UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 7

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ANNOUNCEMENTS

- Welcome Back!

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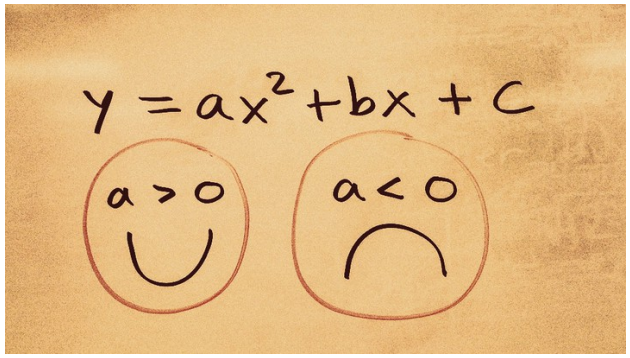


$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

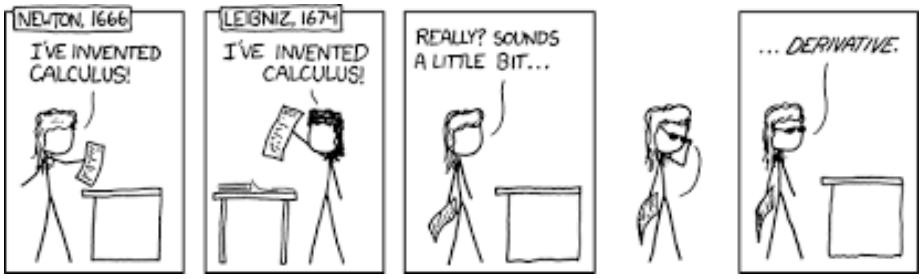
$$\int_a^b f(x)dx = F(b) - F(a)$$

L

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$

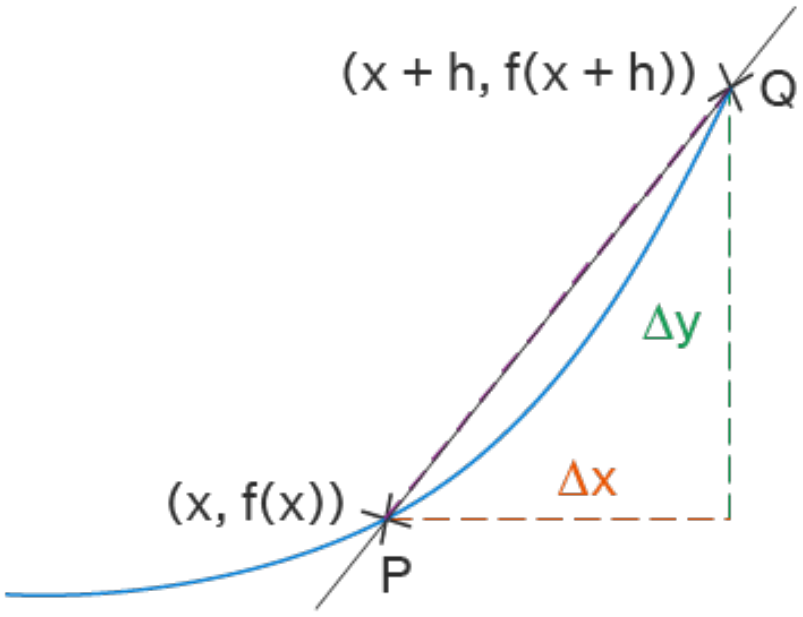


THE DERIVATIVE

- Rate of Change of a function w.r.t a variable

- Definition:

$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



DIFFERENTIATION FROM FIRST PRINCIPLES

Example:

Let $f(x) = \frac{1}{x}$ such that $x > 0$, determine $\frac{df}{dx}$.

EXERCISE

Consider $f(x) = x^3 + x^2$. Determine $f'(x)$ using first principles.

EXERCISE

Let

$$g(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ -\sqrt{-x}, & x < 0 \end{cases}$$

Show that $g(x)$ is continuous at $x = 0$. Is $g(x)$ differentiable at $x = 0$?

CHALLENGE:

From First Principles, show that

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

Hints:

- Binomial Theorem: $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

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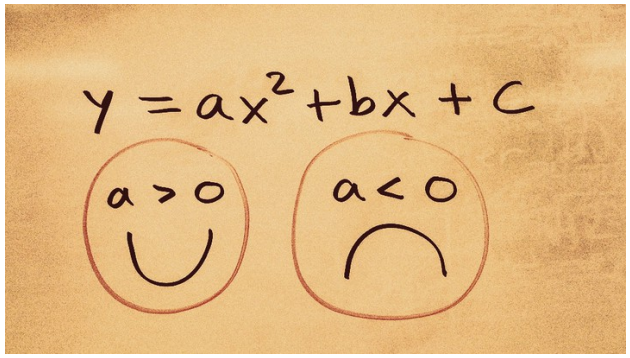

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