Interim Test. 2022

MA1202, Introductory Statistics

19 May 2022, 10:00AM

Please do not turn this page over until you are told you can start.

General information

Please read this information before you start.

- You have 60 minutes for this test.
- Please write your student number on each page of your work.
- Please number all your pages.
- There are 2 questions.
- Overall there are 20 marks.
- You can use recommended calculator.
- There is a statistical table at the end of the document that might be useful for the test, you do not need any other tables.
  Please read the description of the table before you use it.
- Advice:
  - Read the questions carefully.
  - Read through your answers at the end.

Question 1 [10 points]

The Army Physical Fitness Test consists of two minutes of push-ups followed by two minutes of sit-ups, and is completed with a two-mile run.

Table 1 presents 34 run times in seconds for a group of male army officers.

Table 1. Data set of run times in seconds (sorted in ascending order).

707	712	719	720	740	763	774
778	780	785	791	801	816	821
828	832	836	837	845	861	870
875	879	880	881	886	919	921
934	935	939	954	961	973	

i) Construct a relative frequency distribution for the data, taking as your grouping intervals the following 6 intervals :

$$[0,750), [750,800), [800,850), [850,900), [900,950), [950,\infty).$$

Describe the main characteristics of the obtained distribution: modality, symmetry, skewness.

ii) On the basis of the frequency table, find the grouped mean and the grouped variance; use 725 and 975 as the mid-point values for the first and the last intervals.

iii) Assuming that the average run time has a normal distribution:

- explain in your words, how we can find a confidence interval for the population mean on the basis of the sample;
  state all assumptions that should be taken into account;
- write the pivot random variable and state its distribution;
- if $\sum_{i=1}^{34} x_i = 28553$ and $\sum_{i=1}^{34} x_i^2 = 24172899$, find the sample variance for X;
- find the 90% confidence interval for the population mean.

Round your answers to 3 d.p. where it is applicable.

Question 2 [10 points]

1. [6 points] Suppose that $X \sim Bin(12, p)$ (a binomial distribution) and consider the point estimator for the parameter p:

$$\hat{p} = \frac{X}{14}$$

i) What is the bias of this point estimator?

ii) What is the variance of this point estimator?

iii) Show that this point estimator has a mean square error of

$$\frac{3p - 2p^2}{49}$$

2. [**4 points**] If X_1 and X_2 are independent random variables having binomial distributions with the parameters n_1 and θ ($X_1 \sim Bin(n_1, \theta)$) and n_2 and θ ($X_2 \sim Bin(n_2, \theta)$), show that

$$\hat{\theta} = \frac{X_1 + X_2}{n_1 + n_2}$$

is a sufficient estimator of θ.

APPENDIX

The table below is extracted from the book
Reliability Engineering, First Edition. Kailash C. Kapur. John Wiley & Sons, Inc. (2014)

The following table gives values for cumulative standard normal distribution function. The probability density function for the standard normal random variable, z, is:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \ -\infty < z < \infty.$$

The cumulative distribution function is given by:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} dx, \ -\infty < z < \infty.$$

