Problem Sheet 11 for the Tutorial, December 8.

(Infinite Sequences and Series)

Problem 1. Which of the sequences $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence.

a) $\{a_n\} = \frac{1-n^3}{70-4n^2}$,

b) $\{a_n\} = \frac{\sin^2 n}{2^n}$,

c) $\{a_n\} = \frac{n}{2^n}$.

Solution:

Problem 2. Use the nth-Term Test for divergence to show that the series is divergent, or state that the test is inconclusive.

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$,

b) $\sum_{n=0}^{\infty} \frac{e^n}{e^n+n}$,

c) $\sum_{n=0}^{\infty} \cos(n\pi)$.

Solution:

Problem 3. Which series converge, and which diverge? Give reasons for your answers. If a series converges, find its sum.

a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3}{2^n}$,

b) $\sum_{n=1}^{\infty}\ln\frac{1}{3^n}$,

c) $\sum_{n=1}^{\infty}\frac{2^n+4^n}{3^n+4^n}$.

Solution:

Problem 4. Use the Integral Test to determine if the series converge or diverge. Be sure to check that the conditions of the Integral Test are satisfied.

a) $\sum_{n=1}^{\infty} \frac{1}{n+4}$,

b) $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$,

c) $\sum_{n=1}^{\infty} \frac{7}{\sqrt{n+4}}$.

Solution:

Problem 5. Use the Limit Comparison Test to determine if each series converges or diverges.

a) $\sum_{n=2}^{\infty} \frac{n\,(n+1)}{(n^2+1)(n-1)}$,

b) $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}\;4^n}$,

c) $\sum_{n=1}^{\infty} \left(\frac{2n+3}{5n+4}\right)^n$.

Solution: