MA2261 - DLI, Linear Statistical Models, Year 2022-2023

Solutions of exercises for feedback class 3

(Note: the exercise number refers to the workbook)

EXERCISE 3.3 i)

i) $\hat{b} = S_{xy}/S_{xx} = 83.723/628.672 = 0.133, \quad \hat{a} = \bar{y} - \hat{b}\bar{x} = 6.467 - 0.133 \times 23.048 = 3.40$ The regression line is $\hat{y} = 3.40 + 0.133x$.

EXERCISE 3.4 i) - iv)

i) The sample correlation coefficient is

$$\hat{\rho} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{83.723}{\sqrt{628.672 \times 17.367}} = 0.8013$$

ii) To test the hypothesis $\rho = 0$ we calculate

$$\hat{\rho}\sqrt{\frac{n-2}{1-\hat{\rho}^2}} = 0.8013\sqrt{\frac{19}{1-0.8013^2}} = 5.838 \sim t_{19}$$

The critical region is $(-\infty, -2.093) \cup (2.093, +\infty)$ and the statistic value is inside the critical region. Therefore we reject the hypothesis $\rho = 0$.

iii)

$$\hat{\rho}^2 = 0.6421$$

Hence the 64.21% of the variation of y is explained by x.

iv) From above $\hat{\rho} = 0.8013$. Therefore

$$l_1 = \frac{e^{-\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)} \cdot \frac{1+\hat{\rho}}{1-\hat{\rho}} - 1}{e^{-\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)} \cdot \frac{1+\hat{\rho}}{1-\hat{\rho}} + 1} = \frac{e^{-\left(\frac{2\times 1.96}{\sqrt{21-3}}\right)} \times \frac{1+0.8013}{1-0.8013} - 1}{e^{-\left(\frac{2\times 1.96}{\sqrt{21-3}}\right)} \times \frac{1+0.8013}{1-0.8013} + 1} = 0.565\ ,$$

$$l_2 = \frac{e^{\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)} \cdot \frac{1+\hat{\rho}}{1-\hat{\rho}} - 1}{e^{\left(\frac{2\times 1.96}{\sqrt{n-3}}\right)} \cdot \frac{1+\hat{\rho}}{1-\hat{\rho}} + 1} = \frac{e^{\left(\frac{2\times 1.96}{\sqrt{21-3}}\right)} \times \frac{1+0.8013}{1-0.8013} - 1}{e^{\left(\frac{2\times 1.96}{\sqrt{21-3}}\right)} \times \frac{1+0.8013}{1-0.8013} + 1} = 0.916$$

The 95% confidence interval for ρ is (0.565, 0.916).