Problem Sheet 4 for the Tutorial, October 20.

(Partial Derivatives.)

Problem 1. Find and sketch the domain for each function

a) $f(x, y) = \ln(x^2 + y^2 - 4)$.

b) $f(x, y) = \frac{1}{\ln(4 - x^2 - y^2)}$

Solution:

Problem 2. Given a function $f(x, y) = e^{-(x^2+y^2)}$,

(a) find the function's domain,

(b) find the function's range,

(c) describe the function's level curves,

(d) find the boundary of the function's domain,

(e) determine if the domain is an open region, a closed region, or neither, and

(f) decide if the domain is bounded or unbounded.

Solution:

Problem 3. Find an equation for and sketch the graph of the level curve of the function $f(x,y) = \frac{2y-x}{x+y+1}$ that passes through the point $(-1, 1)$.

Solution:

Problem 4. Show that the limits do not exist

a)

$$\lim_{(x,y)\to(1,1)} \frac{xy^2 - 1}{y - 1}$$

b)

$$\lim_{(x,y)\to(0,1)} \frac{x \ln y}{x^2 + (\ln y)^2}$$

Solution:

Problem 5. At what points (x, y, z) in space is the functions continuous

a) $f(x, y, z) = \ln(xyz)$,

b) $f(x, y, z) = e^{x+y} \cos z$?

Solution: