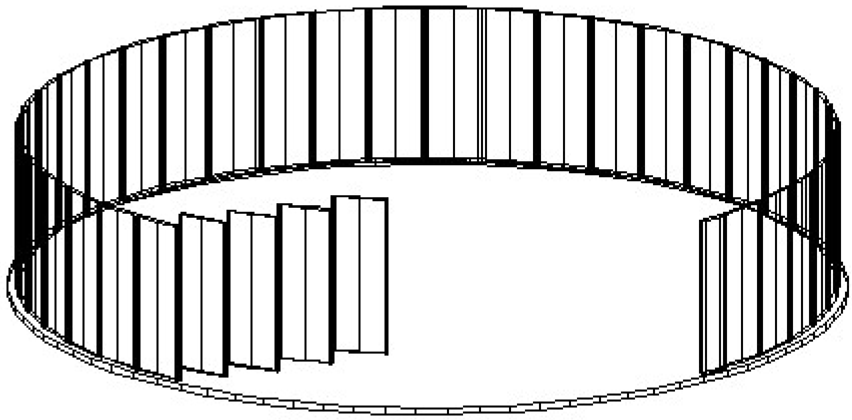
A-CONSULT Ltd

PCC Tank Static Calculation
Project 430773 Knapp Mill

Tank size:
N4500 32+2B
Diameter: 21.79m
Height: 4.50m



Aqua-Tank Precast Concrete Tanks
with Horizontal / Vertical Prestress & Elastic (EPDM) Sealing Strips in Wall Joints - Orthotropic Shell Design

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1.0 INTRODUCTION TO CALCULATIONS

1.1 General Description

These calculations cover the structural analysis, and design of reinforcement to DS EN 1992-1-1 Eurocode 2, for an Aqua / Agri -Tank. Bending moments, hoop tensions and shear forces are derived from first principles using formulae from Timoshenko. The tank construction comprises precast panel segments placed on a concrete base slab with vertical joints. Fabric mesh reinforcement is used to reinforce both sides of the panels. The joints are sealed using a patented proprietary EPDM strip. The calculations take account of the orthotropic behaviour of the tank system resulting from behaviour of these vertical joints. The tank panels are horizontally prestressed with the base of the panels allowed to slide horizontally (a free base condition). Tendons are located centrally in ducts within the wall panels. Two buttress panels are used for the horizontal post-tensioning of the tank structure. Following tensioning the base of the panels is concreted in (pinned base condition). Long term relaxation effects are also considered and the effect of this on increasing vertical bending moment with time is accounted for in the calculation of crack widths for the serviceability limit state.

The basic equation defined (and solved) in Timoshenko for circular cylindrical shells is as follows :

$$\frac{d^4}{dx^4} w + 4 \cdot \beta^4 \cdot w = \frac{Z}{D}$$

Where :

w = Radial displacement at height x above base of wall

x = Height above base of wall

β = a constant (defined later in these calculations) for a given tank radius, height and Poisson's ratio

Z = Load component perpendicular to wall per unit area, positive radially inwards

D = Flexural rigidity of shell wall

for linear load cases the membrane solution (i.e. with Z=0 so the right side of the equation reduces to 0) can be used.

These calculations should not be regarded as a computer program but as a series of proforma calculations requiring specific checking as to their validity for each tank situation to which they are applied.
The worksheet uses the assumptions in these papers (e.g. that for short shells an orthotropy factor corresponding to increasing load on the sealing strips is applicable to prestress relaxation, while for long shells an orthotropy ratio corresponding to decreasing load on the sealing strips is used). The validity of this design basis to the tanks as constructed is the responsibility of A-Consult.

1.1.1 Load cases Considered

The following load cases are considered in the calculations :

Ultimate Limit State

1.0 x Maximum Prestress (with no hydrostatic load) ULS for inside face vertical bending reinforcement
1.0 x Maximum Prestress + 1.5 x Maximum external soil and water loading

1.5 x Maximum Hydrostatic - 1.0 x Minimum Prestress ULS for outside face vertical bending reinforcement

Serviceability Limit State - Flexural

Initial Prestress after initial losses and prior to relaxation	Case A
Prestress after all losses and relaxation have taken place	Case B
Initial Prestress with Hydrostatic Loading	Case C
Prestress after relaxation with Hydrostatic Loading	Case D
Initial Prestress and External Soil and Water Loading	Case E
All External Loads after losses and relaxation	Case F
Initial Prestress, External Loads and Hydrostatic Loading	Case G
All loads including Hydrostatic after relaxation	Case H

Serviceability Limit State - Thermal

Minimum critical reinforcement required to be effective in distributing cracking
Crack widths due to temperature changes from hydration temperature to seasonal ambient temperature

1.2 Constants and Material Properties

1.2.1 Design lifetime

$t := 50\ yr$

1.2.2 Material Properties for Concrete

Characteristic Cube Strength $F_{ck.cube} := 50 \cdot MPa$ Poisson's Ratio $\nu := 0.2$

Cylinder strengths $F_{ck} := 40 \cdot MPa$

Strain modulus of concrete (short term) $E_{cs} := 9500 \cdot \left(\frac{F_{ck}}{MPa} + 8\right)^{\frac{1}{3}} \cdot \frac{N}{mm^2} = 34.5\ kN \cdot mm^{-2}$ = Ecm

Strain modulus of concrete (long term) (see later creep factor calculation) $E_{cl} := 13.4 \cdot kN \cdot mm^{-2}$ Density of concrete $\gamma_c := 2500 \cdot \frac{kg}{m^3}$

Material partial safety factor for concrete $\gamma_{mc} = 1.5$

Coefficient of thermal expansion $\alpha_c := 10 \cdot 10^{-6}$ per degree C

Tensile strength (modulus of rupture) $F_{ct} := 0.3 \cdot \left(\frac{F_{ck}}{MPa}\right)^{\frac{2}{3}} \cdot MPa = 3.509\ MPa$

1.2.3 Material Properties for Reinforcement

Characteristic yield strength of fabric reinforcement $f_{yf} := 500\ MPa$ = f_{yk}

Material partial safety factor for reinforcement $\gamma_{mr} = 1.15$ DIN 1045-1 5.3.3 & EC2 2.4.2.4

Strain modulus of reinforcement $E_s := 200\ kN \cdot mm^{-2}$ Poisson's Ratio $\nu_s := 0.3$

Coefficient of Thermal Expansion $\alpha_s := 10 \cdot 10^{-6}$ Density of steel $\gamma_s := 7850\ \frac{kg}{m^3}$

1.2.4 Material Properties for Prestressing Strand

Horizontal seven strand tendons - size and nominal area: $A_t :=$ Tendon: 15.3 mm $= 140\ mm^2$

Vertical seven strand tendons - size and nominal area: $A_{tv} :=$ Tendon: 12.5 mm $= 93\ mm^2$

Strain modulus of strand $E_p := 195\ kN \cdot mm^{-2}$

Linear loss (profile) coefficient $K := 0.025\ m^{-1}$

Coefficient of friction along tendon $\mu := 0.07$

Nominal tensile strength of strand

Horizontal $\begin{bmatrix} f_{yp} \\ f_{p01} \end{bmatrix} :=$ 1860 $= \begin{bmatrix} 1860 \\ 1660 \end{bmatrix} MPa$

Vertical $f_{ypv} := 1860\ MPa$ $f_{p01v} := 1660\ MPa$

Material partial safety factor for prestressing strand $\gamma_{ms} = 1.15$

Anchorage slip $\Delta x := 5\ mm$

1.2.5 Material Properties for Joint Sealing Strips (EPDM Ethylene Propylene Diene Monomer)

Strain modulus of sealing strips for increasing compressive load on strip $S_i := 47.3\ MPa$

Strain modulus of sealing strips for decreasing compressive load on strip $S_d := 170\ MPa$

A-Consult test data

1.2.6 Modular Ratios for Tendons

Modular ratio between tendons and concrete, initial(taken as zero for increasing load applied by the tendons to the concrete) $\alpha_{i.d} := \dfrac{E_p}{E_{cs}} = 5.648$ $\quad \alpha_{i.i} := 0$

Modular ratio between tendons and concrete, long term $\alpha_{t.d} := \dfrac{E_p}{E_{cl}} = 14.552$ $\quad \alpha_{t.i} := 0$

1.3 Tank Geometry, Contained Liquid and Reinforcement

1.3.1 Tank Geometric data

Depth of tank	$d := 4.5\ m$	Min. internal tank diameter	$D_m := 22.097\ m$
Depth of tank contents	$d_h := 4.2\ m$	Depth of Water External to tank	$d_w := 2.7\ m$
Depth of Earth External to tank	$d_e := 3.1\ m$	Tank regular wall panel width	$b := 2.11\ m$
Minimum panel thickness	$h_{min} \equiv 155\ mm$	Tank buttress panel width	$b_b := 1.05\ m$
Maximum panel thickness	$h_{max} \equiv h_{min} + 60\ mm = 215\ mm$		
Tank panel end thickening	$eb_t := h_{min} + 45\ mm = 200\ mm$		
Panel end thickening width	$eb_w := 120\ mm$		

Vertical load applied at top of tank from tank roof $P_{vr} := 0.0\ kN \cdot m^{-1}$

Surcharge pressure on ground outside tank wall $q_{es} := 5\ kN \cdot m^{-2}$

Tank wall thickness (average for calculations) $h \equiv \dfrac{h_{min} + h_{max}}{2} = 0.185\ m$

Radius on wall centre $r := \sqrt{\left(\dfrac{D_m}{2}\right)^2 + \left(\dfrac{b}{2}\right)^2} + \dfrac{h_{min}}{2} = 11.176\ m$

Max. internal radius $\sqrt{\left(\dfrac{D_m}{2}\right)^2 + \left(\dfrac{b}{2}\right)^2} = 11.099\ m$

Buttress panel thickness $h_b := h_{min} + 175\ mm = 330\ mm$

Number of standard panels $n_p := 32$

Number of buttress panels $n_b := 2$

Checks on panel numbers $2 \cdot n_p \cdot \operatorname{atan}\left(\dfrac{b}{D_m}\right) + 2 \cdot n_b \cdot \operatorname{atan}\left(\dfrac{b_b}{D_m}\right) = 360\ deg$ $\approx 360°$

$\dfrac{\pi}{\operatorname{atan}\left(\dfrac{b}{D_m}\right)} - \dfrac{n_b}{\operatorname{round}\left(\dfrac{2.11\ m}{b_b}\right)} = 32$ $\approx n_p$

1.3.2 Standard Panel Properties (see formulae in 17.0)

Area	$A_{pn} := A_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w\right)$	$A_{pn} = 3939.5\ cm^2$	$A_{pn} \cdot b^{-1} = 1867.1\ \dfrac{cm^2}{m}$
Centroid	$C_{pn} := C_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w\right)$	$C_{pn} = 94.126\ mm$	
Modulus	$I_{pn} := I_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w, C_{pn}\right)$	$I_{pn} = 120022.9\ cm^4$	$I_{pn} \cdot b^{-1} = 56882.9\ \dfrac{cm^4}{m}$

1.3.2 Sealing Strip Geometry

Width of sealing strips $a_s := 80\ mm$ Thickness of sealing strips $s_s := 6\ mm$

1.3.3 Properties of Contained Liquid, and Soil and Water Pressures External to tank

	Tank contents	External soil	External water
Specific Gravity (Density)	$\gamma := 10.3\ \frac{kN}{m^3}$	$\gamma_e := 19.0\ \frac{kN}{m^3}$	$\gamma_{ew} := 10.0\ \frac{kN}{m^3}$
		Dry density	

Partial safety factor for ULS load $\gamma_f := 1.5$ Earth Pressure Coefficient $k_{es} := 0.4$

1.3.4 Reinforcement Geometric Data

Cover to Main Reinforcement $cv_o := 55\ mm$ DC4, XF4, XC4

$cv_i := 55\ mm$ DC4, XF4, XC4

Vertical Reinforcement mesh in panels

Inside Face	Bar Diameter	$\phi_{vi} := 10\ mm$	Bar Spacing (pitch)	$p_{vi} := 100\ mm$
Outside Face	Bar Diameter	$\phi_{vo} := 10\ mm$	Bar Spacing (pitch)	$p_{vo} := 100\ mm$

Vertical loose bars

Inside Face	Bar Diameter	$\phi_{vli} := [8]^T\ mm$	Total no. of bars	$n_{vi} := [4]^T$
Outside Face	Bar Diameter	$\phi_{vlo} := [8]^T\ mm$	Total no. of bars	$n_{vo} := [2]^T$

Reinforcement Areas

$$\gamma_{ei} := \frac{p_{vi}}{b} \cdot \left(\text{floor}\left(\frac{b - 2 \cdot cv_i}{p_{vi}}\right) + 1\right) \qquad \gamma_{eo} := \frac{p_{vo}}{b} \cdot \left(\text{floor}\left(\frac{b - 2 \cdot cv_o}{p_{vo}}\right) + 1\right)$$

$$ki := \text{length}\left(\phi_{vli}\right) - 1 = 0$$

$$ko := \text{length}\left(\phi_{vlo}\right) - 1 = 0$$

$\gamma_{ei} = 0.995$ $\gamma_{eo} = 0.995$

$$As_{vi} := \frac{\pi \cdot \phi_{vi}^{\ 2}}{4} \cdot \frac{\gamma_{ei}}{p_{vi}} + \sum_{kk=0}^{ki} \left(\frac{\pi \cdot {\phi_{vli}}_{kk,0}^{\ 2}}{4} \cdot \frac{{n_{vi}}_{kk,0}}{b}\right) = 877\ \frac{mm^2}{m} \qquad As_{vi} \cdot b = 1850.4\ mm^2 \qquad \frac{As_{vi}}{h} = 0.47\%$$

$$As_{vo} := \frac{\pi \cdot \phi_{vo}^{\ 2}}{4} \cdot \frac{\gamma_{eo}}{p_{vo}} + \sum_{kk=0}^{ko} \left(\frac{\pi \cdot {\phi_{vlo}}_{kk,0}^{\ 2}}{4} \cdot \frac{{n_{vo}}_{kk,0}}{b}\right) = 829.3\ \frac{mm^2}{m} \qquad As_{vo} \cdot b = 1749.9\ mm^2 \qquad \frac{As_{vo}}{h} = 0.45\%$$

Distance from point on surface with maximum crack width to surface of nearest longitudinal bar

$$a_{cr.vi} := \sqrt{\left(\frac{p_{vi}}{2}\right)^2 + \left(cv_i + \frac{\phi_{vi}}{2}\right)^2} - \frac{\phi_{vi}}{2} = 73.1\ mm$$

$$a_{cr.vo} := \sqrt{\left(\frac{p_{vo}}{2}\right)^2 + \left(cv_o + \frac{\phi_{vo}}{2}\right)^2} - \frac{\phi_{vo}}{2} = 73.1\ mm$$

Horizontal Reinforcement mesh in panels (checked later against minimum to suit crack control criteria)

Inside Face	Bar Diameter	$\phi_{hi} := 8\ mm$	Bar Spacing (pitch)	$p_{hi} := 200\ mm$
Outside Face	Bar Diameter	$\phi_{ho} := 8\ mm$	Bar Spacing (pitch)	$p_{ho} := 200\ mm$

1.4 Tendon Arrangement and Vertical Prestress

1.4.1 Horizontal Tendon Levels and Vertical Tendon Prestress Force

The height of the tendons above the base of the tank are tabulated as follows (distances from bottom of wall panel):

$$tx_A := \begin{bmatrix} 0.2 & 0.4 & 0.5 & 2.0 & 2.1 & 2.4 & 2.7 & 2.9 & 3.2 & 4.1 \end{bmatrix}^T \, m$$

$$tx := \text{stack}\left(tx_A\right)$$

Number of tendons in height of wall $tn := \text{length}(tx) = 10$ $\quad i := 0 .. tn - 1$

Number of vertical tendons in each panel $tnv := 0$

Jack force as % of tendon capacity $jf1 := 75\%$ $\quad jf2 := 85\%$

Loss in vertical tendons % $lv := 12\%$

Anchorage configuration: $Bp_{config} :=$ Through−going $= 1$

Vertical prestress loading applied to tank panels (after losses) $P_{vp} := tnv \cdot min\left(\begin{bmatrix} jf1 \cdot f_{ypv} \\ jf2 \cdot f_{p01v} \end{bmatrix}\right) \cdot A_{tv} \cdot (1 - lv) \cdot b^{-1} = 0 \, \dfrac{kN}{m}$

Vertical tendon area per metre of wall $A_{vp} := tnv \cdot A_{tv} \cdot b^{-1} = 0 \, \dfrac{mm^2}{m}$

Depth to vertical tendons from inside face of panel $d_{pi} := 74 \, mm$ $\quad C_{pn} = 94.1 \, mm$ $\quad d_p := h - d_{pi}$

1.4.2 Mid Height Levels Between Horizontal Tendons

The mid point heights between tendons at which a change in hoop compression will take place are calculated as follows:

$$mx_i := \textbf{if}\left(i \neq tn - 1, \frac{tx_i + tx_{i+1}}{2}, d\right)$$

$$mx := \text{stack}(0, mx)$$

1.4.3 Height of Tank Stressed by Each Horizontal Tendon

The heights of the zones stressed by each of the tendons are calculated as follows :

$$zx_i := mx_{i+1} - mx_i$$

1.4.4 Initial Horizontal Tendon Hoop Forces

Initial Tendon Force $P_{ti} := 205 \, kN$ $\quad f_{yp} \cdot A_t = 260.4 \, kN$ $\quad \dfrac{P_{ti}}{f_{yp} \cdot A_t} = 78.7\%$ <80% ok $\quad \dfrac{P_{ti}}{f_{p01} \cdot A_t} = 88.2\%$ <90% ok

Initial Hoop Forces $tfi_i := \dfrac{P_{ti}}{zx_i}$ $\quad tfi_{tn} := tfi_{tn-1}$ $\quad tfi_{tn+1} := tfi_{tn-1}$

$$tfi_{tn} = 241176.471 \, \frac{kg}{s^2}$$

$$P_i(x) := \left\| \begin{array}{l} ii \leftarrow 0 \\ \text{while } mx_{ii} < x \\ \quad \| \; ii \leftarrow ii + 1 \\ tfi_{ii-1} \end{array} \right.$$

1.5 Number of Wall Elements for Calculations

Number of vertical wall elements for calculation $we := \dfrac{d}{100\ mm} = 45$ $\quad j := 1..we$

Height of Element $he := \dfrac{d}{we}$ $\quad he = 100\ mm$ $\quad th := 0.0001\ mm$

Counters for wall elements $xv := 0, he..d$ $\quad mxs_i := mx_i + th$

Counter for actual prestress levels
(x value in element and at each change of prestress) $xp_{j-1} := he \cdot j$ $\quad pv := \text{csort}(\text{stack}(xp, mx, mxs), 0)$

$jj := 1..2 \cdot \text{length}(pv)$

1.7 Prestress Loss Calculation - due to friction, anchorage slip and tendon steel relaxation

Losses in the horizontal tendons are considered due to linear friction loss, tendon curvature and anchorage slip. Losses will also occur as a result of relaxation of the prestressing steel and elastic deformation, shrinkage and creep of the concrete. The losses resulting from each of these factors are considered in turn below.

Circumferential distance to mid-span between butresses $x_\Delta := \dfrac{1}{2} \cdot \dfrac{2 \cdot \pi \cdot r}{n_b} = 17.56\ m$

Tank radius on wall centre line $r = 11.18\ m$

1.7.1 Loss Due to Linear Friction (assuming tendon is stressed from each buttress)

Maximum loss midway between buttresses

$\theta_\Delta := \dfrac{x_\Delta}{r} = 1.571$ $\quad K = 0.025\ m^{-1}$ $\quad \mu = 0.07$

$\dfrac{1}{\mu \cdot \left(\dfrac{1}{r} + K\right)} = 124.79\ m$

$\phi_{lfc} := 1 - e^{-\mu \cdot (\theta_\Delta + K \cdot x_\Delta)} = 13.12\%$

1.7.2 Loss Due to Anchorage Slip/Lock off (Total at jacking face(s) for each tendon)

Anchorage loss $L_{as} := 2 \cdot P_{ti} - 2 \cdot P_{t0}(x_{sl}) = 29.58\ kN$

Length of anchorage loss $x_{sl} = 9.35\ m$

Elastic shortening of tendon over the anchorage slip length (simplified calculation):

$\dfrac{1}{2} \cdot \dfrac{L_{as}}{A_t \cdot E_p} \cdot x_{sl} = 5.063\ mm$ is equal to the anchorage slip $\Delta x = 5\ mm$

1.7.4 Loss Due to Creep of Low Relaxation Strand (time dependent loss)

$\rho_{1000} := 2.5$ (EN10138 Class 2 strand)

$t_r := min\left(500000 \cdot hr, t\right)$ (EC2 - Long term final relaxation loss after service lift time or max 500000 hours)

$t_r = 50\ yr$ $\mu_r := \dfrac{P_{ti}}{A_t \cdot f_{yp}} = 0.787$

Relative loss of tendon force in relation to the original post-tensioning force:

$$\phi_{sc} := 0.66 \cdot \rho_{1000} \cdot e^{9.1 \cdot \mu_r} \cdot \left(\frac{t_r}{1000 \cdot hr}\right)^{0.75 \cdot (1-\mu_r)} \cdot 10^{-5} = 5.63\%$$ (EC2 3.3.2)

2.0 CALCULATION OF HOOP FORCES AND BENDING MOMENTS

2.1 Basic Constants

The following constants are required as defined in references 1 and 2:

$\eta := \sqrt{12 \cdot (1-\nu^2)}$ $\qquad \eta = 3.4$

$\beta := \sqrt[4]{\dfrac{3 \cdot (1-\nu^2)}{r^2 \cdot h^2}}$ $\qquad \beta = 0.906\ m^{-1}$ $\qquad$ (Isotropic value of) $\qquad \beta \cdot d = 4.077$

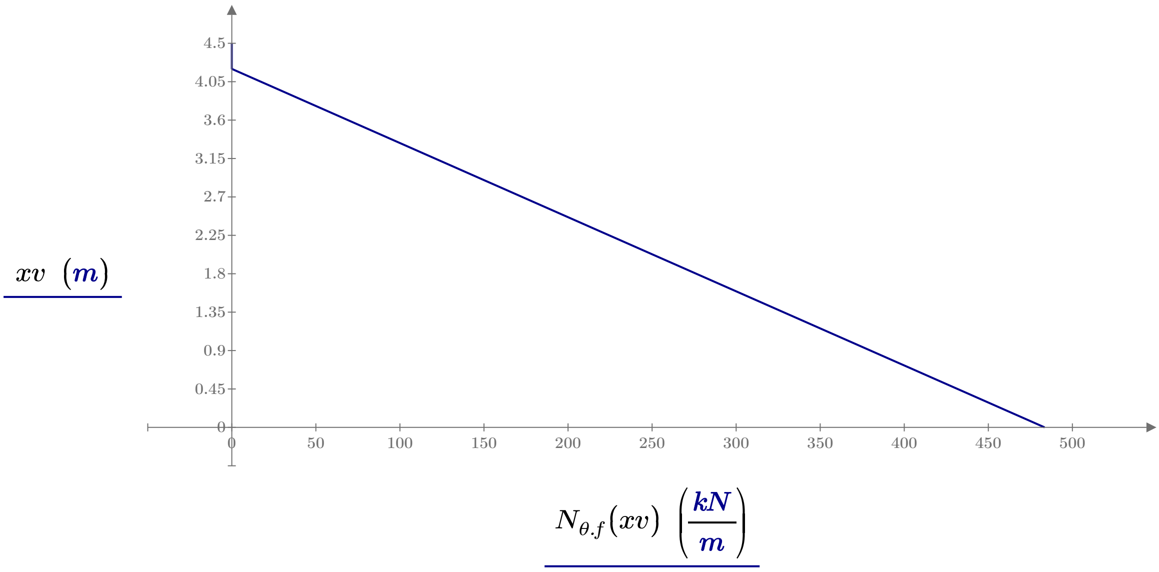
$\delta_r := 1 - \dfrac{1}{\beta \cdot d_h}$ $\qquad \delta_r = 0.737$

$D := \dfrac{E_{cs} \cdot h^3}{12 \cdot (1-\nu^2)}$ $\qquad D = 18975.8\ kN \cdot m$

2.2 Free Hoop Forces due to Hydrostatic Load

$$N_{\theta.f}(x) := \max\left(\gamma \cdot r \cdot d_h \cdot \left(1 - \frac{x}{d_h}\right), 0 \cdot \frac{kN}{m}\right)$$

$$N_{\theta.f}(0\ m) = 483.5\ \frac{kN}{m}$$



2.3 Pinned Base Hoop Force and Moment due to Hydrostatic Load (Isotropic Case)

2.3.1 Pinned Base Hydrostatic Hoop Force

for functions θ and ζ see Appendix 9

$$N_{\theta.p}(x) := \max\left(\gamma \cdot r \cdot d_h \cdot \left(1 - \frac{x}{d_h} - \theta(\beta, x)\right), 0 \cdot \frac{kN}{m}\right)$$

Guess Values

$$x := \frac{d}{4}$$

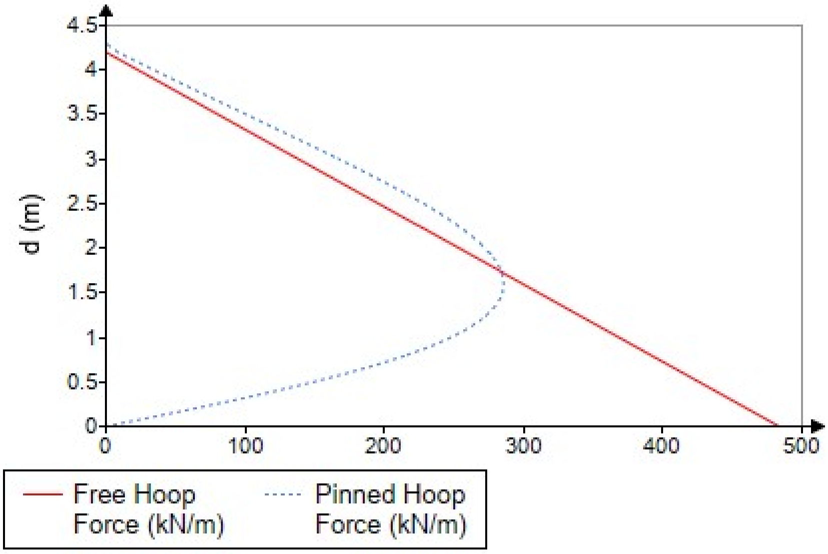
Constraints

$$x > 0\ m$$
$$x < d$$

Solver

$$x_{max.h.p} := \textbf{maximize}\left(N_{\theta.p}, x\right)$$

$$x_{max.h.p} = 1596.519\ mm$$



$$N_{max.h.p} := N_{\theta.p}\left(x_{max.h.p}\right)$$

$$N_{max.h.p} = 285.6\ \frac{kN}{m}$$

2.3.2 Pinned Base Hydrostatic Vertical Bending Moment

$$M_{x.p}(x) := \frac{\gamma \cdot r \cdot d_h \cdot h \cdot -\zeta(\beta, x)}{\eta}$$

Guess Values

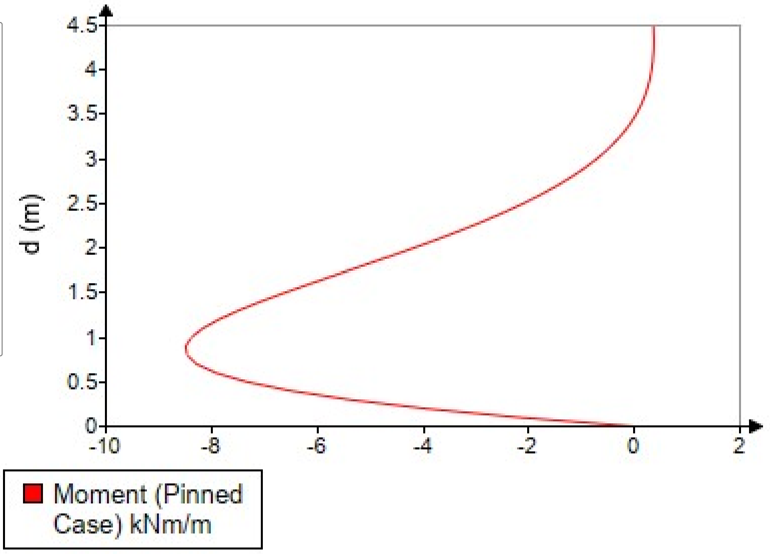
$$x := \frac{d}{4}$$

Constraints

$$x > 0\ m$$
$$x < d$$

Solver

$$x_{max.m.p} := \textbf{minimize}\left(M_{x.p}, x\right)$$



$$x_{max.m.p} = 866.914\ mm$$

$$M_{max.m.p} := M_{x.p}\left(x_{max.m.p}\right)$$

$$M_{max.m.p} = -8.496\ \frac{kN \cdot m}{m}$$

2.4 Fixed Base Hoop Force and Moment due to Hydrostatic Load (Isotropic Case)

This case is not considered to occur in practice due to the detailing of the tank, however the calculation is carried out so that a comparison of the resulting hoop stress distribution and bending can be made.

2.4.1 Fixed Base Hydrostatic Hoop Force

$$N_{\theta.fi}(x) := \max\left(\gamma \cdot r \cdot d_h \cdot \left(1 - \frac{x}{d_h} - \theta(\beta, x) - \delta_r \cdot \zeta(\beta, x)\right), 0\ \frac{kN}{m}\right)$$

Guess Values

$$x := \frac{d}{4}$$

Constraints

$x > 0\ m$

$x < d$

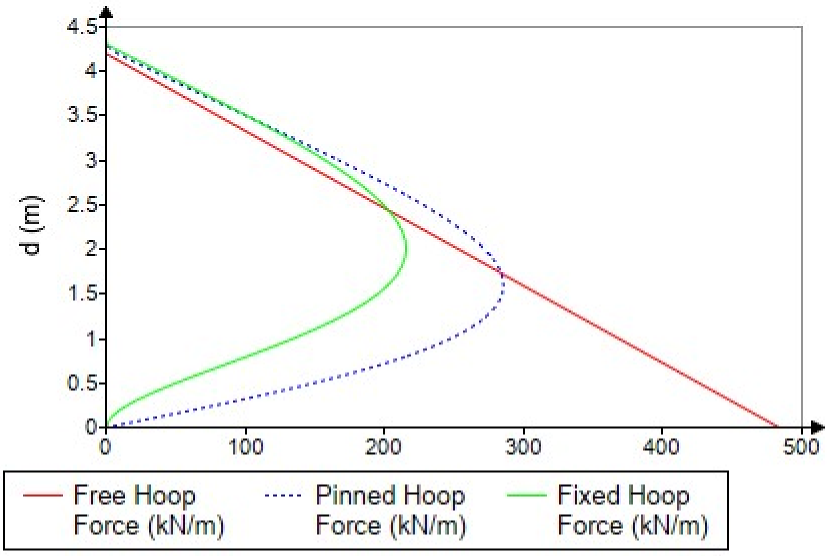
Solver

$$x_{max.h.fi} := \textbf{maximize}\left(N_{\theta.fi}, \mathbf{x}\right)$$

$$x_{max.h.fi} = 2007.317\ mm$$

$$N_{max.h.fi} := N_{\theta.fi}\left(x_{max.h.fi}\right)$$

$$N_{max.h.fi} = 215.6\ \frac{kN}{m}$$



2.4.2 Fixed Base Hydrostatic Vertical Bending Moment

$$M_{x.i}(x) := \frac{\gamma \cdot r \cdot d_h \cdot h \cdot \left(\delta_r \cdot \theta(\beta, x) - \zeta(\beta, x)\right)}{\eta}$$

Guess Values

$$x := \frac{d}{4}$$

Constraints

$x > 0\ m$

$x < d$

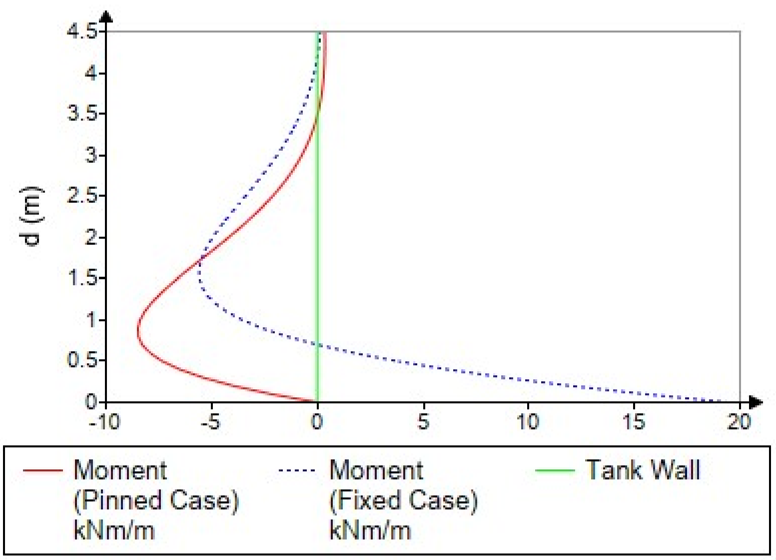
Solver

$$x_{max.m.fi} := \textbf{maximize}\left(M_{x.i}, \mathbf{x}\right)$$

$$x_{max.m.fi} = 0\ mm$$

$$M_{max.m.fi} := M_{x.i}\left(x_{max.m.fi}\right)$$

$$M_{max.m.fi} = 19.427\ \frac{kN \cdot m}{m}$$



Guess Values

$$x := \frac{d}{4}$$

Constraints

$x > 0\ m$

$x < d$

Solver

$$x_{min.m.fi} := \textbf{minimize}\left(M_{x.i}, x\right)$$

$$x_{min.m.fi} = 1.568\ m$$

$$M_{min.m.fi} := M_{x.i}\left(x_{min.m.fi}\right)$$

$$M_{min.m.fi} = -5.592\ \frac{kN \cdot m}{m}$$

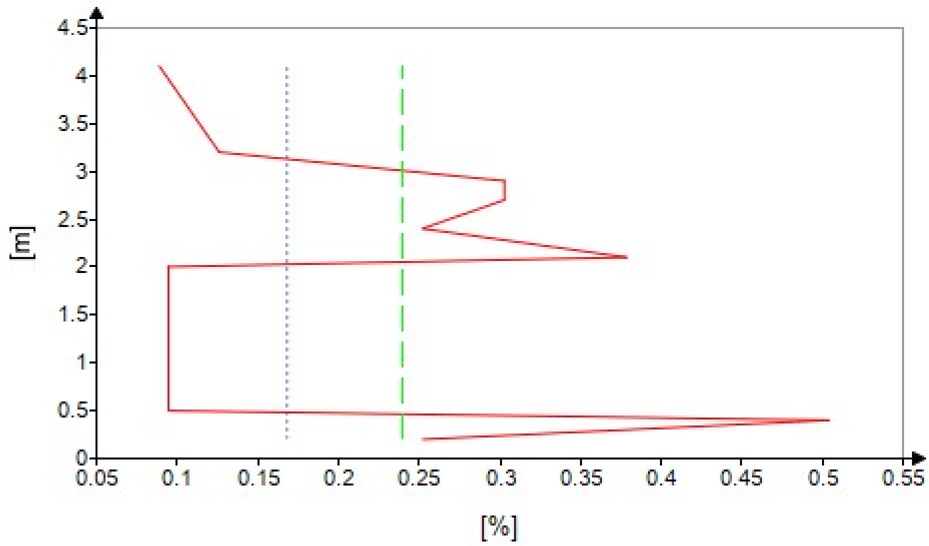
2.5 Factors to Account for Orthotropic Behaviour and Creep Stress Redistribution

2.5.1 Geometric Reinforcement Ratio of Tendons (Reference 6.)

Average geometric tendon reinforcement ratio over tank height $\rho_t := \dfrac{tn \cdot A_t}{d \cdot h}$ $\rho_t = 0.168\%$ $\rho_t = 0.0017$

Geometric Tendon Ratios as varying through height of tank $\rho t := \dfrac{A_t}{zx \cdot h}$

Mean reinforcing ratio over tank height $\rho_m := \text{mean}(\rho t)$ $\rho_m = 0.24\%$



plot of percentage geometric tendon reinforcement ratio with tank height (also showing average of total prestress area over height of tank and mean reinforcing ratio)

2.5.2 Factors to Account for Orthotropic Tank Behaviour Due to Joint Stiffness (Reference 6.)

According to reference 6 for an orthotropic tank (where the properties of the tank are not the same irrespective of direction) the value of will be modified. This modification factor depends on the relative thickness and stiffnesses of the concrete and the joint sealing strips, the width of the joints relative to the panel sizes and the relative contribution of the area of tendon as a proportion of the stiffness (proportioned by orthogonal ratio). The EPDM seals used have different stiffnesses depending on whether load on them is increasing or decreasing.

Initial condition with increasing load on sealing strips

$\delta_{o.i.inc} := \delta\left(h, b, s_s, a_s, E_{cs}, S_i, \alpha_{i.i}, \rho_t\right)$ $\delta_{o.i.inc} = 0.172$

refer to formulae in Appendix 14 for derivation of δ

Applies to the initial prestressing or external load

$$\beta_{o.i.inc} := \sqrt[4]{\frac{\delta_{o.i.inc} \cdot 3 \cdot \left(1 - \nu^2\right)}{r^2 \cdot h^2}} \qquad \beta_{o.i.inc} = 0.584\ \frac{1}{m}$$

$$\frac{\beta_{o.i.inc}}{\beta} = 64.428\%$$

Initial condition with decreasing load on sealing strips

$\delta_{o.i.dec} := \delta_b\left(h, b, s_s, a_s, E_{cs}, S_d, \alpha_{i.d}, \rho_t\right)$ $\delta_{o.i.dec} = 0.438$

refer to formulae in Appendix 14 for derivation of δ_b

Applies to the initial application of hydrostatic load

$$\beta_{o.i.dec} := \sqrt[4]{\frac{\delta_{o.i.dec} \cdot 3 \cdot \left(1 - \nu^2\right)}{r^2 \cdot h^2}} \qquad \beta_{o.i.dec} = 0.737\ \frac{1}{m}$$

$$\frac{\beta_{o.i.dec}}{\beta} = 81.337\%$$

Long term condition with increasing load on sealing strips

$\delta_{o.t.inc} := \delta\left(h, b, s_s, a_s, E_{cl}, S_i, \alpha_{t.i}, \rho_t\right)$ $\delta_{o.t.inc} = 0.349$

Applies to the prestressing or external load after relaxation

$$\beta_{o.t.inc} := \sqrt[4]{\frac{\delta_{o.t.inc} \cdot 3 \cdot (1-\nu^2)}{r^2 \cdot h^2}}$$

$\beta_{o.t.inc} = 0.696 \ \frac{1}{m}$

$\frac{\beta_{o.t.inc}}{\beta} = 76.872\%$

Long term with decreasing load on sealing strips load

$$\delta_{o.t.dec} := \delta_b (h, b, s_s, a_s, E_{cl}, S_d, \alpha_{t.d}, \rho_t)$$

$\delta_{o.t.dec} = 0.683$

Applies to the hydrostatic load after relaxation load

$$\beta_{o.t.dec} := \sqrt[4]{\frac{\delta_{o.t.dec} \cdot 3 \cdot (1-\nu^2)}{r^2 \cdot h^2}}$$

$\beta_{o.t.dec} = 0.824 \ \frac{1}{m}$

$short := \text{if} (\beta_{o.t.dec} \cdot d < 4, 1, 0)$ $short = 1$ (=1 if tank is a short shell)

$\frac{\beta_{o.t.dec}}{\beta} = 90.912\%$

2.5.3 Creep Factor Calculation

Calculation of creep coefficient to EC2.