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-x^2/2} dx.$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	$0.9^{2}0097$	$0.9^{2}0358$	$0.9^{2}0613$	$0.9^{2}0863$	$0.9^{2}1106$	$0.9^{2}1344$	$0.9^{2}1576$
2.4	$0.9^{2}1802$	$0.9^{2}2024$	$0.9^{2}2240$	$0.9^{2}2451$	$0.9^{2}2656$	$0.9^{2}2857$	$0.9^{2}3053$	$0.9^{2}3244$	$0.9^{2}3431$	$0.9^{2}3613$
2.5	$0.9^{2}3790$	$0.9^{2}3963$	$0.9^{2}4132$	$0.9^{2}4297$	$0.9^{2}4457$	$0.9^{2}4614$	$0.9^{2}4766$	$0.9^{2}4915$	$0.9^{2}5060$	$0.9^{2}5201$
2.6	$0.9^{2}5339$	$0.9^{2}5473$	$0.9^{2}5604$	$0.9^{2}5731$	$0.9^{2}5855$	$0.9^{2}5975$	$0.9^{2}6093$	$0.9^{2}6207$	$0.9^{2}6319$	$0.9^{2}6427$
2.7	$0.9^{2}6533$	$0.9^{2}6636$	$0.9^{2}6736$	$0.9^{2}6833$	$0.9^{2}6928$	$0.9^{2}7020$	$0.9^{2}7110$	$0.9^{2}7197$	$0.9^{2}7282$	$0.9^{2}7365$
2.8	$0.9^{2}7445$	$0.9^{2}7523$	$0.9^{2}7599$	$0.9^{2}7673$	$0.9^{2}7744$	$0.9^{2}7814$	$0.9^{2}7882$	$0.9^{2}7948$	$0.9^{2}8012$	$0.9^{2}8074$
2.9	$0.9^{2}8134$	$0.9^{2}8193$	$0.9^{2}8250$	$0.9^{2}8305$	$0.9^{2}8359$	$0.9^{2}8411$	$0.9^{2}8462$	$0.9^{2}8511$	$0.9^{2}8559$	$0.9^{2}8605$
3.0	$0.9^{2}8650$	$0.9^{2}8694$	$0.9^{2}8736$	$0.9^{2}8777$	$0.9^{2}8817$	$0.9^{2}8856$	$0.9^{2}8893$	$0.9^{2}8930$	$0.9^{2}8965$	$0.9^{2}8999$
3.1	$0.9^{3}0324$	$0.9^{3}0646$	$0.9^{3}0957$	$0.9^{3}1260$	$0.9^{3}1553$	$0.9^{3}1836$	$0.9^{3}2112$	$0.9^{3}2378$	$0.9^{3}2636$	$0.9^{3}2886$
3.2	$0.9^{3}3129$	$0.9^{3}3363$	$0.9^{3}3590$	$0.9^{3}3810$	$0.9^{3}4024$	$0.9^{3}4230$	$0.9^{3}4429$	$0.9^{3}4623$	$0.9^{3}4810$	$0.9^{3}4991$
3.3	$0.9^{3}5166$	$0.9^{3}5335$	$0.9^{3}5499$	$0.9^{3}5658$	$0.9^{3}5811$	$0.9^{3}5959$	$0.9^{3}6103$	$0.9^{3}6242$	$0.9^{3}6376$	$0.9^{3}6505$
3.4	$0.9^{3}6631$	$0.9^{3}6752$	$0.9^{3}6869$	$0.9^{3}6982$	$0.9^{3}7091$	$0.9^{3}7197$	$0.9^{3}7299$	$0.9^{3}7398$	$0.9^{3}7493$	$0.9^{3}7585$
3.5	$0.9^{3}7674$	$0.9^{3}7759$	$0.9^{3}7842$	$0.9^{3}7922$	$0.9^{3}7999$	$0.9^{3}8074$	$0.9^{3}8146$	$0.9^{3}8215$	$0.9^{3}8282$	$0.9^{3}8347$
3.6	$0.9^{3}8409$	$0.9^{3}8469$	$0.9^{3}8527$	$0.9^{3}8583$	$0.9^{3}8637$	$0.9^{3}8689$	$0.9^{3}8739$	$0.9^{3}8787$	$0.9^{3}8834$	$0.9^{3}8879$
3.7	$0.9^{3}8922$	$0.9^{3}8964$	$0.9^{4}0039$	$0.9^{4}0426$	$0.9^{4}0799$	$0.9^{4}1158$	$0.9^{4}1504$	$0.9^{4}1838$	$0.9^{4}2159$	$0.9^{4}2568$
3.8	$0.9^{4}2765$	$0.9^{4}3052$	$0.9^{4}3327$	$0.9^{4}3593$	$0.9^{4}3848$	$0.9^{4}4094$	$0.9^{4}4331$	$0.9^{4}4558$	$0.9^{4}4777$	$0.9^{4}4988$
3.9	$0.9^{4}5190$	$0.9^{4}5385$	$0.9^{4}5573$	$0.9^{4}5753$	$0.9^{4}5926$	$0.9^{4}6092$	$0.9^{4}6253$	$0.9^{4}6406$	$0.9^{4}6554$	$0.9^{4}6696$
4.0	$0.9^{4}6833$	$0.9^{4}6964$	$0.9^{4}7090$	$0.9^{4}7211$	$0.9^{4}7327$	$0.9^{4}7439$	$0.9^{4}7546$	$0.9^{4}7649$	$0.9^{4}7748$	$0.9^{4}7843$
4.1	$0.9^{4}7934$	$0.9^{4}8022$	$0.9^{4}8106$	$0.9^{4}8186$	$0.9^{4}8263$	$0.9^{4}8338$	$0.9^{4}8409$	$0.9^{4}8477$	$0.9^{4}8542$	$0.9^{4}8605$
4.2	$0.9^{4}8665$	$0.9^{4}8723$	$0.9^{4}8778$	$0.9^{4}8832$	$0.9^{4}8882$	$0.9^{4}8931$	$0.9^{4}8978$	$0.9^{5}0226$	$0.9^{5}0655$	$0.9^{5}1066$
4.3	$0.9^{5}1460$	$0.9^{5}1837$	$0.9^{5}2199$	$0.9^{5}2545$	$0.9^{5}2876$	$0.9^{5}3193$	$0.9^{5}3497$	$0.9^{5}3788$	$0.9^{5}4066$	$0.9^{5}4332$
4.4	$0.9^{5}4587$	$0.9^{5}4831$	$0.9^{5}5065$	$0.9^{5}5288$	$0.9^{5}5502$	$0.9^{5}5706$	$0.9^{5}5902$	$0.9^{5}6089$	$0.9^{5}6268$	$0.9^{5}6439$
4.5	$0.9^{5}6602$	$0.9^{5}6759$	$0.9^{5}6908$	$0.9^{5}7051$	$0.9^{5}7187$	$0.9^{5}7318$	$0.9^{5}7442$	$0.9^{5}7561$	$0.9^{5}7675$	$0.9^{5}7784$
4.6	$0.9^{5}7888$	$0.9^{5}7987$	$0.9^{5}8081$	$0.9^{5}8172$	$0.9^{5}8258$	$0.9^{5}8340$	$0.9^{5}8419$	$0.9^{5}8494$	$0.9^{5}8566$	$0.9^{5}8634$
4.7	$0.9^{5}8699$	$0.9^{5}8761$	$0.9^{5}8821$	$0.9^{5}8877$	$0.9^{5}8931$	$0.9^{5}8983$	$0.9^{6}0320$	$0.9^{6}0789$	$0.9^{6}1235$	$0.9^{6}1661$
4.8	$0.9^{6}2067$	$0.9^{6}2453$	$0.9^{6}2822$	$0.9^{6}3173$	$0.9^{6}3508$	$0.9^{6}3827$	$0.9^{6}4131$	$0.9^{6}4420$	$0.9^{6}4696$	$0.9^{6}4958$
4.9	$0.9^{6}5208$	$0.9^{6}5446$	$0.9^{6}5673$	$0.9^{6}5889$	$0.9^{6}6094$	$0.9^{6}6289$	$0.9^{6}6475$	$0.9^{6}6652$	$0.9^{6}6821$	$0.9^{6}6981$

Example: $\Phi(3.39) = 0.9996505$, $\Phi(0.98) = 0.8365$.