

Formulas Sheet for Final Exam

Sheet 1. Vectors and the Geometry of Space. Vector-Valued Functions and Motion in Space.

Equation for a Plane

The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ has

Vector equation: $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$

Component equation: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

Component equation simplified: $Ax + By + Cz = D,$ where
 $D = Ax_0 + By_0 + Cz_0$

Parametric Equations for a Line

The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty \quad (3)$$

The graph $y = f(x)$ in the xy -plane automatically has the parametrization $x = x, y = f(x)$, and the vector formula $\mathbf{r}(x) = x\mathbf{i} + f(x)\mathbf{j}$. If f is a twice-differentiable function of x , then its curvature $\kappa(x)$ can be expressed by

$$\kappa(x) = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}.$$

Sheet 2. Partial Derivatives.

THEOREM 8—A Formula for Implicit Differentiation

Suppose that $F(x, y)$ is differentiable and that the equation $F(x, y) = 0$ defines y as a differentiable function of x . Then at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = -\frac{F_x}{F_y}. \quad (1)$$

THEOREM 9—The Directional Derivative Is a Dot Product

If $f(x, y)$ is differentiable in an open region containing $P_0(x_0, y_0)$, then

$$\left(\frac{df}{ds}\right)_{\mathbf{u}, P_0} = \nabla f|_{P_0} \cdot \mathbf{u}, \quad (4)$$

the dot product of the gradient ∇f at P_0 with the vector \mathbf{u} . In brief, $D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$.

Coordinate Conversion Formulas

CYLINDRICAL TO RECTANGULAR

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

SPHERICAL TO RECTANGULAR

$$x = \rho \sin \phi \cos \theta$$

$$y = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

SPHERICAL TO CYLINDRICAL

$$r = \rho \sin \phi$$

$$z = \rho \cos \phi$$

$$\theta = \theta$$

Corresponding formulas for dV in triple integrals:

$$\begin{aligned} dV &= dx \, dy \, dz \\ &= dz \, r \, dr \, d\theta \\ &= \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \end{aligned}$$

THEOREM 4—Green's Theorem (Circulation-Curl or Tangential Form)

Let C be a piecewise smooth, simple closed curve enclosing a region R in the plane. Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ be a vector field with M and N having continuous first partial derivatives in an open region containing R . Then the counterclockwise circulation of \mathbf{F} around C equals the double integral of $(\text{curl } \mathbf{F}) \cdot \mathbf{k}$ over R .

$$\oint_C \mathbf{F} \cdot \mathbf{T} \, ds = \oint_C M \, dx + N \, dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx \, dy \quad (3)$$

Counterclockwise circulation Curl integral

THEOREM 5—Green's Theorem (Flux-Divergence or Normal Form)

Let C be a piecewise smooth, simple closed curve enclosing a region R in the plane. Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ be a vector field with M and N having continuous first partial derivatives in an open region containing R . Then the outward flux of \mathbf{F} across C equals the double integral of $\text{div } \mathbf{F}$ over the region R enclosed by C .

$$\oint_C \mathbf{F} \cdot \mathbf{n} \, ds = \oint_C M \, dy - N \, dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx \, dy \quad (4)$$

Outward flux Divergence integral

THEOREM 13—The Ratio Test

Let $\sum a_n$ be any series and suppose that

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \rho.$$

Then (a) the series *converges absolutely* if $\rho < 1$, (b) the series *diverges* if $\rho > 1$ or ρ is infinite, (c) the test is *inconclusive* if $\rho = 1$.

THEOREM 14—The Root Test

Let $\sum a_n$ be any series and suppose that

$$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \rho.$$

Then (a) the series *converges absolutely* if $\rho < 1$, (b) the series *diverges* if $\rho > 1$ or ρ is infinite, (c) the test is *inconclusive* if $\rho = 1$.

THEOREM 15—The Alternating Series Test

The series

$$\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if the following conditions are satisfied:

1. The u_n 's are all positive.
2. The u_n 's are eventually nonincreasing: $u_n \geq u_{n+1}$ for all $n \geq N$, for some integer N .
3. $u_n \rightarrow 0$.

1. **The n th-Term Test for Divergence:** Unless $a_n \rightarrow 0$, the series diverges.
2. **Geometric series:** $\sum ar^n$ converges if $|r| < 1$; otherwise it diverges.
3. **p -series:** $\sum 1/n^p$ converges if $p > 1$; otherwise it diverges.
4. **Series with nonnegative terms:** Try the Integral Test or try comparing to a known series with the Direct Comparison Test or the Limit Comparison Test. Try the Ratio or Root Test.
5. **Series with some negative terms:** Does $\sum |a_n|$ converge by the Ratio or Root Test, or by another of the tests listed above? Remember, absolute convergence implies convergence.
6. **Alternating series:** $\sum a_n$ converges if the series satisfies the conditions of the Alternating Series Test.