Time of loading (assumed for prestressing to be when 28 day strength achieved) $t_0 \equiv 28 \cdot day$

relative humidity of ambient environment $RH \equiv 85\%$

For Initial loads long term (this will be a maximum value for long term effects on the basis of all load being applied at 28 days. In practice this would be the earliest application of prestress and is conservative for the hydrostatic loadcase which will be cyclic and applied later.)

$$h_o \equiv \frac{2 \cdot h \cdot 1000 \ mm}{2 \cdot 1000 \ mm}$$

$h_o = 185 \ mm$

$$f_{cm} := F_{ck} + 8 \ MPa$$

$f_{cm} = 48 \ MPa$

$$\alpha_1 := \left(\frac{35 \ MPa}{f_{cm}}\right)^{0.7} \qquad \alpha_2 := \left(\frac{35 \ MPa}{f_{cm}}\right)^{0.2} \qquad \alpha_3 := \left(\frac{35 \ MPa}{f_{cm}}\right)^{0.5}$$

$\alpha_1 = 0.8$ $\alpha_2 = 0.94$ $\alpha_3 = 0.85$

$$\phi_{RH} := \left(1 + \frac{1-RH}{\sqrt[3]{0.001 \cdot \frac{h_o}{mm}}} \cdot \alpha_1\right) \cdot \alpha_2$$

$\phi_{RH} = 1.137$

$$\beta_{fcm} := \frac{5.3}{\sqrt{0.1 \cdot \frac{f_{cm}}{MPa}}}$$

$\beta_{fcm} = 2.419$

$$\beta_{to} := \frac{1}{0.1 + \left(\frac{t_0}{day}\right)^{0.2}}$$

$\beta_{to} = 0.488$

$$\phi_o := \phi_{RH} \cdot \beta_{fcm} \cdot \beta_{to}$$

$\phi_o = 1.343$

$$\beta_H := min\left(1.5 \cdot (1+0.012 \cdot RH)^{18} \cdot \frac{h_o}{mm} + 250 \cdot \alpha_3 , 1500 \cdot \alpha_3\right) \qquad \beta_H = 546.594$$

$$\beta_{c.t} := \left(\frac{\frac{70\ yr - t_0}{day}}{\beta_H + \frac{70\ yr - t_0}{day}}\right)^{0.3} \qquad \beta_{c.t} = 0.994$$

$$\phi_t := \phi_o \cdot \beta_{c.t} \qquad \phi_t = 1.335$$

$$E_{cs} \cdot \frac{1}{1+\phi_t} = 14.8\ kN \cdot mm^{-2} \qquad E_{cl} = 13.4\ kN \cdot mm^{-2}$$

The design value of E_{cl} correlates with the calculated creep factor

2.5.4 Factor to Account for Stress Redistribution following Relaxation of Prestress (Reference 7.)

It is considered reasonable to assume that the strain modulus at the time of loading is equal to the 28 day strain modulus so that has a value of 1.0

The aging coefficient(ref CEB FIP model code clause 2.1.6.1)

$$t_1 = 1\ day \qquad \chi := \frac{\sqrt{\frac{t_0}{t_1}}}{1+\sqrt{\frac{t_0}{t_1}}}$$

$$\mu_r := 1.0 \qquad \chi = 0.841 \qquad \chi := 0.8$$

Relaxation ratio appropriate to cyclic hydrostatic loading

$$\kappa := 1 - \left(\frac{\mu_r}{\phi_t} + \chi\right)^{-1} \qquad \kappa = 0.35 \qquad 1-\kappa = 0.65$$

2.6 Idealised Theoretical Tendon Stressing Profile

2.6.1 Idealised Tendon Stressing Profile for a low (Short Shell) tank (reference 5)

reference 5 (distribution of circumferential prestress) gives a profile for the optimum variation of circumferential
prestress through the height of the tank. The actual prestressing is matched as closely as possible to this theoretical curve.

$t_p(x) := 0.462 + (2.58 \cdot 4) \cdot x - (0.506 \cdot 4^2) \cdot x^2$

note : this formula was derived for a 4m tank height and has been scaled to non dimensional values of x from 0 to 1

Solver Guess Values Constraints

$x := 0.5$

$x > 0 \quad x < 1$

$x_{max.tp} := \mathbf{maximize}\left(t_p, \mathrm{x}\right)$

$x_{max.tp} = 0.637$

$tp_{max} := t_p\left(x_{max.tp}\right) \qquad tp_{max} = 3.751$

$tp_{av} := \int_0^1 t_p(x)\, \mathrm{d}x \qquad tp_{av} = 2.923$

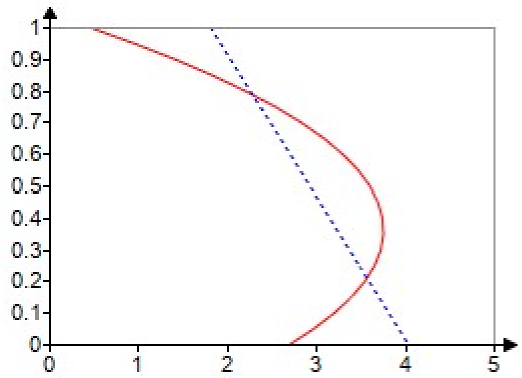
$t_p(0.5) = 3.598 \qquad k_o := \dfrac{t_p(0.5) - tp_{av}}{tp_{av}} \qquad k_o = 0.231$

$t_p(0) = 0.462 \qquad k_t := \dfrac{t_p(0) + 2 \cdot \left(t_p(0.5) - tp_{av}\right)}{tp_{av}} \qquad k_t = 0.62$

$t_p(1) = 2.686 \qquad k_b := \dfrac{t_p(1) + 2 \cdot \left(t_p(0.5) - tp_{av}\right)}{tp_{av}} \qquad k_b = 1.38$

$t_{pl}(x) := tp_{av} \cdot \left(k_t + x \cdot \left(k_b - k_t\right)\right)$

$tc := 0, 0.05 .. 1$



THE FORMULA PRESENTED ON THIS PAGE ARE CONFIDENTIAL AND ARE NOT TO BE ISSUED WITHOUT THE PRIOR CONSENT OF A-CONSULT

$t_a := 320.2 \dfrac{kN}{m} \qquad \dfrac{t_a \cdot d_h}{tn} = 134.484\ kN \qquad t_o := k_o \cdot t_a \qquad t_b := k_b \cdot t_a \qquad t_t := k_t \cdot t_a$

$t_a \cdot d_h = 1344.84\ kN$ avg. per tendon $\qquad t_o = 73.9\ kN \cdot m^{-1} \qquad t_b = 442\ kN \cdot m^{-1} \qquad t_t = 198.4\ kN \cdot m^{-1}$

2.6.1 Idealised Tendon Stressing Profile for a tall (Long Shell) tank (reference 11)

$\rho_p \equiv 1$ $\qquad f := \frac{8.5\ m}{8\ m} \cdot d_h$ $\qquad f = 4.463\ m$

$\beta = 0.906\ \frac{1}{m}$ $\qquad \xi := 0.6$ $\qquad \chi_p := 0.6\ m^{-1}$

$t_{ls}(x) := \rho_p \cdot \gamma \cdot d_h \cdot \left(1 - \frac{x}{f} - \xi \cdot \psi(\chi_p, x)\right)$ $\qquad tv := 0,25\ mm..d$

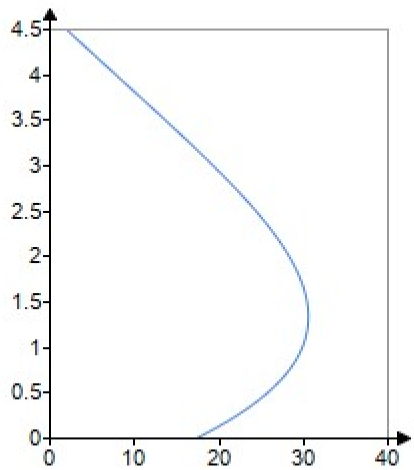
$x := \frac{d_h}{2}$

$x > 0\ m \qquad x < d_h$

$x_{max.ls} := \text{maximize}(t_{ls}, \text{x})$

$x_{max.ls} = 1.338\ m$

$t_{ls.max} := t_{ls}(x_{max.ls})$ $\qquad t_{ls.max} = 30.575\ \frac{kN}{m^2}$



THE FORMULA PRESENTED ON THIS PAGE ARE CONFIDENTIAL AND ARE NOT TO BE ISSUED WITHOUT THE PRIOR CONSENT OF A-CONSULT

Total tendon force required with the above profile

$p_{tt} := \rho_p \cdot \gamma \cdot d_h^{\,2} \cdot r \cdot \left(1 - \frac{d_h}{2 \cdot f} - \frac{\xi}{\chi_p \cdot d_h} \cdot \zeta(\chi_p, d_h)\right)$ $\qquad p_{tt} = (1.1 \cdot 10^3)\ kN$

Check : $\quad r \cdot \int_{0\ m}^{d_h} t_{ls}(x)\, \mathrm{d}x = (1.1 \cdot 10^3)\ kN$

With:

$\rho_p = 1$ $\qquad d_h = 4.2\ m$ $\qquad \gamma = 10.3\ kN \cdot m^{-3}$ $\qquad r = 11.176\ m$

$f = 4.5\ m$ $\qquad \xi = 0.6$ $\qquad \chi_p = 0.6\ m^{-1}$

2.7 Hydrostatic Load Cases for Orthotropic Shell (calculations as reference 5.)

2.7.1 Initial Hydrostatic Load only on Orthotropic Shell (Without Prestress)

Hoop Force

$N_{\theta.H}(x) := \text{if}\left(short, N_{\theta.H.s}(x, d_h, r, \gamma, \beta_{o.i.dec}), N_{\theta.H.l}(x, d_h, r, \gamma, \beta_{o.i.dec})\right)$

$\frac{\beta_{o.i.dec}}{\beta} = 81.337\%$

Plotted for comparison with the isotropic hydrostatic cases.

Guess Values

$x := \frac{d}{4}$

Constraints

$x > 0\ m \qquad x < d$

Solver

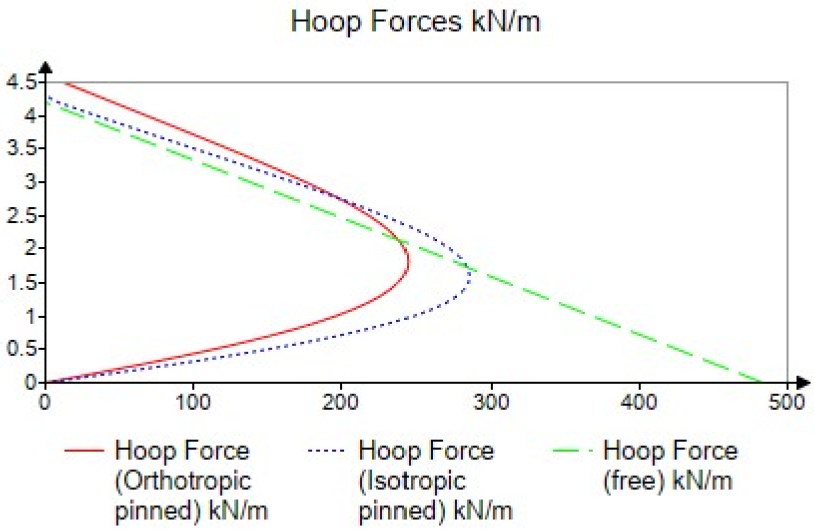
$x_{max.H.o} := \textbf{maximize}\left(N_{\theta.H}, \mathrm{x}\right)$

$x_{max.H.o} = 1.811\ m$

$N_{max.H.o} := N_{\theta.H}\left(x_{max.H.o}\right)$

$N_{max.H.o} = 244.333\ \frac{kN}{m}$

$\frac{N_{max.H.o}}{N_{max.h.p}} = 85.6\%$



Vertical bending moments

$M_{x.H}(x) := \text{if}\left(short, M_{x.H.s}(x, d_h, \gamma, \beta_{o.i.dec}), M_{x.H.l}(x, d_h, \gamma, \beta_{o.i.dec})\right)$

Guess Values

$x := \frac{d}{4}$

Constraints

$x > 0\ m \qquad x < d$

Solver

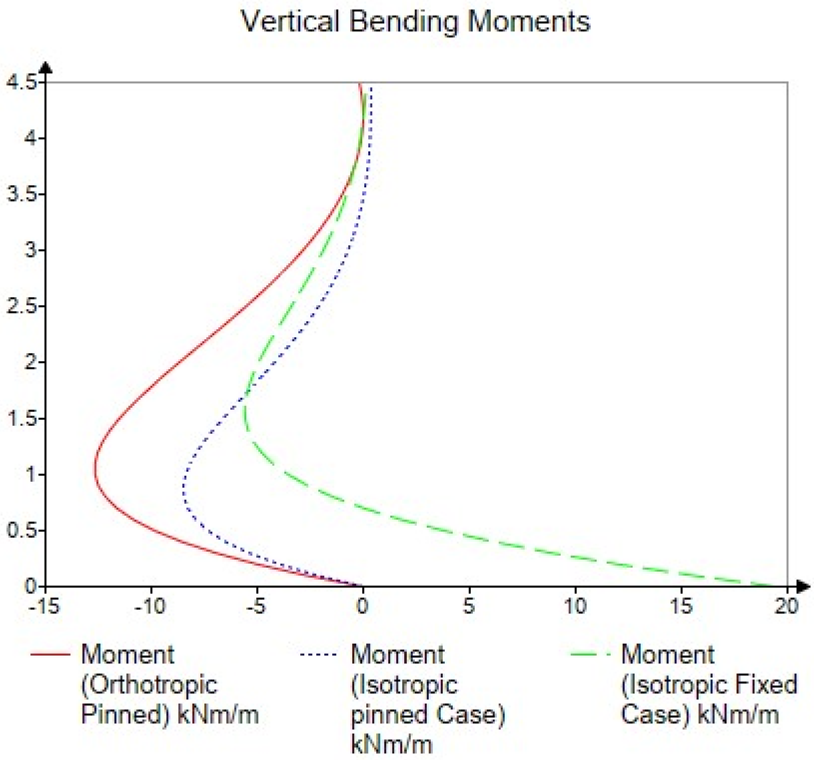
$x_{max.H.o} := \textbf{minimize}\left(M_{x.H}, x\right)$

$x_{max.H.o} = \left(1.1 \cdot 10^3\right)\ mm$

$M_{max.H.o} := M_{x.H}\left(x_{max.H.o}\right)$

$M_{max.H.o} = -12.7\ \frac{kN \cdot m}{m}$

$\frac{M_{max.H.o}}{M_{max.m.p}} = 149.2\%$



2.7.2 Long Term Hydrostatic Load only on Orthotropic Shell (Without Prestress)

Hoop Force

$N_{\theta.Ht}(x) := \text{if}\left(short, N_{\theta.H.s}\left(x, d_h, r, \gamma, \beta_{o.t.dec}\right), N_{\theta.H.l}\left(x, d_h, r, \gamma, \beta_{o.t.dec}\right)\right)$ $\qquad \dfrac{\beta_{o.t.dec}}{\beta} = 90.9\%$

Plotted for comparison with the isotropic hydrostatic cases.

Guess Values

$x := \dfrac{d}{4}$

Constraints

$x > 0\ m \qquad x < d$

Solver

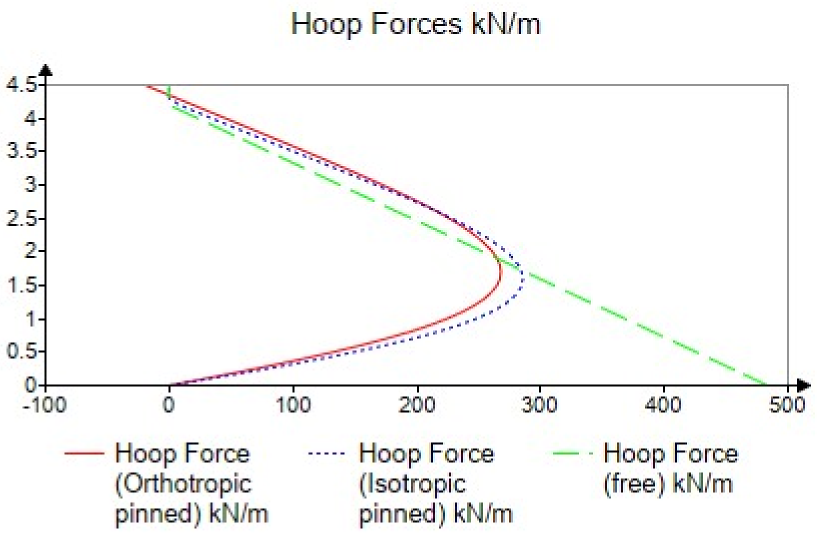
$x_{max.Ht.o} := \textbf{maximize}\left(N_{\theta.Ht}, x\right)$

$x_{max.Ht.o} = 1694.6\ mm$

$N_{max.Ht.o} := N_{\theta.Ht}\left(x_{max.Ht.o}\right)$

$N_{max.Ht.o} = 267.8\ \dfrac{kN}{m}$

$\dfrac{N_{max.Ht.o}}{N_{max.h.p}} = 93.8\% \qquad \dfrac{N_{max.Ht.o}}{N_{max.H.o}} = 109.6\%$



Vertical bending moments

$M_{x.Ht}(x) := \text{if}\left(short, M_{x.H.s}\left(x, d_h, \gamma, \beta_{o.t.dec}\right), M_{x.H.l}\left(x, d_h, \gamma, \beta_{o.t.dec}\right)\right)$

Guess Values

$x := \dfrac{d}{4}$

Constraints

$x > 0\ m \qquad x < d$

Solver

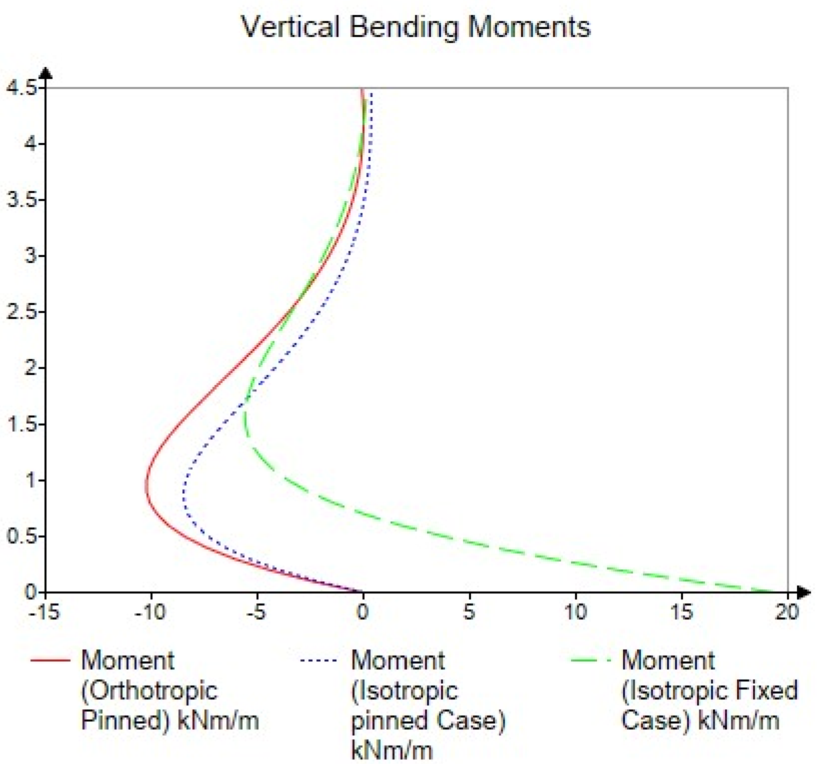
$x_{max.Ht.o} := \textbf{minimize}\left(M_{x.Ht}, x\right)$

$x_{max.Ht.o} = 951\ mm$

$M_{max.Ht.o} := M_{x.Ht}\left(x_{max.Ht.o}\right)$

$M_{max.Ht.o} = -10.248\ \dfrac{kN \cdot m}{m}$

$\dfrac{M_{max.Ht.o}}{M_{max.m.p}} = 120.6\%$



2.8 Serviceability Load Cases for Orthotropic Shell (calculations as reference 5.)

2.8.1 Case A : Condition Immediately after Tensioning (No hydrostatic Pressure or Prestress Losses)

$$\frac{\beta_{o.i.inc}}{\beta} = 64.4\%$$

Hoop Force

$$N_{\theta.A}(xt) := \mathbf{if}\left(short, N_{\theta.A.s}\left(xt, d_h, t_o, t_b, t_t, \beta_{o.i.inc}\right), N_{\theta.A.l}\left(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.inc}\right)\right)$$

Maximum moment at mid span

$$x := \frac{d}{4}$$

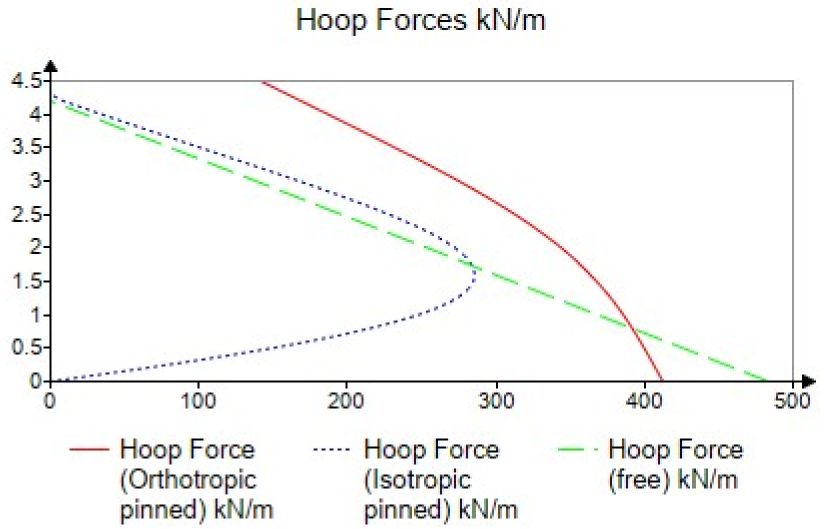
$$x > 0\ m \qquad x < d$$

$$x_{max.A.o} := \mathbf{maximize}\left(N_{\theta.A}, \mathbf{x}\right)$$

$$x_{max.A.o} = 0\ m$$

$$N_{max.A.o} := N_{\theta.A}\left(x_{max.A.o}\right)$$

$$N_{max.A.o} = 412.386\ \frac{kN}{m}$$



Vertical bending moments

$$M_{x.A}(xt) := \mathbf{if}\left(short, M_{x.A.s}\left(xt, d_h, r, t_o, \beta_{o.i.inc}\right), M_{x.A.l}\left(xt, d_h, \gamma, \rho_p, \xi, \chi_p, \beta_{o.i.inc}\right)\right)$$

$$x := \frac{d}{4}$$

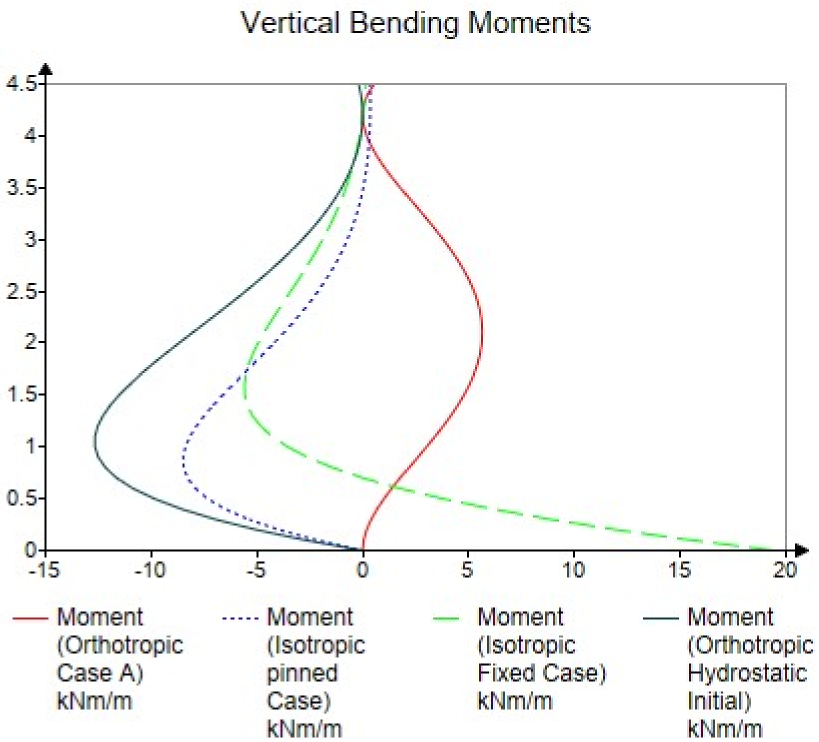
$$x > 0\ m \qquad x < d$$

$$x_{max.A.o} := \mathbf{maximize}\left(M_{x.A}, x\right)$$

$$x_{max.A.o} = 2100\ mm$$

$$M_{max.A.o} := M_{x.A}\left(x_{max.A.o}\right)$$

$$M_{max.A.o} = 5.6\ \frac{kN \cdot m}{m}$$



2.8.2 Case B : Prestress only after Stress Relaxation

$$\beta_{o.t.lon} := \text{if}\left(inclong, \beta_{o.t.inc}, \beta_{o.t.dec}\right) \qquad \frac{\beta_{o.t.inc}}{\beta} = 76.9\% \qquad \frac{\beta_{o.t.dec}}{\beta} = 90.9\%$$

Hoop Force

$$N_{\theta.B}(xt) := \begin{array}{l} \text{if } short \\ \quad N_{\theta.B.s}\left(xt, d_h, t_o, t_b, t_t, \beta_{o.i.inc}, \beta_{o.t.inc}, 1, 1-\kappa\right) \\ \text{else} \\ \quad N_{\theta.B.l}\left(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.t.lon}, \kappa, 1-\kappa\right) \end{array}$$

Plotted positive for comparison with the isotropic hydrostatic cases.

Guess Values

$$x := \frac{d}{4}$$

Constraints

$$x > 0\ m \qquad x < d$$

Solver

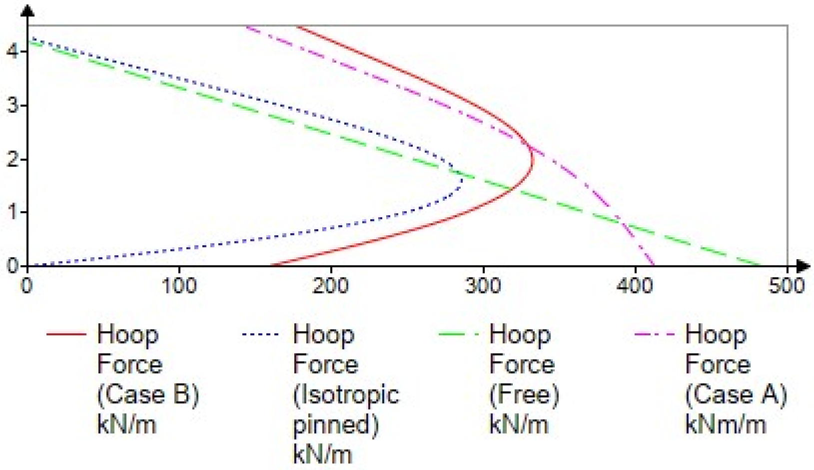
$$x_{max.B.o} := \textbf{maximize}\left(N_{\theta.B}, \mathbf{x}\right)$$

$$x_{max.B.o} = 1979.353\ mm$$

$$N_{max.B.o} := N_{\theta.B}\left(x_{max.B.o}\right)$$

$$N_{max.B.o} = 332.2\ \frac{kN}{m}$$

Hoop Forces kN/m



Vertical bending moments

$$M_{x.B}(xt) := \begin{array}{l} \text{if } short \\ \quad M_{x.B.s}\left(xt, d_h, r, t_o, t_b, \beta_{o.i.inc}, \beta_{o.t.inc}, 1, 1-\kappa\right) \\ \text{else} \\ \quad M_{x.B.l}\left(xt, d_h, \gamma, \rho_p, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.t.lon}, \kappa, 1-\kappa\right) \end{array}$$

Guess Values

$$x := \frac{d}{4}$$

Constraints

$$x > 0\ m \qquad x < d$$

Solver

$$x_{max.B.o} := \textbf{maximize}\left(M_{x.B}, \mathbf{x}\right)$$

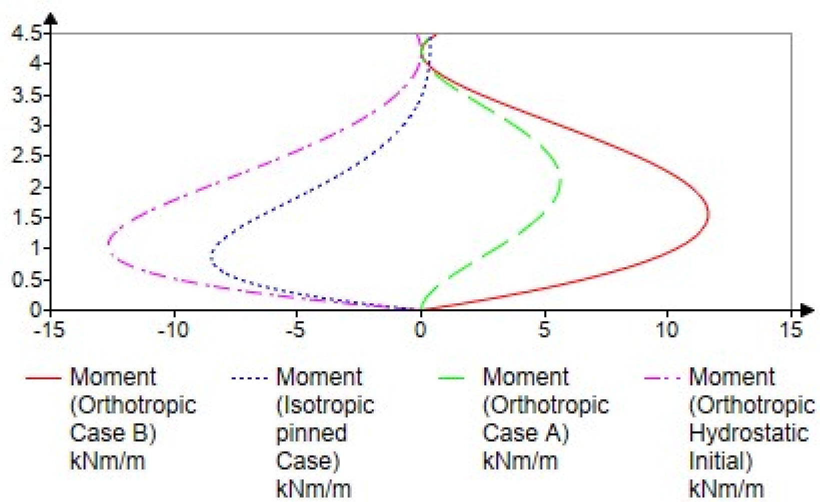
$$x_{max.B.o} = 1557\ mm$$

$$M_{max.B.o} := M_{x.B}\left(x_{max.B.o}\right)$$

$$M_{max.B.o} = 11.6\ \frac{kN \cdot m}{m}$$

$$\frac{M_{max.B.o}}{M_{max.A.o}} = 206.4\%$$

Vertical Bending Moments



2.8.3 Case C : Short Term Prestress and Hydrostatic Load before Stress Relaxation

Hoop Force (compression)

$N_{\theta.C}(xt) := N_{\theta.A}(xt) - N_{\theta.H}(xt)$

Plotted positive for comparison with the isotropic hydrostatic cases.

Solver

Guess Values

$x := \frac{d}{4}$

Constraints

$x > 0\ m \qquad x < d$

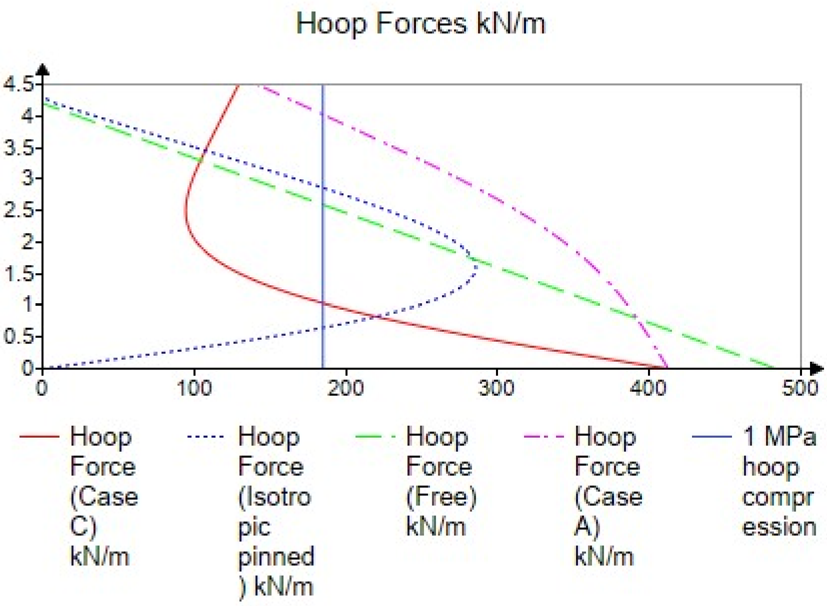
$x_{min.C.o} := \textbf{minimize}\left(N_{\theta.C}, \mathbf{x}\right)$

$x_{min.C.o} = 2493.1\ mm$

$N_{min.C.o} := N_{\theta.C}\left(x_{min.C.o}\right)$

$N_{min.C.o} = 94.5\ \frac{kN}{m}$

$\frac{N_{min.C.o}}{h} = 0.511\ MPa$



Vertical bending moments

$M_{x.C}(x) := M_{x.A}(x) + M_{x.H}(x)$

Solver

Guess Values

$x := \frac{d}{4}$

Constraints

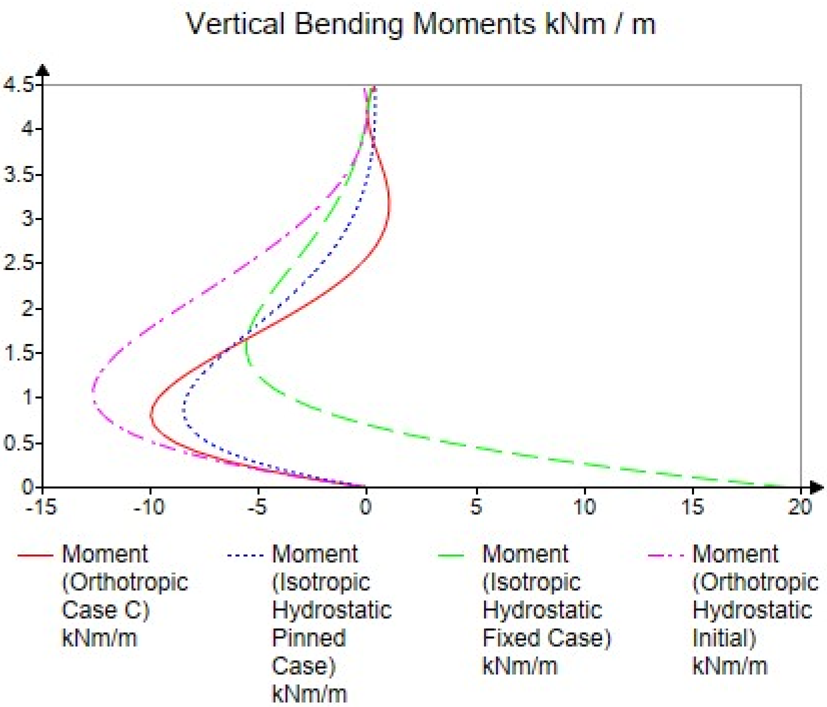
$x > 0\ m \qquad x < d$

$\boxed{x_{min.C.o}} := \textbf{minimize}\left(M_{x.C}, x\right)$

$x_{min.C.o} = 809.4\ mm$

$M_{min.C.o} := M_{x.C}\left(x_{min.C.o}\right)$

$M_{min.C.o} = -10\ \frac{kN \cdot m}{m}$



2.8.4 Case D : Long Term Prestress and Hydrostatic Load after Stress Relaxation

Hoop Force (compression)

$$N_{\theta.D}(x) := N_{\theta.B}(x) - N_{\theta.H}(x)$$

Plotted positive for comparison with the isotropic hydrostatic cases.

$$x := \frac{d}{4}$$

$$x > 0\ m \qquad x < d$$

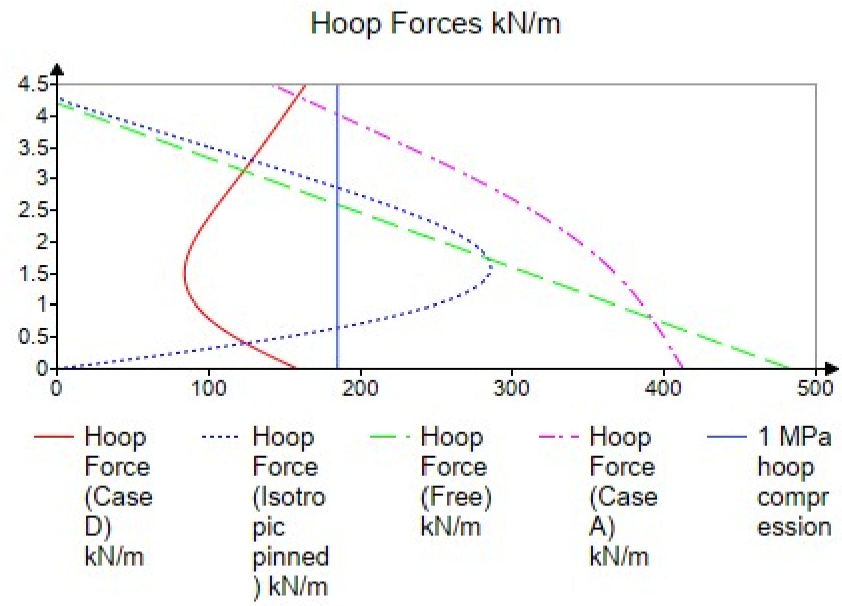
$$x_{min.D.o} := \textbf{minimize}\left(N_{\theta.D}, x\right)$$

$$x_{min.D.o} = 1495.6\ mm$$

$$N_{min.D.o} := N_{\theta.D}\left(x_{min.D.o}\right)$$

$$N_{min.D.o} = 84.1\ \frac{kN}{m}$$

$$\frac{N_{min.D.o}}{h} = 0.455\ MPa$$



Vertical bending moments

$$M_{x.D}(x) := M_{x.B}(x) + M_{x.Ht}(x)$$

$$x := \frac{d}{4}$$

$$x > -1\ m \quad x < d$$

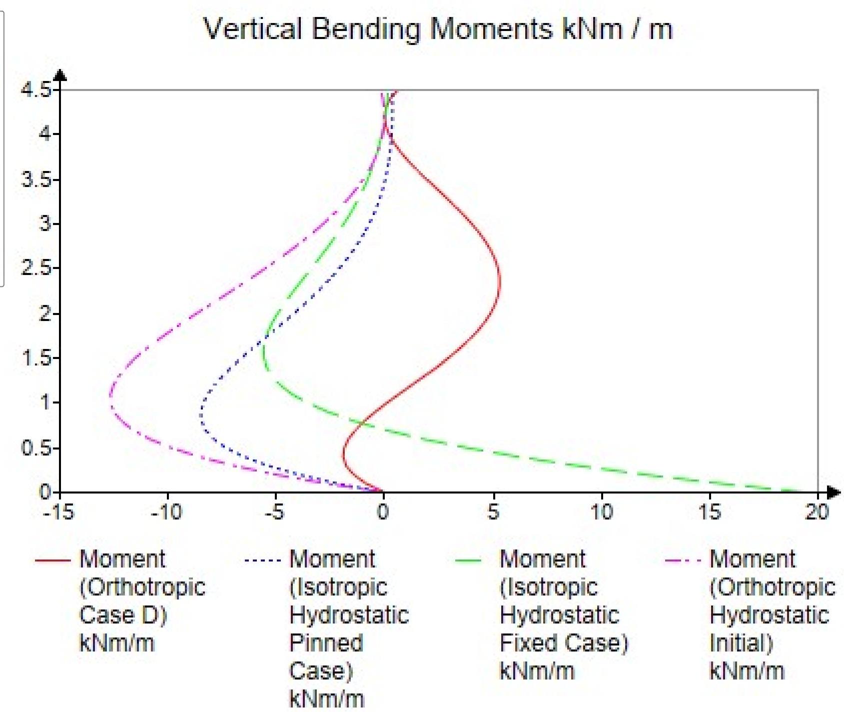
$$x_{min.D.o} := \textbf{minimize}\left(M_{x.D}, \mathrm{x}\right)$$

$$x_{min.D.o} = 420.662\ mm$$

$$M_{min.D.o} := M_{x.D}\left(x_{min.D.o}\right)$$

$$M_{min.D.o} = -1.9\ \frac{kN \cdot m}{m}$$

$$\frac{M_{min.D.o}}{M_{min.C.o}} = 19.1\%$$



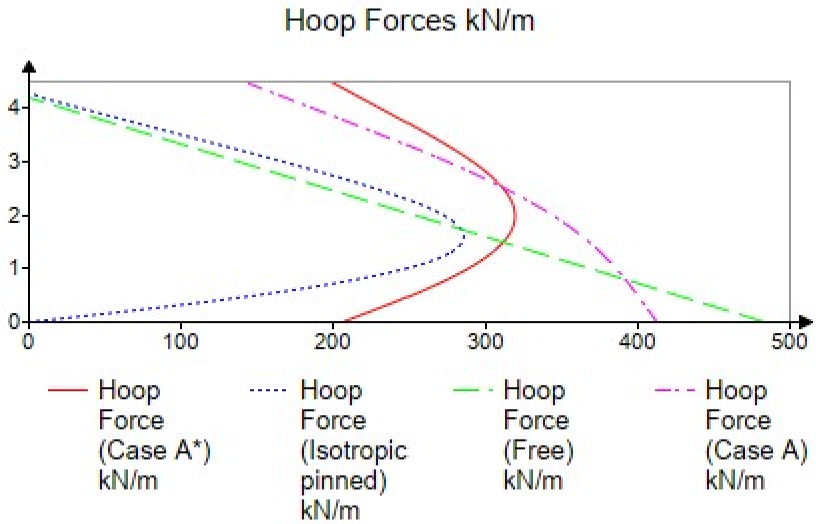
2.9 Partially Pinned Case

2.9.1 Case A* : Initial Prestress only assuming 50% partial pin effect at base

Hoop Force

$$N_{\theta.AS}(x) := \begin{Vmatrix} \text{if } short \\ \quad \| N_{\theta.B.s}(x, d_h, t_o, t_b, t_t, \beta_{o.i.inc}, \beta_{o.i.inc}, 1, 0.5) \\ \text{else} \\ \quad \| N_{\theta.B.l}(x, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.i.inc}, 0.5, 0.5) \end{Vmatrix}$$

Plotted positive for comparison with the isotropic hydrostatic cases.



