Probability

Random Varibales

- For a given sample space S of some experiment, a random variable (rv) is any rule that associates a number with each outcome in S.

Discrete

Bernoulli random variable

- Any random variable whose only possible values are 0 and 1 is called a Bernoulli random variable

PMS

- The probability mass function (pmf，概率质量函数) of a discrete $rv\ X$ is defined for every number x by

$$p(x) = P(X = x) = P(all s \in S : X(s) = x).$$

- Proposition

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

CDF

- The cumulative distribution function (cdf) $F(x)$ of a discrete rv variable X with pmf $p(x)$ is defined for every number x by

$$F(x) = P(X \leq x) = \sum_{y:y\leq x} p(y)$$

Binomial experiment 二项试验

- An experiment is called a binomial experiment if it satisfies

1. The experiment consists of a sequence of n smaller experiments called trials, where n is fixed in advance of the experiment.

2. Each trial can result in one of the same two possible outcomes (dichotomous trials), which we denote by success (S) and failure (F).
3. The trials are independent, so that the outcome on any particular trial does not influence the outcome on any other trial.
4. The probability of success is constant from trial to trial; we denote this probability by p.

binomial random variable（二项随机变量）

- The binomial random variable X associated with a binomial experiment consisting of n trials is defined as
  X=the number of S's among the n trials
  We often write X~Bin(n, p) to indicate that X is a binomial rv based on n trials with success probability p.
- Notation:
  Because the pmf of a binomial rv X depends on the two parameters n and p, we denote the pmf by b(x;n, p).

$$b(x;n,p)=\begin{cases} \binom{n}{x}p^x(1-p)^(n-x), x=0,1,2,\ldots,n \\ 0, otherwise \end{cases}$$

- $E[x]=np$
- $V[x]=np(1-p$

Poisson distribution （泊松分布）

- A random variable X is said to have a Poisson distribution with parameter(λ>0) if the pmf of X is

$$P(X;\lambda)=\frac{e^{-\lambda}\lambda^x}{x!}, x=0,1,\ldots$$

- According to this proposition, in any binomial experiment in which n is large and p is small, b(x;n, p)≈p(x;λ), where λ=np. (二项分布n→∞, p→0, 近似为泊松分布)
- $E[x]=V[x]=\lambda$
- Proof:

The Geometric Distribution（几何分布）

- Suppose that independent trials, each having a probability p, 0<p<1, of being a success, are performed until a success occurs. If we let X equal the number of trials required, then(成功一次)
- $$p\left(x\right)=\begin{cases} p(1-p)^{x-1}, x=1,2,3,\ldots \\ 0, otherwise \end{cases}$$
- $E[x]=\frac{1}{p}$

- $V[x] = \frac{1-p}{p^2}$

Negative Binomial Random Variable（负二项分布）

- Suppose that independent trials, each having probability p, 0<p<1, of being a success are performed until a total of r successes is accumulated(成功r次)
- $E[x] = \frac{r}{p}$
- $V[x] = \frac{r(1-p)}{p^2}$

Hypergeometric Random Variable（超几何分布）

- Suppose that a sample of size n is to be chosen randomly (without replacement) from an urn containing N balls, of which m are white and N−m are black. If we let X denote the number of white balls selected, then(成功抽出指定种类的物件的次数-质检)

$$p(x) = P\{X = i\} = \frac{\binom{m}{i}\binom{N-m}{n-i}}{\binom{N}{n}}, i = 0, 1, \ldots, n$$

Continuous Random Variables(连续型随机变量)

- We say that X is a continuous random variable if there exists a nonnegative function f, defined for all real x∈(−∞, ∞), having the property that for any set B of real numbers,

$$P(X \in B) = \int_B f(x)\,dx$$

The Normal Distribution（正态分布）

- A continuous rv X is said to have a normal distribution with parameters μ and σ (μ and σ^2), where −∞<μ<+∞ and σ>0, if the pdf of X is

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Notation: X~N(μ,σ^2)

Standard Normal Distribution（标准正态分布）

- The normal distribution with parameter values μ=0 and σ=1 is called the standard normal distribution. A random variable having a standard normal distribution is called a standard normal random variable and will be denoted by Z. The pdf of Z is

$$f = (z; 0, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Nonstandard Normal Distribution

- If X has a normal distribution with mean μ and standard deviation σ, then

$$Z = \frac{X - \mu}{\sigma}$$

The DeMoivre-Laplace limit theorem（棣莫弗一拉普拉斯局部极限定理）

- If S_n denotes the number of successes that occur when n independent trials, each resulting in a success with probability p, are performed, then, for any a<b,(S_n代表的是Bernoulli Expermint成功的次数)

$$P\left\{a \leq \frac{S_n - np}{\sqrt{np(1-p)}} \leq b\right\} \rightarrow \Phi\left(b\right) - \Phi\left(a\right)$$

The Exponential Distribution（指数分布）

X is said to have an exponential distribution with parameter λ(λ>0) if the pdf of X is

$$f\left(x;\lambda\right) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, otherwise \end{cases}$$

- $E[x] = \frac{1}{\lambda}$
- $V[x] = \frac{1}{\lambda^2}$

The Gamma Distribution（伽马分布）

- A random variable is said to have a gamma distribution with parameters (α, λ), λ>0,α>0, if its density function is given by

$$f\left(x;\alpha,\lambda\right) = \begin{cases} \frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)} \end{cases}$$

- Gamma function

$$\Gamma\left(\alpha\right) = \int_0^\infty e^{-y} y^{\alpha-1} dy$$

- $E[x] = \frac{\alpha}{\lambda}$
- $V[x] = \frac{\alpha}{\lambda^2}$