MA2261 - DLI, Linear Statistical Models, Year 2022-2023

Solutions of exercises for feedback class 6
(Note: the exercise number refers to the workbook)

EXERCISE 3.11

i) We calculate

$$\hat{b} = \frac{S_{xy}}{S_{xx}} = \frac{1754}{114} = 15.386$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} = 96.1429 - 15.386 \times 6 = 3.8269 \ .$$

Therefore the fitted regression line equation is

$$y = 3.8269 + 15.386x \ .$$

ii) Given $SSB = 27272.71$, we have

$$SST = S_{yy} = 27577.7$$

$$SSE = RSS = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = 27577.7 - \frac{1754^2}{114} = 590.72$$

$$SSM = SST - SSE = 27577.7 - 590.72 = 26986.98$$

$$SSW = SST - SSB = 27577.7 - 27272.71 = 305$$

$$SSL = SSE - SSW = 590.72 - 305 = 285.72$$

The corresponding mean squares are given by

$$MSE = \frac{SSE}{N-2} = \frac{590.72}{12} = 49.23$$

$$MSM = SSM$$

$$MSW = \frac{SSW}{N-k} = \frac{305}{4} = 76.25$$

$$MSB = \frac{SSB}{k-1} = \frac{27272.71}{9} = 3030.3$$

$$MSL = \frac{SSL}{k-2} = \frac{285.72}{8} = 35.72$$

In summary, the ANOVA table is

	Source	df	SS	MS	F
SSM	Regression (Model)	1	26986.98	26986.98	
SSL	Lack of fit	8	285.72	35.72	0.468
SSW	Pure Error (Residual)	4	305	76.25	
SST	Total	13	27577.7		

iii) $F = \frac{MSB}{MSW} = \frac{3030.3}{76.25} = 39.742 \sim F_{9,4}$. The critical region is $(5.999, +\infty)$. In conclusion the hypothesis $H_0 : b = 0$ is rejected. Hence, the number of components repaired is statistically significant in determining the length of service call.

iv) $F = \dfrac{SSL/(k-2)}{SSW/(N-k)} = \dfrac{MSL}{MSW} = 0.468 \sim F_{8,4}$, p-value=0.83.

The critical region is $(6.041, +\infty)$. In conclusion the hypothesis H_0 that the model is true is accepted.

EXERCISE 3.12

i) We calculate

$$\hat{b} = \frac{S_{xy}}{S_{xx}} = \frac{207.23}{251.46} = 0.8241$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} = 12.479 - 0.8241 \times 10.458 = 3.8606 \ .$$

Therefore the fitted regression line equation is

$$y = 3.8606 + 0.8241x \ .$$

ii) Given $SSW = 5.605$, we have

$$SST = S_{yy} = 182.74$$

$$SSE = RSS = S_{yy} - \frac{S_{xy}^2}{S_{xx}} = 182.74 - \frac{207.23^2}{251.46} = 11.96$$

$$SSM = SST - SSE = 182.74 - 11.96 = 170.78$$

$$SSB = SST - SSW = 182.74 - 5.604 = 177.136$$

$$SSL = SSE - SSW = 11.96 - 5.605 = 6.355$$

The corresponding mean squares are given by

$$MSE = \frac{SSE}{N-2} = \frac{11.96}{22} = 0.544$$
$$MSM = SSM$$
$$MSW = \frac{SSW}{N-k} = \frac{5.605}{12} = 0.467$$
$$MSB = \frac{SSB}{k-1} = \frac{177.135}{11} = 16.103$$
$$MSL = \frac{SSL}{k-2} = \frac{6.355}{10} = 0.6355$$

In summary, the ANOVA table is

	Source	df	SS	MS	F
SSM	Regression (Model)	1	170.78	170.78	
SSL	Lack of fit	10	6.355	0.6355	1.3606
SSW	Pure Error (Residual)	12	5.604	0.467	
SST	Total	23	182.74		

iii) $F = \frac{MSB}{MSW} = \frac{16.103}{0.467} = 34.482 \sim F_{11,12}$. The critical region is $(2.717, +\infty)$. In conclusion the hypothesis $H_0 : b = 0$ is rejected. Hence, the variable 'Group length' is statistically significant in explaining the response 'Width'.

iv)

$$F = \frac{SSL/(k-2)}{SSW/(N-k)} = \frac{MSL}{MSW} = \frac{6.355/10}{5.605/12} = 1.3606 \sim F_{10,12}, \text{ p-value} = 0.302$$

The critical region is $(2.753, +\infty)$. In conclusion the hypothesis H_0 that the model is true cannot be rejected.