Vertical bending moments

$$M_{x.AS}(x) := \begin{Vmatrix} \text{if } short \\ \quad \| M_{x.B.s}(x, d_h, r, t_o, t_b, \beta_{o.i.inc}, \beta_{o.i.inc}, 1, 0.5) \\ \text{else} \\ \quad \| M_{x.B.l}(x, d_h, \gamma, \rho_p, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.i.inc}, 0.5, 0.5) \end{Vmatrix}$$

Guess Values

$$x := \frac{d}{4}$$

Constraints

$$x > -1\ m \quad x < d$$

Solver

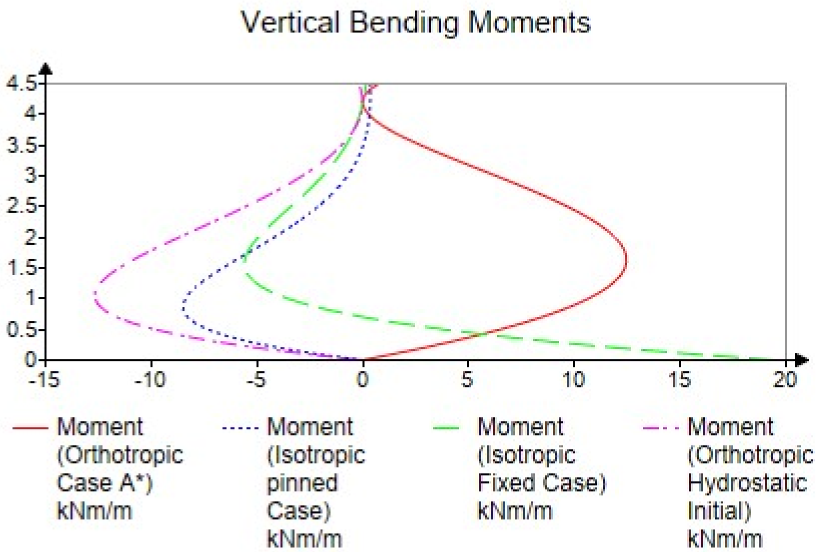
$$x_{max.AS.o} := \mathbf{maximize}\left(M_{x.AS}, \mathbf{x}\right)$$

$$x_{max.AS.o} = 1632.516\ mm$$

$$M_{max.AS.o} := M_{x.AS}\left(x_{max.AS.o}\right)$$

$$M_{max.AS.o} = 12.5\ \frac{kN \cdot m}{m}$$

$$\frac{M_{max.AS.o}}{M_{max.A.o}} = 221\%$$



3.0 PRESTRESS FORCES REQUIRED TO ACHIEVE DESIGN DISTRIBUTION

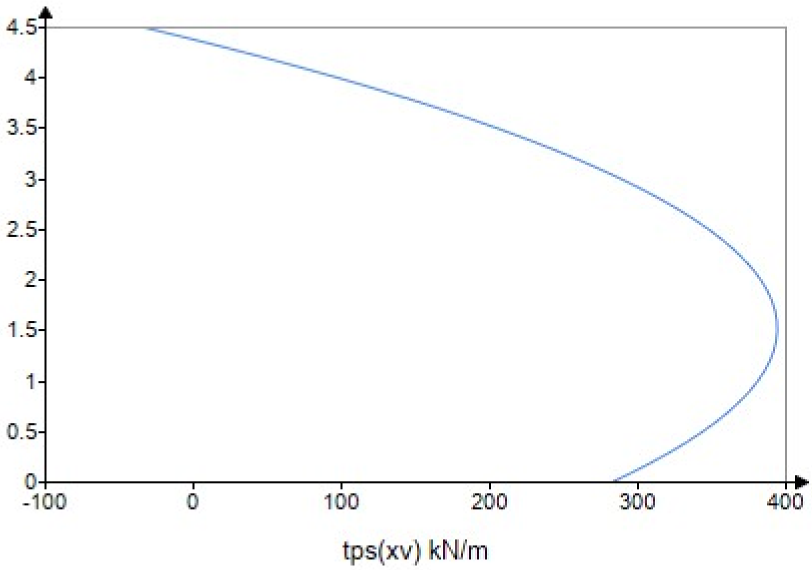
3.1 Prestress Forces Required after Losses

$$tps(x) := \text{if}\left(short, \left(t_a \cdot \frac{1+k_o}{tp_{max}} \cdot t_p\left(\frac{d_h - x}{d_h}\right)\right), r \cdot t_{ls}(x)\right)$$

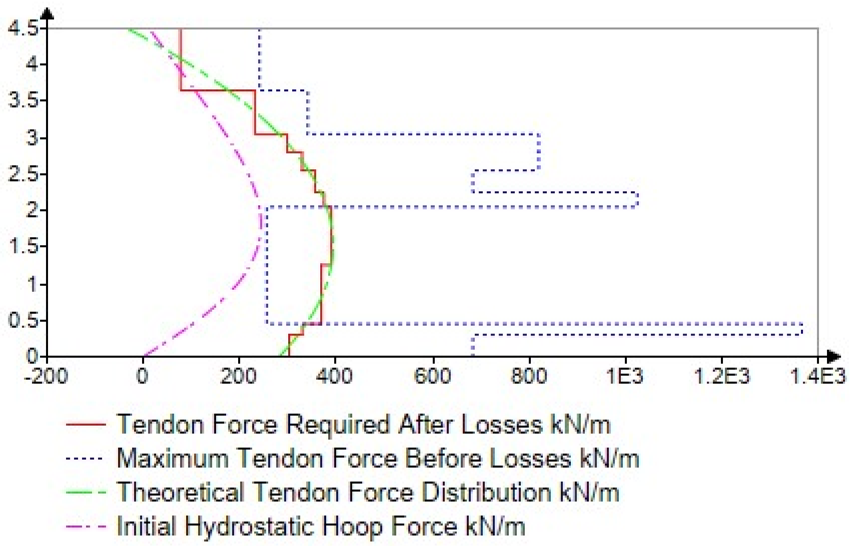
$$P_{tr_i} := \int_{mx_i}^{mx_{i+1}} tps(x)\,dx$$

$$\int_{0\,m}^{d} tps(x)\,dx = 1292.587\ kN$$

$$\sum P_{tr} = 1292.587\ kN$$



$$P_{ir}(x) := \left\| \begin{array}{l} ii \leftarrow 0 \\ \text{while } mx_{ii} < x \\ \quad \| \ ii \leftarrow ii + 1 \\ \dfrac{P_{tr_{ii-1}}}{zx_{ii-1}} \end{array} \right.$$



3.2 Optimum Tendon Levels For the Number of Tendons Provided

3.2.1 Short Shell Optimum Distribution (Reference 5)

$tp_{av} = 2.923$ $\quad tn = 10$ $\quad k_a := \dfrac{tp_{av}}{tn}$ $\quad k_a = 0.292$ $\quad q := 1 .. tn - 1$

optimum average tendon force (after losses) $\quad \dfrac{t_a \cdot d_h}{tn} = 134.484\ kN$

Calculate x values between division lines if theoretical prestress distribution is split into (tn) number sections of equal area

$$x := 0$$

$$xo_q := \text{root}\left(\int_0^x t_p(x)\, dx - q \cdot k_a, x, 0, 1\right) \qquad xo := \text{stack}(xo, 1)$$

Calculate x values of centroids of each area

Scale unit x values to actual tank height

$$xc_i := \frac{1}{k_a} \cdot \int_{xo_i}^{xo_{i+1}} x \cdot t_p(x)\, dx \qquad xh := xc \cdot d_h \qquad xb_i := d_h - xh_i$$

Optimum short shell tendon levels for $tn = 10$ number tendons over height of tank.

3.2.2 Long Shell Optimum Distribution (Reference 11)

$$A_{tg} := \left(\int_0^d t_{ls}(x)\, dx\right) \qquad A_{tg} = 95.275\ kN \cdot m^{-1} \qquad \frac{A_{tg}}{tn} = 9.53\ \frac{kN}{m}$$

$$y := 0\ m$$

$$xol_q := \text{root}\left(\int_{0\,m}^y t_{ls}(y)\, dy - q \cdot \frac{A_{tg}}{tn}, y, 0\ m, d_h\right) \qquad xol := \text{stack}(xol, d)$$

$$xcl_i := \frac{tn}{A_{tg}} \cdot \int_{xol_i}^{xol_{i+1}} x \cdot t_{ls}(x)\, dx \qquad xt_i := d_h - tx_i \qquad xf_i := xcl_i$$

Optimum long shell tendon levels for $tn = 10$ number tendons over height of tank.

$\beta_{o.t.dec} \cdot d = 3.71$

If greater than 4 tank is a long shell otherwise tank is a short shell.

$xt_{optimum} := \text{if}\left(short = 0, xf, xb\right)$

Actual Tendon Levels

Theoretical Optimum Tendon Levels

$$d_h - xt = \begin{bmatrix} 200 \\ 400 \\ 500 \\ 2000 \\ 2100 \\ 2400 \\ 2700 \\ 2900 \\ 3200 \\ 4100 \end{bmatrix} mm$$

$$xt_{optimum} = \begin{bmatrix} 3674.2 \\ 3085.7 \\ 2661.1 \\ 2291.4 \\ 1948.2 \\ 1617.3 \\ 1289.1 \\ 954.3 \\ 602 \\ 214 \end{bmatrix} mm$$

3.3 Prestress Forces Required Prior to Losses

3.3.1 Losses due to Concrete Contraction, Shrinkage and Concrete Creep

Maximum initial hoop tension in concrete after losses $N_{max.A.o} = 412.4\ \frac{kN}{m}$

Concrete stress $\sigma_{ci} := \frac{N_{max.A.o}}{h}$ $\sigma_{ci} = 2.23\ MPa$

Sealing strip initial stress $\sigma_{si} := \frac{N_{max.A.o}}{a_s}$ $\sigma_{si} = 5.15\ MPa$

Concrete strain $\varepsilon_{ci} := \frac{\sigma_{ci}}{E_{cs}}$ $\varepsilon_{ci} = 0.000065$

$\varepsilon_{si} := \frac{\sigma_{si}}{S_i}$ $\varepsilon_{si} = 0.108982$

Sealing strip initial strain $\varepsilon_{so} := \frac{\sigma_{si}}{S_d}$ $\varepsilon_{so} = 0.030323$

Loss of stress (hence force) in strand due to elastic shortening in the concrete and sealing strips (average loss). The elastic loss occurs immediately when the tendonds are post-tensioned.

$$L_{es} := \frac{1}{2} \cdot \frac{\varepsilon_{ci} \cdot (b - s_s) + \varepsilon_{so} \cdot s_s}{b} \cdot E_p \cdot A_t \qquad L_{es} = 2.056\ kN$$

Strain from concrete shrinkage (time dependent loss) - EC2, EN 1992-1-1 3.1.4 (6)

$$\beta_{RH} := 1.55 \cdot \left(1 - \left(\frac{RH}{100\%}\right)^3\right) = 0.598$$

$\alpha_{ds1} := 4$ (Cement: S=3, N=4, R=6) $\alpha_{ds2} := 0.12$ (Cement: S=0.13, N=0.12, R=0.11)

$$\varepsilon_{cd0} := 0.85 \cdot \left((220 + 110 \cdot \alpha_{ds1}) \cdot e^{-\alpha_{ds2} \cdot \frac{f_{cm}}{10\ MPa}} \right) \cdot 10^{-6} \cdot \beta_{RH} = 0$$

$$\beta_{dstts} := \frac{\frac{t}{day}}{\frac{t}{day} + 0.04 \cdot \sqrt{\left(\frac{h}{mm}\right)^3}} = 0.995 \qquad k_h := 0.95 \quad \text{(conservative value)}$$

$$\varepsilon_{ca\omega} := 2.5 \cdot \left(\frac{F_{ck}}{MPa} - 10\right) \cdot 10^{-6} = 0.000075 \qquad \beta_{ast} := 1 - e^{\left(-0.2 \cdot \left(\frac{t}{day}\right)^{0.5}\right)} = 1$$

$\varepsilon_{cdt} := \beta_{dstts} \cdot k_h \cdot \varepsilon_{cd0} = 0.0001782$ Shrinkage from drying

$\varepsilon_{cat} := \beta_{ast} \cdot \varepsilon_{ca\omega} = 0.000075$ Autogenous shrinkage

$\varepsilon_{cs.t} := \varepsilon_{cdt} + \varepsilon_{cat} = 0.0002532$ Total shrinkage

Loss of stress (hence force) in strand due to concrete shrinkage and creep plus tendon relaxation (time dependent)

$\phi_t = 1.3$ $h = 185\ mm$ $A_t = 140\ mm^2$ $\varepsilon_{cs.t} = 253.21 \cdot 10^{-6}$

$P_{ti} = 205\ kN$

$$\alpha_p := \frac{E_p}{E_{cs}}$$ $\alpha_p = 5.65$

$$\Delta\sigma_{pr} := \frac{P_{ti} - P_{ti} \cdot (1 - \phi_{sc})}{A_t}$$ $\Delta\sigma_{pr} = 82.4\ MPa$

$$N_{max.A.o} = 412.39\ \frac{kN}{m}$$

$$N_{mean.A.o} := \left\| \begin{array}{l} total \leftarrow 0 \\ count \leftarrow 0 \\ \text{for } in \in 0\ m, 0.1\ m .. d \\ \quad \left\| \begin{array}{l} total \leftarrow N_{\theta.A}(in) + total \\ count \leftarrow count + 1 \end{array} \right. \\ \dfrac{total}{count} \end{array} \right. = 308.5\ \frac{kN}{m}$$

$$\sigma_{cpo} := \frac{N_{mean.A.o}}{h} = 1.67\ MPa$$

EC2 5.10.5.3(3) For stresses mean values should be used when using unbonded tendons

$$\Delta\sigma := \frac{\text{abs}(\varepsilon_{cs.t}) \cdot E_p + 0.8 \cdot \Delta\sigma_{pr} + \alpha_p \cdot \phi_t \cdot (\sigma_{cpo})}{1 + \alpha_p \cdot \dfrac{A_t}{\dfrac{d}{tn} \cdot h} \cdot (1 + 0.8 \cdot \phi_t)}$$ $\Delta\sigma = 125.4\ MPa$

$$L_{shcr} := \Delta\sigma \cdot A_t$$ $L_{shcr} = 17.56\ kN$

3.3.2 Prestress Jack Forces Required to match idealised profile

Prestress forces required before anchorage losses and tendon creep $P_{il_i} := \dfrac{P_{tr_i} + L_{as}}{1 - \phi_{sc}}$

Elastic shortening, immediate loss $L_{es} = 2.056\ kN$

Anchorage loss due to slip in at buttresses $L_{as} = 29.6\ kN$

Creep, shrinkage and relaxation (time dependent losses) $L_{shcr} = 17.6\ kN$ $L_{es} + L_{shcr} = 19.6\ kN$

Prestress forces for theoretic curve with maximum linear friction and curvature losses $P_{jf_i} := \dfrac{P_{il_i}}{1 - \phi_{lfc}} + L_{es} + L_{shcr}$

$Bp_{config} = 1$

Losses between buttresses due to friction $\phi_{lfc} = 13.1\%$

Initial jack force at buttress anchorages $P_{ti} = 205\ kN$ at jacking

In case of non-through-going tendons (hence, if Bp_config = 0) before time dependent losses:

$$Pnon_{min} := \text{if}\left(x_{sl} < x_\Delta, min\left(P_{ti} - L_{as}, P_{t0}\left(x_\Delta\right)\right), min\left(P_{ti} - L_{as}, P_{t1}\left(x_\Delta\right)\right)\right) - L_{es} = 173.4\ kN$$

In case of through-going tendons (hence, if Bp_config = 1) before time dependent losses:

$$Pthrough_{min} := \frac{1}{2} \cdot \text{if}\left(x_{sl} < x_\Delta, min\left(P_{ti} - L_{as} + P_{t0}\left(2 \cdot x_\Delta\right), 2 \cdot P_{t0}\left(x_\Delta\right), P_{t0}\left(x_{sl}\right) + P_{t0}\left(2 \cdot x_\Delta - x_{sl}\right)\right), min\left(P_{ti} - L_{as} + P_{t0}\right.\right.$$

$$Pthrough_{min} = 163\ kN$$

Actual minimum tendon force calculated after time dependent losses

$$P_{tl} := \text{if}\left(Bp_{config} = 0, Pnon_{min}, Pthrough_{min}\right) - L_{shcr} \qquad P_{tl} = 145.5\ kN$$

Total theoretical tendon force to optimised distribution $\sum P_{tr} = \left(1.293 \cdot 10^3\right)\ kN$ $\dfrac{\sum P_{tr}}{tn} = 129.26\ kN$ required per tendon

Tendon force per meter after losses $\dfrac{P_{tl} \cdot tn}{d} = 323.2\ \dfrac{kN}{m}$

Total tendon force applied after losses $P_{tl} \cdot tn = \left(1.5 \cdot 10^3\right)\ kN$ $\dfrac{P_{tl} \cdot tn}{\sum P_{tr}} = 1.13$

$$\text{if}\left(\frac{P_{tl} \cdot tn}{\sum P_{tr}} > 1, \text{“ok”}, \text{“!!”}\right) = \text{“ok”}$$

Required to be greater than 1.0 for tendon force applied after losses to exceed theoretical required tendon force to optimised distribution

Ultimate limit state control of the number of tendons:

Total hydrostatic hoop force: $THHF := \frac{1}{2} \cdot {d_h}^2 \cdot \gamma \cdot r = 1015.3\ kN$

Total breaking load of tendons: $Ult_t := tn \cdot f_{yp} \cdot A_t = 2604\ kN$

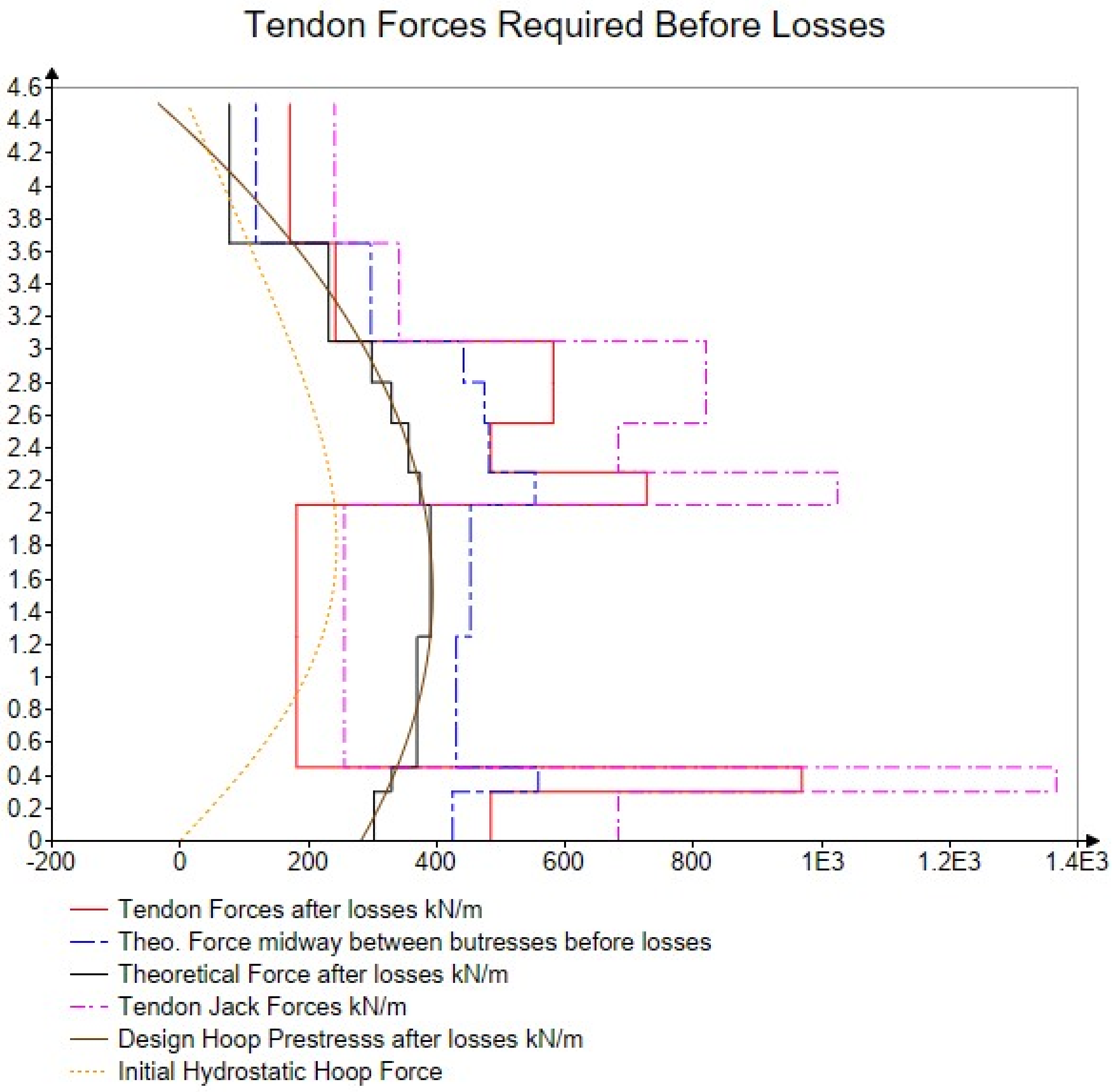
$$\text{if}\left(\frac{Ult_t}{\gamma_{ms}} > \gamma_f \cdot THHF, \text{“ok”}, \text{“!!”}\right) = \text{“ok”}$$

$$\frac{1}{\gamma_f \cdot THHF} \cdot \frac{Ult_t}{\gamma_{ms}} = 1.49$$

$$P_{tr} = \begin{bmatrix} 90.8 \\ 49.6 \\ 295.7 \\ 312.6 \\ 75 \\ 107 \\ 82.5 \\ 74.8 \\ 139 \\ 65.6 \end{bmatrix} kN \qquad P_{il} = \begin{bmatrix} 127.6 \\ 83.9 \\ 344.7 \\ 362.6 \\ 110.8 \\ 144.7 \\ 118.7 \\ 110.6 \\ 178.7 \\ 100.8 \end{bmatrix} kN \qquad P_{jf} = \begin{bmatrix} 166.5 \\ 116.1 \\ 416.4 \\ 437 \\ 147.2 \\ 186.2 \\ 156.3 \\ 146.9 \\ 225.3 \\ 135.7 \end{bmatrix} kN \qquad \frac{P_{tr_i}}{P_{jf_i}} = \begin{bmatrix} 0.546 \\ 0.427 \\ 0.71 \\ 0.715 \\ 0.51 \\ 0.575 \\ 0.528 \\ 0.509 \\ 0.617 \\ 0.483 \end{bmatrix}$$

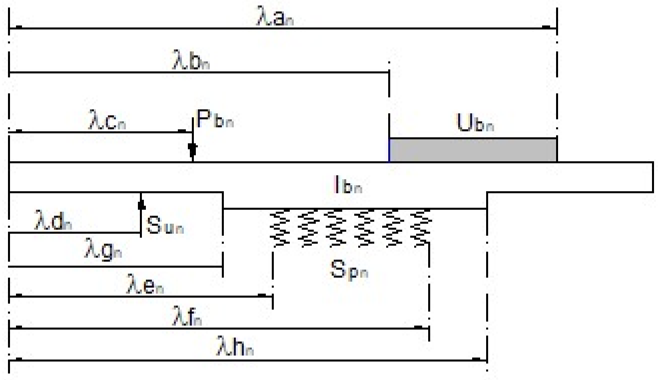
$$P_{lr}(x) := \left\| \begin{array}{l} ii \leftarrow 0 \\ \text{while } mx_{ii} < x \\ \quad \| \ ii \leftarrow ii + 1 \\ \dfrac{P_{il_{ii-1}}}{zx_{ii-1}} \end{array} \right. \qquad P_{jr}(x) := \left\| \begin{array}{l} ii \leftarrow 0 \\ \text{while } mx_{ii} < x \\ \quad \| \ ii \leftarrow ii + 1 \\ \dfrac{P_{tl}}{zx_{ii-1}} \end{array} \right.$$

3.3.3 Determination of prestressing force before losses



4.0 TANK WALL MODELLED AS A BEAM ON A FLEXIBLE ELASTIC FOUNDATION

4.1 General Description and Diagram Showing Definitions



The solution uses the method of finite differences. There may be any number of changes of section, point loads, whole or patch UDL's, point (rigid or spring) supports, or lengths of spring support. Using a large number of nodes reduces inaccuracies due to the re-distribution of load and support positions to the nearest node. Ensuring loads coincide with nodes eliminates small errors due to the process of allocating the loads to the nearest nodes.

Number of nodes for finite difference calculation $n := d \cdot (25\ mm)^{-1}$ $n = 180$

Tank wall stiffness per m width $w := 1\ m$ $I_w := \dfrac{h^3 \cdot w}{12 \cdot (1 - \nu^2)}$ $I_w = 54962\ cm^4$

Semi pinned base stiffness $S_{pn} := 1.2\ kN \cdot mm^{-1}$

Semi rotational fixity factor $semi := 0.0134$

4.1.1 Orthotropic Spring Stiffnesses

Isotropic spring stiffnesses

$k_{cs} := \dfrac{w \cdot h \cdot E_{cs}}{r^2}$ $k_{cs} = 51.135\ MPa$ short term

$k_{cl} := \dfrac{w \cdot h \cdot E_{cl}}{r^2}$ $k_{cl} = 19.846\ MPa$ long term

$k_{s.ip} := \delta_{o.i.inc} \cdot k_{cs}$ $k_{s.ip} = 8.811\ MPa$

$k_{s.tp} := \delta_{o.t.inc} \cdot k_{cl}$ $k_{s.tp} = 6.93\ MPa$

$k_{s.ih} := \delta_{o.i.dec} \cdot k_{cs}$ $k_{s.ih} = 22.38\ MPa$

$k_{s.th} := \delta_{o.t.dec} \cdot k_{cl}$ $k_{s.th} = 13.557\ MPa$

4.1.2 Beam Stiffness Details

$I_b := \begin{bmatrix} I_w \\ 0\ mm^4 \end{bmatrix}$ stiffness of unit width of wall $\lambda_g := \begin{bmatrix} 0\ mm \\ 0\ mm \end{bmatrix}$ $\lambda_h := \begin{bmatrix} d \\ 0\ mm \end{bmatrix}$

4.1.3 Support Details [kN & mm] :

a large stiffness is used to signify a rigid support : e.g. $10^{20}\ kN \cdot mm^{-1}$ **)**

$S_{u.p} := \begin{bmatrix} 10^{15} \\ 0 \end{bmatrix} \cdot kN \cdot mm^{-1}$ stiff restraint at left hand end

$S_{u.s} := \begin{bmatrix} S_{pn} \\ 0 \cdot kN \cdot mm^{-1} \end{bmatrix}$ spring restraint at left hand end

$S_{u.f} := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot kN \cdot mm^{-1}$ free at left hand end

$S_{p.ip} := \begin{bmatrix} k_{s.ip} \\ 0 \cdot kN \cdot mm^{-2} \end{bmatrix}$ uniform spring stiffness for initial prestress state

$\lambda_d := \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot mm$ Support location

$S_{p.tp} := \begin{bmatrix} k_{s.tp} \\ 0 \cdot kN \cdot mm^{-2} \end{bmatrix}$ uniform spring stiffness for relaxed prestress state

$\lambda_e := \begin{bmatrix} 0 \cdot mm \\ 0 \cdot mm \end{bmatrix}$ Start point for spring support

$S_{p.ih} := \begin{bmatrix} k_{s.ih} \\ 0 \cdot kN \cdot mm^{-2} \end{bmatrix}$ uniform spring stiffness for initial hydrostatic state

$\lambda_f := \begin{bmatrix} d \\ 0 \cdot mm \end{bmatrix}$ End point for spring support

$S_{p.th} := \begin{bmatrix} k_{s.th} \\ 0 \cdot kN \cdot mm^{-2} \end{bmatrix}$ uniform spring stiffness for relaxed hydrostatic state

4.2 Definition of Constants

$\varepsilon 1 := \frac{d}{n}$ $\quad x := \frac{n}{2}$ $\quad \varepsilon 2 := \frac{d}{x}$ $\quad x_w := \frac{d_h}{\varepsilon 2}$ $\quad \kappa_s := \frac{E_{cs}}{\varepsilon 1^3}$ $\quad \kappa_l := \frac{E_{cl}}{\varepsilon 1^3}$

$x = 90$

$\varepsilon 1 = 25\ mm$ $\quad a := 1 .. x_w$ $\quad \varepsilon 2 = 50\ mm$ $\quad x_w = 84$ $\quad \kappa_s = 2.2\ N \cdot mm^{-5}$ $\quad \kappa_l = 0.9\ N \cdot mm^{-5}$

4.4 Loading Details (kN & mm)

UDL Hydrostatic loading (load direction -ve)

$\gamma = 10.3\ \frac{kN}{m^3}$ $\quad d_h = 4.2\ m$

$H_m := \gamma \cdot d_h$ $\quad H_m = 43.26\ \frac{kN}{m^2}$

$U_{b_a} := -\frac{\varepsilon 2 \cdot \left(\left(x_w - a\right) + 0.5\right) \cdot w}{d_h} \cdot H_m$

$\lambda_{b_a} := \varepsilon 2 \cdot (a - 1)$ $\quad \lambda_{c_a} := \varepsilon 2 \cdot a$

$U_{b_1} = -43.003\ \frac{kN}{m}$ $\quad \lambda_{b_1} = 0\ mm$ $\quad \lambda_{c_1} = 50\ mm$

$U_{b_{x_w}} = -0.258\ \frac{kN}{m}$ $\quad \lambda_{b_{x_w}} = 4150\ mm$ $\quad \lambda_{c_{x_w}} = 4200\ mm$

$\sum_a \left(U_{b_a} \cdot \left(\lambda_{c_a} - \lambda_{b_a}\right)\right) = -90.85\ kN$ $\quad \frac{H_m \cdot d_h \cdot w}{2} = 90.85\ kN$

Point loads from prestressing tendons

Change of tendon load during relaxation

$P_{b_i} := P_{tl} \cdot \frac{w}{r}$ $\quad P_{b_1} \cdot \frac{r}{w} = 145.46\ kN$ $\quad \lambda_a := tx$ $\quad P_{br_i} := (1 - \kappa) \cdot P_{b_i}$ $\quad P_{br_1} \cdot \frac{r}{w} = 93.9\ kN$

UDL External loading

$x_e := \dfrac{d_e}{\varepsilon 2}$ $\qquad a_e := 1 .. \max(x_e, 1)$ $\qquad x_e = 62$

$x_{ew} := \dfrac{d_w}{\varepsilon 2}$ $\qquad a_{ew} := 1 .. \max(x_{ew}, 1)$ $\qquad x_{ew} = 54$

$E_m := k_{es} \cdot \gamma_e \cdot d_e$ $\qquad E_m = 23.56 \dfrac{kN}{m^2}$

$E_{mw} := k_{es} \cdot \gamma_e \cdot (d_e - d_w) + k_{es} \cdot (\gamma_e - \gamma_{ew}) \cdot d_w + \gamma_{ew} \cdot d_w$ $\qquad E_{mw} = 39.76 \dfrac{kN}{m^2}$

$U_{e_{a_e}} := \text{if}\left(d_e \neq 0\ m, \left(\dfrac{\varepsilon 2 \cdot ((x_e - a_e) + 0.5) \cdot w}{d_e} \cdot E_m + q_{es} \cdot k_{es} \cdot w\right), 0 \dfrac{kN}{m}\right)$ $\qquad q_{es} \cdot k_{es} = 2 \dfrac{kN}{m^2}$

$\lambda_{be_{a_e}} := \varepsilon 2 \cdot (a_e - 1)$ $\qquad d_e = 3.1\ m$

$\lambda_{ce_{a_e}} := \varepsilon 2 \cdot a_e$

$\sum_{a_e} \left(U_{e_{a_e}} \cdot \left(\lambda_{ce_{a_e}} - \lambda_{be_{a_e}}\right)\right) = 42.718\ kN$ $\qquad \dfrac{d_e^{\ 2} \cdot k_{es} \cdot \gamma_e \cdot w}{2} + q_{es} \cdot k_{es} \cdot d_e \cdot w = 42.718\ kN$

$U_{ew_{a_{ew}}} := \text{if}\left(d_w \neq 0\ m, \dfrac{\varepsilon 2 \cdot ((x_{ew} - a_{ew}) + 0.5) \cdot w}{d_w} \cdot E_{mw}, 0 \dfrac{kN}{m}\right)$ $\qquad \dfrac{d_w^{\ 2} \cdot \gamma_{ew} \cdot w}{2} = 36.45\ kN$

$\lambda_{bw_{a_{ew}}} := \varepsilon 2 \cdot (a_{ew} - 1)$ $\qquad d_w = 2.7\ m$

$\lambda_{cw_{a_{ew}}} := \varepsilon 2 \cdot a_{ew}$ $\qquad \sum_{a_{ew}} \left(U_{ew_{a_{ew}}} \cdot \left(\lambda_{cw_{a_{ew}}} - \lambda_{bw_{a_{ew}}}\right)\right) = 53.676\ kN$ $\qquad \dfrac{E_{mw} \cdot d_w \cdot w}{2} = 53.676\ kN$

$$\sum_{a_e} \left(U_{e_{a_e}} \cdot \left(\lambda_{ce_{a_e}} - \lambda_{be_{a_e}}\right)\right) + \sum_{a_{ew}} \left(U_{ew_{a_{ew}}} \cdot \left(\lambda_{cw_{a_{ew}}} - \lambda_{bw_{a_{ew}}}\right)\right) = 96.394\ kN$$

$$\left(\frac{E_m \cdot d_e}{2} + \frac{E_{mw} \cdot d_w}{2} + q_{es} \cdot k_{es} \cdot d_e\right) \cdot w = 96.394\ kN$$

4.5 Function Calls to Initialise and Set Up the Load, Support and Stiffness Vectors

The Loads, the Support conditions and the Beam Inertias, which are specified dimensionally above, are distributed to the uniformly spaced nodes of the finite difference grid, and then the stiffness matrix is assembled and inverted. The functions called in this section are globally defined at the end of the calculation

Initialise load and stiffness matrices to zero

$i := 0..n$ $\quad L_{i_i} := 0\ kN$ $\quad I_i := 1.0\ mm^4$ $\quad x_i := \frac{d \cdot (i)}{n}$

Load matrices for prestressing and hydrostatic loads *ip - initial prestress, tp - relaxed prestress, h - hydrostatic*

$L_{ip} := L_i + F_p(n, P_b, \lambda_a, d)$ $\quad L_{tp} := L_i + F_p(n, P_{br}, \lambda_a, d)$

$L_h := L_i + F_u(n, U_b, \lambda_b, \lambda_c, \varepsilon 1)$ $\quad L_e := L_i + F_u(n, U_e, \lambda_{be}, \lambda_{ce}, \varepsilon 1) + F_u(n, U_{ew}, \lambda_{bw}, \lambda_{cw}, \varepsilon 1)$

Flag values for rotational end fixity

$rot := 0$ $\quad semi = 0.0134$ $\quad fix := 1$ $\quad$ 0 - rotates , 1 - fixed

Spring support matrices for each condition

$S_{u.s}$ Base lateral spring restraint $\quad S_{u.p}$ Base lateral rigid restraint (pin) $\quad S_{u.f}$ Base free to move laterally

Rotational Fixity end Flags
L_f - left end, R_f - right end

Initial prestress free base

$S_{ip} := (F_{su}(n, S_{u.f}, \lambda_d, d) + F_{sp}(n, S_{p.ip}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.ip} := rot$ $\quad R_{f.ip} := rot$

Initial prestress assuming semi pinned base

$S_{sp} := (F_{su}(n, S_{u.s}, \lambda_d, d) + F_{sp}(n, S_{p.ip}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.sp} := rot$ $\quad R_{f.sp} := rot$

Relaxed prestress, initial hydrostatic and relaxed hydrostatic with pinned base

$S_{tp} := (F_{su}(n, S_{u.p}, \lambda_d, d) + F_{sp}(n, S_{p.tp}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.tp} := rot$ $\quad R_{f.tp} := rot$

$S_{ih} := (F_{su}(n, S_{u.p}, \lambda_d, d) + F_{sp}(n, S_{p.ih}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.ih} := L_{f.tp}$ $\quad R_{f.ih} := R_{f.tp}$

$S_{th} := (F_{su}(n, S_{u.p}, \lambda_d, d) + F_{sp}(n, S_{p.th}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.th} := L_{f.tp}$ $\quad R_{f.th} := R_{f.tp}$

$S_{ie} := (F_{su}(n, S_{u.p}, \lambda_d, d) + F_{sp}(n, S_{p.ip}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.ie} := rot$ $\quad R_{f.ie} := rot$

$S_{te} := (F_{su}(n, S_{u.p}, \lambda_d, d) + F_{sp}(n, S_{p.tp}, \lambda_e, \lambda_f, \varepsilon 1))$ $\quad L_{f.te} := rot$ $\quad R_{f.te} := rot$

Relaxed prestress, initial hydrostatic and relaxed hydrostatic with semi rotational base fixity

$L_{f.tps} := semi$ $\quad R_{f.tps} := rot$

$L_{f.ihs} := L_{f.tps}$ $\quad R_{f.ihs} := R_{f.tps}$

$L_{f.ths} := L_{f.tps}$ $\quad R_{f.ths} := R_{f.tps}$

Relaxed prestress, initial hydrostatic and relaxed hydrostatic with full rotational base fixity

$L_{f.tpf} := fix$ $\quad R_{f.tpf} := rot$

$L_{f.ihf} := L_{f.tpf}$ $\quad R_{f.ihf} := R_{f.tpf}$

$L_{f.thf} := L_{f.tpf}$ $\quad R_{f.thf} := R_{f.tpf}$

Wall stiffness matrix

$I := I + F_i(n, I_b, \lambda_g, \lambda_h, \varepsilon 1)$ $\quad S_{tl} := \textbf{if}(inclong, S_{tp}, S_{th})$

4.6 Set Up Stiffness Matrices for each Loadcase, Invert and Solve

Compile stiffness matrices and modify for supports

$A_{ip} := F_{sm}\left(n, I, \kappa_s, L_{f.ip}, R_{f.ip}\right)$ $\quad \boxed{A_{ip_{i,i}}} := A_{ip_{i,i}} + S_{ip_i}$

Similarly for other loading and restraint cases

$A_{tp} := F_{sm}\left(n, I, \kappa_l, L_{f.tp}, R_{f.tp}\right)$ $\quad \boxed{A_{tp_{i,i}}} := A_{tp_{i,i}} + S_{tpv_i}$

$A_{ih} := F_{sm}\left(n, I, \kappa_s, L_{f.ih}, R_{f.ih}\right)$ $\quad \boxed{A_{ih_{i,i}}} := A_{ih_{i,i}} + S_{ih_i}$

$A_{th} := F_{sm}\left(n, I, \kappa_l, L_{f.th}, R_{f.th}\right)$ $\quad \boxed{A_{th_{i,i}}} := A_{th_{i,i}} + S_{th_i}$

$A_{sp} := F_{sm}\left(n, I, \kappa_s, L_{f.sp}, R_{f.sp}\right)$ $\quad \boxed{A_{sp_{i,i}}} := A_{sp_{i,i}} + S_{sp_i}$

