UNIVERSITY OF LEICESTER

MA1014
CALCULUS AND ANALYSIS
TUTORIAL 4

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ANNOUNCEMENTS

- No Tutorials: 1/10-7/10

LIMITS

Let $f(x) = e^x$, what would happen if I let x get very large?

Try different values for x. What happens?

Now, what would happen if you let x get large but imposed that $x < 0$?

It should be clear that the following seem to happen,

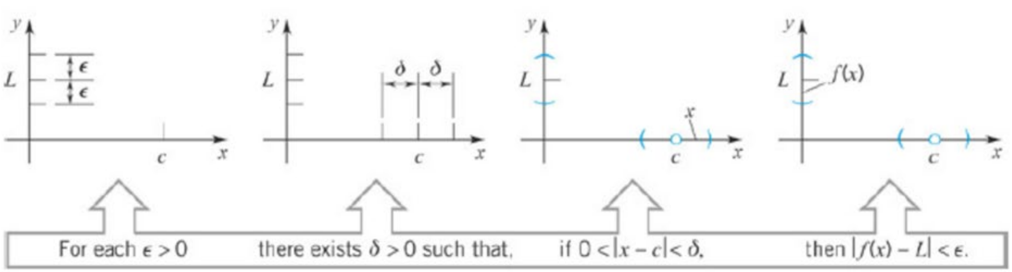
$$\lim_{x \to \infty} f(x) = \infty \ \& \ \lim_{x \to -\infty} f(x) = 0$$

FORMAL DEFINITION OF THE LIMIT

A function, f, has a limit, L, at $x = c \in (a, b) \subset \mathbb{R}$, if

$$\boxed{\forall \varepsilon > 0, \exists \delta : |f(x) - L| < \varepsilon, \forall |x - c| < \delta}$$

and $f(x)$ is well defined on $x \in (a, b) \backslash \{c\}$



EXAMPLE

Let $f(x) = x^2 - 4x + 5$. Show that $\lim_{x \to 2} f(x) = 1$ by using an $\varepsilon - \delta$ argument.

EXERCISE

Let $f(x) = -2x - 5$.

a) Find $A > 0$ such that if $0 < |x + 2| < A$ then $|f(x) + 1| < \frac{1}{200}$

b) Given $\varepsilon > 0$ find $\delta > 0$ so that if $0 < |x + 2| < \delta$ then $|f(x) + 1| < \varepsilon$.

c) Hence, prove $\lim_{x \to -2} f(x) = -1$.

EXERCISE

Let $h(x) = x^2 - 2x + 3$. Show that

$$\lim_{x \to -1} h(x) = 6.$$

EXERCISE

Let

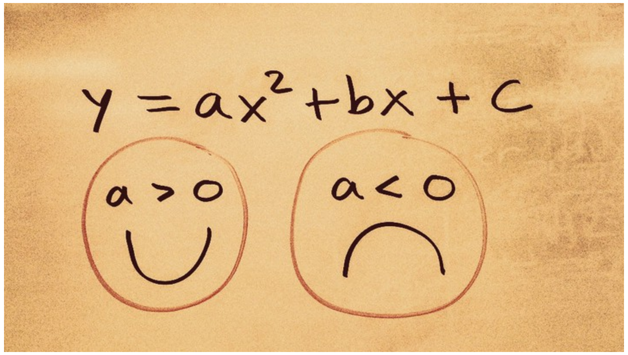
$$g(x) = \begin{cases} x^2 \quad , & x < 1 \\ 2 - x, & x \geq 1 \end{cases}$$

Show that $\lim_{x \to 1} g(x) = 1$.

EXTRA TIME

Can you modify the definition of the limit to accommodate limits at infinity (i.e. $\lim_{x \to \pm\infty} f(x)$)?

$$\boxed{\forall \varepsilon > 0, \exists \delta : |f(x) - L| < \varepsilon, \forall |x - c| < \delta}$$



$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

ANY QUESTIONS?

$$m\frac{d^2x}{dt^2} = -kx$$

$$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + C$$