Homework 2: Operational Semantics for WHILE

CS 252: Advanced Programming Languages
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$e ::=$		*Expressions*
	x	variables/addresses
	v	values
	$x := e$	assignment
	$e; e$	sequential expressions
	$e\ op\ e$	binary operations
	$e\ AND\ e$	AND operation
	$e\ OR\ e$	OR operation
	$e\ NOT\ e$	NOT operation
	if e then e else e	conditional expressions
	while $(e)\ e$	while expressions
$v ::=$		*Values*
	i	integer values
	b	boolean values
$op ::=$	$+\ \|\ -\ \|\ *\ \|\ /\ \|\ >\ \|\ >=\ \|\ <\ \|\ <=$	*Binary operators*

Figure 1: The WHILE language

Runtime Syntax:

$$\sigma \quad \in \quad Store \quad = \quad variable \ \rightarrow \ v$$

Evaluation Rules: $\boxed{e, \sigma \rightarrow e', \sigma'}$

$[\text{SS-ACCESS-RED}]$
$$\frac{x \in domain(\sigma) \qquad \sigma(x) = v}{x, \sigma \rightarrow v, \sigma}$$

$[\text{SS-ASSIGN-CONTEXT}]$
$$\frac{e, \sigma \rightarrow e', \sigma'}{x := e, \sigma \rightarrow x := e', \sigma'}$$

$[\text{SS-ASSIGN-RED}]$
$$\frac{}{x := v, \sigma \rightarrow v, \sigma[x := v]}$$

$[\text{SS-SEQ-CONTEXT}]$
$$\frac{e_1, \sigma \rightarrow e_1', \sigma'}{e_1; e_2, \sigma \rightarrow e_1'; e_2, \sigma'}$$

$[\text{SS-SEQ-RED}]$
$$\frac{}{v; e, \sigma \rightarrow e, \sigma}$$

$[\text{SS-IF-CONTEXT}]$
$$\frac{e_1, \sigma \rightarrow e_1', \sigma'}{\texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3, \sigma \rightarrow \texttt{if } e_1' \texttt{ then } e_2 \texttt{ else } e_3, \sigma'}$$

$[\text{SS-IFTRUE-RED}]$
$$\frac{}{\texttt{if true then } e_1 \texttt{ else } e_2, \sigma \rightarrow e_1, \sigma}$$

$[\text{SS-IFFALSE-RED}]$
$$\frac{}{\texttt{if false then } e_1 \texttt{ else } e_2, \sigma \rightarrow e_2, \sigma}$$

$[\text{SS-OP-CONTEXT-1}]$
$$\frac{e_1, \sigma \rightarrow e_1', \sigma'}{e_1 \ op \ e_2, \sigma \rightarrow e_1' \ op \ e_2, \sigma'}$$

$[\text{SS-OP-CONTEXT-2}]$
$$\frac{e, \sigma \rightarrow e', \sigma'}{v \ op \ e, \sigma \rightarrow v \ op \ e', \sigma'}$$

$[\text{SS-OP-RED}]$
$$\frac{v = v_1 \ op \ v_2}{v_1 \ op \ v_2, \sigma \rightarrow v, \sigma}$$

$[\text{SS-AND-CONTEXT}]$
$$\frac{e_1, \sigma \rightarrow e_1', \sigma'}{e_1 \ AND \ e_2, \sigma \rightarrow e_1' \ AND \ e_2, \sigma'}$$

$[\text{SS-AND-RED-1}]$
$$\frac{e, \sigma \rightarrow e', \sigma'}{\texttt{true} \ AND \ e, \sigma \rightarrow e', \sigma'}$$

$[\text{SS-AND-RED-2}]$
$$\frac{}{\texttt{false} \ AND \ e, \sigma \rightarrow \texttt{false}, \sigma'}$$

$[\text{SS-OR-CONTEXT}]$
$$\frac{e_1, \sigma \rightarrow e_1', \sigma'}{e_1 \ OR \ e_2, \sigma \rightarrow e_1' \ OR \ e_2, \sigma'}$$

$[\text{SS-OR-RED-1}]$
$$\frac{e, \sigma \rightarrow e', \sigma'}{\texttt{false} \ or \ e, \sigma \rightarrow e', \sigma'}$$

$[\text{SS-OR-RED-2}]$
$$\frac{}{\texttt{true} \ OR \ e, \sigma \rightarrow \texttt{true}, \sigma'}$$

$[\text{SS-NOT-CONTEXT}]$
$$\frac{e, \sigma \rightarrow e', \sigma'}{NOT \ e, \sigma \rightarrow NOT \ e', \sigma'}$$

$[\text{SS-NOT-RED-1}]$
$$\frac{}{NOT \ \texttt{true}, \sigma \rightarrow \texttt{false}, \sigma'}$$

$[\text{SS-NOT-RED-2}]$
$$\frac{}{NOT \ \texttt{false}, \sigma \rightarrow \texttt{true}, \sigma'}$$

$[\text{SS-WHILE}]$
$$\frac{}{\texttt{while } (e_1) \ e_2, \sigma \rightarrow \texttt{if } e_1 \texttt{ then } e_2; \texttt{while } (e_1) \ e_2 \texttt{ else false}, \sigma}$$

Figure 2: Small-step semantics for WHILE