$A_{tps} := F_{sm}\left(n, I, \kappa_l, L_{f.tps}, R_{f.tps}\right)$ $\quad \boxed{A_{tps_{i,i}}} := A_{tps_{i,i}} + S_{tpv_i}$

$A_{ihs} := F_{sm}\left(n, I, \kappa_s, L_{f.ihs}, R_{f.ihs}\right)$ $\quad \boxed{A_{ihs_{i,i}}} := A_{ihs_{i,i}} + S_{ih_i}$

$A_{ths} := F_{sm}\left(n, I, \kappa_l, L_{f.ths}, R_{f.ths}\right)$ $\quad \boxed{A_{ths_{i,i}}} := A_{ths_{i,i}} + S_{th_i}$

$A_{tpf} := F_{sm}\left(n, I, \kappa_l, L_{f.tpf}, R_{f.tpf}\right)$ $\quad \boxed{A_{tpf_{i,i}}} := A_{tpf_{i,i}} + S_{tpv_i}$

$A_{ihf} := F_{sm}\left(n, I, \kappa_s, L_{f.ihf}, R_{f.ihf}\right)$ $\quad \boxed{A_{ihf_{i,i}}} := A_{ihf_{i,i}} + S_{ih_i}$

$A_{thf} := F_{sm}\left(n, I, \kappa_l, L_{f.thf}, R_{f.thf}\right)$ $\quad \boxed{A_{thf_{i,i}}} := A_{thf_{i,i}} + S_{th_i}$

$S_{tfp} := \left(F_{su}\left(n, S_{u.f}, \lambda_d, d\right) + F_{sp}\left(n, S_{p.tp}, \lambda_e, \lambda_f, \varepsilon 1\right)\right)$

$A_{tfp} := F_{sm}\left(n, I, \kappa_l, L_{f.tp}, R_{f.tp}\right)$ $\quad \boxed{A_{tfp_{i,i}}} := A_{tfp_{i,i}} + S_{tfp_i}$

$A_{ie} := F_{sm}\left(n, I, \kappa_s, L_{f.ie}, R_{f.ie}\right)$ $\quad \boxed{A_{ie_{i,i}}} := A_{ie_{i,i}} + S_{ie_i}$

$A_{te} := F_{sm}\left(n, I, \kappa_l, L_{f.te}, R_{f.te}\right)$ $\quad \boxed{A_{te_{i,i}}} := A_{te_{i,i}} + S_{te_i}$

Invert and Solve for deflections and reactions

$\delta_{ip} := A_{ip}^{-1} \cdot L_{ip}$ $\quad R_{ip} := \overrightarrow{\left(S_{ip} \cdot \delta_{ip}\right)}$

$S_{tpv} := \textbf{if}\left(short, S_{tp}, S_{tl}\right)$

$\delta_{tp} := A_{tp}^{-1} \cdot L_{tp}$ $\quad R_{tp} := \overrightarrow{\left(S_{tpv} \cdot \delta_{tp}\right)}$

$\delta_{ih} := A_{ih}^{-1} \cdot L_{h}$ $\quad R_{ih} := \overrightarrow{\left(S_{ih} \cdot \delta_{ih}\right)}$

$\delta_{th} := A_{th}^{-1} \cdot L_{h}$ $\quad R_{th} := \overrightarrow{\left(S_{th} \cdot \delta_{th}\right)}$

$\delta_{sp} := A_{sp}^{-1} \cdot L_{ip}$ $\quad R_{sp} := \overrightarrow{\left(S_{sp} \cdot \delta_{sp}\right)}$

$\delta_{tps} := A_{tps}^{-1} \cdot L_{tp}$ $\quad R_{tps} := \overrightarrow{\left(S_{tpv} \cdot \delta_{tps}\right)}$

$\delta_{ihs} := A_{ihs}^{-1} \cdot L_{h}$ $\quad R_{ihs} := \overrightarrow{\left(S_{ih} \cdot \delta_{ihs}\right)}$

$\delta_{ths} := A_{ths}^{-1} \cdot L_{h}$ $\quad R_{ths} := \overrightarrow{\left(S_{th} \cdot \delta_{ths}\right)}$

$\delta_{tpf} := A_{tpf}^{-1} \cdot L_{tp}$ $\quad R_{tpf} := \overrightarrow{\left(S_{tpv} \cdot \delta_{tpf}\right)}$

$\delta_{ihf} := A_{ihf}^{-1} \cdot L_{h}$ $\quad R_{ihf} := \overrightarrow{\left(S_{ih} \cdot \delta_{ihf}\right)}$

$\delta_{thf} := A_{thf}^{-1} \cdot L_{h}$ $\quad R_{thf} := \overrightarrow{\left(S_{th} \cdot \delta_{thf}\right)}$

$\delta_{tfp} := A_{tfp}^{-1} \cdot L_{tp}$ $\quad R_{tfp} := \overrightarrow{\left(S_{tfp} \cdot \delta_{tfp}\right)}$

$\delta_{ie} := A_{ie}^{-1} \cdot L_{e}$ $\quad R_{ie} := \overrightarrow{\left(S_{ie} \cdot \delta_{ie}\right)}$

$\delta_{te} := A_{te}^{-1} \cdot L_{e}$ $\quad R_{te} := \overrightarrow{\left(S_{te} \cdot \delta_{te}\right)}$

	Prestress initial free added to relaxed pinned	Initial prestress and hydrostatic	Relaxed prestress and hydrostatic
Combined deflections (pinned, semi fixed & fixed starting from free base condition during stressing)	$\delta_{rp} := \overrightarrow{\left(\delta_{ip} + \delta_{tp}\right)}$	$\delta_{if} := \overrightarrow{\left(\delta_{ip} + \delta_{ih}\right)}$	$\delta_{tf} := \overrightarrow{\left(\delta_{rp} + \delta_{th}\right)}$
	$\delta_{rps} := \overrightarrow{\left(\delta_{ip} + \delta_{tps}\right)}$	$\delta_{ifs} := \overrightarrow{\left(\delta_{ip} + \delta_{ihs}\right)}$	$\delta_{tfs} := \overrightarrow{\left(\delta_{rps} + \delta_{ths}\right)}$
	$\delta_{rpf} := \overrightarrow{\left(\delta_{ip} + \delta_{tpf}\right)}$	$\delta_{iff} := \overrightarrow{\left(\delta_{ip} + \delta_{ihf}\right)}$	$\delta_{tff} := \overrightarrow{\left(\delta_{rpf} + \delta_{thf}\right)}$

	Prestress initial free added to relaxed pinned	Initial prestress and hydrostatic	Relaxed prestress and hydrostatic
Combined deflections (starting from partial pin case during prestressing)	$\delta_{srp} := \overrightarrow{\left(\delta_{sp} + \delta_{tp}\right)}$	$\delta_{sif} := \overrightarrow{\left(\delta_{sp} + \delta_{ih}\right)}$	$\delta_{stf} := \overrightarrow{\left(\delta_{srp} + \delta_{th}\right)}$
	$\delta_{srps} := \overrightarrow{\left(\delta_{sp} + \delta_{tps}\right)}$	$\delta_{sifs} := \overrightarrow{\left(\delta_{sp} + \delta_{ihs}\right)}$	$\delta_{stfs} := \overrightarrow{\left(\delta_{srps} + \delta_{ths}\right)}$
	$\delta_{srpf} := \overrightarrow{\left(\delta_{sp} + \delta_{tpf}\right)}$	$\delta_{siff} := \overrightarrow{\left(\delta_{sp} + \delta_{ihf}\right)}$	$\delta_{stff} := \overrightarrow{\left(\delta_{srpf} + \delta_{thf}\right)}$

Combined deflections (with external earth and water loadings)	$\delta_{ipe} := \overrightarrow{\left(\delta_{ip} + \delta_{ie}\right)}$		
	$\delta_{rpe} := \overrightarrow{\left(\delta_{rp} + \delta_{te}\right)}$	$\delta_{ife} := \overrightarrow{\left(\delta_{if} + \delta_{ie}\right)}$	$\delta_{tfe} := \overrightarrow{\left(\delta_{tf} + \delta_{te}\right)}$

4.7 Check Balance of Vertical Forces and Reactions

Sum of reactions and imposed load for initial prestress cases

$\sum R_{ip} = 130.15\ kN$ $\sum R_{sp} = 130.15\ kN$ $\sum L_{ip} = 130.15\ kN$ $\sum P_b = 130.15\ kN$

Sum of reactions and imposed load for relaxed prestress cases

$\sum R_{tp} = 84.01\ kN$ $\sum R_{tps} = 84.01\ kN$ $\sum R_{tpf} = 84.01\ kN$ $\sum L_{tp} = 84.01\ kN$ $\sum P_{br} = 84.01\ kN$

Sum of reactions and imposed load for initial hydrostatic case

$\sum R_{ih} = -97.38\ kN$ $\sum R_{ihs} = -97.38\ kN$ $\sum R_{ihf} = -97.38\ kN$ $\sum L_h = -97.38\ kN$

$$\frac{H_m \cdot d_h \cdot w}{2} = 90.85\ kN$$

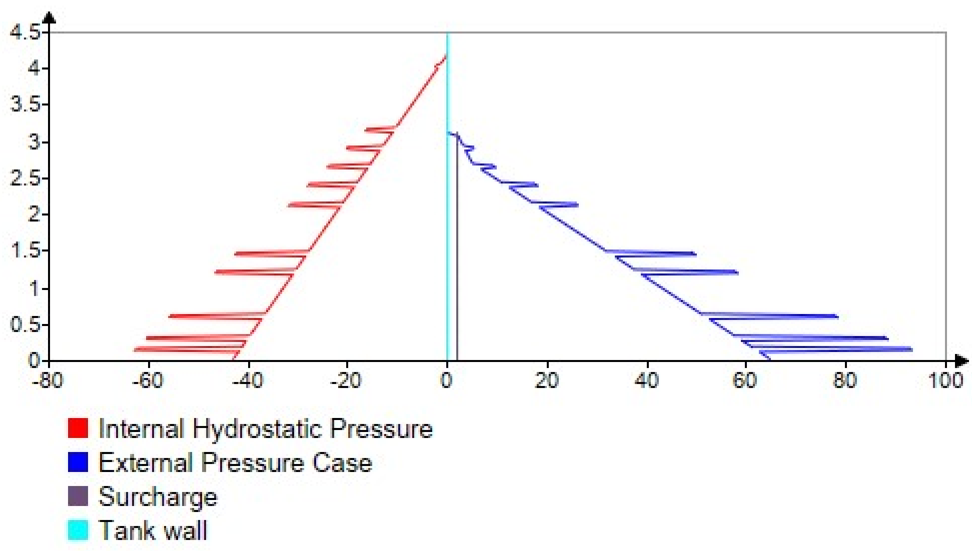
Sum of reactions and imposed load for relaxed hydrostatic case

$\sum R_{th} = -97.38\ kN$ $\sum R_{ths} = -97.38\ kN$ $\sum R_{thf} = -97.38\ kN$ $\sum L_h = -97.38\ kN$

Sum of reactions and imposed load for external earth & water pressure cases

$\sum R_{ie} = 103.55\ kN$ $\sum R_{te} = 103.55\ kN$ $\sum L_e = 103.55\ kN$

Plot of applied loads $er := 0 .. \dfrac{d_e}{\varepsilon 1} + 1$ $tw := 0 .. \dfrac{d_h}{\varepsilon 1} + 1$



$\lambda := 0, n..n$

$M_{ip} := M_f\left(\delta_{ip}, E_{cs}, I, n, \varepsilon 1, L_{f.ip}, R_{f.ip}\right)$ $\quad V_{ip} := V_f\left(M_{ip}, R_{ip_0}, n, \varepsilon 1\right)$

$M_{tp} := M_f\left(\delta_{tp}, E_{cl}, I, n, \varepsilon 1, L_{f.tp}, R_{f.tp}\right)$ $\quad V_{tp} := V_f\left(M_{tp}, R_{tp_0}, n, \varepsilon 1\right)$

$M_{rp} := M_f\left(\delta_{rp}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{rp}}_{\lambda} := M_{ip_\lambda} + M_{tp_\lambda}$ $\quad V_{rp} := V_f\left(M_{rp}, R_{ip_0} + R_{tp_0}, n, \varepsilon 1\right)$

$M_{ih} := M_f\left(\delta_{ih}, E_{cs}, I, n, \varepsilon 1, L_{f.ih}, R_{f.ih}\right)$ $\quad V_{ih} := V_f\left(M_{ih}, R_{ih_0}, n, \varepsilon 1\right)$

$M_{th} := M_f\left(\delta_{th}, E_{cl}, I, n, \varepsilon 1, L_{f.th}, R_{f.th}\right)$ $\quad V_{th} := V_f\left(M_{th}, R_{th_0}, n, \varepsilon 1\right)$

$M_{if} := M_f\left(\delta_{if}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{if}}_{\lambda} := M_{ip_\lambda} + M_{ih_\lambda}$ $\quad V_{if} := V_f\left(M_{if}, R_{ip_0} + R_{ih_0}, n, \varepsilon 1\right)$

$M_{tf} := M_f\left(\delta_{tf}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{tf}}_{\lambda} := M_{ip_\lambda} + M_{tp_\lambda} + M_{th_\lambda}$ $\quad V_{tf} := V_f\left(M_{tf}, R_{ip_0} + R_{tp_0} + R_{th_0}, n, \varepsilon 1\right)$

$M_{sp} := M_f\left(\delta_{sp}, E_{cs}, I, n, \varepsilon 1, L_{f.sp}, R_{f.sp}\right)$ $\quad V_{sp} := V_f\left(M_{sp}, R_{sp_0}, n, \varepsilon 1\right)$

$M_{tps} := M_f\left(\delta_{tps}, E_{cl}, I, n, \varepsilon 1, L_{f.tps}, R_{f.tps}\right)$ $\quad V_{tps} := V_f\left(M_{tps}, R_{tps_0}, n, \varepsilon 1\right)$

$M_{rps} := M_f\left(\delta_{rps}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{rps}}_{\lambda} := M_{ip_\lambda} + M_{tps_\lambda}$ $\quad V_{rps} := V_f\left(M_{rps}, R_{ip_0} + R_{tps_0}, n, \varepsilon 1\right)$

$M_{tpf} := M_f\left(\delta_{tpf}, E_{cl}, I, n, \varepsilon 1, L_{f.tpf}, R_{f.tpf}\right)$ $\quad V_{tpf} := V_f\left(M_{tpf}, R_{tpf_0}, n, \varepsilon 1\right)$

$M_{rpf} := M_f\left(\delta_{rpf}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{rpf}}_{\lambda} := M_{ip_\lambda} + M_{tpf_\lambda}$ $\quad V_{rpf} := V_f\left(M_{rpf}, R_{ip_0} + R_{tpf_0}, n, \varepsilon 1\right)$

$M_{ihs} := M_f\left(\delta_{ihs}, E_{cs}, I, n, \varepsilon 1, L_{f.ihs}, R_{f.ihs}\right)$ $\quad V_{ihs} := V_f\left(M_{ihs}, R_{ihs_0}, n, \varepsilon 1\right)$

$M_{ths} := M_f\left(\delta_{ths}, E_{cl}, I, n, \varepsilon 1, L_{f.ths}, R_{f.ths}\right)$ $\quad V_{ths} := V_f\left(M_{ths}, R_{ths_0}, n, \varepsilon 1\right)$

$M_{ihf} := M_f\left(\delta_{ihf}, E_{cs}, I, n, \varepsilon 1, L_{f.ihf}, R_{f.ihf}\right)$ $\quad V_{ihf} := V_f\left(M_{ihf}, R_{ihf_0}, n, \varepsilon 1\right)$

$M_{thf} := M_f\left(\delta_{thf}, E_{cl}, I, n, \varepsilon 1, L_{f.thf}, R_{f.thf}\right)$ $\quad V_{thf} := V_f\left(M_{thf}, R_{thf_0}, n, \varepsilon 1\right)$

$M_{ifs} := M_f\left(\delta_{ifs}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{ifs}}_{\lambda} := M_{ip_\lambda} + M_{ihs_\lambda}$ $\quad V_{ifs} := V_f\left(M_{ifs}, R_{ip_0} + R_{ihs_0}, n, \varepsilon 1\right)$

$M_{tfs} := M_f\left(\delta_{tfs}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{tfs}}_{\lambda} := M_{ip_\lambda} + M_{tps_\lambda} + M_{ths_\lambda}$ $\quad V_{tfs} := V_f\left(M_{tfs}, R_{ip_0} + R_{tps_0} + R_{ths_0}, n, \varepsilon 1\right)$

$M_{iff} := M_f\left(\delta_{iff}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{iff}}_{\lambda} := M_{ip_\lambda} + M_{ihf_\lambda}$ $\quad V_{iff} := V_f\left(M_{iff}, R_{ip_0} + R_{ihf_0}, n, \varepsilon 1\right)$

$M_{tff} := M_f\left(\delta_{tff}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{tff}}_{\lambda} := M_{ip_\lambda} + M_{tpf_\lambda} + M_{thf_\lambda}$ $\quad V_{tff} := V_f\left(M_{tff}, R_{ip_0} + R_{tpf_0} + R_{thf_0}, n, \varepsilon 1\right)$

$M_{srp} := M_f\left(\delta_{srp}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{srp}}_{\lambda} := M_{sp_\lambda} + M_{tp_\lambda}$ $\quad V_{srp} := V_f\left(M_{srp}, R_{sp_0} + R_{tp_0}, n, \varepsilon 1\right)$

$M_{srps} := M_f\left(\delta_{srps}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{srps}}_{\lambda} := M_{sp_\lambda} + M_{tps_\lambda}$ $\quad V_{srps} := V_f\left(M_{srps}, R_{sp_0} + R_{tps_0}, n, \varepsilon 1\right)$

$M_{srpf} := M_f\left(\delta_{srpf}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad \boxed{M_{srpf}}_{\lambda} := M_{sp_\lambda} + M_{tpf_\lambda}$ $\quad V_{srpf} := V_f\left(M_{srpf}, R_{sp_0} + R_{tpf_0}, n, \varepsilon 1\right)$

$M_{sif} := M_f\left(\delta_{sif}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{sif}}_{\lambda} := M_{sp_{\lambda}} + M_{ih_{\lambda}}$ $\quad$ $V_{sif} := V_f\left(M_{sif}, R_{sp_0} + R_{ih_0}, n, \varepsilon 1\right)$

$M_{stf} := M_f\left(\delta_{stf}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{stf}}_{\lambda} := M_{sp_{\lambda}} + M_{tp_{\lambda}} + M_{th_{\lambda}}$ $\quad$ $V_{stf} := V_f\left(M_{stf}, R_{sp_0} + R_{tp_0} + R_{th_0}, n, \varepsilon 1\right)$

$M_{sifs} := M_f\left(\delta_{sifs}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{sifs}}_{\lambda} := M_{sp_{\lambda}} + M_{ihs_{\lambda}}$ $\quad$ $V_{sifs} := V_f\left(M_{sifs}, R_{sp_0} + R_{ihs_0}, n, \varepsilon 1\right)$

$M_{stfs} := M_f\left(\delta_{stfs}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{stfs}}_{\lambda} := M_{sp_{\lambda}} + M_{tps_{\lambda}} + M_{ths_{\lambda}}$ $\quad$ $V_{stfs} := V_f\left(M_{stfs}, R_{sp_0} + R_{tps_0} + R_{ths_0}, n, \varepsilon 1\right)$

$M_{siff} := M_f\left(\delta_{siff}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{siff}}_{\lambda} := M_{sp_{\lambda}} + M_{ihf_{\lambda}}$ $\quad$ $V_{siff} := V_f\left(M_{siff}, R_{sp_0} + R_{ihf_0}, n, \varepsilon 1\right)$

$M_{stff} := M_f\left(\delta_{stff}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{stff}}_{\lambda} := M_{sp_{\lambda}} + M_{tpf_{\lambda}} + M_{thf_{\lambda}}$ $\quad$ $V_{stff} := V_f\left(M_{stff}, R_{sp_0} + R_{tpf_0} + R_{thf_0}, n, \varepsilon 1\right)$

$M_{ie} := M_f\left(\delta_{ie}, E_{cs}, I, n, \varepsilon 1, L_{f.ie}, R_{f.ie}\right)$ $\quad$ $V_{ie} := V_f\left(M_{ie}, R_{ie_0}, n, \varepsilon 1\right)$

$M_{te} := M_f\left(\delta_{te}, E_{cl}, I, n, \varepsilon 1, L_{f.te}, R_{f.te}\right)$ $\quad$ $V_{te} := V_f\left(M_{te}, R_{te_0}, n, \varepsilon 1\right)$

$M_{ipe} := M_f\left(\delta_{ipe}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{ipe}}_{\lambda} := M_{ip_{\lambda}} + M_{ie_{\lambda}}$ $\quad$ $V_{ipe} := V_f\left(M_{ipe}, R_{ip_0} + R_{ie_0}, n, \varepsilon 1\right)$

$M_{rpe} := M_f\left(\delta_{rpe}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{rpe}}_{\lambda} := M_{ip_{\lambda}} + M_{tp_{\lambda}} + M_{te_{\lambda}}$ $\quad$ $V_{rpe} := V_f\left(M_{rpe}, R_{ip_0} + R_{tp_0} + R_{te_0}, n, \varepsilon 1\right)$

$M_{ife} := M_f\left(\delta_{ife}, E_{cs}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{ife}}_{\lambda} := M_{ip_{\lambda}} + M_{ih_{\lambda}} + M_{ie_{\lambda}}$ $\quad$ $V_{ife} := V_f\left(M_{ife}, R_{ip_0} + R_{ih_0} + R_{ie_0}, n, \varepsilon 1\right)$

$M_{tfe} := M_f\left(\delta_{tfe}, E_{cl}, I, n, \varepsilon 1, 0, 0\right)$ $\quad$ $\boxed{M_{tfe}}_{\lambda} := M_{ip_{\lambda}} + M_{tp_{\lambda}} + M_{th_{\lambda}} + M_{te_{\lambda}}$

$V_{tfe} := V_f\left(M_{tfe}, R_{ip_0} + R_{tp_0} + R_{th_0} + R_{te_0}, n, \varepsilon 1\right)$

combined reactions

$R_{rp} := \overrightarrow{\left(R_{ip} - \left(R_{tfp} - R_{tp}\right)\right)}$ $\quad$ $R_{if} := \overrightarrow{\left(R_{ip} + R_{ih}\right)}$ $\quad$ $R_{tf} := \overrightarrow{\left(R_{rp} + R_{th}\right)}$

$R_{rps} := \overrightarrow{\left(R_{ip} - \left(R_{tfp} - R_{tps}\right)\right)}$ $\quad$ $R_{ifs} := \overrightarrow{\left(R_{ip} + R_{ihs}\right)}$ $\quad$ $R_{tfs} := \overrightarrow{\left(R_{rps} + R_{ths}\right)}$

$R_{rpf} := \overrightarrow{\left(R_{ip} - \left(R_{tfp} - R_{tpf}\right)\right)}$ $\quad$ $R_{iff} := \overrightarrow{\left(R_{ip} + R_{ihf}\right)}$ $\quad$ $R_{tff} := \overrightarrow{\left(R_{rpf} + R_{thf}\right)}$

$R_{srp} := \overrightarrow{\left(R_{sp} - \left(R_{tfp} - R_{tp}\right)\right)}$ $\quad$ $R_{sif} := \overrightarrow{\left(R_{sp} + R_{ih}\right)}$ $\quad$ $R_{stf} := \overrightarrow{\left(R_{srp} + R_{th}\right)}$

$R_{srps} := \overrightarrow{\left(R_{sp} - \left(R_{tfp} - R_{tps}\right)\right)}$ $\quad$ $R_{sifs} := \overrightarrow{\left(R_{sp} + R_{ihs}\right)}$ $\quad$ $R_{stfs} := \overrightarrow{\left(R_{srps} + R_{ths}\right)}$

$R_{srpf} := \overrightarrow{\left(R_{sp} - \left(R_{tfp} - R_{tpf}\right)\right)}$ $\quad$ $R_{siff} := \overrightarrow{\left(R_{sp} + R_{ihf}\right)}$ $\quad$ $R_{stff} := \overrightarrow{\left(R_{srpf} + R_{thf}\right)}$

$R_{ipe} := \overrightarrow{\left(R_{ip} + R_{ie}\right)}$ $\quad$ $R_{rpe} := \overrightarrow{\left(R_{ip} - \left(R_{tfp} - R_{tp}\right) + R_{te}\right)}$

$R_{ife} := \overrightarrow{\left(R_{ip} + R_{ih} + R_{ie}\right)}$ $\quad$ $R_{tfe} := \overrightarrow{\left(R_{rp} + R_{th} + R_{te}\right)}$

4.8 Plots of Results and Maximum and Minimum values

4.8.1 Initial Prestress with Free (or Semi Pinned) Base

$min(\delta_{ip}) = 2.123\ mm$

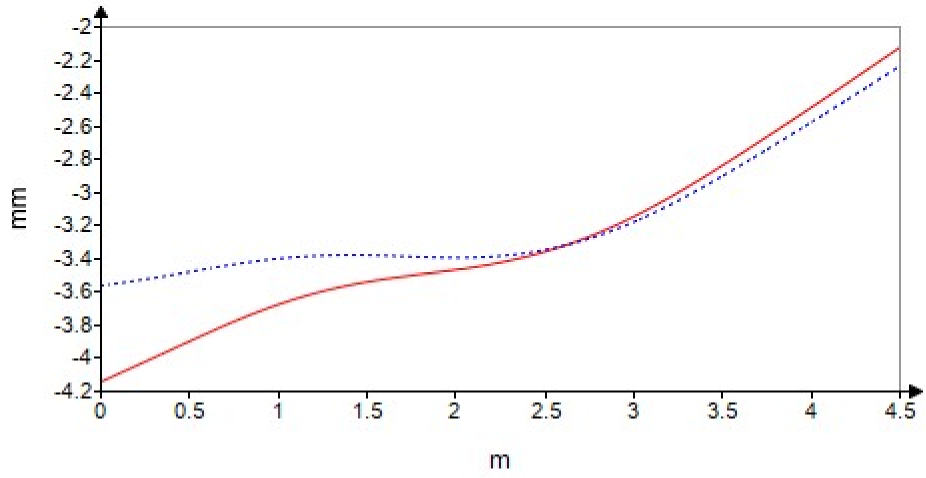
$\max(\delta_{ip}) = 4.141\ mm$

$min(\delta_{sp}) = 2.236\ mm$

$\max(\delta_{sp}) = 3.561\ mm$

$\delta_{sp_0} = 3.561\ mm$

$\dfrac{\delta_{ip_0}}{2} = 2.071\ mm$



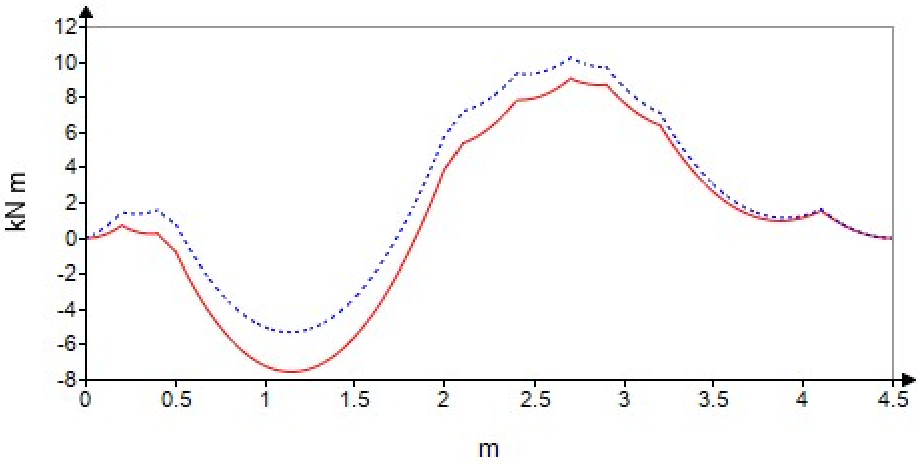
DEFLECTION

$min(M_{ip}) = -7.541\ kN \cdot m$

$\max(M_{ip}) = 9.068\ kN \cdot m$

$min(M_{sp}) = -5.288\ kN \cdot m$

$\max(M_{sp}) = 10.256\ kN \cdot m$



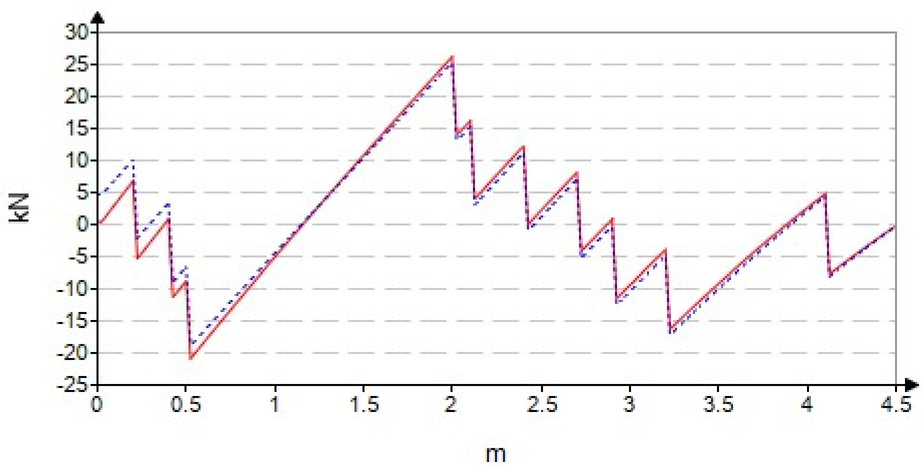
BENDING MOMENT

$min(V_{ip}) = -20.911\ kN$

$\max(V_{ip}) = 26.205\ kN$

$min(V_{sp}) = -18.877\ kN$

$\max(V_{sp}) = 25.339\ kN$



SHEAR FORCE

4.8.2 Relaxed Prestress with Restrained Base (Pinned and with or without rotational restraint)

$min\left(\delta_{rp}\right)=4.004\ mm$

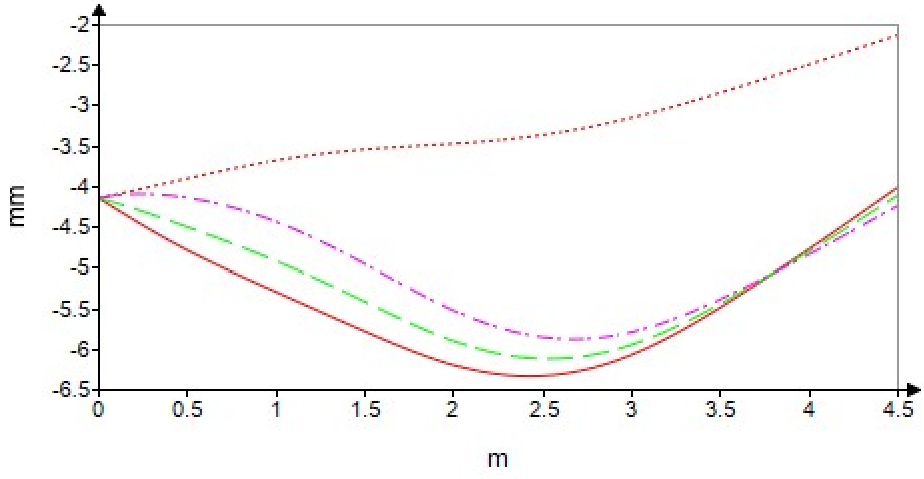
$\max\left(\delta_{rp}\right)=6.329\ mm$

$min\left(\delta_{rps}\right)=4.104\ mm$

$\max\left(\delta_{rps}\right)=6.115\ mm$

$min\left(\delta_{rpf}\right)=4.089\ mm$

$\max\left(\delta_{rpf}\right)=5.872\ mm$



DEFLECTION

$min\left(M_{rp}\right)=-0.01\ kN\cdot m$

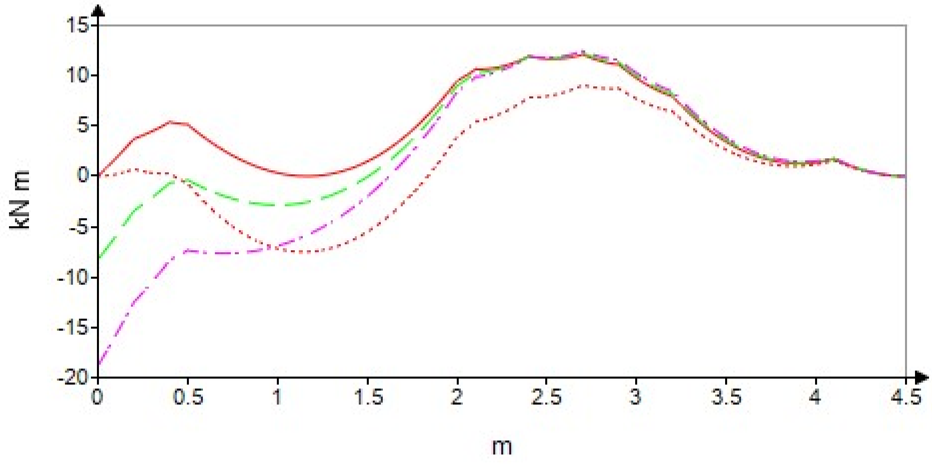
$\max\left(M_{rp}\right)=12.1\ kN\cdot m$

$min\left(M_{rps}\right)=-8.31\ kN\cdot m$

$\max\left(M_{rps}\right)=12.26\ kN\cdot m$

$min\left(M_{rpf}\right)=-18.84\ kN\cdot m$

$\max\left(M_{rpf}\right)=12.45\ kN\cdot m$



BENDING MOMENT

$min\left(V_{rp}\right)=-19.12\ kN$

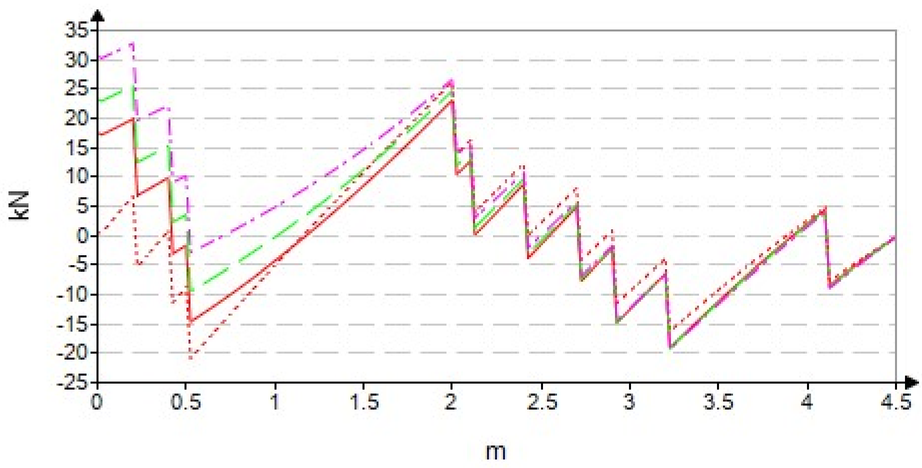
$\max\left(V_{rp}\right)=23.1\ kN$

$min\left(V_{rps}\right)=-19.19\ kN$

$\max\left(V_{rps}\right)=25.57\ kN$

$min\left(V_{rpf}\right)=-19.28\ kN$

$\max\left(V_{rpf}\right)=32.76\ kN$



SHEAR FORCE

4.8.3 Initial Hydrostatic with Restrained Base (Pinned and with or without rotational restraint)

$min\left(\delta_{ih}\right) = -1.043\ mm$

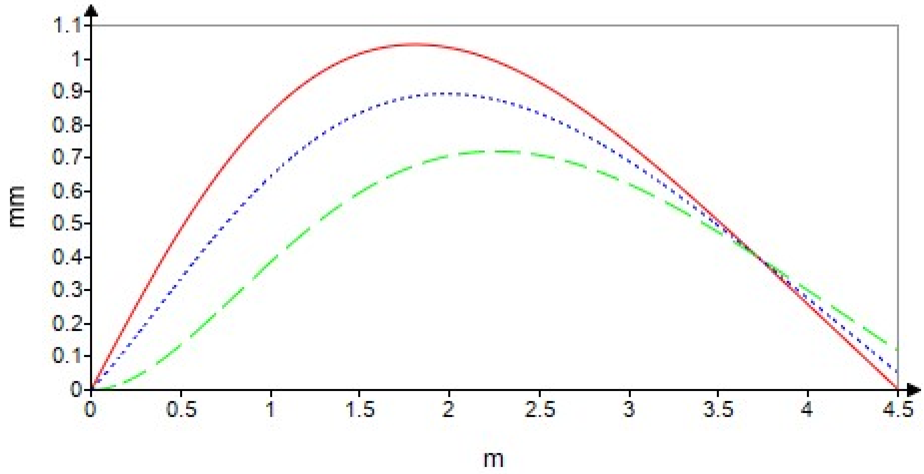
$\max\left(\delta_{ih}\right) = 0\ mm$

$min\left(\delta_{ihs}\right) = -0.893\ mm$

$\max\left(\delta_{ihs}\right) = 0\ mm$

$min\left(\delta_{ihf}\right) = -0.72\ mm$

$\max\left(\delta_{ihf}\right) = 0\ mm$



DEFLECTION

$min\left(M_{ih}\right) = -13.51\ kN \cdot m$

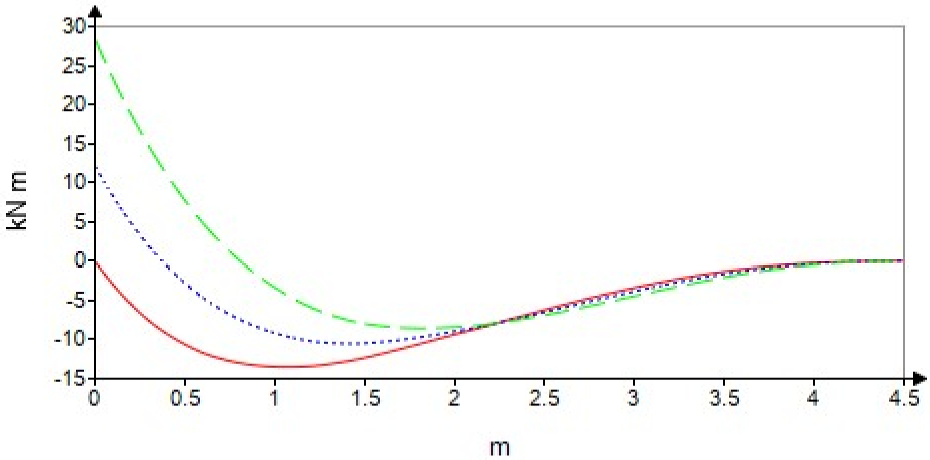
$\max\left(M_{ih}\right) = 0\ kN \cdot m$

