Computer Assignment 3

1. Using MATLAB, find a solution (if it exists) to the following system of equations:

$$\begin{aligned} x - 2y + z &= -1 \\ 2x - 4y + z &= -1 \\ x + 2y - 2z &= 6 \\ x - 6y + 4z &= -8 \end{aligned} \tag{1}$$

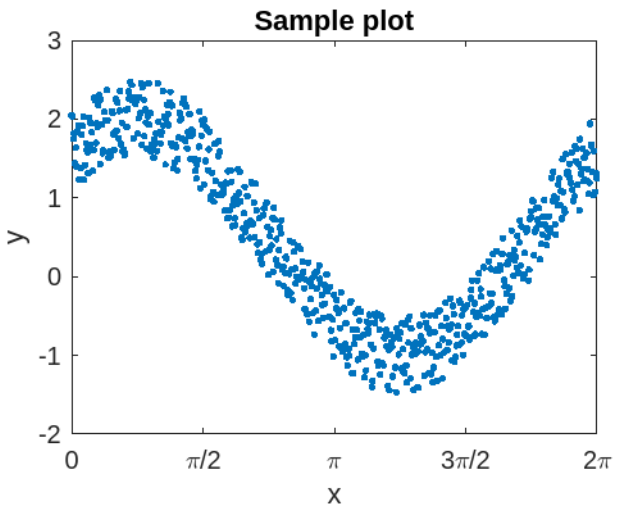
   Is your solution unique?

2. Find the general solution to the system of equations:

$$\begin{aligned} 2x - y + 2z - w &= 0 \\ x + y - z - w &= 0 \\ x - y - z + w &= 0 \end{aligned} \tag{2}$$

3. A matrix is called 'lower triangular' if all the elements above the main diagonal are zero. Let L be an nxn lower triangular matrix and b be a nx1 column vector. Then the system of equations Lx=b can be solved using 'forward substitution'. Write a function forwsubs(L,b) to perform this calculation. Use your function to solve this system when

$$\mathrm{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 1 & -1 & 3 & 0 \\ 2 & 1 & 4 & -2 \end{bmatrix} \text{ and } \mathrm{b} = \begin{bmatrix} 1 \\ 5 \\ 13 \\ 22 \end{bmatrix}.$$



4. Write a script file which plots a regression curve using least squares regression for the data shown in the above figure. The figure file sample_plot.fig is provided in the Blackboard. Assume the estimation function of the form $\hat{y}(x) = af_1(x)+bf_2(x)+cf_3(x)$ for appropriate choice of basis functions. Also, write the equation of the regression curve.

5. Write a function with header [yq]=linear_interpol(x,y,xq) where yq is the value at a query x-data point xq calculated using linear interpolation for given vectors x and y containing data points.

   Find the value of yq for the test case: x=[0 1 2 3], y=[1 4 3 6] and xq=1.2. You should not use MATLAB's interp1() function. You may use MATLAB's length() function to find the number of elements of a vector.

6. Write a script file which uses cubic spline interpolation to find the value of yq for the test case of Q5. You can use MATLAB's interp1() function for this question. Also plot the data points, the cubic spline curve and the query data point.

7. Write a function with header [yq]=Lagrange_interpol(x,y,xq) where yq is the value at a query x-data point xq calculated using Lagrange interpolation for given vectors x and y containing data points. (**Hint:** You may use nested for-loops and if-statement.) Find the value of yq for the same test case as in Q5.

8. Write a function with header out=pi_approx(k) which approximates the value of π using kth order Taylor polynomial of $sin^{-1}x$ centered at $x = 0$. Using your function, find the approximate value of π for k=21. You should use format long. You may assume the fact that Maclaurin series of $sin^{-1}x$ converges in the interval $-1 \leq x \leq 1$.

9. Write a script file which uses bisection method to find the roots of the equation $\frac{1}{x-2} + e^x = 0$. Explain what happens if one chooses the guess interval [1.9,2.1]. Provide reason for your answer.

10. Write a script file to find the maximum value of the function $f(x) = x^3 - e^x$ for tolerance 0.001 using Newton-Raphson method.