

Homework 2: Operational Semantics for WHILE

CS 252: Advanced Programming Languages

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$e ::=$	x v $x := e$ $e; e$ $e \text{ op } e$ $e \text{ AND } e$ $e \text{ OR } e$ $e \text{ NOT } e$ $\text{if } e \text{ then } e \text{ else } e$ $\text{while } (e) \text{ } e$	<i>Expressions</i> variables/addresses values assignment sequential expressions binary operations AND operation OR operation NOT operation conditional expressions while expressions
$v ::=$	i b	<i>Values</i> integer values boolean values
$\text{op} ::=$	$+ \mid - \mid * \mid / \mid > \mid >= \mid < \mid <=$	<i>Binary operators</i>

Figure 1: The WHILE language

Runtime Syntax:

$$\sigma \in Store = variable \rightarrow v$$

Evaluation Rules:

$$e, \sigma \rightarrow e', \sigma'$$

[SS-ACCESS-RED]	$\frac{x \in domain(\sigma) \quad \sigma(x) = v}{x, \sigma \rightarrow v, \sigma}$
[SS-ASSIGN-CONTEXT]	$\frac{e, \sigma \rightarrow e', \sigma'}{x := e, \sigma \rightarrow x := e', \sigma'}$
[SS-ASSIGN-RED]	$\frac{}{x := v, \sigma \rightarrow v, \sigma[x := v]}$
[SS-SEQ-CONTEXT]	$\frac{e_1, \sigma \rightarrow e'_1, \sigma'}{e_1; e_2, \sigma \rightarrow e'_1; e_2, \sigma'}$
[SS-SEQ-RED]	$\frac{}{v; e, \sigma \rightarrow e, \sigma}$
[SS-IF-CONTEXT]	$\frac{e_1, \sigma \rightarrow e'_1, \sigma'}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3, \sigma \rightarrow \text{if } e'_1 \text{ then } e_2 \text{ else } e_3, \sigma'}$
[SS-IFTRUE-RED]	$\frac{}{\text{if true then } e_1 \text{ else } e_2, \sigma \rightarrow e_1, \sigma}$
[SS-IFFALSE-RED]	$\frac{}{\text{if false then } e_1 \text{ else } e_2, \sigma \rightarrow e_2, \sigma}$
[SS-OP-CONTEXT-1]	$\frac{e_1, \sigma \rightarrow e'_1, \sigma'}{e_1 \text{ op } e_2, \sigma \rightarrow e'_1 \text{ op } e_2, \sigma'}$
[SS-OP-CONTEXT-2]	$\frac{e, \sigma \rightarrow e', \sigma'}{v \text{ op } e, \sigma \rightarrow v \text{ op } e', \sigma'}$
[SS-OP-RED]	$\frac{v = v_1 \text{ op } v_2}{v_1 \text{ op } v_2, \sigma \rightarrow v, \sigma}$
[SS-AND-CONTEXT]	$\frac{e_1, \sigma \rightarrow e'_1, \sigma'}{e_1 \text{ AND } e_2, \sigma \rightarrow e'_1 \text{ AND } e_2, \sigma'}$
[SS-AND-RED-1]	$\frac{e, \sigma \rightarrow e', \sigma'}{\text{true AND } e, \sigma \rightarrow e', \sigma'}$
[SS-AND-RED-2]	$\frac{}{\text{false AND } e, \sigma \rightarrow \text{false}, \sigma}$
[SS-OR-CONTEXT]	$\frac{e_1, \sigma \rightarrow e'_1, \sigma'}{e_1 \text{ OR } e_2, \sigma \rightarrow e'_1 \text{ OR } e_2, \sigma'}$
[SS-OR-RED-1]	$\frac{e, \sigma \rightarrow e', \sigma'}{\text{false or } e, \sigma \rightarrow e', \sigma'}$
[SS-OR-RED-2]	$\frac{}{\text{true OR } e, \sigma \rightarrow \text{true}, \sigma}$
[SS-NOT-CONTEXT]	$\frac{e, \sigma \rightarrow e', \sigma'}{NOT \ e, \sigma \rightarrow NOT \ e', \sigma'}$
[SS-NOT-RED-1]	$\frac{}{NOT \ \text{true}, \sigma \rightarrow \text{false}, \sigma}$
[SS-NOT-RED-2]	$\frac{}{NOT \ \text{false}, \sigma \rightarrow \text{true}, \sigma}$
[SS-WHILE]	$\frac{}{\text{while } (e_1) \ e_2, \sigma \rightarrow \text{if } e_1 \text{ then } e_2; \text{while } (e_1) \ e_2 \text{ else false}, \sigma}$

Figure 2: Small-step semantics for WHILE