$min\left(M_{ihs}\right) = -10.52\ kN \cdot m$

$\max\left(M_{ihs}\right) = 12.12\ kN \cdot m$

$min\left(M_{ihf}\right) = -8.54\ kN \cdot m$

$\max\left(M_{ihf}\right) = 28.41\ kN \cdot m$



BENDING MOMENT

$min\left(V_{ih}\right) = -31.74\ kN$

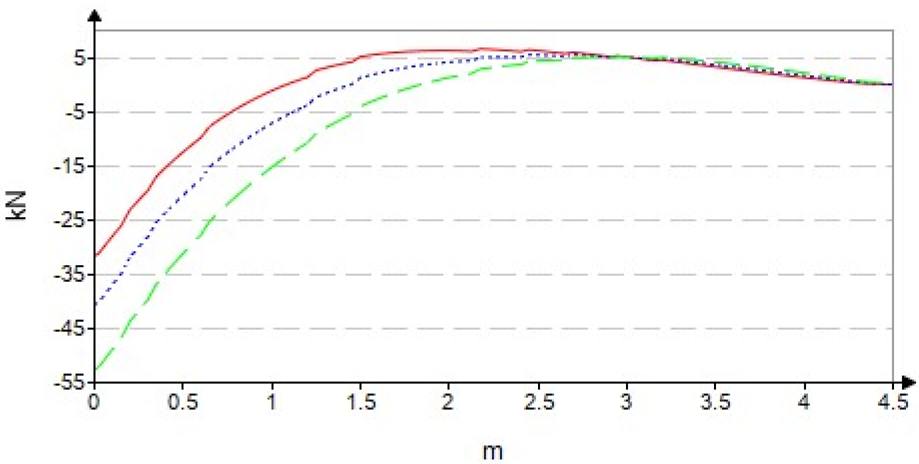
$\max\left(V_{ih}\right) = 6.59\ kN$

$min\left(V_{ihs}\right) = -40.66\ kN$

$\max\left(V_{ihs}\right) = 5.55\ kN$

$min\left(V_{ihf}\right) = -52.64\ kN$

$\max\left(V_{ihf}\right) = 5.21\ kN$



SHEAR FORCE

4.8.4 Relaxed Hydrostatic with Restrained Base (Pinned and with or without rotational restraint)

$min\left(\delta_{th}\right)=-1.882\ mm$

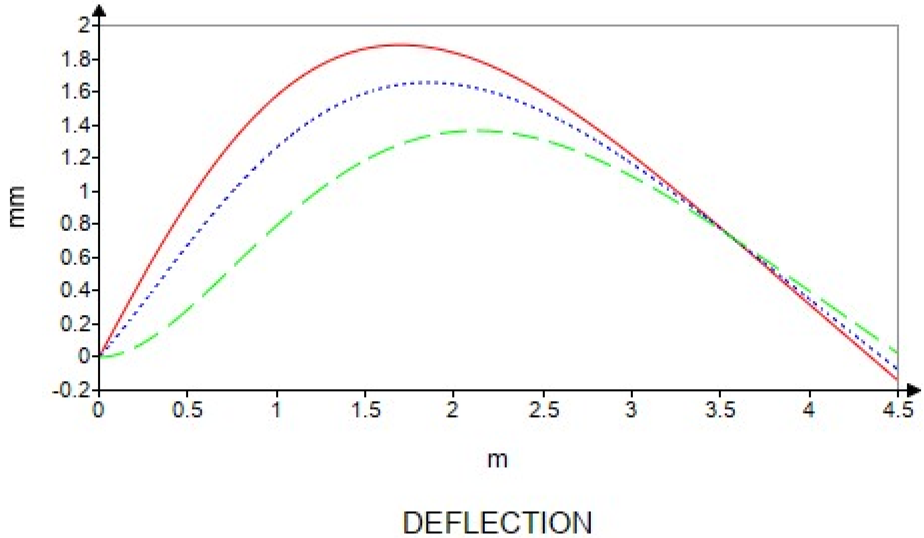
$\max\left(\delta_{th}\right)=0.14\ mm$

$min\left(\delta_{ths}\right)=-1.655\ mm$

$\max\left(\delta_{ths}\right)=0.075\ mm$

$min\left(\delta_{thf}\right)=-1.365\ mm$

$\max\left(\delta_{thf}\right)=0\ mm$



$min\left(M_{th}\right)=-10.85\ kN \cdot m$

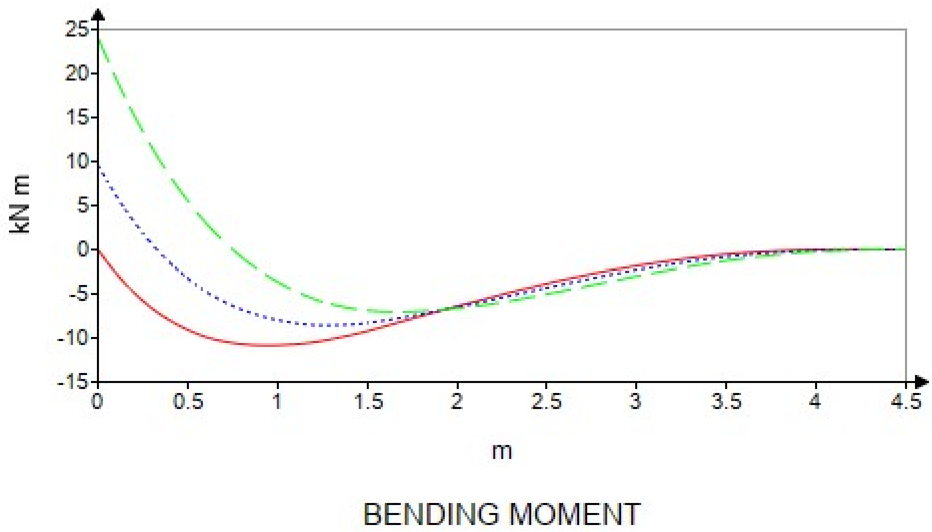
$\max\left(M_{th}\right)=0.03\ kN \cdot m$

$min\left(M_{ths}\right)=-8.56\ kN \cdot m$

$\max\left(M_{ths}\right)=9.6\ kN \cdot m$

$min\left(M_{thf}\right)=-7.02\ kN \cdot m$

$\max\left(M_{thf}\right)=24.09\ kN \cdot m$



$min\left(V_{th}\right)=-28.5\ kN$

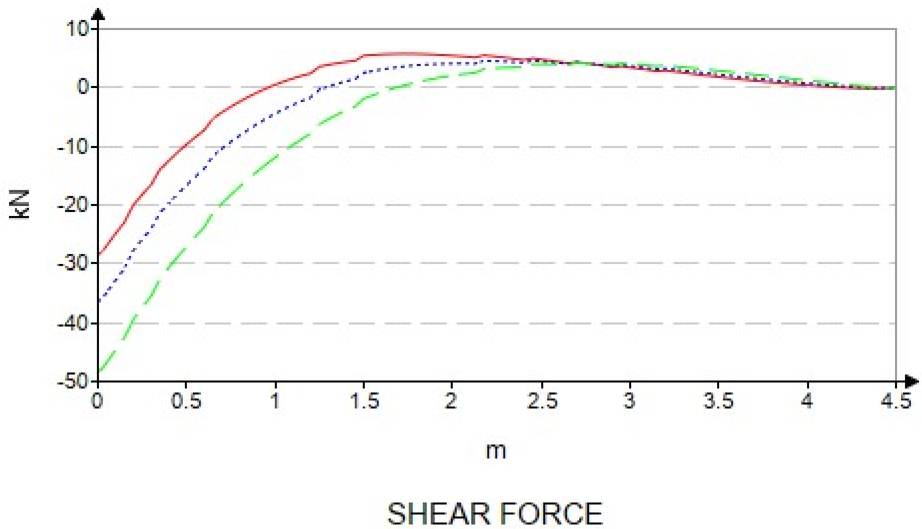
$\max\left(V_{th}\right)=5.76\ kN$

$min\left(V_{ths}\right)=-36.41\ kN$

$\max\left(V_{ths}\right)=4.6\ kN$

$min\left(V_{thf}\right)=-48.35\ kN$

$\max\left(V_{thf}\right)=4.26\ kN$



4.8.5 Initial Prestress and Hydrostatic with Restrained Base

$min\left(\delta_{if}\right)=2.121\ mm$

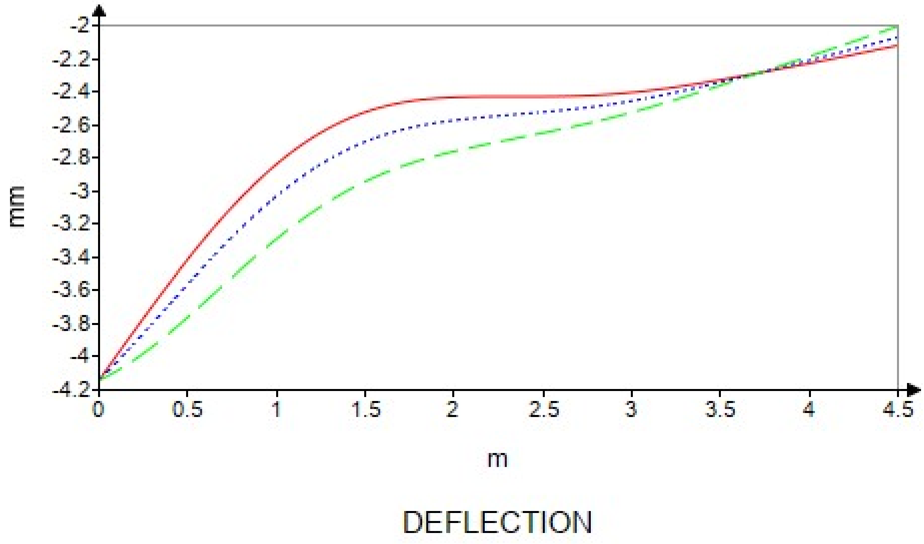
$\max\left(\delta_{if}\right)=4.141\ mm$

$min\left(\delta_{ifs}\right)=2.071\ mm$

$\max\left(\delta_{ifs}\right)=4.141\ mm$

$min\left(\delta_{iff}\right)=2.005\ mm$

$\max\left(\delta_{iff}\right)=4.141\ mm$



$min\left(M_{if}\right)=-21.02\ kN\cdot m$

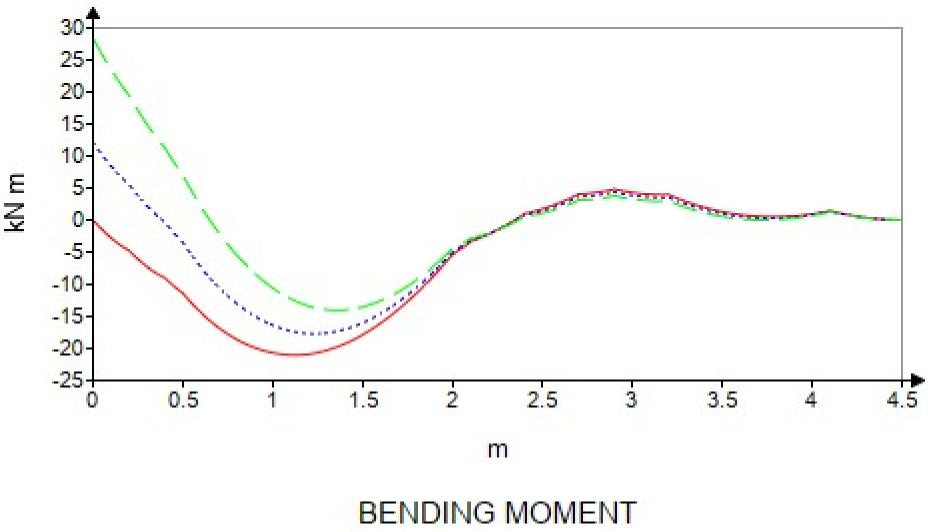
$\max\left(M_{if}\right)=4.82\ kN\cdot m$

$min\left(M_{ifs}\right)=-17.71\ kN\cdot m$

$\max\left(M_{ifs}\right)=12.12\ kN\cdot m$

$min\left(M_{iff}\right)=-14.01\ kN\cdot m$

$\max\left(M_{iff}\right)=28.41\ kN\cdot m$



$min\left(V_{if}\right)=-32.69\ kN$

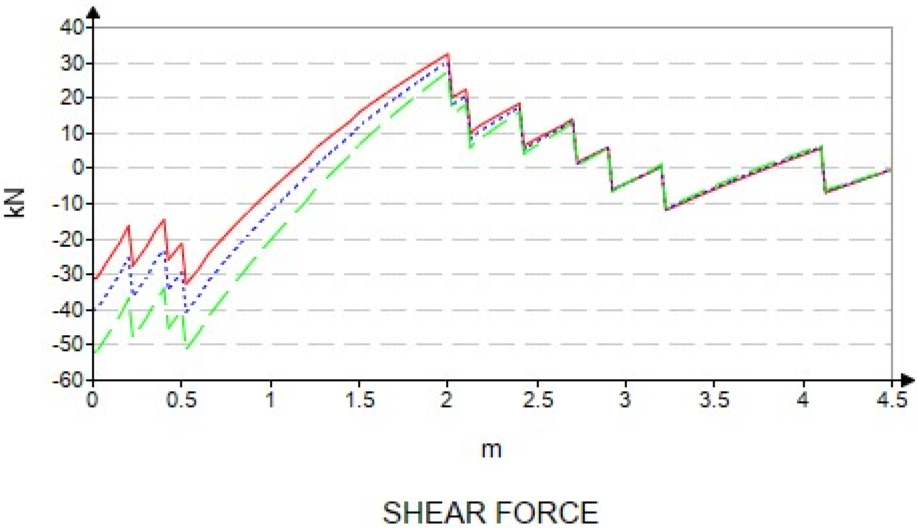
$\max\left(V_{if}\right)=32.45\ kN$

$min\left(V_{ifs}\right)=-40.62\ kN$

$\max\left(V_{ifs}\right)=30.34\ kN$

$min\left(V_{iff}\right)=-52.19\ kN$

$\max\left(V_{iff}\right)=27.51\ kN$



4.8.6 Relaxed Prestress and Hydrostatic with Restrained Base

$min\left(\delta_{tf}\right)=3.718\ mm$

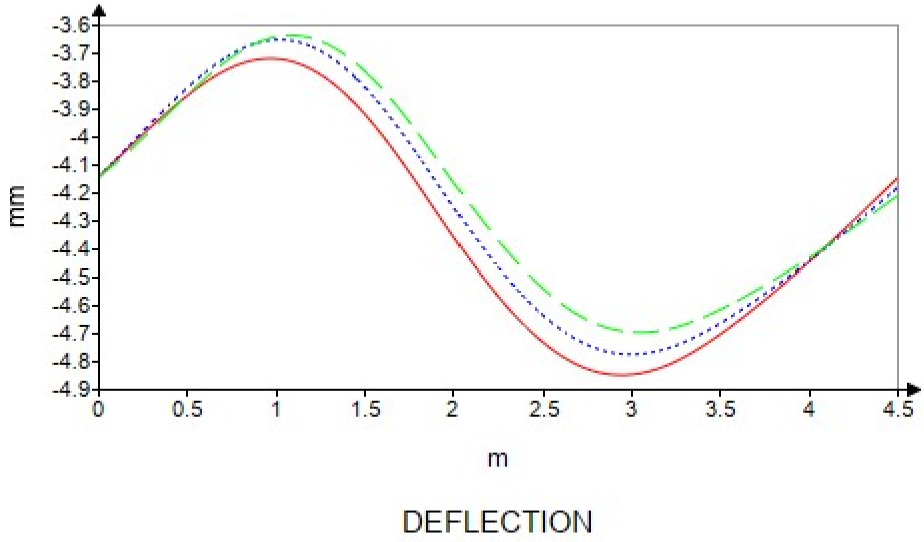
$\max\left(\delta_{tf}\right)=4.848\ mm$

$min\left(\delta_{tfs}\right)=3.651\ mm$

$\max\left(\delta_{tfs}\right)=4.773\ mm$

$min\left(\delta_{tff}\right)=3.635\ mm$

$\max\left(\delta_{tff}\right)=4.695\ mm$



$min\left(M_{tf}\right)=-10.66\ kN\cdot m$

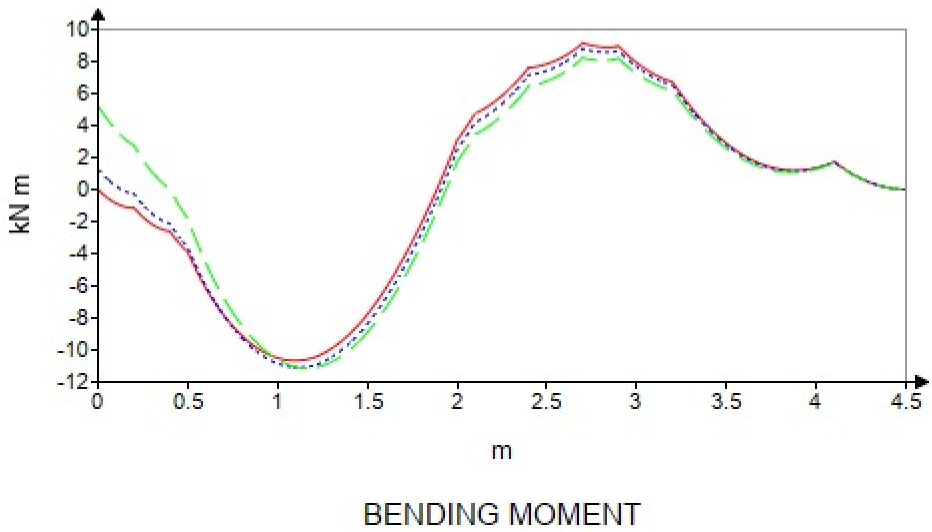
$\max\left(M_{tf}\right)=9.15\ kN\cdot m$

$min\left(M_{tfs}\right)=-11.09\ kN\cdot m$

$\max\left(M_{tfs}\right)=8.8\ kN\cdot m$

$min\left(M_{tff}\right)=-11.13\ kN\cdot m$

$\max\left(M_{tff}\right)=8.23\ kN\cdot m$



$min\left(V_{tf}\right)=-23.67\ kN$

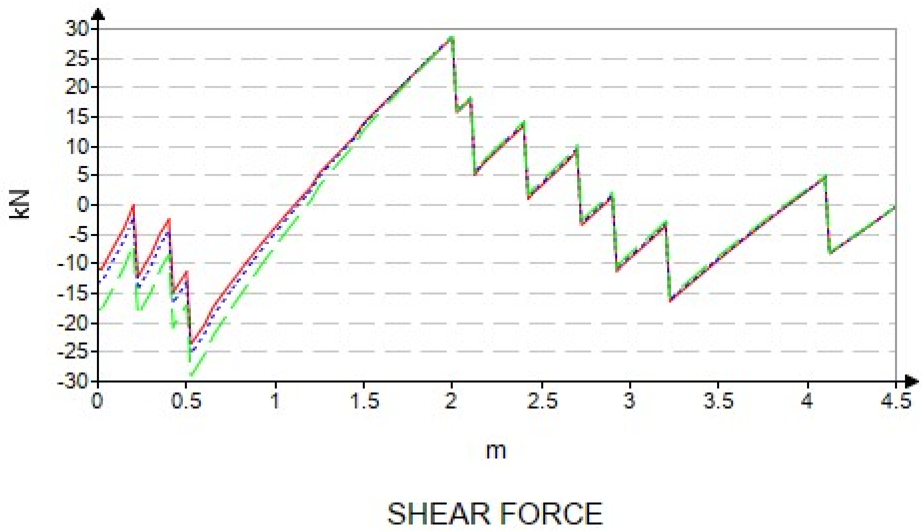
$\max\left(V_{tf}\right)=28.56\ kN$

$min\left(V_{tfs}\right)=-25.35\ kN$

$\max\left(V_{tfs}\right)=28.77\ kN$

$min\left(V_{tff}\right)=-29.12\ kN$

$\max\left(V_{tff}\right)=28.71\ kN$



4.8.7 Relaxed Prestress with Restrained Base (After Partial Restraint during Stressing)

$min\left(\delta_{srp}\right)=3.561\ mm$

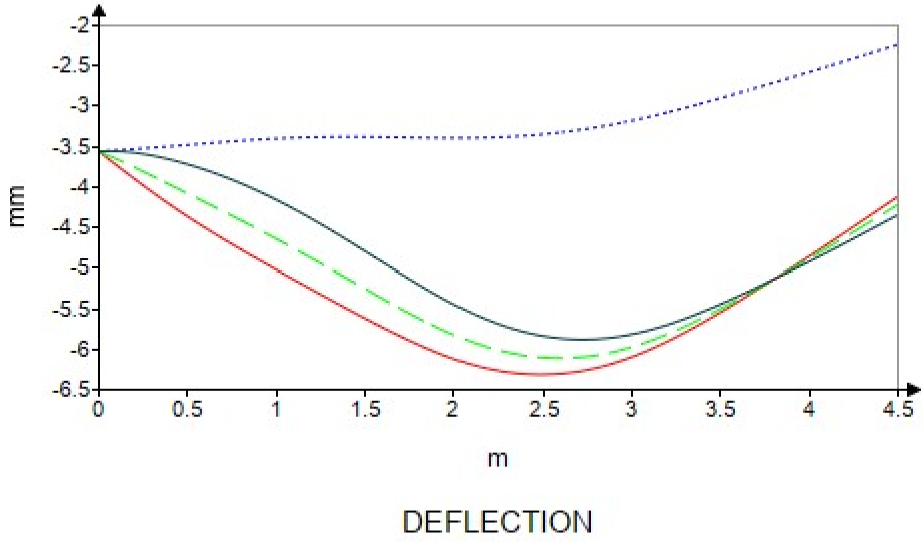
$\max\left(\delta_{srp}\right)=6.313\ mm$

$min\left(\delta_{srps}\right)=3.561\ mm$

$\max\left(\delta_{srps}\right)=6.11\ mm$

$min\left(\delta_{srpf}\right)=3.556\ mm$

$\max\left(\delta_{srpf}\right)=5.88\ mm$



$min\left(M_{srp}\right)=0\ kN\cdot m$

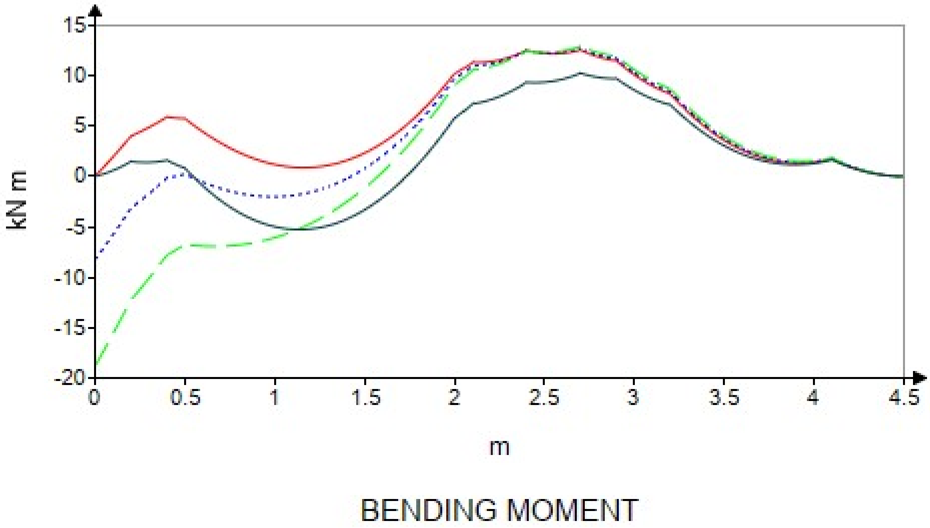
$\max\left(M_{srp}\right)=12.56\ kN\cdot m$

$min\left(M_{srps}\right)=-8.31\ kN\cdot m$

$\max\left(M_{srps}\right)=12.72\ kN\cdot m$

$min\left(M_{srpf}\right)=-18.84\ kN\cdot m$

$\max\left(M_{srpf}\right)=12.91\ kN\cdot m$



$min\left(V_{srp}\right)=-19.48\ kN$

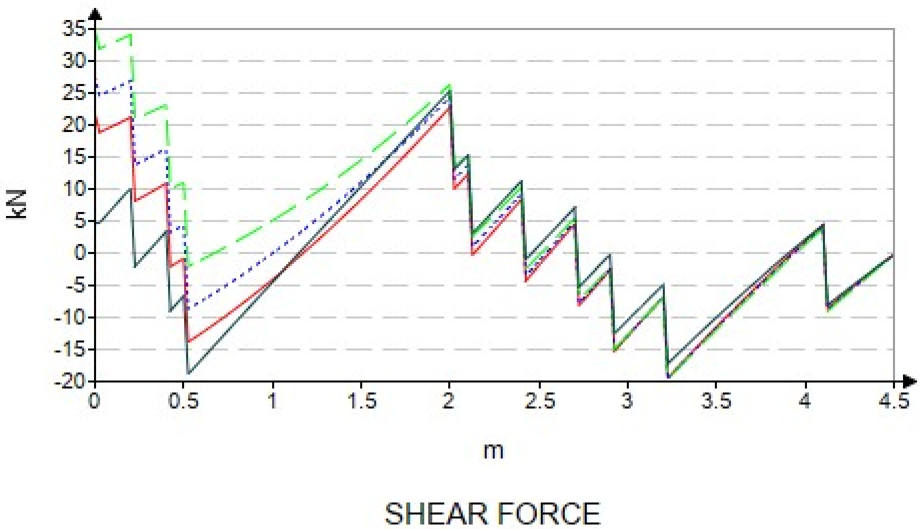
$\max\left(V_{srp}\right)=22.76\ kN$

$min\left(V_{srps}\right)=-19.55\ kN$

$\max\left(V_{srps}\right)=27.48\ kN$

$min\left(V_{srpf}\right)=-19.63\ kN$

$\max\left(V_{srpf}\right)=34.78\ kN$



4.8.8 Initial Prestress and Hydrostatic with Restrained Base (After Partial Restraint during Stressing)

$min\left(\delta_{sif}\right)=2.234\ mm$

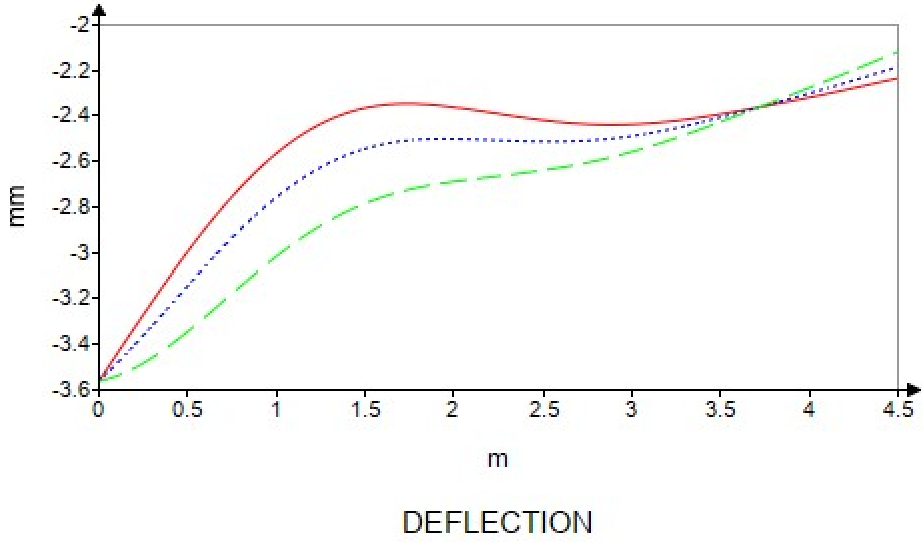
$\max\left(\delta_{sif}\right)=3.561\ mm$

$min\left(\delta_{sifs}\right)=2.184\ mm$

$\max\left(\delta_{sifs}\right)=3.561\ mm$

$min\left(\delta_{siff}\right)=2.118\ mm$

$\max\left(\delta_{siff}\right)=3.561\ mm$



$min\left(M_{sif}\right)=-18.78\ kN\cdot m$

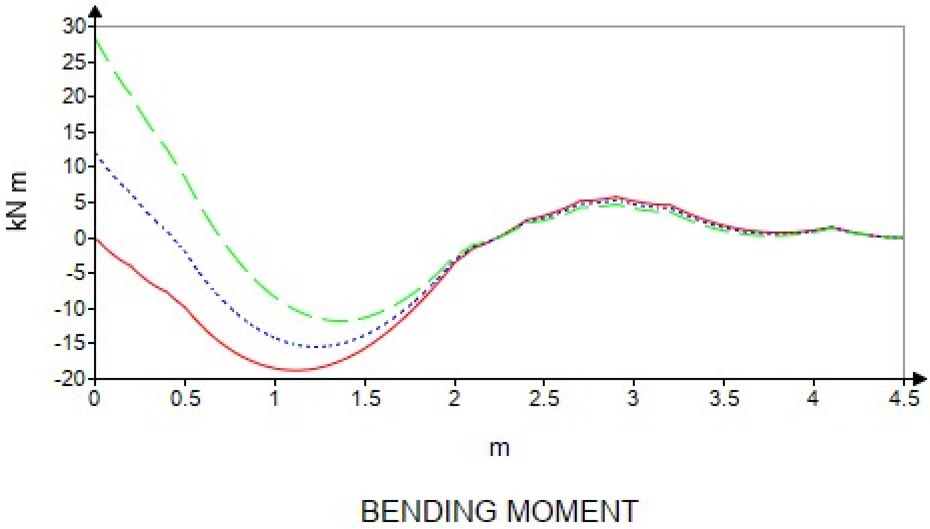
$\max\left(M_{sif}\right)=5.8\ kN\cdot m$

$min\left(M_{sifs}\right)=-15.45\ kN\cdot m$

$\max\left(M_{sifs}\right)=12.12\ kN\cdot m$

$min\left(M_{siff}\right)=-11.75\ kN\cdot m$

$\max\left(M_{siff}\right)=28.41\ kN\cdot m$



$min\left(V_{sif}\right)=-30.66\ kN$

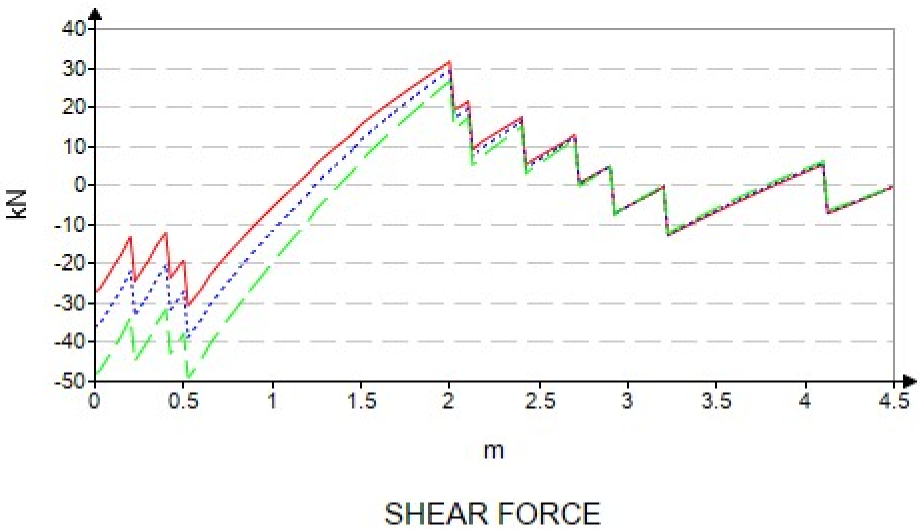
$\max\left(V_{sif}\right)=31.59\ kN$

$min\left(V_{sifs}\right)=-38.58\ kN$

$\max\left(V_{sifs}\right)=29.48\ kN$

$min\left(V_{siff}\right)=-49.23\ kN$

$\max\left(V_{siff}\right)=26.64\ kN$



4.8.9 Relaxed Prestress and Hydrostatic with Restrained Base (After Partial Restraint during Stressing)

$min\left(\delta_{stf}\right) = 3.407\ mm$

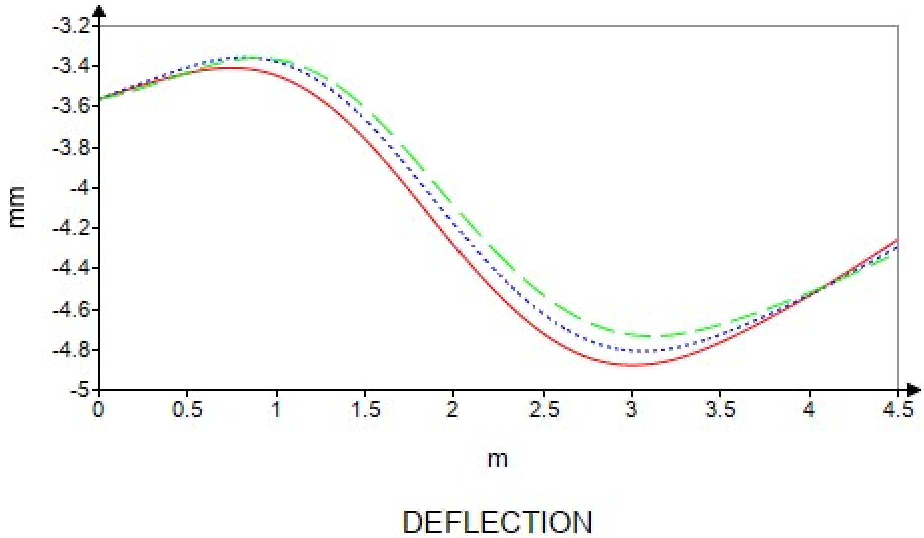
$\max\left(\delta_{stf}\right) = 4.878\ mm$

$min\left(\delta_{stfs}\right) = 3.356\ mm$

$\max\left(\delta_{stfs}\right) = 4.808\ mm$

$min\left(\delta_{stff}\right) = 3.359\ mm$

$\max\left(\delta_{stff}\right) = 4.733\ mm$



$min\left(M_{stf}\right) = -9.8\ kN \cdot m$

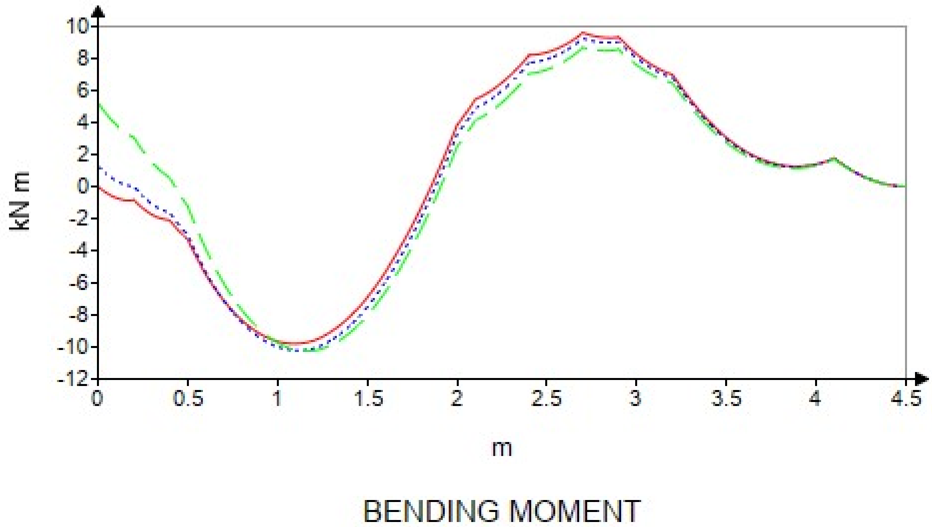
$\max\left(M_{stf}\right) = 9.61\ kN \cdot m$

$min\left(M_{stfs}\right) = -10.22\ kN \cdot m$

$\max\left(M_{stfs}\right) = 9.26\ kN \cdot m$

$min\left(M_{stff}\right) = -10.26\ kN \cdot m$

$\max\left(M_{stff}\right) = 8.69\ kN \cdot m$



$min\left(V_{stf}\right) = -22.88\ kN$

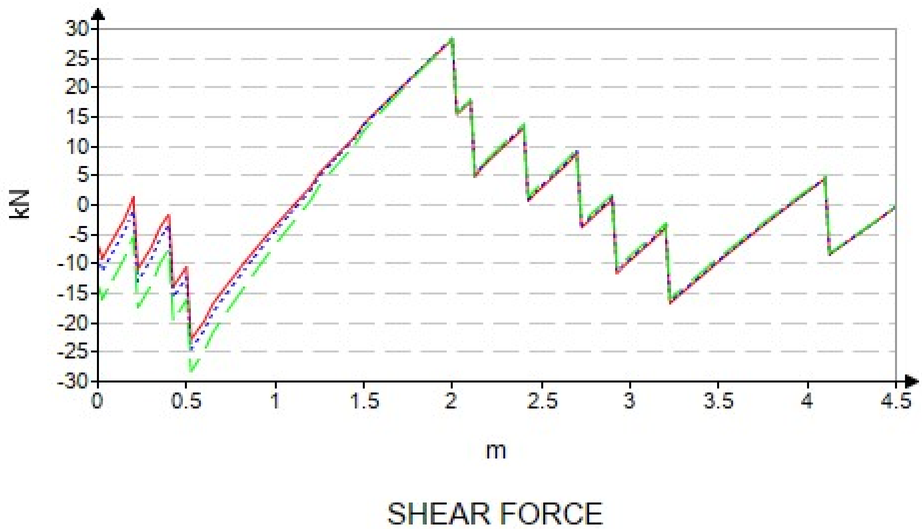
$\max\left(V_{stf}\right) = 28.22\ kN$

$min\left(V_{stfs}\right) = -24.56\ kN$

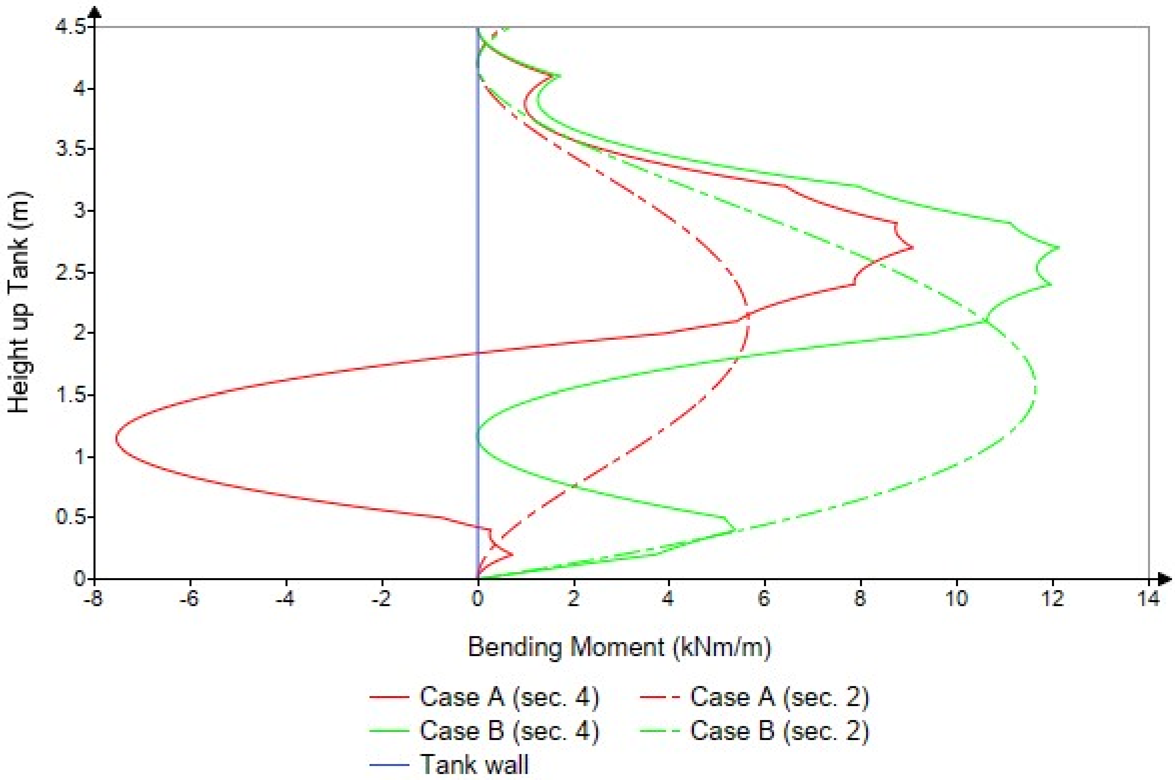
$\max\left(V_{stfs}\right) = 28.43\ kN$

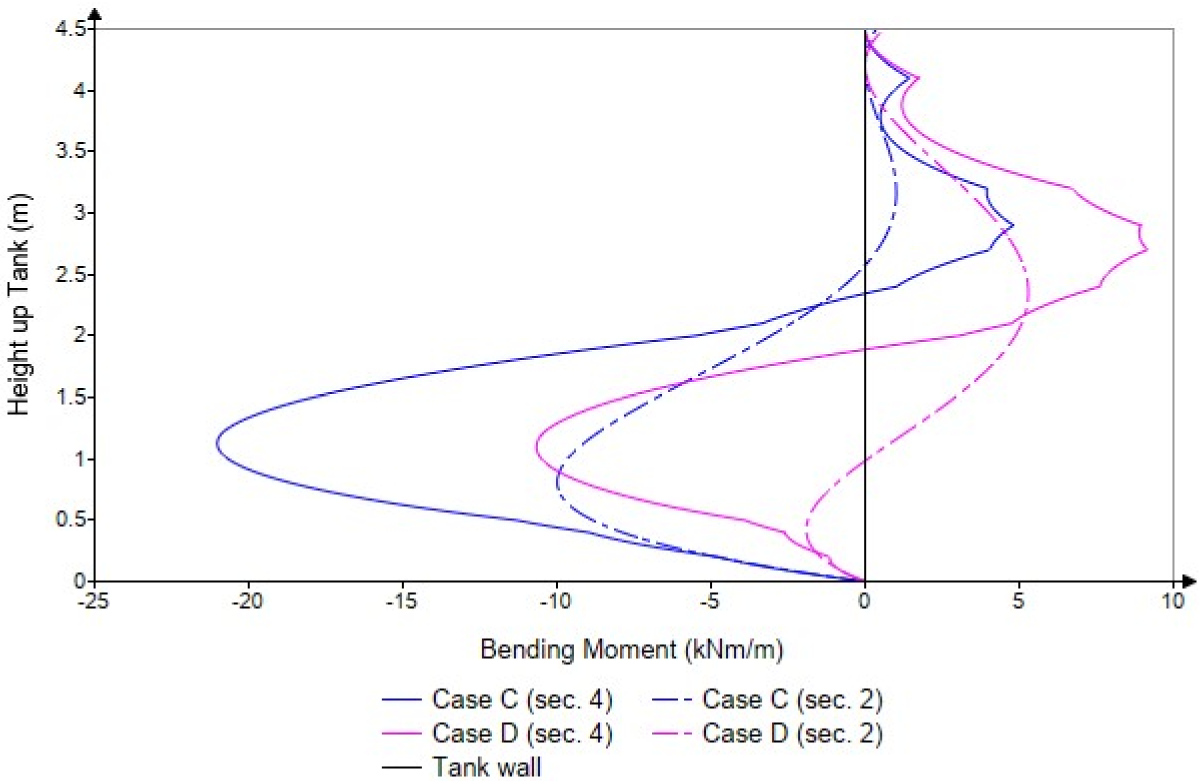
$min\left(V_{stff}\right) = -28.33\ kN$

$\max\left(V_{stff}\right) = 28.37\ kN$



4.8.10 Comparison of Bending Moments Calculated in Sections 2.0 and 4.0





4.8.11 Theoretical Prestress Distribution Check using Finite Difference Method

$U_{t_a} := tps(a \cdot \varepsilon 2) \cdot \dfrac{w}{r}$

Load matrices for udl prestress to theoretical curve

$S_{tpp} := \text{if}\,(short, S_{tp}, S_{tl})$

$L_{ipp} := L_i + F_u(n, U_t, \lambda_b, \lambda_c, \varepsilon 1)$ $\quad L_{tpp} := L_i + F_u(n, (1-\kappa) \cdot U_t, \lambda_b, \lambda_c, \varepsilon 1)$

$A_{ipp} := F_{sm}(n, I, \kappa_s, L_{f.ip}, R_{f.ip})$ $\quad A_{ipp_{i,i}} := A_{ipp_{i,i}} + S_{ip_i}$ $\quad \delta_{ipp} := A_{ipp}^{-1} \cdot L_{ipp}$ $\quad R := \overrightarrow{(S_{ip} \cdot \delta_{ipp})}$

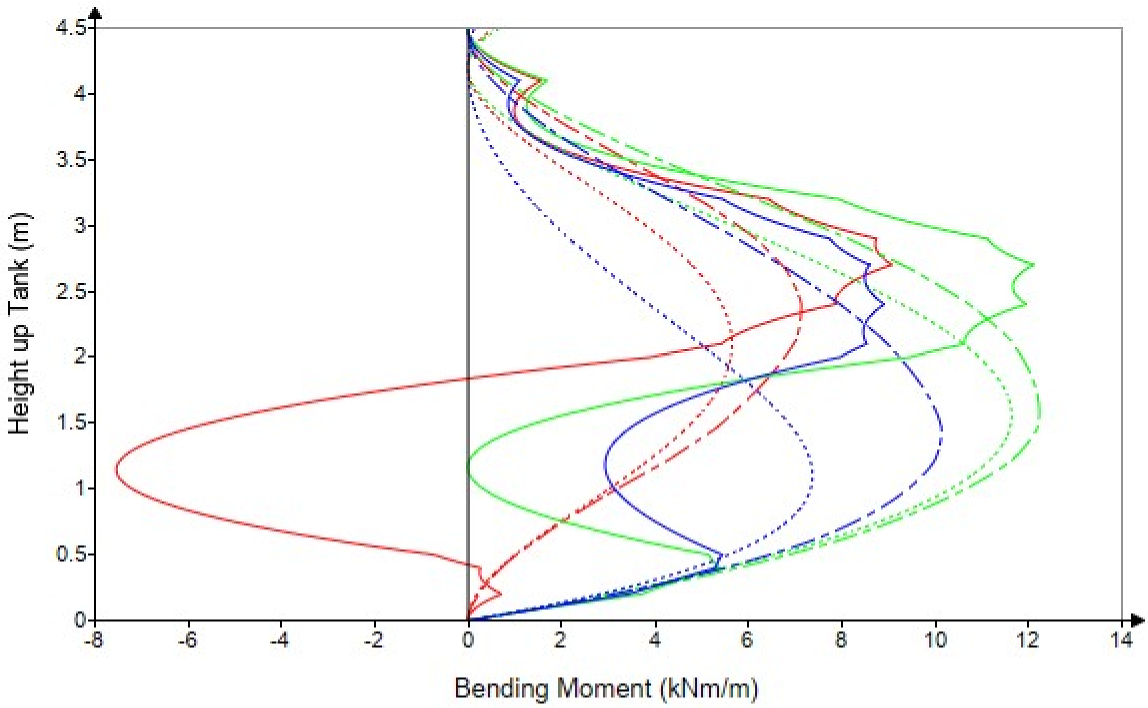
$A_{tpp} := F_{sm}(n, I, \kappa_l, L_{f.ip}, R_{f.ip})$ $\quad A_{tpp_{i,i}} := A_{tpp_{i,i}} + S_{tpp_i}$ $\quad \delta_{tpp} := A_{tpp}^{-1} \cdot L_{tpp}$ $\quad R := \overrightarrow{(S_{tp} \cdot \delta_{tpp})}$

$M_{ipp} := M_f(\delta_{ipp}, E_{cs}, I, n, \varepsilon 1, L_{f.ip}, R_{f.ip})$ $\quad \delta_{rpp} := \overrightarrow{(\delta_{ipp} + \delta_{tpp})}$

$M_{tpp} := M_f(\delta_{tpp}, E_{cl}, I, n, \varepsilon 1, L_{f.tp}, R_{f.tp})$

$M_{rpp} := M_f(\delta_{rpp}, E_{cl}, I, n, \varepsilon 1, 0, 0)$ $\quad M_{rpp_\lambda} := M_{ipp_\lambda} + M_{tpp_\lambda}$

$$M_{x.R}(x) := \begin{array}{|l} \text{if } short \\ \quad \| M_{x.B.s}(x, d_h, r, t_o, t_b, \beta_{o.i.inc}, \beta_{o.t.inc}, 0, 1-\kappa) \\ \text{else} \\ \quad \| M_{x.B.l}(x, d_h, \gamma, \rho_p, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.t.lon}, 0, 1-\kappa) \end{array}$$



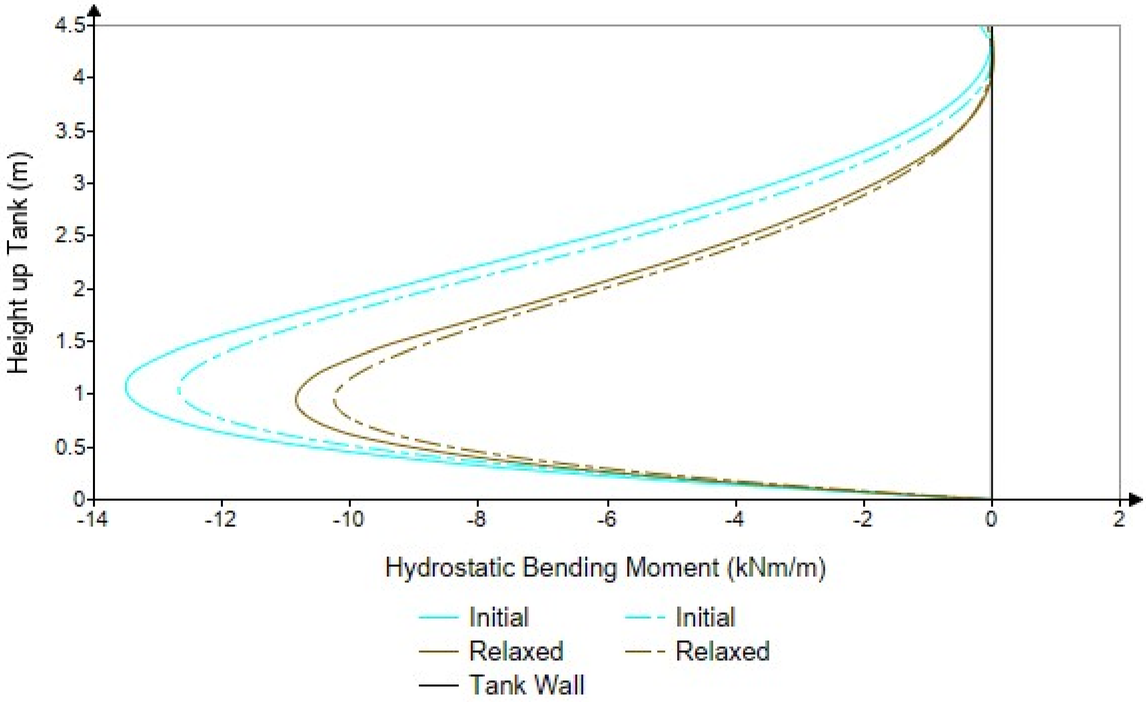
Case A - Red , Case B - Green , Redistribution - Blue
dotted line - theoretical distribution using Finite Difference Method
dashed line - theoretical distribution using Brondum Nielsen Formulae
solid line - actual point loads at tendon positions using Finite Difference

$\max(M_{ip}) = 9.068\ kN \cdot m$ $\quad \max(M_{ipp}) = 7.122\ kN \cdot m$ $\quad M_{max.A.o} = 5.638\ \dfrac{kN \cdot m}{m}$

$\max(M_{tp}) = 8.902\ kN \cdot m$ $\quad \max(M_{tpp}) = 10.131\ kN \cdot m$

$\max(M_{rp}) = 12.103\ kN \cdot m$ $\quad \max(M_{rpp}) = 12.225\ kN \cdot m$ $\quad M_{max.B.o} = 11.634\ \dfrac{kN \cdot m}{m}$

4.8.10 Comparison of Bending Moments Calculated in Sections 2.0 and 4.0 (continued):



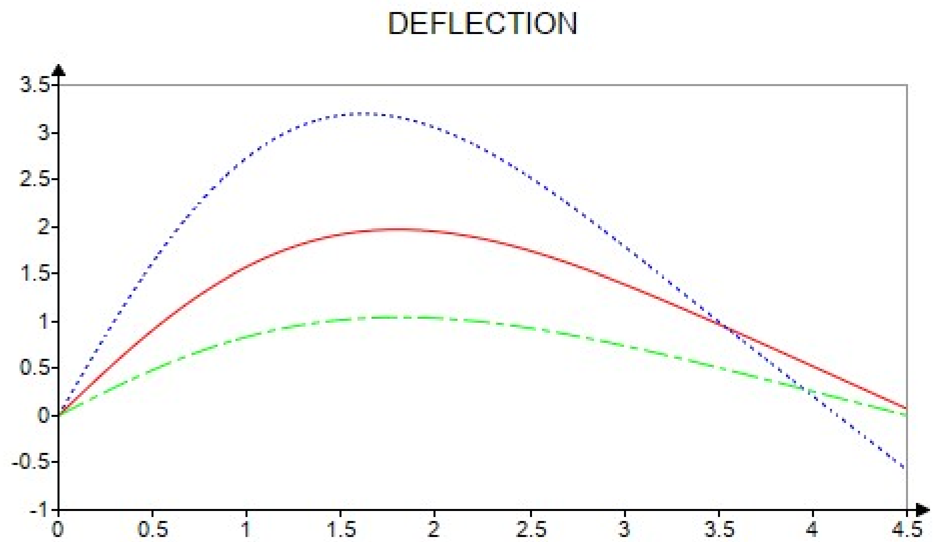
4.8.10 Initial and Relaxed External Earth and Water Pressures with Restrained Base (Pinned)

$min\left(\delta_{ie}\right)=0\ mm$

$\max\left(\delta_{ie}\right)=1.968\ mm$

$min\left(\delta_{te}\right)=-0.575\ mm$

$\max\left(\delta_{te}\right)=3.195\ mm$

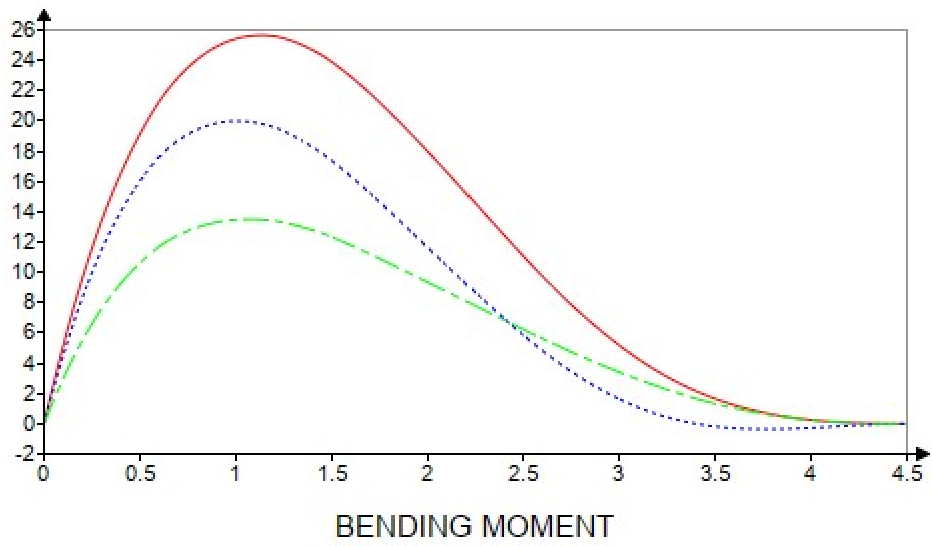


$min\left(M_{ie}\right)=0\ kN\cdot m$

$\max\left(M_{ie}\right)=25.653\ kN\cdot m$

$min\left(M_{te}\right)=-0.36\ kN\cdot m$

$\max\left(M_{te}\right)=19.988\ kN\cdot m$

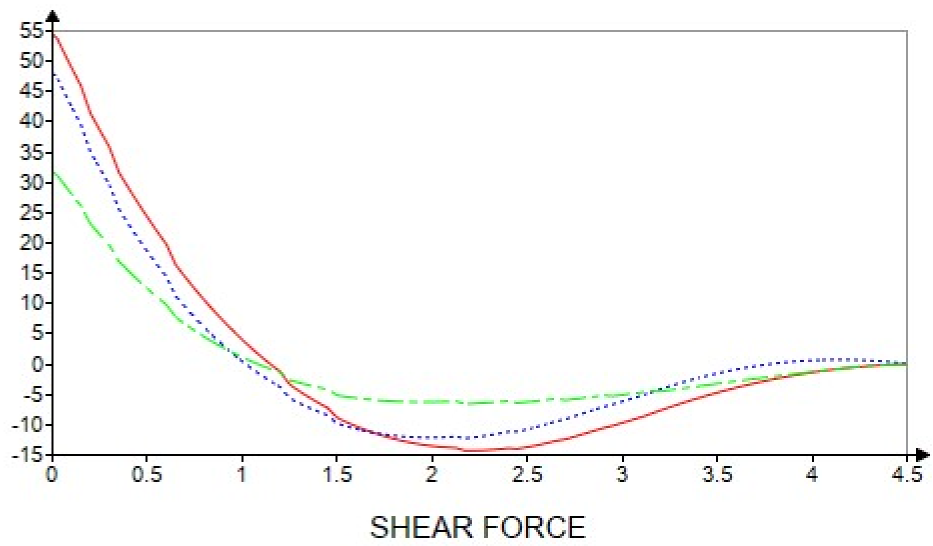


$min\left(V_{ie}\right)=-14.286\ kN$

$\max\left(V_{ie}\right)=54.542\ kN$

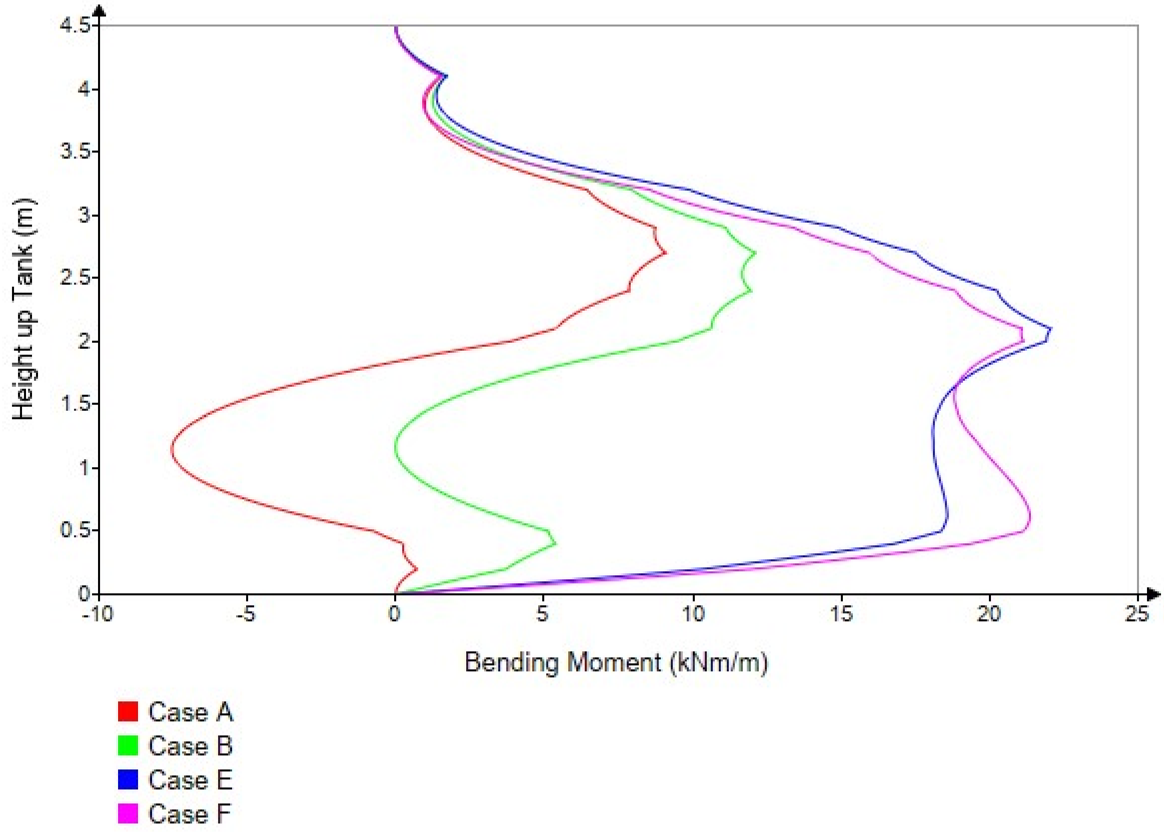
$min\left(V_{te}\right)=-12.242\ kN$

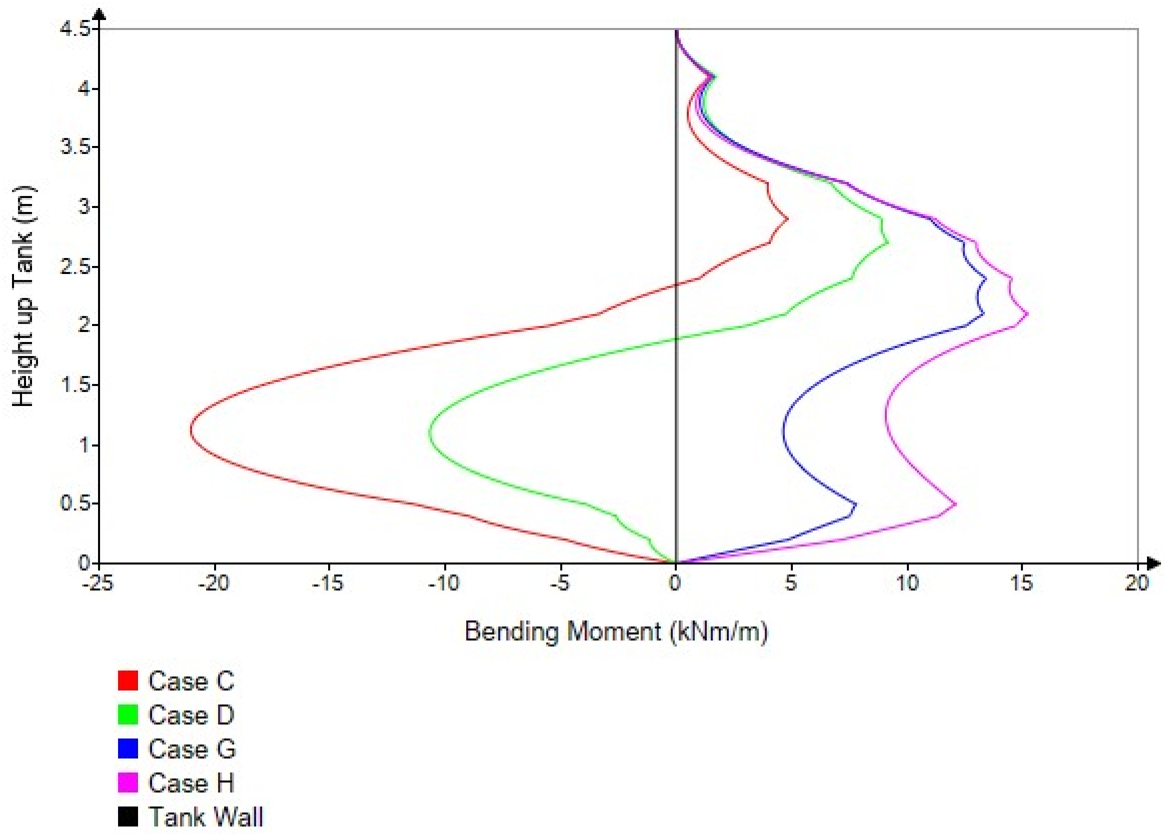
$\max\left(V_{te}\right)=48.049\ kN$



Initial (solid) and relaxed (dotted) earth pressure cases plotted against the hydrostatic (dashed) case for comparison

4.8.11 Bending Moment Diagrams for Serviceability Cases A - H



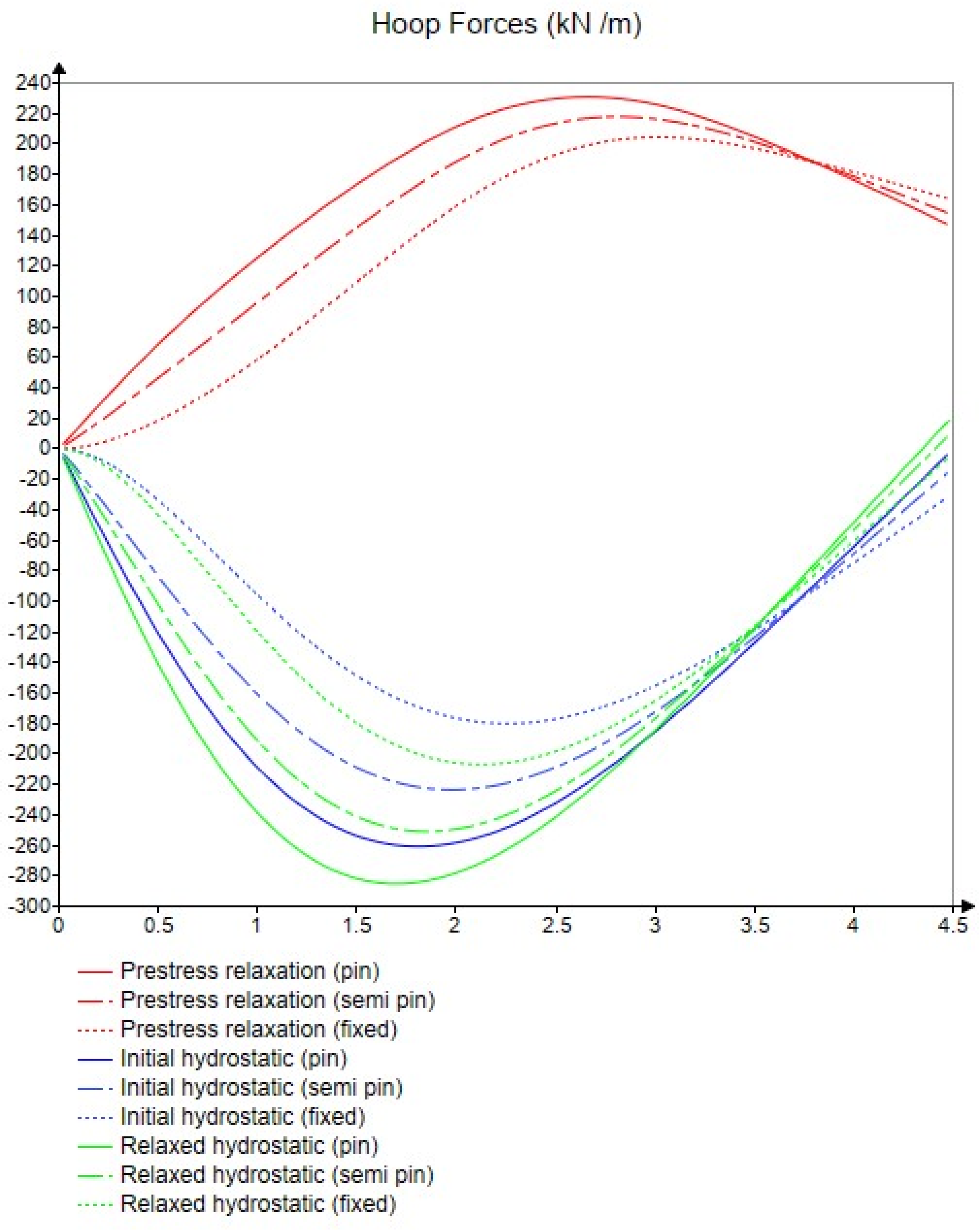


4.9 Hoop Force Diagrams

hoop forces derived directly from spring reactions R

$kh := r \cdot (w \cdot \varepsilon 1)^{-1}$ $\quad kh = 447.05\ m^{-1}$ $\quad j := 1..n-1$

4.9.1 Hydrostatic / Prestress Relaxation (with pinned or fixed base, initial and relaxed hydrostatic)



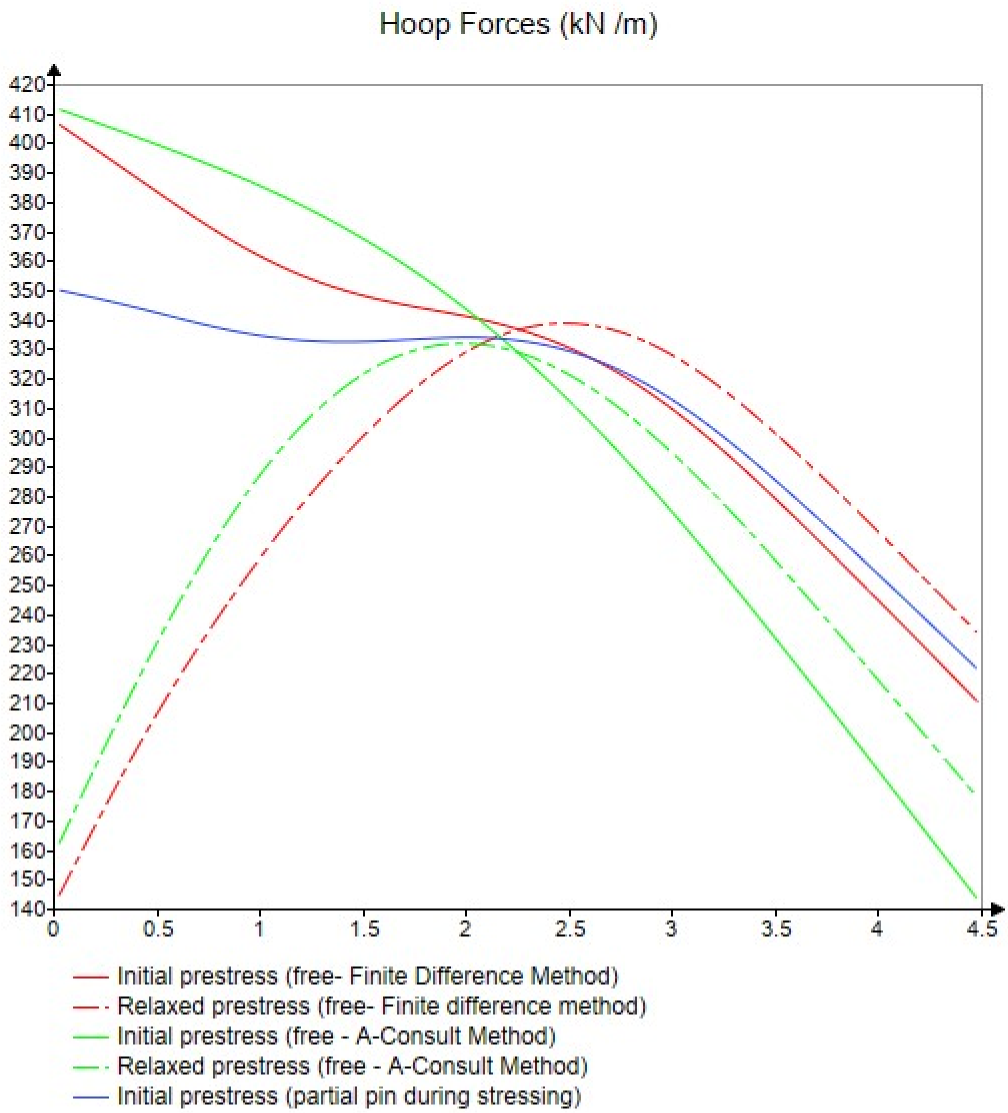
$R_{ih.max} := -min\left(\text{submatrix}\left(R_{ih}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{ih.max} = 260.8\ \frac{kN}{m}$ $\quad N_{max.H.o} = 244.3\ \frac{kN}{m}$

$R_{th.max} := -min\left(\text{submatrix}\left(R_{th}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{th.max} = 285.2\ \frac{kN}{m}$ $\quad N_{max.Ht.o} = 267.8\ \frac{kN}{m}$

$R_{ih_0} = -31.7\ kN$ $\quad R_{th_0} = -28.5\ kN$

4.9 Hoop Force Diagrams (Continued)

4.9.2 Initial and Relaxed Prestress (with pinned or fixed base)



$R_{ip.max} := \max\left(\text{submatrix}\left(R_{ip}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{ip.max} = 406.6\ \dfrac{kN}{m}$ $\quad \dfrac{R_{ip.max}}{h} = 2.2\ MPa$

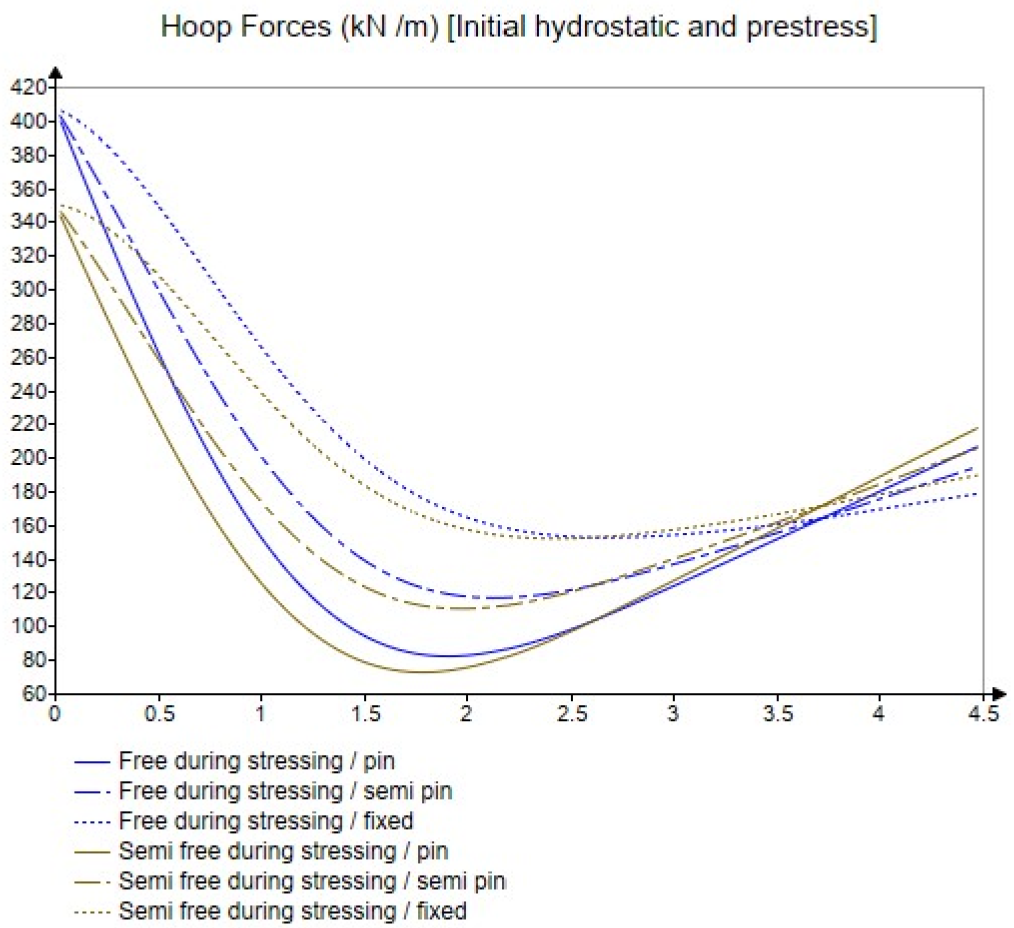
$N_{max.A.o} = 412.4\ \dfrac{kN}{m}$ $\quad \dfrac{R_{ip.max}}{a_s} = 5.1\ MPa$

$R_{rp_0} = 17.2\ kN$

$R_{rp.max} := \max\left(\text{submatrix}\left(R_{rp}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{rp.max} = 339\ \dfrac{kN}{m}$ $\quad N_{max.B.o} = 332.2\ \dfrac{kN}{m}$

4.9 Hoop Force Diagrams (Continued)

4.9.3 Initial Hydrostatic and Prestress (with pinned or fixed base, and initial hydrostatic)



$R_{if.min} := min\left(\text{submatrix}\left(R_{if}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{if.min} = 82.5\ \frac{kN}{m}$ $\quad \frac{R_{if.min}}{h} = 0.45\ MPa$

$\frac{R_{if.min}}{a_s} = 1\ MPa$ $\quad N_{min.C.o} = 94.5\ \frac{kN}{m}$ $\quad R_{if_0} = -31.3\ kN$

$R_{iff.max} := \max\left(\text{submatrix}\left(R_{iff}, 1, n-1, 0, 0\right)\right) \cdot kh$ $\quad R_{iff.max} = 406.5\ \frac{kN}{m}$ $\quad \frac{R_{iff.max}}{h} = 2.2\ MPa$

$\frac{R_{iff.max}}{a_s} = 5.1\ MPa$

$R_{sif.min} := min\left(\text{submatrix}\left(R_{sif}, 1, n-1, 0, 0\right)\right) \cdot kh$

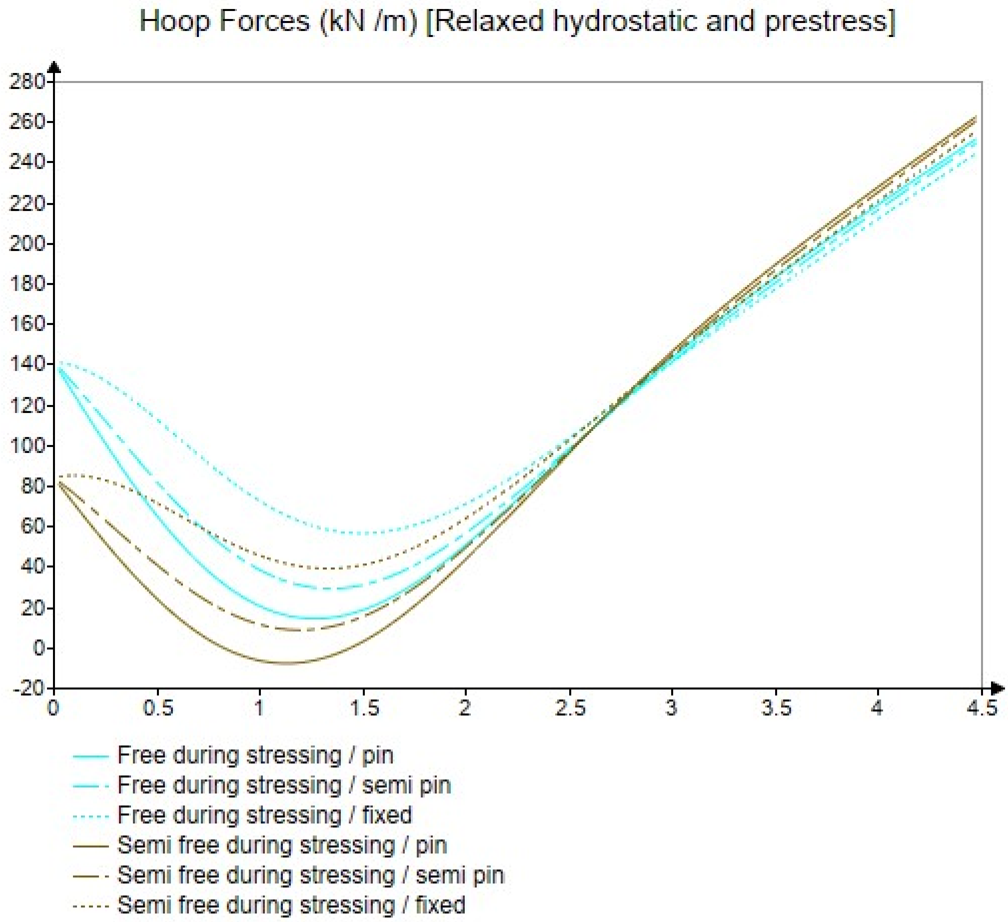
$R_{sif.min} = 72.997\ \frac{kN}{m}$ $\quad \frac{R_{sif.min}}{h} = 0.395\ MPa$ $\quad \frac{R_{sif.min}}{a_s} = 0.912\ MPa$

$R_{sif.max} := \max\left(\text{submatrix}\left(R_{sif}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{sif.max} = 343.858\ \frac{kN}{m}$ $\quad \frac{R_{sif.max}}{h} = 1.859\ MPa$ $\quad \frac{R_{sif.max}}{a_s} = 4.298\ MPa$

4.9 Hoop Force Diagrams (Continued)

4.9.4 Relaxed Hydrostatic and Prestress (with pinned or fixed base, and relaxed hydrostatic)



$R_{tf.min} := min\left(\text{submatrix}\left(R_{tf}, 1, n-1, 0, 0\right)\right) \cdot kh \quad R_{tf.min} = 14.6\ \frac{kN}{m} \quad \frac{R_{tf.min}}{h} = \left(7.9 \cdot 10^{-2}\right)\ MPa$

$\frac{R_{tf.min}}{a_s} = 0.2\ MPa \quad N_{min.D.o} = 84.1\ \frac{kN}{m} \quad R_{tf_0} = -11.3\ kN$

$R_{tff.max} := \max\left(\text{submatrix}\left(R_{tff}, 1, n-1, 0, 0\right)\right) \cdot kh \quad R_{tff.max} = 244.8\ \frac{kN}{m} \quad \frac{R_{tff.max}}{h} = 1.3\ MPa$

$\frac{R_{tff.max}}{a_s} = 3.1\ MPa$

$R_{stf.min} := min\left(\text{submatrix}\left(R_{stf}, 1, n-1, 0, 0\right)\right) \cdot kh$

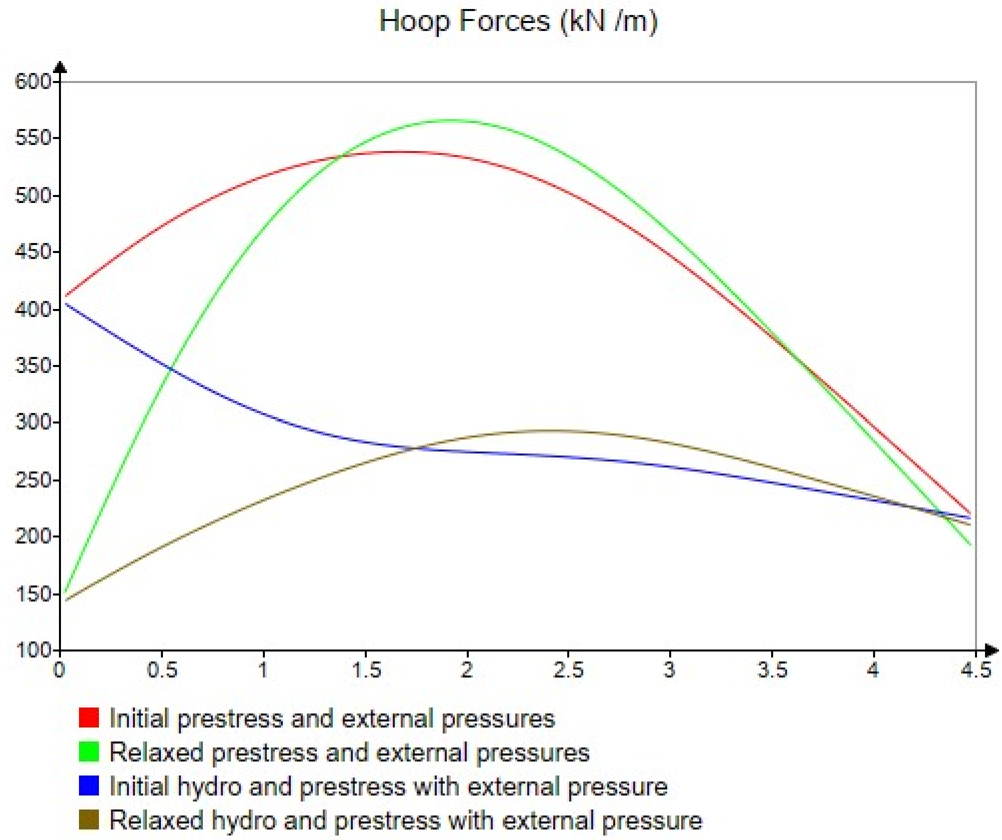
$R_{stf.min} = -7.591\ \frac{kN}{m} \quad \frac{R_{stf.min}}{h} = -0.041\ MPa \quad \frac{R_{stf.min}}{a_s} = -0.095\ MPa$

$R_{stf.max} := \max\left(\text{submatrix}\left(R_{stf}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{stf.max} = 262.984\ \frac{kN}{m} \quad \frac{R_{stf.max}}{h} = 1.422\ MPa \quad \frac{R_{stf.max}}{a_s} = 3.287\ MPa$

4.9 Hoop Force Diagrams (Continued)

4.9.4 Hoop Stresses for Cases with External Earth and Water Pressure)



$R_{ife.min} := min\left(\text{submatrix}\left(R_{ife}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{tfe.min} := min\left(\text{submatrix}\left(R_{tfe}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{tfe.min} = 144 \ \frac{kN}{m}$ $\qquad \frac{R_{tfe.min}}{h} = 0.8 \ MPa$ $\qquad \frac{R_{tfe.min}}{a_s} = 1.8 \ MPa$

$R_{ipe.max} := \max\left(\text{submatrix}\left(R_{ipe}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{ipe.max} = 538.6 \ \frac{kN}{m}$ $\qquad \frac{R_{ipe.max}}{h} = 2.9 \ MPa$ $\qquad \frac{R_{ipe.max}}{a_s} = 6.7 \ MPa$

$R_{rpe.max} := \max\left(\text{submatrix}\left(R_{rpe}, 1, n-1, 0, 0\right)\right) \cdot kh$

$R_{rpe.max} = 566.037 \ \frac{kN}{m}$ $\qquad \frac{R_{rpe.max}}{h} = 3.06 \ MPa$ $\qquad \frac{R_{rpe.max}}{a_s} = 7.075 \ MPa$

5.0 Ultimate Capacity and Crack Width Calculations

5.1 Summary of Design Moments

$M_{max.H.o} = -12.676 \frac{kN \cdot m}{m}$ $M_{max.Ht.o} = -10.248 \frac{kN \cdot m}{m}$

5.1.1 Ultimate Limit State

$min(M_{ih}) = -13.505\ kN \cdot m$ $min(M_{th}) = -10.846\ kN \cdot m$

Maximum ultimate bending on inside face (prestress only)

$$Mu_p := \gamma_f \cdot \left(\max\left(M_{max.A.o}, M_{max.B.o}, \frac{\max(M_{ip})}{m}, \frac{\max(M_{rp})}{m}, \frac{\max(M_{ipe})}{m}, \frac{\max(M_{rpe})}{m}\right)\right)$$

$Mu_p = 33.1 \frac{kN \cdot m}{m}$

Maximum ultimate bending on outside face (prestress and hydrostatic) - conservative

$$maxH := min\left(M_{max.H.o}, min(M_{ih}) \cdot m^{-1}\right) \qquad maxHt := min\left(M_{max.Ht.o}, min(M_{th}) \cdot m^{-1}\right)$$

$$Mu_h := min\left(1.0 \cdot M_{max.A.o} + \gamma_f \cdot maxH, 1.0 \cdot M_{max.B.o} + \gamma_f \cdot maxHt\right)$$

$Mu_h = -14.62 \frac{kN \cdot m}{m}$

5.1.2 Serviceability Limit State (negative moments are tension on outside face)

Case A (Initial prestress before stress relaxation) $\max(M_{ip}) = 9.1\ kN \cdot m$ $M_{max.A.o} = 5.6 \frac{kN \cdot m}{m}$

Case B (Prestress after stress relaxation) $\max(M_{rp}) = 12.1\ kN \cdot m$ $M_{max.B.o} = 11.6 \frac{kN \cdot m}{m}$

Case C (Initial prestress & hydrostatic before relaxation) $min(M_{if}) = -21\ kN \cdot m$ $M_{min.C.o} = -10 \frac{kN \cdot m}{m}$

Case D (Prestress and hydrostatic after stress relaxation) $min(M_{tf}) = -10.7\ kN \cdot m$ $M_{min.D.o} = -1.9 \frac{kN \cdot m}{m}$

Case A* (Semi Free to slide base restraint) $\max(M_{sp}) = 10.256\ kN \cdot m$ $M_{max.AS.o} = 12.5 \frac{kN \cdot m}{m}$

$$\boxed{M_{max.A.o}} := \max\left(M_{max.A.o}, \max(M_{ip}) \cdot m^{-1}\right) \qquad M_{max.A.o} = 9.1 \frac{kN \cdot m}{m}$$

$$\boxed{M_{max.B.o}} := \max\left(M_{max.B.o}, \max(M_{rp}) \cdot m^{-1}\right) \qquad M_{max.B.o} = 12.1 \frac{kN \cdot m}{m}$$

$$\boxed{M_{min.C.o}} := min\left(M_{min.C.o}, min(M_{if}) \cdot m^{-1}\right) \qquad M_{min.C.o} = -21 \frac{kN \cdot m}{m}$$

$$\boxed{M_{min.D.o}} := min\left(M_{min.D.o}, min(M_{tf}) \cdot m^{-1}\right) \qquad M_{min.D.o} = -10.7 \frac{kN \cdot m}{m}$$

$$M_{max.As.o} := \max\left(M_{max.AS.o}, \max(M_{sp}) \cdot m^{-1}\right) \qquad M_{max.As.o} = 12.5 \frac{kN \cdot m}{m}$$

5.2 Ultimate Capacities of Section

$\alpha_{cc} := 1$ $\lambda_{cc} := 0.8$ $(f_{ck} < 50\ MPa)$ (EC2 3.1.6) $F_{ck} = 40\ MPa$ $f_{cd} := \frac{\alpha_{cc} \cdot F_{ck}}{\gamma_{mc}}$

$f_{yf} = 500\ MPa$ $\gamma_{mr} = 1.15$ $f_{yd} := \frac{f_{yf}}{\gamma_{mr}}$ $f_{yd} = 434.8\ MPa$ EC2 3.1.6 $f_{cd} = 26.7\ MPa$

5.2.1 Outside Face Reinforcement (without prestress)

$h = 185\ mm$ $cv_o = 55\ mm$ $d_{vo} := h - cv_o - \frac{\phi_{vo}}{2}$ $d_{vo} = 125\ mm$

$As_{vo} = 829.3\ \frac{mm^2}{m}$ $Tr_{vo} := f_{yd} \cdot As_{vo}$ $Tr_{vo} = 360.6\ \frac{kN}{m}$

From Cc (compression in concrete) = Tr (tension in steel)

$f_{cd} = 26.7\ MPa$

k factor for simplified rectangular stress block

$x_{vo} := \frac{Tr_{vo}}{f_{cd} \cdot \lambda_{cc}}$ $x_{vo} = 16.9\ mm$ $\left(\frac{\alpha_{cc}}{\gamma_{mc}}\right) \cdot \lambda_{cc} = 0.533$

$Mu_{vo} := Tr_{vo} \cdot \left(d_{vo} - \frac{\lambda_{cc} \cdot x_{vo}}{2}\right)$ $Mu_{vo} = 42.6\ \frac{kN \cdot m}{m}$ $-Mu_h = 14.6\ \frac{kN \cdot m}{m}$

if $\left(Mu_{vo} > -Mu_h, \text{“satisfactory”}, \text{“low”}\right) = \text{“satisfactory”}$

5.2.2 Inside Face Reinforcement (without prestress)

$h = 185\ mm$ $cv_i = 55\ mm$ $d_{vi} := h - cv_i - \frac{\phi_{vi}}{2}$ $d_{vi} = 125\ mm$

$As_{vi} = 877\ \frac{mm^2}{m}$ $Tr_{vi} := f_{yd} \cdot As_{vi}$ $Tr_{vi} = 381.3\ \frac{kN}{m}$

From Cc (compression in concrete) = Tr (tension in steel)

$f_{cd} = 26.7\ MPa$

$x_{vi} := \frac{Tr_{vi}}{f_{cd} \cdot \lambda_{cc}}$ $x_{vi} = 17.9\ mm$

$Mu_{vi} := Tr_{vi} \cdot \left(d_{vi} - \frac{\lambda_{cc} \cdot x_{vi}}{2}\right)$ $Mu_{vi} = 44.9\ m \cdot \frac{kN}{m}$ $Mu_p = 33.1\ \frac{kN \cdot m}{m}$

if $\left(Mu_{vi} > Mu_p, \text{“satisfactory”}, \text{“low”}\right) = \text{“satisfactory”}$

5.2.1 Outside Face Reinforcement (with prestress)

$As_{vo} = 829.321\ \frac{mm^2}{m}$ $\quad f_{yf} = 500\ MPa$ $\quad \gamma_{mr} = 1.15$ $\quad h = 185\ mm$ $\quad d_{vo} = 125\ mm$

$A_{vp} = 0\ \frac{mm^2}{m}$ $\quad f_{ypv} = 1860\ MPa$ $\quad \gamma_{mr} = 1.15$ $\quad d_p = 111\ mm$ $\quad h - d_p = 74\ mm$

$$\boxed{A_{vp}} := \text{if}\left(A_{vp} = 0\ \frac{mm^2}{m}, 1 \cdot 10^{-14} \cdot \frac{mm^2}{m}, A_{vp}\right)$$

$$\boxed{Tr_{vo}} := \frac{f_{yf}}{\gamma_{mr}} \cdot As_{vo} \qquad Tr_{vo} = 360.6\ \frac{kN}{m}$$

$$T_{pr} := \frac{f_{ypv}}{\gamma_{mr}} \cdot A_{vp} \qquad T_{pr} = \left(1.6 \cdot 10^{-14}\right)\ \frac{kN}{m}$$

Concrete deformation in the final limit state (ULS) $\quad \varepsilon_{cu2} := 0.0035 \quad f_{cd} = 26.7\ MPa$

From Cc (compression in concrete) = Tr (tension in steel) $\quad$ table 3.1 $\quad \gamma_{mc} = 1.5$

Stress strain relationship to EC2 for prestressing tendon

$$\varepsilon_f := \frac{f_{ypv}}{\gamma_{mr} \cdot E_p} \qquad \varepsilon_f = 0.008$$

$$f_p\left(\varepsilon_p\right) := \max\left(\varepsilon_p \cdot E_p, \frac{f_{ypv}}{\gamma_{mr}}\right)$$

Concrete strain at ULS $\quad C_{vc}(x) := \lambda_{cc} \cdot f_{cd} \cdot x$

Tension in prestress at ULS $\quad T_{vop}(x) := min\left(f_p\left(\varepsilon_{cu2} \cdot \left(\frac{\left(h - d_p\right) - x}{x}\right)\right) \cdot A_{vp}, T_{pr}, Tr_{vo} \cdot \frac{\left(h - d_p\right) - x}{d_{vo} - x} \cdot \frac{E_p}{E_s} \cdot \frac{A_{vp}}{As_{vo}}\right)$

Tension in reinf't at ULS $\quad T_{vos}(x) := min\left(\varepsilon_{cu2} \cdot \frac{d_{vo} - x}{x} \cdot E_s \cdot As_{vo}, Tr_{vo}, T_{pr} \cdot \frac{d_{vo} - x}{\left(h - d_p\right) - x} \cdot \frac{E_s}{E_p} \cdot \frac{As_{vo}}{A_{vp}}\right)$

Balance of forces :ULS $\quad F_{uos}(x) := C_{vc}(x) - T_{vop}(x) - T_{vos}(x) - P_{vp}$

$F_{uosx}(xg) := \textbf{root}\left(F_{uos}(xg), xg\right)$ $\quad x_{vop} := F_{uosx}(25\ mm)$ $\quad x_{vop} = 16.9\ mm$ $\quad F_{uos}\left(x_{vop}\right) = 0\ \frac{kN}{m}$

Concrete Compression at ULS	Tension in prestress at ULS	Tension in reinf't at ULS
$C_{vc}\left(x_{vop}\right) = 360.6\ \frac{kN}{m}$	$T_{vop}\left(x_{vop}\right) = 0\ \frac{kN}{m}$	$T_{vos}\left(x_{vop}\right) = 360.574\ \frac{kN}{m}$

Bending Capacity taking moments about the neutral axis

$$Mu_{vop} := C_{vc}\left(x_{vop}\right) \cdot \left(\left(1 - 0.5 \cdot \lambda_{cc}\right) \cdot x_{vop}\right) + \left(T_{vop}\left(x_{vop}\right) + P_{vp}\right) \cdot \left(\left(h - d_p\right) - x_{vop}\right) + T_{vos}\left(x_{vop}\right) \cdot \left(d_{vo} - x_{vop}\right)$$

$Mu_{vop} = 42.6\ \frac{kN \cdot m}{m}$ $\quad \frac{Mu_{vop}}{Mu_{vo}} = 100\%$ $\quad -Mu_h = 14.6\ \frac{kN \cdot m}{m}$ $\quad \text{if}\left(Mu_{vop} > -Mu_h, \text{“OK”}, \text{“low”}\right) = \text{“OK”}$

5.2.2 Inside Face Reinforcement (with prestress)

$As_{vi} = 876.966\ \frac{mm^2}{m}$ $f_{yf} = 500\ MPa$ $\gamma_{mr} = 1.15$ $d_{vi} = 125\ mm$

$A_{vp} = (1 \cdot 10^{-14})\ \frac{mm^2}{m}$ $f_{ypv} = 1860\ MPa$ $\gamma_{mr} = 1.15$ $d_p = 111\ mm$

$Tr_{vi} := \frac{f_{yf}}{\gamma_{mr}} \cdot As_{vi}$ $Tr_{vi} = 381.3\ \frac{kN}{m}$

$T_{pr} := \frac{f_{ypv}}{\gamma_{mr}} \cdot A_{vp}$ $T_{pr} = (1.6 \cdot 10^{-14})\ \frac{kN}{m}$

Concrete strain at ULS $\varepsilon_{cu2} = 0.0035$ $F_{ck} = 40\ MPa$ $\gamma_{mc} = 1.5$

Tension in prestress at ULS $T_{vip}(x) := min\left(f_p\left(\varepsilon_{cu2} \cdot \frac{d_p - x}{x}\right) \cdot A_{vp}, T_{pr}, Tr_{vi} \cdot \frac{d_p - x}{d_{vi} - x} \cdot \frac{E_p}{E_s} \cdot \frac{A_{vp}}{As_{vi}}\right)$

Tension in reinf't at ULS $T_{vis}(x) := min\left(\varepsilon_{cu2} \cdot \frac{d_{vi} - x}{x} \cdot E_s \cdot As_{vi}, Tr_{vi}, T_{pr} \cdot \frac{d_{vi} - x}{d_p - x} \cdot \frac{E_s}{E_p} \cdot \frac{As_{vi}}{A_{vp}}\right)$

Balance of forces : $F_{uis}(x) := \lambda_{cc} \cdot f_{cd} \cdot x - T_{vip}(x) - T_{vis}(x) - P_{vp}$

$F_{uisx}(xg) := \mathbf{root}\left(F_{uis}(xg), xg\right)$ $x_{vip} := F_{uisx}(25\ mm)$ $x_{vip} = 17.873\ mm$ $F_{uis}(x_{vip}) = 0\ \frac{kN}{m}$

Concrete Compression at ULS $C_{vc}(x_{vip}) = 381.3\ \frac{kN}{m}$

Tension in prestress at ULS $T_{vip}(x_{vip}) = (3.7 \cdot 10^{-15})\ \frac{kN}{m}$

Tension in reinf't at ULS $T_{vis}(x_{vip}) = 381.3\ \frac{kN}{m}$

Bending Capacity taking moments about the neutral axis

$$Mu_{vip} := C_{vc}(x_{vip}) \cdot (1 - 0.5 \cdot \lambda_{cc}) \cdot x_{vip} + (T_{vip}(x_{vip}) + P_{vp}) \cdot (d_p - x_{vip}) + T_{vis}(x_{vip}) \cdot (d_{vi} - x_{vip})$$

$Mu_{vip} = 44.9\ m \cdot \frac{kN}{m}$ $\frac{Mu_{vip}}{Mu_{vi}} = 100\%$ $Mu_p = 33.1\ \frac{kN \cdot m}{m}$ if $(Mu_{vip} > Mu_p, \text{“OK”}, \text{“low”}) = \text{“OK”}$

5.2.3 Maximum tension pressure (ULS)

$\left(h_{min}\right)=155\ mm$ (minimum wall thickness)

$R_{max}:=\max\left(R_{ipe.max},R_{ip.max},R_{rp.max},R_{rpe.max}\right)$

$R_{max}=566.037\ \frac{kN}{m}$ $\gamma_{fp}:=1.2$ (EN 1992-1-1 2.4.2.2)

$\sigma_{ED}:=\frac{\gamma_{fp}\cdot R_{max}}{h_{min}}=4.382\ MPa$ $f_{cd.1}:=0.8\cdot 0.6\cdot F_{ck}=19.2\ MPa$ According to EN 1992-1-1 5.42 and assuming 80% strength at tensioning

$\text{if}\left(f_{cd.1}>\sigma_{ED},\text{“OK”},\text{“low”}\right)=\text{“OK”}$

Expansion of EPDM - EN 1992-1-1 (10.5):

$a_s=80\ mm$ (width of EPDM)

$s_s=6\ mm$ (thickness of EPDM)

$A_{se.req}:=0.25\cdot\left(\frac{s_s}{a_s}\right)\cdot\gamma_{fp}\cdot\frac{R_{max}}{f_{yd}}$ $A_{se.req}=29.3\ \frac{mm^2}{m}$

Splitting reinforcement - EN 1992-1-1 (6.58)

$A_{ss.req}:=\frac{h_{min}-a_s}{h_{min}\cdot 4}\cdot\gamma_{fp}\cdot\frac{R_{max}}{f_{yd}}$ $A_{ss.req}=189\ \frac{mm^2}{m}$

Reinforcement chosen: 1 Ø6/100 mm at 90 degrees

Diameter: $\phi_w:=6\ mm$ Spacing $\rho_w:=100\ mm$

$A_{ss}:=\frac{1}{\rho_w}\cdot\frac{\pi}{4}\cdot\left(\phi_w\right)^2$ $A_{ss}=282.7\ \frac{mm^2}{m}$

$\text{if}\left(A_{ss}>A_{se.req}\wedge A_{ss}>A_{ss.req},\text{“OK”},\text{“Low”}\right)=\text{“OK”}$

Concentrated loading - EN 1992-1-1 (6.63)

Minimum area (Ø23mm tube every 100 mm):

Spacing $\rho_{tube}:=100\ mm$

$A_{c0}:=a_s-\frac{\pi}{\rho_{tube}}\cdot\left(\frac{23}{2}\ mm\right)^2$ $A_{c0}=75845.2\ \frac{mm^2}{m}$

$F_{rdu}:=A_{c0}\cdot f_{cd.1}\cdot\sqrt{\frac{h_{min}}{A_{c0}}}$ $F_{rdu}=2081.8\ \frac{kN}{m}$ $>\gamma_{fp}\cdot R_{max}=679.244\ \frac{kN}{m}$

$\text{if}\left(F_{rdu}>\gamma_{fp}\cdot R_{max},\text{“OK”},\text{“Low”}\right)=\text{“OK”}$

5.3 Crack Width Calculations (Reference 4.)

5.3.1 Formulae and Constants for Crack Width calculations

Modular ratios for reinforcement

$E_s = 200 \; \frac{kN}{mm^2}$ $\qquad E_{cs} = 34.525 \; \frac{kN}{mm^2}$ $\qquad E_{cl} = 13.4 \; \frac{kN}{mm^2}$

$\alpha_{es} := \frac{E_s}{E_{cs}} = 5.793$ $\qquad \alpha_{el} := \frac{E_s}{E_{cl}} = 14.925$

$F_{ck} = 40 \; MPa$ $\qquad f_{ctm} := 0.3 \cdot \left(\frac{F_{ck}}{MPa}\right)^{\frac{2}{3}} \cdot MPa$ $\qquad F_{ck} \leq 50 \; MPa$

$f_{ct.eff} := f_{ctm}$ $\qquad f_{ctm} = 3.509 \; MPa$

5.3.2 Limiting Crack Width for Serviceability Limit State

$\omega_{lim} := \text{if}\left(tnv > 0, 0.2 \; mm, 0.3 \; mm\right) = 0.3 \; mm$ (severe/aggressive environment)

5.3.3 Crack Width Calculations - Case A

Panel cross sectional area

$A_{pn} = 393950\ mm^2$

Panel modulus

$I_{pn} = 120022.9\ cm^4$

$M_{max.A.o} = 9.07\ \dfrac{kN \cdot m}{m}$

5.3.3.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_A := \dfrac{As_{vi}}{d_{vi}}$ $\quad \rho_A = 0.007 \quad d_{vi} = 125\ mm \quad \alpha_{es} = 5.8$

Depth to neutral axis $\quad x_A := x_{na}\left(\alpha_{es}, \rho_A, d_{vi}\right) \quad x_A = 30.918\ mm$

5.3.3.2 Depth to neutral axis, stresses and strains - with vertical prestress

Modular ratios for prestressing

$\alpha_{ps} := \dfrac{E_p}{E_{cs}} \quad \alpha_{ps} = 5.6 \qquad \alpha_{pl} := \dfrac{E_p}{E_{cl}} \quad \alpha_{pl} = 14.6$

$x_{A.p} := x_{cr}\left(As_{vi}, As_{vo}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vi}, h - d_{vo}, h - d_p, M_{max.A.o}, P_{vp}, 40\ mm\right) \quad x_{A.p} = 34.209\ mm$

$\varepsilon_{c.A.p} := \varepsilon_{cr}\left(x_{A.p}, As_{vi}, As_{vo}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vi}, h - d_{vo}, h - d_p, M_{max.A.o}, P_{vp}, E_{cs}\right) \quad \varepsilon_{c.A.p} = 0.0001538$

$\text{if}\left(\varepsilon_{c.A.p} \cdot \dfrac{h - x_{A.p}}{x_{A.p}} \cdot E_{cs} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”} \qquad \varepsilon_{c.A.p} \cdot \dfrac{h - x_{A.p}}{x_{A.p}} \cdot E_{cs} = 23.406\ MPa$

Strain in the reinforcement $\quad \varepsilon_{r.A.p} := \dfrac{d_{vi} - x_{A.p}}{x_{A.p}} \cdot \varepsilon_{c.A.p} \quad \varepsilon_{r.A.p} = 0.000408$

Strain at the level of the prestress (tensile opposing prestress) $\quad \varepsilon_{p.A.p} := \dfrac{\left(h - d_p\right) - x_{A.p}}{x_{A.p}} \cdot \varepsilon_{c.A.p} \quad \varepsilon_{p.A.p} = 0.000179$

Strain at the concrete tension face $\quad \varepsilon_{1.A.p} := \dfrac{h - x_{A.p}}{x_{A.p}} \cdot \varepsilon_{c.A.p} \quad \varepsilon_{1.A.p} = 677.95 \cdot 10^{-6}$

Tensile stress in the reinforcement $\quad \sigma_{r.A.p} := \varepsilon_{r.A.p} \cdot E_s \quad \sigma_{r.A.p} = 81.64\ MPa$

Compressive stress in the concrete $\quad \sigma_{c.A.p} := \varepsilon_{c.A.p} \cdot E_{cs} \quad \sigma_{c.A.p} = 5.31\ MPa$

Tensile stress in the prestressing $\quad \sigma_{p.A.p} := \dfrac{P_{vp}}{A_{vp}} + \varepsilon_{p.A.p} \cdot E_p \quad \sigma_{p.A.p} = 34.9\ MPa$

Stiffening effect of the concrete $\quad \varepsilon_{2.A.p} := \varepsilon_2\left(h, x_{A.p}, h, d_{vi}, As_{vi}, E_s, \omega_{lim}\right) \quad \varepsilon_{2.A.p} = 0.000476$

5.3.3.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{A.0} := \text{if}\left(tnv > 0, x_{A.p}, x_A\right) = 30.918\ mm$

$h = 185\ mm$ $\phi_{vi} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

EC2

$\frac{h - x_{A.0}}{3} = 51.361\ mm$ $\frac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vi}\right) = 150\ mm$

$A_{c.eff.A.0} := A_{c.eff}\left(h, d_{vi}, x_{A.0}\right) = 51360.8\ \frac{mm^2}{m}$

$As_{vi} = 877\ \frac{mm^2}{m}$ $p_{vi} = 100\ mm$

$\rho_{p.eff.A.0} := \rho_{p.eff}\left(As_{vi}, \phi_{vi}, A_{vp}, \phi_{vp}, A_{c.eff.A.0}\right) = 1.7\%$ $5 \cdot \left(cv_i + \frac{\phi_{vi}}{2}\right) = 300\ mm$

$S_{r.max.A.0} := s_{r.max}\left(cv_i, \phi_{vi}, \rho_{p.eff.A.0}\right) = 286.6\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{A.0}\right) = 200.3\ mm$

Tensile stress in the reinforcement

with prestress: $\sigma_{r.A.p} = 81.6\ MPa$

without prestress: $\sigma_{s.A} := \frac{M_{max.A.o}}{\left(d_{vi} - \frac{x_A}{3}\right) \cdot As_{vi}} = 90.2\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

Crack width

$\varepsilon_{sm.cm.A.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.A.p}, f_{ct.eff}, \rho_{p.eff.A.0}, E_s, E_{cs}\right), \varepsilon_{sm.cm}\left(\sigma_{s.A}, f_{ct.eff}, \rho_{p.eff.A.0}, E_s, E_{cs}\right)\right)$

$\varepsilon_{sm.cm.A.0} = 0$ EC2 EQ 7.9

$\frac{0.6 \cdot \sigma_{r.A.p}}{E_s} = 244.9 \cdot 10^{-6}$

EC2 - $w_{k.A} := \max\left(S_{r.max.A.0} \cdot \varepsilon_{sm.cm.A.0}, 0\right) = 0.078\ mm$

$\text{if}\left(S_{r.max.A.0} \cdot \varepsilon_{sm.cm.A.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.4 Crack Width Calculations - Case B

5.3.4.1 Depth to neutral axis - without vertical prestress

$M_{max.B.o} = 12.1 \ \frac{kN \cdot m}{m}$

Reinforcement Ratio

$\rho_B := \frac{As_{vi}}{d_{vi}}$ $\quad \rho_B = 0.007$ $\quad d_{vi} = 125 \ mm$ $\quad \alpha_{el} = 14.93$

Depth to neutral axis $\quad x_B := x_{na}\left(\alpha_{el}, \rho_B, d_{vi}\right)$ $\quad x_B = 45.59 \ mm$

5.3.4.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{B.p} := x_{cr}\left(As_{vi}, As_{vo}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vi}, h - d_{vo}, h - d_p, M_{max.B.o}, P_{vp}, 50 \ mm\right)$ $\quad x_{B.p} = 48.06 \ mm$

$\varepsilon_{c.B.p} := \varepsilon_{cr}\left(x_{B.p}, As_{vi}, As_{vo}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vi}, h - d_{vo}, h - d_p, M_{max.B.o}, P_{vp}, E_{cl}\right)$ $\quad \varepsilon_{c.B.p} = 0.0003734$

$\text{if}\left(\varepsilon_{c.B.p} \cdot \frac{h - x_{B.p}}{x_{B.p}} \cdot E_{cl} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”}$ $\quad \varepsilon_{c.B.p} \cdot \frac{h - x_{B.p}}{x_{B.p}} \cdot E_{cl} = 14.257 \ MPa$

Strain in the reinforcement $\quad \varepsilon_{r.B.p} := \frac{d_{vi} - x_{B.p}}{x_{B.p}} \cdot \varepsilon_{c.B.p}$ $\quad \varepsilon_{r.B.p} = 0.000598$

Strain at the level of the prestress
(tensile opposing prestress) $\quad \varepsilon_{p.B.p} := \frac{\left(h - d_p\right) - x_{B.p}}{x_{B.p}} \cdot \varepsilon_{c.B.p}$ $\quad \varepsilon_{p.B.p} = 0.000202$

Strain at the concrete tension face $\quad \varepsilon_{1.B.p} := \frac{h - x_{B.p}}{x_{B.p}} \cdot \varepsilon_{c.B.p}$ $\quad \varepsilon_{1.B.p} = 1.06 \cdot 10^{-3}$

Tensile stress in the reinforcement $\quad \sigma_{r.B.p} := \varepsilon_{r.B.p} \cdot E_s$ $\quad \sigma_{r.B.p} = 119.56 \ MPa$

Compressive stress in the concrete $\quad \sigma_{c.B.p} := \varepsilon_{c.B.p} \cdot E_{cl}$ $\quad \sigma_{c.B.p} = 5 \ MPa$

Tensile stress in the prestressing $\quad \sigma_{p.B.p} := \frac{P_{vp}}{A_{vp}} + \varepsilon_{p.B.p} \cdot E_p$ $\quad \sigma_{p.B.p} = 39.3 \ MPa$

Stiffening effect of the concrete $\quad \varepsilon_{2.B.p} := \varepsilon_2\left(h, x_{B.p}, h, d_{vi}, As_{vi}, E_s, \omega_{lim}\right)$ $\quad \varepsilon_{2.B.p} = 0.000463$

5.3.4.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{B.0} := \text{if}\left(tnv > 0, x_{B.p}, x_B\right) = 45.59\ mm$

$h = 185\ mm$ $\phi_{vi} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

EC2

$\dfrac{h - x_{B.0}}{3} = 46.469\ mm$ $\dfrac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vi}\right) = 150\ mm$

$A_{c.eff.B.0} := A_{c.eff}\left(h, d_{vi}, x_{B.0}\right) = 46469\ \dfrac{mm^2}{m}$

$As_{vi} = 877\ \dfrac{mm^2}{m}$ $p_{vi} = 100\ mm$

$\rho_{p.eff.B.0} := \rho_{p.eff}\left(As_{vi}, \phi_{vi}, A_{vp}, \phi_{vp}, A_{c.eff.B.0}\right) = 1.89\%$ $5 \cdot \left(cv_i + \dfrac{\phi_{vi}}{2}\right) = 300\ mm$

$S_{r.max.B.0} := s_{r.max}\left(cv_i, \phi_{vi}, \rho_{p.eff.B.0}\right) = 277.1\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{B.0}\right) = 181.2\ mm$

Tensile stress in the reinforcement with prestress: $\sigma_{r.B.p} = 119.6\ MPa$

without prestress: $\sigma_{s.B} := \dfrac{M_{max.B.o}}{\left(d_{vi} - \dfrac{x_B}{3}\right) \cdot As_{vi}} = 125.7\ MPa$

$\sigma_{r.B.p} = 119.559\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cl} = 13.4\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\dfrac{0.6 \cdot \sigma_{r.B.p}}{E_s} = 358.7 \cdot 10^{-6}$ $\dfrac{0.6 \cdot \sigma_{s.B}}{E_s} = 377.1 \cdot 10^{-6}$

Crack width

$\varepsilon_{sm.cm.B.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.B.p}, f_{ct.eff}, \rho_{p.eff.B.0}, E_s, E_{cl}\right), \varepsilon_{sm.cm}\left(\sigma_{s.B}, f_{ct.eff}, \rho_{p.eff.B.0}, E_s, E_{cl}\right)\right)$

$\varepsilon_{sm.cm.B.0} = 3.771 \cdot 10^{-4}$ EC2 EQ 7.9

EC2 - $w_{k.B} := \max\left(S_{r.max.B.0} \cdot \varepsilon_{sm.cm.B.0}, 0\ mm\right) = 0.104\ mm$

$\text{if}\left(S_{r.max.B.0} \cdot \varepsilon_{sm.cm.B.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.5 Crack Width Calculations - Case C

Consider case C moment on a standard panel

$M_{cp} := M_{min.C.o} \cdot b$ $\qquad M_{cp} = -44.4\ kN \cdot m$

$N_p := P_{vp} \cdot b$ $\qquad N_p = 0\ kN$

Average vertical prestress on panel

$\frac{P_{vp}}{h} = 0\ MPa$

$M_{min.C.o} = -21.02\ \frac{kN \cdot m}{m}$

5.3.5.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_C := \frac{As_{vo}}{d_{vo}}$ $\qquad \rho_C = 0.0066$ $\qquad d_{vo} = 125\ mm$ $\qquad \alpha_{es} = 5.8$

Depth to neutral axis $\qquad x_C := x_{na}\left(\alpha_{es}, \rho_C, d_{vo}\right)$ $\qquad x_C = 30.18\ mm$

5.3.5.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{C.p} := x_{cr}\left(As_{vo}, As_{vi}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vo}, h - d_{vi}, d_p, -M_{min.C.o}, P_{vp}, 40\ mm\right)$ $\qquad x_{C.p} = 33.8\ mm$

$\varepsilon_{c.C.p} := \varepsilon_{cr}\left(x_{C.p}, As_{vo}, As_{vi}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vo}, h - d_{vi}, d_p, -M_{min.C.o}, P_{vp}, E_{cs}\right)$ $\qquad \varepsilon_{c.C.p} = 0.0003654$

$\text{if}\left(\varepsilon_{c.C.p} \cdot \frac{h - x_{C.p}}{x_{C.p}} \cdot E_{cs} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”}$ $\qquad \varepsilon_{c.C.p} \cdot \frac{h - x_{C.p}}{x_{C.p}} \cdot E_{cs} = 56.438\ MPa$

Strain in the reinforcement $\qquad \varepsilon_{r.C.p} := \frac{d_{vo} - x_{C.p}}{x_{C.p}} \cdot \varepsilon_{c.C.p}$ $\qquad \varepsilon_{r.C.p} = 0.000986$

Strain at the level of the prestress (tensile opposing prestress) $\qquad \varepsilon_{p.C.p} := \frac{d_p - x_{C.p}}{x_{C.p}} \cdot \varepsilon_{c.C.p}$ $\qquad \varepsilon_{p.C.p} = 0.000835$

Strain at the concrete tension face $\qquad \varepsilon_{1.C.p} := \frac{h - x_{C.p}}{x_{C.p}} \cdot \varepsilon_{c.C.p}$ $\qquad \varepsilon_{1.C.p} = 1.63 \cdot 10^{-3}$

Tensile stress in the reinforcement $\qquad \sigma_{r.C.p} := \varepsilon_{r.C.p} \cdot E_s$ $\qquad \sigma_{r.C.p} = 197.2\ MPa$

Compressive stress in the concrete $\qquad \sigma_{c.C.p} := \varepsilon_{c.C.p} \cdot E_{cs}$ $\qquad \sigma_{c.C.p} = 12.62\ MPa$

Tensile stress in the prestressing $\qquad \sigma_{p.C.p} := \frac{P_{vp}}{A_{vp}} + \varepsilon_{p.C.p} \cdot E_p$ $\qquad \sigma_{p.C.p} = 162.8\ MPa$

Stiffening effect of the concrete $\qquad \varepsilon_{2.C.p} := \varepsilon_2\left(h, x_{C.p}, h, d_{vo}, As_{vo}, E_s, \omega_{lim}\right)$ $\qquad \varepsilon_{2.C.p} = 0.000504$

5.3.5.3 Crack width calculations with relevant x_n (depth to neutral axis)

EC2

$x_{C.0} := \text{if}\left(tnv > 0, x_{C.p}, x_C\right) = 30.18\ mm$

$h = 185\ mm$ $\phi_{vo} = 10\ mm$ $\boxed{\phi_{vp}} := 12.5\ mm$

EC2

$\dfrac{h - x_{C.0}}{3} = 51.606\ mm$ $\dfrac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vo}\right) = 150\ mm$

$A_{c.eff.C.0} := A_{c.eff}\left(h, d_{vo}, x_{C.0}\right) = 51605.6\ \dfrac{mm^2}{m}$

$As_{vi} = 877\ \dfrac{mm^2}{m}$ $p_{vi} = 100\ mm$

$\rho_{p.eff.C.0} := \rho_{p.eff}\left(As_{vo}, \phi_{vo}, A_{vp}, \phi_{vp}, A_{c.eff.C.0}\right) = 1.61\%$ $5 \cdot \left(cv_o + \dfrac{\phi_{vo}}{2}\right) = 300\ mm$

$S_{r.max.C.0} := s_{r.max}\left(cv_o, \phi_{vo}, \rho_{p.eff.C.0}\right) = 292.8\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{C.0}\right) = 201.3\ mm$

Tensile stress in the reinforcement with prestress: $\sigma_{r.C.p} = 197.2\ MPa$

without prestress: $\sigma_{s.C} := \dfrac{-M_{min.C.o}}{\left(d_{vo} - \dfrac{x_C}{3}\right) \cdot As_{vo}} = 220.5\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\dfrac{0.6 \cdot \sigma_{r.C.p}}{E_s} = 591.6 \cdot 10^{-6}$ $\dfrac{0.6 \cdot \sigma_{s.C}}{E_s} = 661.6 \cdot 10^{-6}$

Crack width

$\varepsilon_{sm.cm.C.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.C.p}, f_{ct.eff}, \rho_{p.eff.C.0}, E_s, E_{cs}\right), \varepsilon_{sm.cm}\left(\sigma_{s.C}, f_{ct.eff}, \rho_{p.eff.C.0}, E_s, E_{cs}\right)\right)$

$\varepsilon_{sm.cm.C.0} = 6.62 \cdot 10^{-4}$ EC2 EQ 7.9

EC2 - $w_{k.C} := \max\left(S_{r.max.C.0} \cdot \varepsilon_{sm.cm.C.0}, 0\ mm\right) = 0.194\ mm$

$\text{if}\left(S_{r.max.C.0} \cdot \varepsilon_{sm.cm.C.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.6 Crack Width Calculations - Case D

$$M_{min.D.o} = -10.66 \ \frac{kN \cdot m}{m}$$

5.3.6.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_D := \frac{As_{vo}}{d_{vo}}$ $\rho_D = 0.0066$ $d_{vo} = 125 \ mm$ $\alpha_{el} = 14.9$

Depth to neutral axis $x_D := x_{na}\left(\alpha_{el}, \rho_D, d_{vo}\right)$ $x_D = 44.61 \ mm$

5.3.6.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{D.p} := x_{cr}\left(As_{vo}, As_{vi}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vo}, h - d_{vi}, d_p, -M_{min.D.o}, P_{vp}, 55 \ mm\right)$ $x_{D.p} = 47.4 \ mm$

