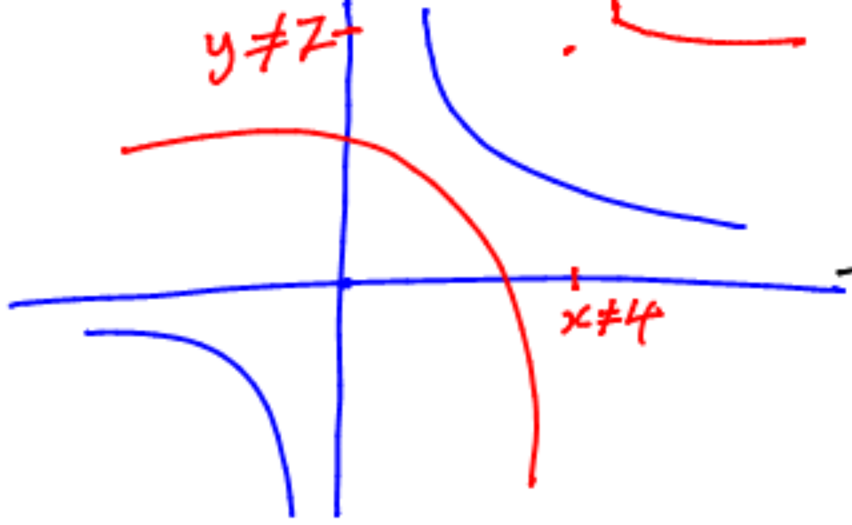
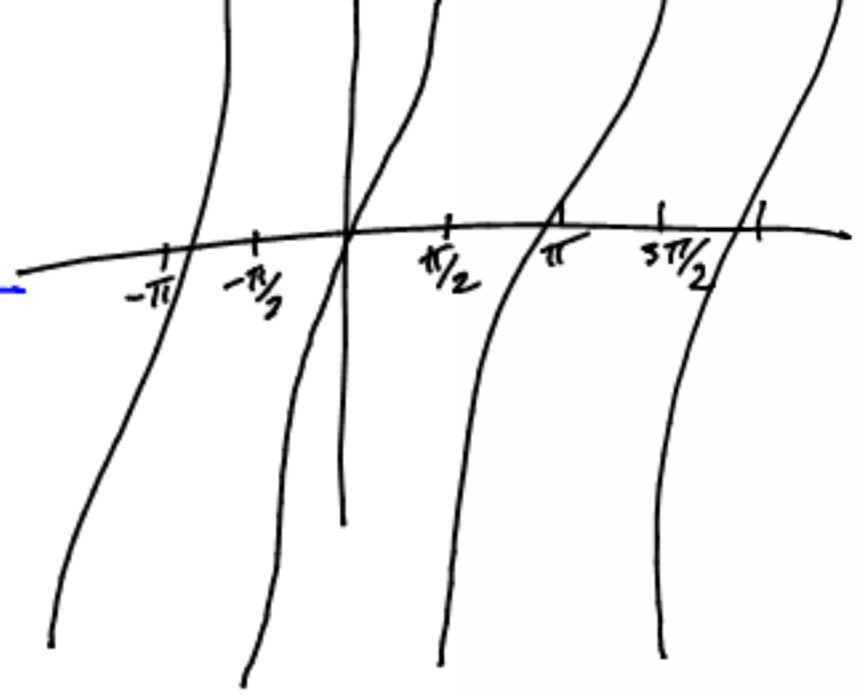
Yesterday : functions. domain & range

$f(x) = \frac{1}{x-4} + 7$ $\mathbb{R} \setminus \{4\}$ $\mathbb{R} \setminus \{7\}$

$f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$ $\{x : x \neq \frac{2k+1}{2}\pi\}$ $\mathbb{R}$





We can only find inverse functions $f^{-1}(x)$ if the function $f(x)$ is ONE-TO-ONE

$$\boxed{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$$

Example : $f(x) = \frac{2x+3}{3x+5}$ $3x+5=0$, $x = -\frac{5}{3}$ · Domain $= \mathbb{R} \setminus \{-\frac{5}{3}\}$

· Prove 1-1 : $f(x_1) = f(x_2) \Rightarrow \frac{2x_1+3}{3x_1+5} = \frac{2x_2+3}{3x_2+5} \Rightarrow (2x_1+3)(3x_2+5) = (2x_2+3)(3x_1+5)$

$\Rightarrow \cancel{6x_1x_2} + 10x_1 + 9x_2 + \cancel{15} = \cancel{6x_1x_2} + 10x_2 + 9x_1 + \cancel{15} \Rightarrow x_1 = x_2$

