

Problem Sheet 2 for the Tutorial, October 6.

(Vectors and the Geometry of Space, Vector-Valued Functions and Motion in Space.)

Problem 1. Find a plane through the points $P_1(1, 2, 3)$, $P_2(3, 2, 1)$ and perpendicular to the plane $4x - y + 2z = 7$.

Solution:

Problem 2. Find the distance from the plane $x + 2y + 6z = 1$ to the plane $x + 2y + 6z = 10$.

Solution:

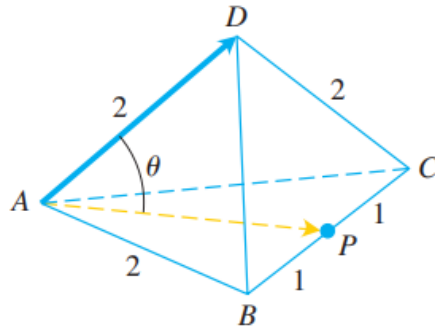
Problem 3. The planes $3x + 6z = 1$ and $2x + 2y - z = 3$ intersect in a line.

- a) Show that the planes are orthogonal.
- b) Find equations for the line of intersection.

Solution:

Problem 4. Consider a regular tetrahedron of side length 2.

- Use vectors to find the angle θ formed by the base of the tetrahedron and any one of its other edges.
- Use vectors to find the angle θ formed by any two adjacent faces of the tetrahedron. This angle is commonly referred to as a dihedral angle.



Solution:

Problem 5. Show that the vector-valued function

$$r(t) = (2i + 2j + k) + \cos t \left(\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j \right) + \sin t \left(\frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k \right)$$

describes the motion of a particle moving in the circle of radius 1 centered at the point $(2, 2, 1)$ and lying in the plane $x + y - 2z = 2$.

Solution: