

$$Q1. E(\hat{p}) = E\left(\frac{X+3}{n+4}\right) = \frac{E(X)}{n+4} + \frac{3}{n+4} = \frac{\mu+3}{n+4} \neq \frac{\mu}{n}$$

Hence, it is a biased estimator of the binomial parameter p

$$B_n(\hat{p}, p) = \frac{\mu+3}{n+4} - \frac{\mu}{n} = \frac{3}{n+4} - \frac{4\mu}{n(n+4)}$$

$$\lim_{n \rightarrow \infty} B_n(\hat{p}, p) = \lim_{n \rightarrow \infty} \left(\frac{3}{n+4} - \frac{4\mu}{n(n+4)} \right) = 0$$

Hence we say that it is an asymptotically unbiased estimator

$$Q2. \frac{d \ln f_X(x, \theta)}{d\theta} = \frac{d}{d\theta} \left(-\ln \theta - \frac{x}{\theta} \right) = -\frac{1}{\theta} + \frac{x}{\theta^2}$$

$$\frac{d^2}{d\theta^2} \ln f_X(x, \theta) = \frac{1}{\theta^2} - \frac{2x}{\theta^3}$$

$$\text{Hence CRLB} = - \frac{1}{n E \left[\frac{d^2}{d\theta^2} \ln f_X(x, \theta) \right]} = \frac{1}{\frac{2n E[X]}{\theta^3} - \frac{n}{\theta^2}}$$

$$\text{Since } E[X] = \theta$$

$$\text{CRLB} = \frac{\theta^2}{n}$$