- Find $f^{-1}(x)$

$$y = \frac{2x+3}{3x+5}$$

$$\Rightarrow 3xy + 5y = 2x+3$$

$$\Rightarrow 3xy - 2x = -5y+3$$

$$\Rightarrow x = \frac{-5y+3}{3y-2}$$

So $f^{-1}(x) = \dfrac{-5x+3}{3x-2}$

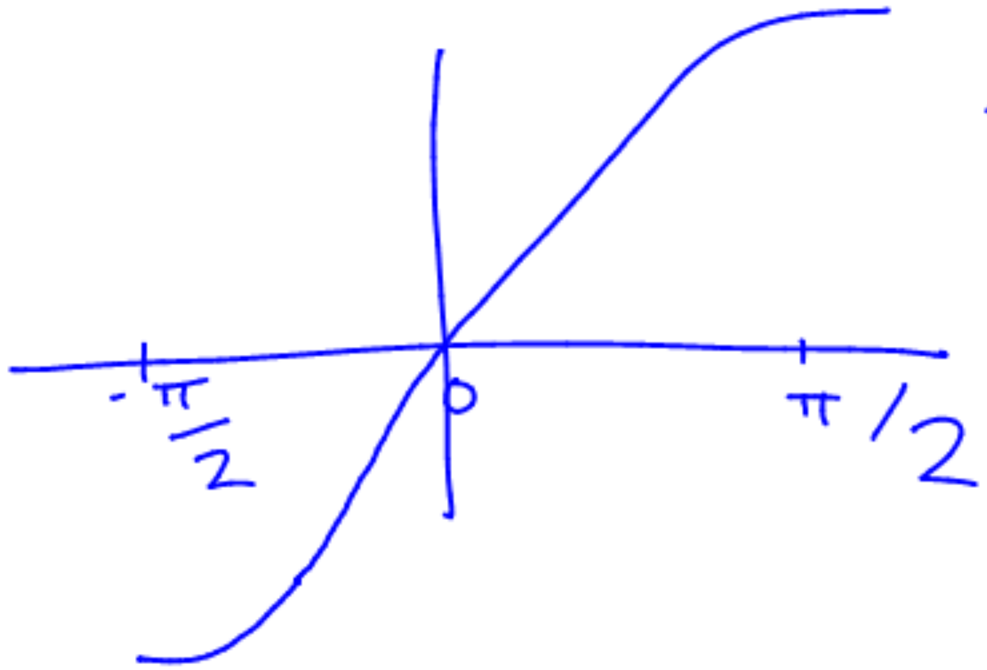
Range of f

$=$ Domain of f^{-1}

$= \mathbb{R} \setminus \{\frac{2}{3}\}$

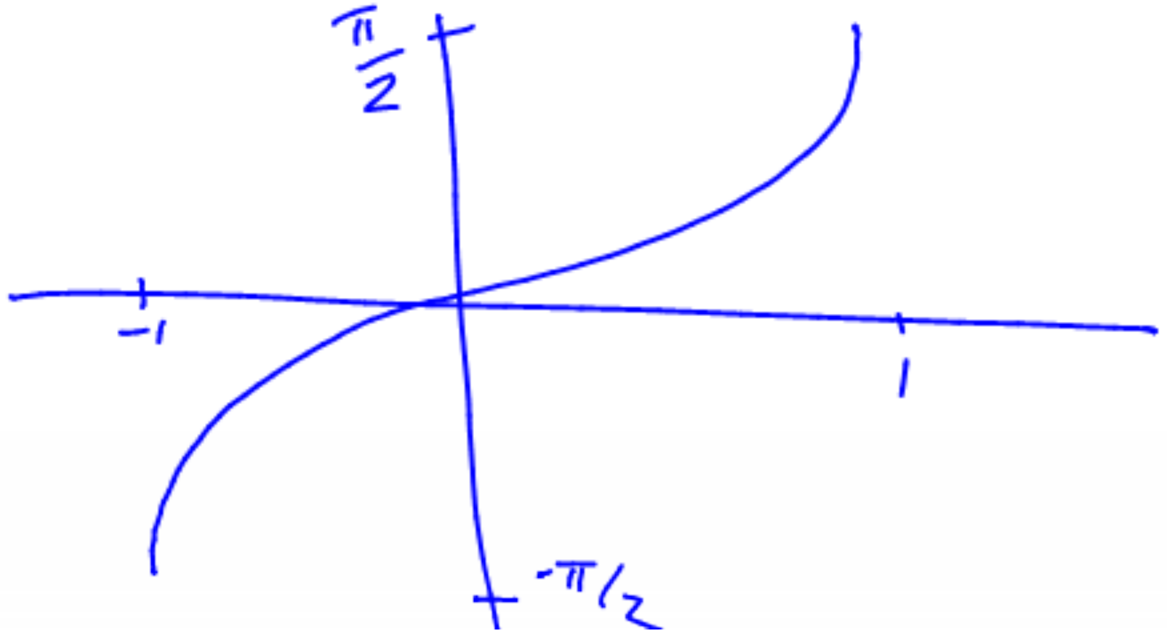
Trig. Examples.

sin : $\underbrace{[-\frac{\pi}{2}, \frac{\pi}{2}]}_{\text{Domain}} \longrightarrow \underbrace{[-1, 1]}_{\text{Range}}$



is one-to-one.

$\sin^{-1}$ or arcsin



Similarly :

$\cos : [0, \pi] \to [-1, 1]$

$\cos^{-1} : [-1, 1] \to [0, \pi]$

$\tan : (-\frac{\pi}{2}, \frac{\pi}{2}) \to \mathbb{R}$

$\tan^{-1} : \mathbb{R} \longrightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

Polynomials

$$f(x) = 3x^2 - 7x + 4$$

or $f(x) = \displaystyle\sum_{k=0}^{n} a_k x^k$

$$= a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

General Polynomial of degree n.

Coefficients $a_i \in \mathbb{R}$

$$f(-x) = f(x) \quad \forall x \in \text{domain.}$$

$$f(x) = 3x^4 + 9x^6 - 7x^2 + \frac{\;}{x^{12}}$$

$$f(x) = \cos(x) \qquad f(x) = x\sin(x)$$

A function f is <u>odd</u>

$$f(-x) = -f(x)$$

$$f(x) = \frac{1}{x^{11}} + 2x^3 \qquad f(x) = \tan^{-1}(x)$$

$$f(x) = \sin(x), \qquad f(x) = x^3\cos(x)$$

Excercise: odd · odd = even

$$f(-x) \cdot g(-x) = (-f(x)) \cdot (-g(x))$$
$$= f(x) \cdot g(x)$$

If g even

Then $h(x) = f(g(x))$

(composition of functions)

$$h(-x) = f(\underbrace{g(-x)}_{g \text{ even}}) = f(\underline{g(x)}) = h(x)$$