More Examples for Lecture 3.

MA2032 Vector Calculus

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Angle Between Vectors

Example 1

Find the measures of the angles between the diagonals of the rectangle whose vertices are $A = (1, 0)$, $B = (0, 3)$, $C = (3, 4)$, and $D = (4, 1)$.

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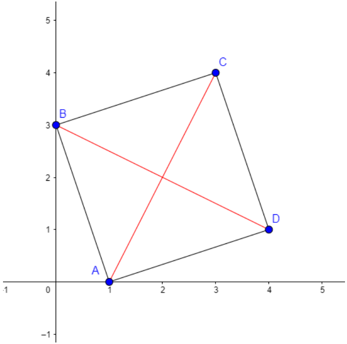
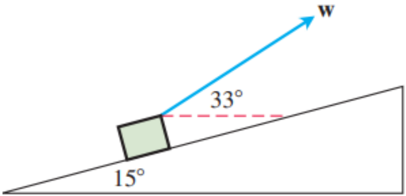
Solution:

$\overrightarrow{AC} = \langle 2, 4 \rangle$ and $\overrightarrow{BD} = \langle 4, -2 \rangle$. $\overrightarrow{AC} \cdot \overrightarrow{BD} = 2(4) + 4(-2) = 0$, so the angle measures are all $90°$.



Example 2

Suppose that a box is being towed up an inclined plane as shown in the figure. Find the force w needed to make the component of the force parallel to the inclined plane equal to 2.5 lb.



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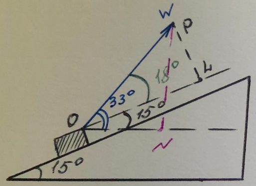

Solution:

$|OL| = 2.5 \Rightarrow \cos 18^\circ = \dfrac{|OL|}{|OP|}$

$|OL| = \cos 18^\circ \cdot |W|$

$|W| = \dfrac{2.5}{\cos 18^\circ}$

$W = |W| \cdot \underbrace{\dfrac{W}{|W|}}_{\text{Unit vector in the direction of } W \text{ (UV)} \Rightarrow} = \dfrac{2.5}{\cos 18^\circ} \cdot \langle \cos 33^\circ, \sin 33^\circ \rangle \approx \langle 2.2, 1.4 \rangle$

UV from $\Delta OWN = \langle \cos 33^\circ, \sin 33^\circ \rangle$

Example 3

Find an equation for the line through $P(2, 1)$ perpendicular to $v = i + 2j$. Then sketch the line. Include v in your sketch as a vector starting at the origin.

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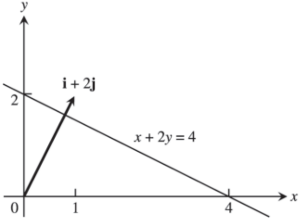
Solution:

$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$ is perpendicular to the line $x + 2y = c$;

$P(2, 1)$ on the line $\Rightarrow 2 + 2 = c \Rightarrow x + 2y = 4$



Equations for Lines in the Plane

Example 4

Find an equation for the line through $P(0, -2)$ parallel to $v = 2i + 3j$. Then sketch the line. Include v in your sketch as a vector starting at the origin.

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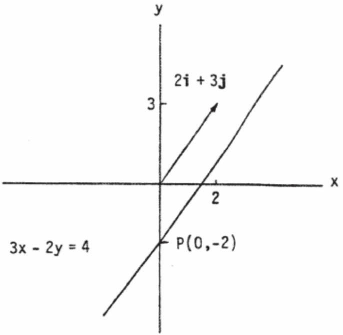
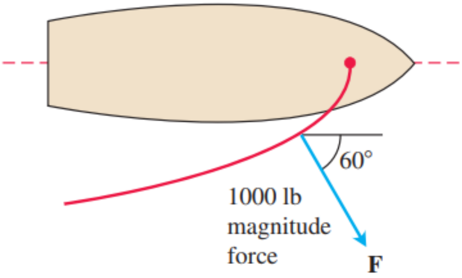
Solution:

$\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ is parallel to the line $3x - 2y = c$;
$P(0, -2)$ on the line $\Rightarrow 0 - 2(-2) = c \Rightarrow 3x - 2y = 4$



Example 5

The wind passing over a boat's sail exerted a 1000-lb magnitude force F as shown here. How much work did the wind perform in moving the boat forward 1 mi? Answer in foot-pounds.



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Solution:

$$\mathbf{W} = |\mathbf{F}|\left|\overrightarrow{PQ}\right|\cos\theta = (1000)(5280)(\cos 60^\circ) = 2,640,000 \text{ ft}\cdot\text{lb}$$