UNIVERSITY OF LEICESTER

MA7404
All Candidates

Semester 1 Examinations 2020

DO NOT OPEN THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE CHIEF INVIGILATOR	
School	Mathematics and Actuarial Sciences
Module Code	MA7404
Module Title	Markov processes
Exam Duration	Two hours
CHECK YOU HAVE THE CORRECT QUESTION PAPER	
Number of Pages	3
Number of Questions	4
Instructions to Candidates	Answer all questions
FOR THIS EXAM YOU ARE ALLOWED TO USE THE FOLLOWING:	
Calculators	Approved calculators may be used
Books/Statutes provided by the University	Formulae and tables for actuarial examinations
Are students permitted to bring their own Books/Statutes/Notes?	No
Additional Stationery	No

1. An insurance company is interested in estimating the total size S of claims they will receive next week. They model it as $S = X_1 + X_2 + \dots X_N$, where the number of claims N follows a negative binomial distribution with parameters $k = 3$ and p, and the claim sizes X_i are independent from each other and from N, identically distributed, and follow an exponential distribution with parameter λ. Parameters λ and p are unknown and should be estimated from the past data.

   (i) [5 marks] Formulate the method of moments for estimating unknown parameters of a distribution.

   (ii) [5 marks] The sizes of the last 10 claims were

   $$100, 250, 150, 400, 200, 230, 170, 100, 500 \text{ and } 400.$$

   Use the method of moments to estimate parameter λ.

   (iii) [5 marks] The numbers of claims for the last 10 weeks are $3, 5, 2, 0, 8, 4, 1, 0, 1$ and 3. Use the method of maximum likelihood to estimate parameter p.

   (iv) [5 marks] Estimate the mean and standard deviation of S.

   (v) [5 marks] What is the most likely number of claims during the next week?

   Total: 25 marks

2. The densities of non-negative random variables X and Y are $f_X(x) = e^{-x}, x \geq 0$, and $f_Y(y) = \frac{1}{(1+y)^2}, y \geq 0$, respectively, while their copula is $C(u,v) = \frac{uv}{u+v-uv}$.

   (i) [5 marks] Calculate the probability $P[X \leq \ln 2 \text{ and } Y \leq 3]$.

   (ii) [5 marks] Calculate the hazard rates for X and Y.

   (iii) [5 marks] By analysing the hazard rates of X and Y, determine which of these random variables has a heavier tail.

   (iv) [5 marks] Define the coefficients of lower and upper tail dependence for 2 random variables with copula $C(u,v)$.

   (v) [5 marks] Calculate the coefficient of lower tail dependence for X and Y.

   Total: 25 marks

3. A motor insurance company offers discounts of 0%, 20% and 50% of the full premium, determined by the following rules:

   (a) All new policyholders start at the 20% level.

   (b) If no claim is made during the current year the policyholder moves to/stays at the 50% level.

   (c) If one or more claims are made the policyholder moves down one discount level, or remains at the 0% level.

   Since this policy was introduced, there were:

   - $2,000$ cases when a policyholder had no discount. In 500 cases, a claim was made.
   - $5,000$ cases when a policyholder had a 20% discount. In $1,000$ cases, a claim was made.
   - $40,000$ cases when a policyholder had a 50% discount. In $2,000$ cases, a claim was made.

   (i) [5 marks] Write down the Chapman-Kolmogorov equations for a Markov chain, explaining all notation used.

   (ii) [5 marks] Model the process above as a Markov chain. Determine the state space and transition matrix.

   (iii) [5 marks] Estimate the probability that a policyholder initially at the 20% level is at the 20% level after 3 years.

   (iv) [5 marks] Calculate the stationary distribution.

   (v) [5 marks] Assuming that this policy operates for a long time, estimate the average discount over all policyholders.

   Total: 25 marks

4. To model mortality in a group of people aged 60 with no severe health problems known, an insurance company uses a simple two-state model with states A (alive) and D (dead), and transition rate λ from A to D. They would like to estimate parameter λ from past data. Each year from 2010 to 2019, they observed $1,000$ individuals in the given age group and found that the number of deaths was $8,7,5,5,6,4,5,3,4$ and 3 respectively.

   (i) [5 marks] List possible types of stochastic processes with respect to state space and time changes. To which type Markov jump process belongs?

   (ii) [5 marks] Give three different examples of problems which can be solved using machine learning techniques.

   (iii) [5 marks] Use linear regression with sum of squares error minimization to estimate the parameter λ to be used in 2020.

   (iv) [5 marks] Using λ obtained in part (iii), (or, if you cannot solve (iii), use $\lambda = 0.003$), estimate the probability that a person will survive during the whole 2020 year.

   (v) [5 marks] Repeat part (iv) assuming that $\lambda = \lambda(t)$ is not a constant but decreases linearly from $\lambda_0 = 0.004$ at the beginning of 2020 to $\lambda_1 = 0.003$ at the end of the year.

   Total: 25 marks

END OF PAPER