UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 13

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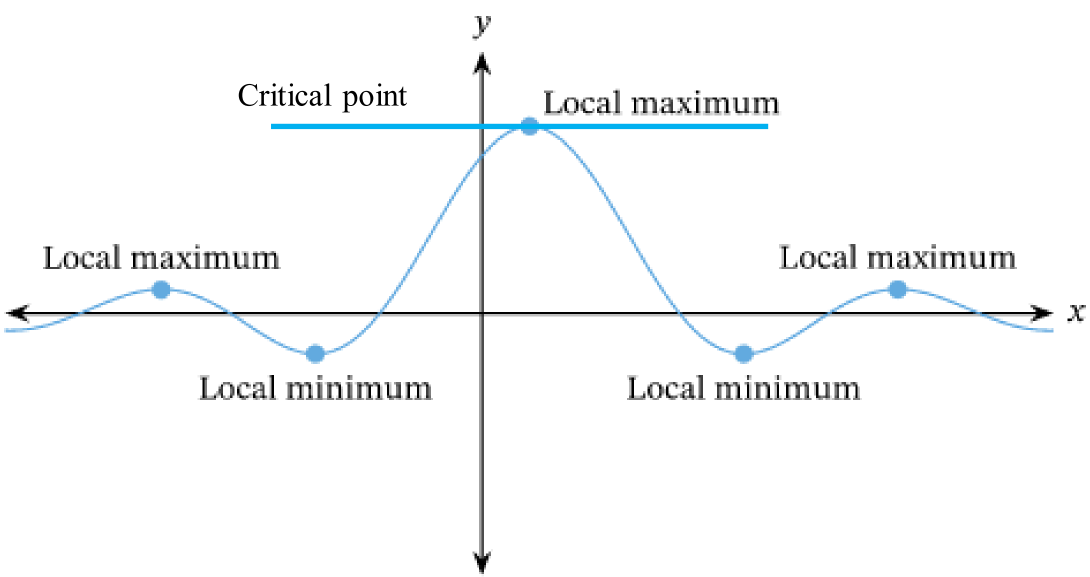
LOCAL EXTREMA

- If $f(c) \leq f(x)\ \forall x$ in the neighbourhood of c, this is called a **local minimum**.
- If $f(c) \geq f(x)\ \forall x$ in the neighbourhood of c, this is called a **local maximum**.
- **Critical point**: A point where $f'(x) = 0$ i.e. the tangent line is horizontal/constant or $f'(x)$ does not exist.

