Problem Sheet 8

MA1202, Introductory Statistics

Due date - 22/05/2022, 23:59 BST

General information

Please upload your work to Blackboard as a single pdf document which is of good quality. Read the **Instructions on Scanning and Uploading handwritten work**. Please name your file *PS8*YourName*Date*.pdf.

Please submit to Blackboard only solutions to questions from Section 1.

Please prepare questions from Section 2 for Feedback Session - you are expected to participate in discussion of these questions, your input will contribute to the participation mark.

Questions in Section 3 are for practice, solutions will be provided.

Section 1. [to be submitted to Blackboard by 22/05/22]

Question 1. Suppose an experimenter wishes to test whether the average number of customers visiting the local shop during one day is 100. Assume that the variable describing the number of visitors in the shop is normally distributed with $\sigma = 14$.

i) State the hypotheses, explain in a few words your motivation for using a particular alternative hypothesis.

ii) Define the test statistics to be used for analysis and define the rejection region, in terms of test statistics and in terms of the sample mean if the risk of rejecting the null hypothesis incorrectly is 0.05

iii) Perform the hypothesis testing if the experimenter observed the shop during one month (30 days) and found that the average number of customers visiting shop in one day is $\bar{X} = 82$. Write the conclusion.

Question 2.

A group of 50 Internet shoppers were asked how much they spent per year on the Internet. The mean of their responses is £304.60, the standard deviation is £101.51. It is desired to test that they spend £325 per year versus it is different from 325. (Assume that the population distribution is normal.)

i) State the hypotheses you need to test.

ii) Write the test statistic (explain your reasoning).

iii) Run the hypothesis testing at 5% significance level. What is your conclusion?

Section 2. [to be discussed in Feedback session on 26/05/22(TBC)]

Question 3.

A supermarket chain is considering two sources A and B for the purchase of 50-pound bags of onions. The following table gives the results of a study.

Test at $\alpha = 0.05$ whether there is a difference in the mean weights.

	Source A	Source B
Number of bags weighed	80	100
Mean weight	105.9	100.5
Sample variance	0.21	0.19

Question 4.

The intelligence quotients (IQs) of 17 students from one area of a city showed a sample mean of 106 with a sample standard deviation of 10, whereas the IQs of 14 students from another area chosen independently showed a sample mean of 109 with a sample standard deviation of 7. Is there a significant difference between the IQs of the two groups at $\alpha = 0.02$? Assume that the population variances are equal.

Section 3. [for practice only]

Question 5.

Suppose $H0 : \mu_X = \mu_Y$ is to be tested against $HA : \mu_X \neq \mu_Y$. X_i and Y_j are two independent samples from normal populations with the sample sizes $n = 6$ and $m = 11$, respectively.

i) If $s_p = 15.3$, what is the smallest value for $|\bar{x} - \bar{y}|$ that will result in $H0$ being rejected at the $\alpha = 0.01$ level of significance?

ii) What is the smallest value for $x - y$ that will lead to the rejection of $H0 : \mu_X = \mu_Y$ in favour of $HA : \mu_X > \mu_Y$ if $\alpha = 0.05$, $s_p = 214.9$, $n = 13$, and $m = 8$?

Question 6.

A sociologist is studying various aspects of the personal lives of nineteenth-century scholars. A total of 120 subjects in her sample had families consisting of two children. The distribution of the number of boys in those families is summarized in the following table. Can it be concluded that the number of boys in two-child families of the scholars is binomially distributed? Let $\alpha = 0.05$.

Number of boys	0	1	2
Number of families	24	64	32