal distribution with known mean value, $\mu = 5$. Using the result from part i) estimate the sample variance for each sample and find the MSE of the estimator if the variance of the population was equal to 4.

Sample 1	5.0024, 2.0698, 6.1802, 5.7652, 2.2358, 6.7074, 5.8078, 9.2920, 4.9483, 6.2735, 4.5847, 5.1965, 11.2301, 5.8753, 2.9813, 4.6384, 4.4133, 3.7457, 5.4246, 3.7178
Sample 2	4.8549, 5.1086, 2.9260, 6.6856, 2.7769, 1.8983, 5.6530, 3.0692, 4.1723, 3.7690, 9.2426, 6.0379, 3.1005, 3.2474, 7.1208, 4.9098, 5.4191, 6.8643, 2.5230, 5.4775

Sample 3	3.0752, 1.8840, 2.3992, 5.8228, 4.7679, 3.1183, 4.4895, 3.9048, 2.5062, 8.3217, 4.6107, 3.9596, 5.8607, 1.3123, 4.3306, 5.7424, 2.6561, 3.6624, 9.2243, 7.6367
Sample 4	9.0545, 5.9464, 6.8167, 5.0933, 4.6594, 4.4462, 5.1348, 4.6557, 4.3490, 7.8146, 5.2406, 6.0551, 3.1550, 5.3080, 4.8028, 4.3694, 5.4526, 3.8958, 6.2271, 7.3465
Sample 5	6.1225, 1.5537, 6.8121, 8.8529, 6.8716, 6.9193, 4.5798, 0.6852, 3.3033, 1.7611, 8.1563, 5.8597, 2.7283, 7.3687, 9.1776, 3.2510, 3.4971, 5.5957, 4.1962, 6.8137

Section 2. [to be discussed in Tutorials on 14/04/22]

Question 3.

Let $X_1, X_2, ..., X_n$ denote the outcomes of a series of n independent Bernoulli trials, where

$$X_i = \begin{cases} 1 & \text{with probability} \quad p \\ 0 & \text{with probability} \quad (1-p) \end{cases}$$

for $i = 1, 2, ..., n$.

Let $X = X_1 + X_2 + ... + X_n$.

i) Show that $\hat{p}_1 = X_1$ and $\hat{p}_2 = X/n$ are unbiased estimators for p.

ii) Verify that $\hat{p}_2$ is a better estimator than $\hat{p}_1$ because $\hat{p}_1$ by comparing the variances of $\hat{p}_1$ and $\hat{p}_2$, find the relative efficiency of $\hat{p}_1$ with respect to $\hat{p}_2$. Explain your finding from "intuitive" point of view.

Question 4.

Let $X_1, ..., X_n$ be a random sample from a distribution with pdf

$$f_X(x, \alpha) = \frac{2x}{\alpha} e^{-x^2/\alpha}, \quad x > 0$$

Show that $\hat{\alpha} = \sum_{i=1}^{n} X_i^2$ is sufficient for the parameter α.