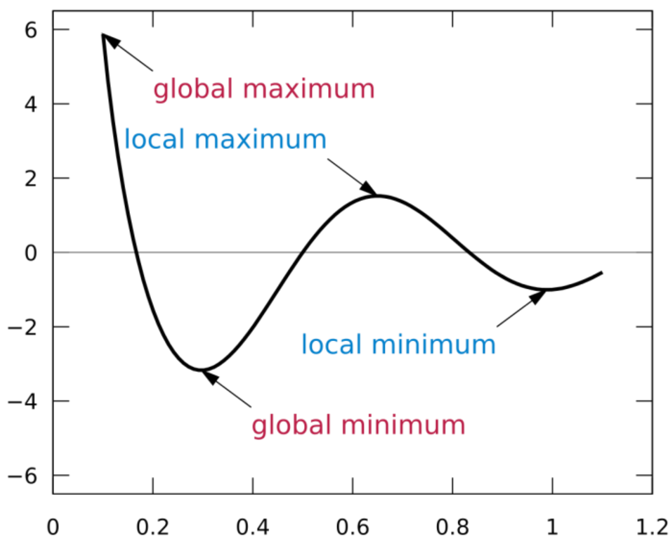


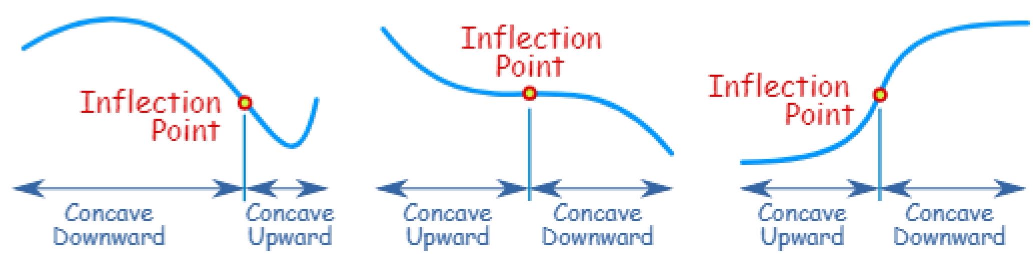
GLOBAL EXTREMA

- If $f(c) \leq f(x)\ \forall x \in \mathrm{dom}(f)$ this is called the **global minimum**

- If $f(c) \geq f(x)\ \forall x \in \mathrm{dom}(f)$ this is called the **global maximum**

- If at $(c, f(c))$ the concavity changes, then this is called an **Inflection point**.





SECOND DERIVATIVE TEST

Suppose $f'(c) = 0$ and $f''(c)$ exists.

i. If $f''(c) < 0$ then f has a local maximum at c
ii. If $f''(c) > 0$ then f has a local minimum at c
iii. If $f''(c) = 0$ then f has an inflection point at c

EXAMPLE

Find and classify the critical points of $f(x) = -3x^5 + 5x^3$.

EXERCISE

Find and classify the critical points of $f(x) = -\frac{(x+2)^2(x-4)^2}{8}$. Find all points of inflection, and, hence, sketch a graph of $f(x)$.

EXERCISE

The deflection D of a beam of length L is

$D = 2x^4 - 5Lx^3 + 3L^2x^2$, where x is the distance from one end of the beam. Find the value of x that yields the maximum deflection.

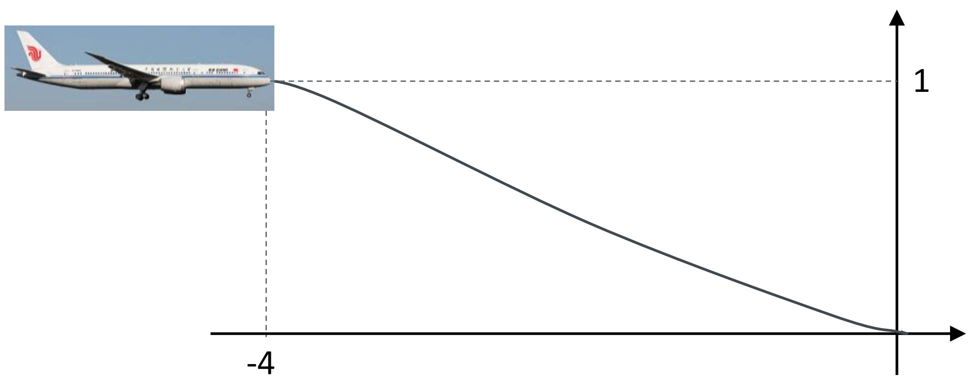
EXERCISE

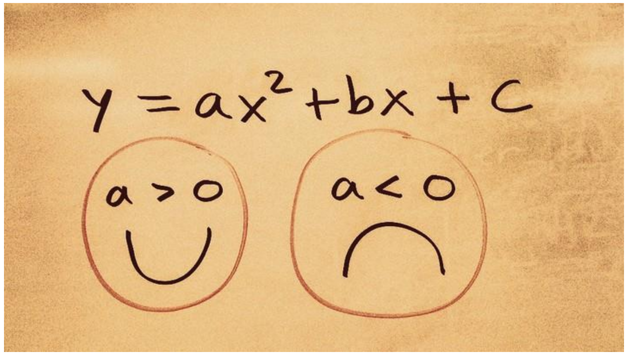
A small aircraft starts its descent from an altitude of 1km, 4km west of the runway.

a) Find the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ where $x \in [-4,0]$ that describes the smooth glide path for the landing.

b) The function $f(x)$ models the glide path of the plane. When would the plane be descending at the greatest rate.

c) If $x(t) = -4 + t^2 : t \in [0,2]$, calculate $f(t)$ and hence determine the velocity of the plane. When does it travel at its fastest?





$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$