Problem Sheet 1

MA1202, Introductory Statistics

Due date - 13/03/2022, 23:59 GMT

General information

Please upload your work to Blackboard as a single pdf document which is of good quality. Read the **Instructions on Scanning and Uploading handwritten work**. Please name your file *PS1*YourName.pdf.

Please submit to Blackboard only solutions to questions from Section 1.

Please prepare questions from Section 2 for Feedback Session - you are expected to participate in discussion of these questions, your input will contribute to the participation mark.

Section 1. [to be submitted to Blackboard by 13/03/22]

Question 1. (Revision on probability distribution functions).
Let X be a random variable with the exponential distribution with parameter 1, which means that the probability density function of X is

$$f_X(x) = \begin{cases} e^{-x}, & \text{for } x \geq 0, \\ 0, & \text{for} x < 0. \end{cases}$$

i) How do you find $P(a \leq X \leq b)$, the probability that a value of X lies between a and b, using the density function?

ii) Compute each of $P(X < 1)$, $P(1 \leq X \leq 2)$ and $P(X \geq 2)$.

iii) Suppose you sample 1000 independent observations of the random variable X. What are the expected numbers of observations lying in each of the intervals $[0, 1)$, $[1, 2)$ and $[2, \infty)$?

Question 2.

Let X be a random variable with pdf

$$f(t) = \begin{cases} \frac{3}{64}(x)^2(4-x), & 0 \leq x \leq 4, \\ 0, & \text{elsewhere} \end{cases}$$

i) Find expected value and variance for X.

ii) Let $Y = 300X + 50$, find expected value and variance for Y.
iii) Find $P(Y > 750)$.

Section 2. [to be discussed in Tutorial on 17/03/22]

Question 3. Here is a simulated data sample:

2.93	2.37	2.19	3.26	3.48	2.09
3.13	2.98	4.03	3.43	2.68	3.50
3.11	2.21	3.12	2.79	2.79	3.24
3.20	3.27				

i) Explain in words how to compute the first quartile Q1 for this data.
ii) Find the sample mean, median, sample variance (unbiased), first quartile, third quartile and interquartile range of the sample above, rounded to 3 decimal points.

Question 4.
Let $X \geq 0$ be an integer-valued random variable such that $P(X = n) = p_n$.
Show that

$$E(X) = \sum_{i=1}^{\infty} P(X \geq i)$$