$\varepsilon_{c.D.p} := \varepsilon_{cr}\left(x_{D.p}, As_{vo}, As_{vi}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vo}, h - d_{vi}, d_p, -M_{min.D.o}, P_{vp}, E_{cl}\right)$ $\varepsilon_{c.D.p} = 0.0003367$

$\text{if}\left(\varepsilon_{c.D.p} \cdot \frac{h - x_{D.p}}{x_{D.p}} \cdot E_{cl} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”}$ $\varepsilon_{c.D.p} \cdot \frac{h - x_{D.p}}{x_{D.p}} \cdot E_{cl} = 13.086 \ MPa$

Strain in the reinforcement $\varepsilon_{r.D.p} := \frac{d_{vo} - x_{D.p}}{x_{D.p}} \cdot \varepsilon_{c.D.p}$ $\varepsilon_{r.D.p} = 0.000551$

Strain at the level of the prestress (tensile opposing prestress) $\varepsilon_{p.D.p} := \frac{d_p - x_{D.p}}{x_{D.p}} \cdot \varepsilon_{c.D.p}$ $\varepsilon_{p.D.p} = 0.000451$

Strain at the concrete tension face $\varepsilon_{1.D.p} := \frac{h - x_{D.p}}{x_{D.p}} \cdot \varepsilon_{c.D.p}$ $\varepsilon_{1.D.p} = 0.000977$

Tensile stress in the reinforcement $\sigma_{r.D.p} := \varepsilon_{r.D.p} \cdot E_s$ $\sigma_{r.D.p} = 110.13 \ MPa$

Compressive stress in the concrete $\sigma_{c.D.p} := \varepsilon_{c.D.p} \cdot E_{cl}$ $\sigma_{c.D.p} = 4.51 \ MPa$

Tensile stress in the prestressing $\sigma_{p.D.p} := \frac{P_{vp}}{A_{vp}} + \varepsilon_{p.D.p} \cdot E_p$ $\sigma_{p.D.p} = 88 \ MPa$

Stiffening effect of the concrete $\varepsilon_{2.D.p} := \varepsilon_2\left(h, x_{D.p}, h, d_{vo}, As_{vo}, E_s, \omega_{lim}\right)$ $\varepsilon_{2.D.p} = 0.00049$

5.3.6.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{D.0} := \text{if}\left(tnv > 0, x_{D.p}, x_D\right) = 44.61\ mm$ EC2

$h = 185\ mm$ $\phi_{vo} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

EC2

$\dfrac{h - x_{D.0}}{3} = 46.796\ mm$ $\dfrac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vo}\right) = 150\ mm$

$A_{c.eff.D.0} := A_{c.eff}\left(h, d_{vo}, x_{D.0}\right) = 46796.5\ \dfrac{mm^2}{m}$

$As_{vo} = 829.3\ \dfrac{mm^2}{m}$ $p_{vo} = 100\ mm$

$\rho_{p.eff.D.0} := \rho_{p.eff}\left(As_{vo}, \phi_{vo}, A_{vp}, \phi_{vp}, A_{c.eff.D.0}\right) = 1.77\%$ $5 \cdot \left(cv_o + \dfrac{\phi_{vo}}{2}\right) = 300\ mm$

$S_{r.max.D.0} := s_{r.max}\left(cv_o, \phi_{vo}, \rho_{p.eff.D.0}\right) = 282.9\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{D.0}\right) = 182.5\ mm$

Tensile stress in the reinforcement with prestress: $\sigma_{r.D.p} = 110.1\ MPa$

without prestress: $\sigma_{s.D} := \dfrac{-M_{min.D.o}}{\left(d_{vo} - \dfrac{x_D}{3}\right) \cdot As_{vo}} = 116.8\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\dfrac{0.6 \cdot \sigma_{r.D.p}}{E_s} = 330.4 \cdot 10^{-6}$ $\dfrac{0.6 \cdot \sigma_{s.D}}{E_s} = 350.3 \cdot 10^{-6}$

Crack width

$\varepsilon_{sm.cm.D.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.D.p}, f_{ct.eff}, \rho_{p.eff.D.0}, E_s, E_{cl}\right), \varepsilon_{sm.cm}\left(\sigma_{s.D}, f_{ct.eff}, \rho_{p.eff.D.0}, E_s, E_{cl}\right)\right)$

$\varepsilon_{sm.cm.D.0} = 3.5 \cdot 10^{-4}$ EC2 EQ 7.9

EC2 - $w_{k.D} := \max\left(S_{r.max.D.0} \cdot \varepsilon_{sm.cm.D.0}, 0\ mm\right) = 0.099\ mm$

$\text{if}\left(S_{r.max.D.0} \cdot \varepsilon_{sm.cm.D.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.7 Crack Width Calculations - Case E

$A_{pn} = 393950\ mm^2$ $I_{pn} = 120022.9\ cm^4$ $M_{max.E.o} := \max(M_{ipe}) \cdot m^{-1}$

$\alpha_{ps} := \frac{E_p}{E_{cs}} = 5.6$ $\alpha_{pl} := \frac{E_p}{E_{cl}} = 14.6$ $M_{max.E.o} = 22.055\ \frac{kN \cdot m}{m}$

5.3.7.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_E := \frac{As_{vi}}{d_{vi}}$ $\rho_E = 0.007$ $d_{vi} = 125\ mm$ $\alpha_{es} = 5.8$

Depth to neutral axis $x_E := x_{na}(\alpha_{es}, \rho_E, d_{vi})$ $x_E = 30.92\ mm$

5.3.7.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{E.p} := x_{cr}(As_{vi}, As_{vo}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vi}, h - d_{vo}, h - d_p, M_{max.E.o}, P_{vp}, 40\ mm)$ $x_{E.p} = 34.2\ mm$

$\varepsilon_{c.E.p} := \varepsilon_{cr}(x_{E.p}, As_{vi}, As_{vo}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vi}, h - d_{vo}, h - d_p, M_{max.E.o}, P_{vp}, E_{cs})$ $\varepsilon_{c.E.p} = 0.0003741$

$\text{if}\left(\varepsilon_{c.E.p} \cdot \frac{h - x_{E.p}}{x_{E.p}} \cdot E_{cs} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”}$ $\varepsilon_{c.E.p} \cdot \frac{h - x_{E.p}}{x_{E.p}} \cdot E_{cs} = 56.933\ MPa$

Strain in the reinforcement $\varepsilon_{r.E.p} := \frac{d_{vi} - x_{E.p}}{x_{E.p}} \cdot \varepsilon_{c.E.p}$ $\varepsilon_{r.E.p} = 0.000993$

Strain at the level of the prestress
(tensile opposing prestress) $\varepsilon_{p.E.p} := \frac{(h - d_p) - x_{E.p}}{x_{E.p}} \cdot \varepsilon_{c.E.p}$ $\varepsilon_{p.E.p} = 0.000435$

Strain at the concrete tension face $\varepsilon_{1.E.p} := \frac{h - x_{E.p}}{x_{E.p}} \cdot \varepsilon_{c.E.p}$ $\varepsilon_{1.E.p} = 0.001649$

Tensile stress in the reinforcement $\sigma_{r.E.p} := \varepsilon_{r.E.p} \cdot E_s$ $\sigma_{r.E.p} = 198.57\ MPa$

Compressive stress in the concrete $\sigma_{c.E.p} := \varepsilon_{c.E.p} \cdot E_{cs}$ $\sigma_{c.E.p} = 12.92\ MPa$

Tensile stress in the prestressing $\sigma_{p.E.p} := \frac{P_{vp}}{A_{vp}} + \varepsilon_{p.E.p} \cdot E_p$ $\sigma_{p.E.p} = 84.9\ MPa$

Stiffening effect of the concrete $\varepsilon_{2.E.p} := \varepsilon_2(h, x_{E.p}, h, d_{vi}, As_{vi}, E_s, \omega_{lim})$ $\varepsilon_{2.E.p} = 4.76 \cdot 10^{-4}$

5.3.7.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{E.0} := \text{if}\left(tnv > 0, x_{E.p}, x_E\right) = 30.92\ mm$ EC2

$h = 185\ mm$ $\phi_{vi} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

$\frac{h - x_{E.0}}{3} = 51.361\ mm$ $\frac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vi}\right) = 150\ mm$

$A_{c.eff.E.0} := A_{c.eff}\left(h, d_{vi}, x_{E.0}\right) = 51360.8\ \frac{mm^2}{m}$

$As_{vi} = 877\ \frac{mm^2}{m}$ $p_{vi} = 100\ mm$

$\rho_{p.eff.E.0} := \rho_{p.eff}\left(As_{vi}, \phi_{vi}, A_{vp}, \phi_{vp}, A_{c.eff.E.0}\right) = 1.71\%$ $5 \cdot \left(cv_i + \frac{\phi_{vi}}{2}\right) = 300\ mm$

$S_{r.max.E.0} := s_{r.max}\left(cv_i, \phi_{vi}, \rho_{p.eff.E.0}\right) = 286.6\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{E.0}\right) = 200.3\ mm$

Tensile stress in the reinforcement

with prestress: $\sigma_{r.E.p} = 198.6\ MPa$

without prestress: $\sigma_{s.E} := \frac{M_{max.E.o}}{\left(d_{vi} - \frac{x_E}{3}\right) \cdot As_{vi}} = 219.3\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\frac{0.6 \cdot \sigma_{r.E.p}}{E_s} = 595.7 \cdot 10^{-6}$ $\frac{0.6 \cdot \sigma_{s.E}}{E_s} = 657.8 \cdot 10^{-6}$

Crack width

$\varepsilon_{sm.cm.E.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.E.p}, f_{ct.eff}, \rho_{p.eff.E.0}, E_s, E_{cs}\right), \varepsilon_{sm.cm}\left(\sigma_{s.E}, f_{ct.eff}, \rho_{p.eff.E.0}, E_s, E_{cs}\right)\right)$

$\varepsilon_{sm.cm.E.0} = 6.58 \cdot 10^{-4}$ EC2 EQ 7.9

EC2 - $w_{k.E} := \max\left(S_{r.max.E.0} \cdot \varepsilon_{sm.cm.E.0}, 0\ mm\right) = 0.189\ mm$

$\text{if}\left(S_{r.max.E.0} \cdot \varepsilon_{sm.cm.E.0} < \omega_{lim}, \text{"OK"}, \text{"!!"}\right) = \text{"OK"}$

5.3.8 Crack Width Calculations - Case F

$M_{max.F.o} := \max\left(M_{rpe}\right) \cdot m^{-1}$ $\quad\quad M_{max.F.o} = 21.366\ \dfrac{kN \cdot m}{m}$

5.3.8.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_F := \dfrac{As_{vi}}{d_{vi}}$ $\quad \rho_F = 0.007$ $\quad d_{vi} = 125\ mm$ $\quad \alpha_{el} = 14.9$

Depth to neutral axis $\quad x_F := x_{na}\left(\alpha_{el}, \rho_F, d_{vi}\right)$ $\quad x_F = 45.59\ mm$

5.3.8.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{F.p} := x_{cr}\left(As_{vi}, As_{vo}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vi}, h-d_{vo}, h-d_p, M_{max.F.o}, P_{vp}, 45\ mm\right)$ $\quad x_{F.p} = 48.1\ mm$

$\varepsilon_{c.F.p} := \varepsilon_{cr}\left(x_{F.p}, As_{vi}, As_{vo}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vi}, h-d_{vo}, h-d_p, M_{max.F.o}, P_{vp}, E_{cl}\right)$ $\quad \varepsilon_{c.F.p} = 0.0006592$

$\text{if}\left(\varepsilon_{c.F.p} \cdot \dfrac{h-x_{F.p}}{x_{F.p}} \cdot E_{cl} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“cracked”}$ $\quad \varepsilon_{c.F.p} \cdot \dfrac{h-x_{F.p}}{x_{F.p}} \cdot E_{cl} = 25.168\ MPa$

Strain in the reinforcement $\quad \varepsilon_{r.F.p} := \dfrac{d_{vi}-x_{F.p}}{x_{F.p}} \cdot \varepsilon_{c.F.p}$ $\quad \varepsilon_{r.F.p} = 0.001055$

Strain at the level of the prestress (tensile opposing prestress) $\quad \varepsilon_{p.F.p} := \dfrac{\left(h-d_p\right)-x_{F.p}}{x_{F.p}} \cdot \varepsilon_{c.F.p}$ $\quad \varepsilon_{p.F.p} = 0.000356$

Strain at the concrete tension face $\quad \varepsilon_{1.F.p} := \dfrac{h-x_{F.p}}{x_{F.p}} \cdot \varepsilon_{c.F.p}$ $\quad \varepsilon_{1.F.p} = 0.002$

Tensile stress in the reinforcement $\quad \sigma_{r.F.p} := \varepsilon_{r.F.p} \cdot E_s$ $\quad \sigma_{r.F.p} = 211.06\ MPa$

Compressive stress in the concrete $\quad \sigma_{c.F.p} := \varepsilon_{c.F.p} \cdot E_{cl}$ $\quad \sigma_{c.F.p} = 8.83\ MPa$

Tensile stress in the prestressing $\quad \sigma_{p.F.p} := \dfrac{P_{vp}}{A_{vp}} + \varepsilon_{p.F.p} \cdot E_p$ $\quad \sigma_{p.F.p} = 69.4\ MPa$

Stiffening effect of the concrete $\quad \varepsilon_{2.F.p} := \varepsilon_2\left(h, x_{F.p}, h, d_{vi}, As_{vi}, E_s, \omega_{lim}\right)$ $\quad \varepsilon_{2.F.p} = 4.632 \cdot 10^{-4}$

5.3.8.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{F.0} := \text{if}\left(tnv > 0, x_{F.p}, x_F\right) = 45.59\ mm$ EC2

$h = 185\ mm$ $\phi_{vi} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

$\frac{h - x_{F.0}}{3} = 46.469\ mm$ $\frac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vi}\right) = 150\ mm$

$A_{c.eff.F.0} := A_{c.eff}\left(h, d_{vi}, x_{F.0}\right) = 46469\ \frac{mm^2}{m}$

$As_{vi} = 877\ \frac{mm^2}{m}$ $p_{vi} = 100\ mm$

$\rho_{p.eff.F.0} := \rho_{p.eff}\left(As_{vi}, \phi_{vi}, A_{vp}, \phi_{vp}, A_{c.eff.F.0}\right) = 1.89\%$ $5 \cdot \left(cv_i + \frac{\phi_{vi}}{2}\right) = 300\ mm$

$S_{r.max.F.0} := s_{r.max}\left(cv_i, \phi_{vi}, \rho_{p.eff.F.0}\right) = 277.1\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{F.0}\right) = 181.2\ mm$

Tensile stress in the reinforcement with prestress: $\sigma_{r.F.p} = 211.1\ MPa$

without prestress: $\sigma_{s.F} := \frac{M_{max.F.o}}{\left(d_{vi} - \frac{x_F}{3}\right) \cdot As_{vi}} = 221.9\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\frac{0.6 \cdot \sigma_{r.F.p}}{E_s} = 633.2 \cdot 10^{-6}$ $\frac{0.6 \cdot \sigma_{s.F}}{E_s} = 665.7 \cdot 10^{-6}$

Crack width

$\varepsilon_{sm.cm.F.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.F.p}, f_{ct.eff}, \rho_{p.eff.F.0}, E_s, E_{cl}\right), \varepsilon_{sm.cm}\left(\sigma_{s.F}, f_{ct.eff}, \rho_{p.eff.F.0}, E_s, E_{cl}\right)\right)$

$\varepsilon_{sm.cm.F.0} = 6.66 \cdot 10^{-4}$ EC2 EQ 7.9

EC2 - $w_{k.F} := \max\left(S_{r.max.F.0} \cdot \varepsilon_{sm.cm.F.0}, 0\ mm\right) = 0.184\ mm$

$\text{if}\left(S_{r.max.F.0} \cdot \varepsilon_{sm.cm.F.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.9 Crack Width Calculations - Case G

$M_{min.G.o} := min\left(M_{ife}\right) \cdot m^{-1}$ $\qquad$ $M_{min.G.o} = 0 \dfrac{kN \cdot m}{m}$

5.3.9.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_G := \dfrac{As_{vo}}{d_{vo}} = 0.0066$ $\qquad$ $d_{vo} = 125\ mm$ $\qquad$ $\alpha_{es} = 5.8$

Depth to neutral axis $\qquad x_G := x_{na}\left(\alpha_{es}, \rho_G, d_{vo}\right) = 30.183\ mm$

5.3.9.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{G.p} := x_{cr}\left(As_{vo}, As_{vi}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vo}, h - d_{vi}, d_p, -M_{min.G.o}, P_{vp}, 45\ mm\right)$ $\qquad$ $x_{G.p} = 45\ mm$

$\varepsilon_{c.G.p} := \varepsilon_{cr}\left(x_{G.p}, As_{vo}, As_{vi}, A_{vp}, \alpha_{es}, \alpha_{ps}, d_{vo}, h - d_{vi}, d_p, -M_{min.G.o}, P_{vp}, E_{cs}\right)$ $\qquad$ $\varepsilon_{c.G.p} = 0$

$\text{if}\left(\varepsilon_{c.G.p} \cdot \dfrac{h - x_{G.p}}{x_{G.p}} \cdot E_{cs} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“uncracked”}$ $\qquad$ $\varepsilon_{c.G.p} \cdot \dfrac{h - x_{G.p}}{x_{G.p}} \cdot E_{cs} = 0\ MPa$

Strain in the reinforcement $\qquad \varepsilon_{r.G.p} := \dfrac{d_{vo} - x_{G.p}}{x_{G.p}} \cdot \varepsilon_{c.G.p}$ $\qquad$ $\varepsilon_{r.G.p} = 0$

Strain at the level of the prestress
(tensile opposing prestress) $\qquad \varepsilon_{p.G.p} := \dfrac{d_p - x_{G.p}}{x_{G.p}} \cdot \varepsilon_{c.G.p}$ $\qquad$ $\varepsilon_{p.G.p} = 0$

Strain at the concrete tension face $\qquad \varepsilon_{1.G.p} := \dfrac{h - x_{G.p}}{x_{G.p}} \cdot \varepsilon_{c.G.p}$ $\qquad$ $\varepsilon_{1.G.p} = 0$

Tensile stress in the reinforcement $\qquad \sigma_{r.G.p} := \varepsilon_{r.G.p} \cdot E_s$ $\qquad$ $\sigma_{r.G.p} = 0\ MPa$

Compressive stress in the concrete $\qquad \sigma_{c.G.p} := \varepsilon_{c.G.p} \cdot E_{cs}$ $\qquad$ $\sigma_{c.G.p} = 0\ MPa$

Tensile stress in the prestressing $\qquad \sigma_{p.G.p} := \dfrac{P_{vp}}{A_{vp}} + \varepsilon_{p.G.p} \cdot E_p$ $\qquad$ $\sigma_{p.G.p} = 0\ MPa$

Stiffening effect of the concrete $\qquad \varepsilon_{2.G.p} := \varepsilon_2\left(h, x_{G.p}, h, d_{vo}, As_{vo}, E_s, \omega_{lim}\right)$ $\qquad$ $\varepsilon_{2.G.p} = 0.000492$

5.3.9.3 Crack width calculations with relevant x_n (depth to neutral axis)

EC2

$x_{G.0} := \text{if}\left(tnv > 0, x_{G.p}, x_G\right) = 30.18\ mm$

$h = 185\ mm$ $\phi_{vo} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

EC2

$\dfrac{h - x_{G.0}}{3} = 51.606\ mm$ $\dfrac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vo}\right) = 150\ mm$

$A_{c.eff.G.0} := A_{c.eff}\left(h, d_{vo}, x_{G.0}\right) = 51605.6\ \dfrac{mm^2}{m}$

$As_{vo} = 829.3\ \dfrac{mm^2}{m}$ $p_{vo} = 100\ mm$

$\rho_{p.eff.G.0} := \rho_{p.eff}\left(As_{vo}, \phi_{vo}, A_{vp}, \phi_{vp}, A_{c.eff.G.0}\right) = 1.61\%$ $5 \cdot \left(cv_o + \dfrac{\phi_{vo}}{2}\right) = 300\ mm$

$S_{r.max.G.0} := s_{r.max}\left(cv_o, \phi_{vo}, \rho_{p.eff.G.0}\right) = 292.8\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{G.0}\right) = 201.3\ mm$

Tensile stress in the reinforcement with prestress: $\sigma_{r.G.p} = 0\ MPa$

without prestress: $\sigma_{s.G} := \dfrac{-M_{min.G.o}}{\left(d_{vo} - \dfrac{x_G}{3}\right) \cdot As_{vo}} = 0\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cs} = 34.525\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\dfrac{0.6 \cdot \sigma_{r.G.p}}{E_s} = 0$ $\dfrac{0.6 \cdot \sigma_{s.G}}{E_s} = 0$

Crack width

$\varepsilon_{sm.cm.G.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.G.p}, f_{ct.eff}, \rho_{p.eff.G.0}, E_s, E_{cs}\right), \varepsilon_{sm.cm}\left(\sigma_{s.G}, f_{ct.eff}, \rho_{p.eff.G.0}, E_s, E_{cs}\right)\right)$

$\varepsilon_{sm.cm.G.0} = 0$ EC2 EQ 7.9

EC2 - $w_{k.G} := \max\left(S_{r.max.G.0} \cdot \varepsilon_{sm.cm.G.0}, 0\ mm\right) = 0\ mm$

$\text{if}\left(S_{r.max.G.0} \cdot \varepsilon_{sm.cm.G.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

5.3.10 Crack Width Calculations - Case H

$M_{min.H.o} := min\left(M_{tfe}\right) \cdot m^{-1}$ $\qquad$ $M_{min.H.o} = 0\ \frac{kN \cdot m}{m}$

5.3.10.1 Depth to neutral axis - without vertical prestress

Reinforcement Ratio

$\rho_H := \frac{As_{vo}}{d_{vo}} = 0.0066$ $\qquad$ $d_{vo} = 125\ mm$ $\qquad$ $\alpha_{el} = 14.9$

Depth to neutral axis $\qquad$ $x_H := x_{na}\left(\alpha_{el}, \rho_H, d_{vo}\right) = 44.611\ mm$

5.3.10.2 Depth to neutral axis, stresses and strains - with vertical prestress

$x_{H.p} := x_{cr}\left(As_{vo}, As_{vi}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vo}, h - d_{vi}, d_p, -M_{min.H.o}, P_{vp}, 50\ mm\right)$ $\qquad$ $x_{H.p} = 50\ mm$

$\varepsilon_{c.H.p} := \varepsilon_{cr}\left(x_{H.p}, As_{vo}, As_{vi}, A_{vp}, \alpha_{el}, \alpha_{pl}, d_{vo}, h - d_{vi}, d_p, -M_{min.H.o}, P_{vp}, E_{cl}\right)$ $\qquad$ $\varepsilon_{c.H.p} = 0$

$\text{if}\left(\varepsilon_{c.H.p} \cdot \frac{h - x_{H.p}}{x_{H.p}} \cdot E_{cl} > F_{ct}, \text{“cracked”}, \text{“uncracked”}\right) = \text{“uncracked”}$ $\qquad$ $\varepsilon_{c.H.p} \cdot \frac{h - x_{H.p}}{x_{H.p}} \cdot E_{cl} = 0\ MPa$

Strain in the reinforcement $\qquad$ $\varepsilon_{r.H.p} := \frac{d_{vo} - x_{H.p}}{x_{H.p}} \cdot \varepsilon_{c.H.p}$ $\qquad$ $\varepsilon_{r.H.p} = 0$

Strain at the level of the prestress (tensile opposing prestress) $\qquad$ $\varepsilon_{p.H.p} := \frac{d_p - x_{H.p}}{x_{H.p}} \cdot \varepsilon_{c.H.p}$ $\qquad$ $\varepsilon_{p.H.p} = 0$

Strain at the concrete tension face $\qquad$ $\varepsilon_{1.H.p} := \frac{h - x_{H.p}}{x_{H.p}} \cdot \varepsilon_{c.H.p}$ $\qquad$ $\varepsilon_{1.H.p} = 0$

Tensile stress in the reinforcement $\qquad$ $\sigma_{r.H.p} := \varepsilon_{r.H.p} \cdot E_s$ $\qquad$ $\sigma_{r.H.p} = 0\ MPa$

Compressive stress in the concrete $\qquad$ $\sigma_{c.H.p} := \varepsilon_{c.H.p} \cdot E_{cl}$ $\qquad$ $\sigma_{c.H.p} = 0\ MPa$

Tensile stress in the prestressing $\qquad$ $\sigma_{p.H.p} := \frac{P_{vp}}{A_{vp}} + \varepsilon_{p.H.p} \cdot E_p$ $\qquad$ $\sigma_{p.H.p} = 0\ MPa$

Stiffening effect of the concrete $\qquad$ $\varepsilon_{2.H.p} := \varepsilon_2\left(h, x_{H.p}, h, d_{vo}, As_{vo}, E_s, \omega_{lim}\right)$ $\qquad$ $\varepsilon_{2.H.p} = 0.000488$

5.3.10.3 Crack width calculations with relevant x_n (depth to neutral axis)

$x_{H.0} := \text{if}\left(tnv > 0, x_{H.p}, x_H\right) = 44.61\ mm$ EC2

$h = 185\ mm$ $\phi_{vo} = 10\ mm$ $\phi_{vp} := 12.5\ mm$

EC2

$\dfrac{h - x_{H.0}}{3} = 46.796\ mm$ $\dfrac{h}{2} = 92.5\ mm$ $2.5 \cdot \left(h - d_{vo}\right) = 150\ mm$

$A_{c.eff.H.0} := A_{c.eff}\left(h, d_{vo}, x_{H.0}\right) = 46796.46\ \dfrac{mm^2}{m}$

$As_{vo} = 829.3\ \dfrac{mm^2}{m}$ $p_{vo} = 100\ mm$

$\rho_{p.eff.H.0} := \rho_{p.eff}\left(As_{vo}, \phi_{vo}, A_{vp}, \phi_{vp}, A_{c.eff.H.0}\right) = 1.77\%$ $5 \cdot \left(cv_o + \dfrac{\phi_{vo}}{2}\right) = 300\ mm$

$S_{r.max.H.0} := s_{r.max}\left(cv_o, \phi_{vo}, \rho_{p.eff.H.0}\right) = 282.9\ mm$ EC2 EQ 7.11 $1.3 \cdot \left(h - x_{H.0}\right) = 182.5\ mm$

Tensile stress in the reinforcement

with prestress: $\sigma_{r.H.p} = 0\ MPa$

without prestress: $\sigma_{s.H} := \dfrac{-M_{min.H.o}}{\left(d_{vo} - \dfrac{x_H}{3}\right) \cdot As_{vo}} = 0\ MPa$

$E_s = 200\ kN \cdot mm^{-2}$ $E_{cl} = 13.4\ kN \cdot mm^{-2}$

$F_{ck} = 40\ MPa$ $f_{ctm} = 3.509\ MPa$ $f_{ct.eff} = 3.509\ MPa$

$\dfrac{0.6 \cdot \sigma_{r.H.p}}{E_s} = 0$ $\dfrac{0.6 \cdot \sigma_{s.H}}{E_s} = 0$

Crack width

$\varepsilon_{sm.cm.H.0} := \text{if}\left(tnv > 0, \varepsilon_{sm.cm}\left(\sigma_{r.H.p}, f_{ct.eff}, \rho_{p.eff.H.0}, E_s, E_{cl}\right), \varepsilon_{sm.cm}\left(\sigma_{s.H}, f_{ct.eff}, \rho_{p.eff.H.0}, E_s, E_{cl}\right)\right)$

$\varepsilon_{sm.cm.H.0} = 0$ EC2 EQ 7.9

EC2 - $w_{k.H} := \max\left(S_{r.max.H.0} \cdot \varepsilon_{sm.cm.H.0}, 0\ mm\right) = 0\ mm$

$\text{if}\left(S_{r.max.H.0} \cdot \varepsilon_{sm.cm.H.0} < \omega_{lim}, \text{“OK”}, \text{“!!”}\right) = \text{“OK”}$

Crack width summary

Inside face of tank:
(post-tensioning + external loading, but no hydrostatic)

$w_{k.A} = 0.078\ mm$

$w_{k.B} = 0.104\ mm$

$w_{k.E} = 0.189\ mm$

$w_{k.F} = 0.184\ mm$

Outside face of tank:
(post-tensioning + hydrostatic, but no external loading)

$w_{k.C} = 0.194\ mm$

$w_{k.D} = 0.099\ mm$

$w_{k.G} = 0\ mm$

$w_{k.H} = 0\ mm$

Max allowed crack width: $\omega_{lim} = 0.3\ mm$

$w_{k.inside} := \text{if}\left(w_{k.A} < \omega_{lim} \wedge w_{k.B} < \omega_{lim} \wedge w_{k.E} < \omega_{lim} \wedge w_{k.F} < \omega_{lim}, \text{“All inside face crack widths are OK”}, \text{“!!”}\right)$

$w_{k.inside} = \text{“All inside face crack widths are OK”}$

$w_{k.outside} := \text{if}\left(w_{k.C} < \omega_{lim} \wedge w_{k.D} < \omega_{lim} \wedge w_{k.G} < \omega_{lim} \wedge w_{k.H} < \omega_{lim}, \text{“All outside face crack widths are OK”}, \text{“!!”}\right)$

$w_{k.outside} = \text{“All outside face crack widths are OK”}$

5.4 Minimum reinforcement

$As_{vi} = 876.966 \ \frac{mm^2}{m}$ $As_{vo} = 829.321 \ \frac{mm^2}{m}$

$As_{hi} := \frac{\pi \cdot \phi_{hi}^2}{4 \cdot p_{hi}} = 251.3 \ \frac{mm^2}{m}$ $As_{ho} := \frac{\pi \cdot \phi_{ho}^2}{4 \cdot p_{ho}} = 251.3 \ \frac{mm^2}{m}$

$As_{min.cr} := \frac{0.4 \cdot f_{ctm} \cdot \frac{h}{2}}{f_{yf}} = 259.7 \ \frac{mm^2}{m}$ (EC2 7.3.2)

$As_{min.br.o} := \max\left(0.26 \cdot \frac{f_{ctm}}{f_{yf}} \cdot \left(h - cv_o - \frac{\phi_{vo}}{2}\right), 0.0013 \cdot \left(h - cv_o - \frac{\phi_{vo}}{2}\right)\right) = 228.1 \ \frac{mm^2}{m}$ (EC2 9.2.1.1)

$As_{min.br.i} := \max\left(0.26 \cdot \frac{f_{ctm}}{f_{yf}} \cdot \left(h - cv_i - \frac{\phi_{vi}}{2}\right), 0.0013 \cdot \left(h - cv_i - \frac{\phi_{vi}}{2}\right)\right) = 228.1 \ \frac{mm^2}{m}$ (EC2 9.2.1.1)

$As_{max} := 0.04 \cdot \frac{h}{2} = 3700 \ \frac{mm^2}{m}$

$\text{if}\left(As_{vi} > As_{min.br.i} \wedge As_{vi} > As_{min.cr} \wedge As_{vi} < As_{max}, \text{“OK”}, \text{“Outside limits”}\right) = \text{“OK”}$

$\text{if}\left(As_{vo} > As_{min.br.o} \wedge As_{vo} > As_{min.cr} \wedge As_{vo} < As_{max}, \text{“OK”}, \text{“Outside limits”}\right) = \text{“OK”}$

$As_{min.hi} := \max\left(0.2 \cdot As_{vi}\right) = 175.393 \ \frac{mm^2}{m}$ (EC2 9.3)

$As_{min.ho} := \max\left(0.2 \cdot As_{vo}\right) = 165.864 \ \frac{mm^2}{m}$ (EC2 9.3)

$\text{if}\left(As_{hi} > As_{min.hi}, \text{“OK”}, \text{“Insufficient”}\right) = \text{“OK”}$

$\text{if}\left(As_{ho} > As_{min.ho}, \text{“OK”}, \text{“Insufficient”}\right) = \text{“OK”}$

5.5 Crack Widths due to Thermal Cracking

5.5.1 Thermal Crack Widths

Assuming bounding case is thermal cracking on the inside

$h = 185\ mm$ $\quad cv_o = 55\ mm$ $\quad \alpha_c = 1 \cdot 10^{-5}$ $\quad k_c := 1$

$s_c := 0.25$ $\quad t_c := 28\ day$ $\quad K_{c1} := 1$ Creep factor $\quad k := 0.5$

$\beta_{cc.t} := e^{s_c \cdot \left(1 - \sqrt{\frac{28\ day}{t_c}}\right)}$ $\quad \Delta T := 20$ Grad C $\quad R := 1$ Pure tension

$f_{cm} = 48\ MPa$

$\varepsilon_{free} := \Delta T \cdot \alpha_c = 2 \cdot 10^{-4}$ Strain from internal restraint

$f_{ctk.0.05} := \beta_{cc.t} \cdot f_{ctm} \cdot 0.7 = 2.46\ MPa$ $\quad E_{cm} := 22000\ MPa \cdot \left(\frac{f_{cm}}{10\ MPa}\right)^{0.3} = 35.2\ GPa$

$$\varepsilon_{ctu} := \frac{f_{ctk.0.05}}{E_{cm} \cdot K_{c1}} = 6.97 \cdot 10^{-5}$$

$\varepsilon_{cr.thermal} := R \cdot K_{c1} \cdot \varepsilon_{free} - 0.5 \cdot \varepsilon_{ctu} = 1.65 \cdot 10^{-4}$ -Crack inducing strain

$A_{ct} := k \cdot k_c \cdot 0.4 \cdot h = 37\ mm$ -Surface zone in tension

$A_{s.min.t} := (1 - 0.5 \cdot R) \cdot k \cdot k_c \cdot \frac{f_{ctk.0.05}}{f_{yf}} \cdot A_{ct} = 45.44\ \frac{mm^2}{m}$ $\quad$ if $\left(A_{s.min.t} < As_{hi}, \text{“OK”}, \text{“Low”}\right) = \text{“OK”}$

Inside

$$h_{ei.ef} := min\left(\frac{h}{2}, 2.5 \cdot \left(cv_i + \frac{\phi_{vi}}{2}\right)\right) = 92.5\ mm$$

$\rho_{eff.ti} := \frac{As_{vi}}{h_{ei.ef}} = 0.009$ $\quad k_1 := 0.8$ -Bonding coefficient

$$S_{ri.max} := 3.4 \cdot cv_i + \frac{0.425 \cdot k_1 \cdot \phi_{vi}}{\rho_{eff.ti}} = 545.62\ mm$$

$W_{k.ti} := \max\left(\varepsilon_{cr.thermal} \cdot S_{ri.max}, 0\ mm\right) = 0.09\ mm$ $\quad$ if $\left(W_{k.ti} < \omega_{lim}, \text{“OK”}, \text{“Low”}\right) = \text{“OK”}$

Outside

$$h_{eo.ef} := min\left(\frac{h}{2}, 2.5 \cdot \left(cv_o + \frac{\phi_{vo}}{2}\right)\right) = 92.5\ mm$$

$\rho_{eff.to} := \frac{As_{vo}}{h_{eo.ef}} = 0.009$ $\quad k_1 := 0.8$ -Bonding coefficient

$$S_{ro.max} := 3.4 \cdot cv_o + \frac{0.425 \cdot k_1 \cdot \phi_{vo}}{\rho_{eff.to}} = 566.23\ mm$$

$W_{k.to} := \max\left(\varepsilon_{cr.thermal} \cdot S_{ro.max}, 0\ mm\right) = 0.094\ mm$ $\quad$ if $\left(W_{k.to} < \omega_{lim}, \text{“OK”}, \text{“Low”}\right) = \text{“OK”}$

6.0 Shear at Base of Tank and between Buttress Panels and Adjacent Standard Panel

6.1 Shear at Base of Tank

6.1.1 Prestress Only

$x := 0\ m$

$V_{x.A} := \frac{d}{dx} M_{x.A}(x)$ $\quad V_{x.A} = 0\ \frac{kN}{m}$ $\quad$ No base shear in case A (as base is free for this case)

$V_{x.B} := \frac{d}{dx} M_{x.B}(x)$ $\quad V_{x.B} = 16.162\ \frac{kN}{m}$ $\quad$ Base shear with prestress only $\quad \max(V_{rp}) = 23.1\ kN$

6.1.2 Prestress and Hydrostatic

$x := 0\ m$

$V_{x.C} := \frac{d}{dx} M_{x.C}(x)$ $\quad V_{x.C} = -29.2\ \frac{kN}{m}$

$V_{x.D} := \frac{d}{dx} M_{x.D}(x)$ $\quad V_{x.D} = -10.1\ \frac{kN}{m}$

$min(V_{ih}) = -31.7\ kN$

$min(V_{if}) = -32.7\ kN$

$min(V_{th}) = -28.5\ kN$

$min(V_{tf}) = -23.7\ kN$

$V_{x.H} := \frac{d}{dx} M_{x.H}(x)$ $\quad V_{x.H} = -29.2\ \frac{kN}{m}$

$V_{x.Ht} := \frac{d}{dx} M_{x.Ht}(x)$ $\quad V_{x.Ht} = -26.2\ \frac{kN}{m}$

Check pinned hydrostatic base shear

free displacement $\quad \delta_f := \frac{\gamma \cdot d \cdot r^2}{E_{cs} \cdot h}$ $\quad \delta_f = 0.906\ mm$

base shear to pin $\quad V_{c.H} := -2 \cdot \beta^3 \cdot D \cdot \delta_f$ $\quad V_{c.H} = -25.6\ \frac{kN}{m}$ $\quad$ of the same order as $V_{x.H}$

$\max(V_{ie}) = 54.5\ kN$ $\max(V_{te}) = 48\ kN$ $\quad min(V_{ife}) = -19.3\ kN$ $\quad min(V_{tfe}) = -20.2\ kN$ $\quad$ with earth pressures

6.1.3 Check Shear Capacity for Maximum Base Shear (Max. shear from all cases checked)

$$V_{max} := \gamma_f \cdot \max\left(\left|V_{x.A}\right|, \left|\frac{\max(V_{ip})}{m}\right|, \left|V_{x.B}\right|, \left|\frac{\max(V_{rp})}{m}\right|, \left|V_{x.C}\right|, \left|\frac{\max(V_{if})}{m}\right|, \left|V_{x.D}\right|, \left|\frac{\max(V_{tf})}{m}\right|\right)$$

$$V_{max} := \max\left(V_{max}, \gamma_f \cdot \max\left(\left|\frac{\max(V_{ie})}{m}\right|, \left|\frac{\max(V_{te})}{m}\right|, \left|\frac{\max(V_{ife})}{m}\right|, \left|\frac{\max(V_{tfe})}{m}\right|\right)\right) \quad V_{max} = 81.8\ \frac{kN}{m} \quad \frac{V_{max}}{\gamma_f}$$

$$d_{mh} := min\left(d_{vo} - \frac{\phi_{vo} + \phi_{ho}}{2}, d_{vi} - \frac{\phi_{vi} + \phi_{hi}}{2}\right) \qquad d_{mv} := min(d_{vo}, d_{vi})$$

Shear stress (maximum) EC2 $\quad v := \frac{V_{max}}{min(d_{mh}, d_{mv})}$ $\quad v = 0.705\ MPa$ $\quad$ maximum absolute base shear (factored for ULS)

$\frac{As_{vo}}{\gamma_{eo}} = 833.3\ \frac{mm^2}{m}$ $\quad \rho_v := \frac{As_{vo}}{\gamma_{eo} \cdot h}$ $\quad \rho_v = 0.45\%$ $\quad \gamma_{mv} := 1.25$

EC 2 $d_{mv}=0.125\ m$ $d_{mh}=0.116\ m$

Extra shear reinforcement (diameter and distance): $d_{shear}:=0\ mm$ $dist_{shear}:=100\ mm$

$$A_{shear}:=\frac{\pi\cdot(0.5\cdot d_{shear})^2}{dist_{shear}} \qquad A_{shear}=0\ \frac{mm^2}{m}$$

$$\rho_{1.v}:=min\left(min\left(\frac{As_{vo}+A_{shear}}{d_{vo}},\frac{As_{vi}+A_{shear}}{d_{vi}}\right),2\%\right)\rho_{1.v}=0.66\% \qquad \frac{As_{vo}}{d_{vo}}=0.66\% \qquad \frac{As_{vi}}{d_{vi}}=0.7\%$$

$$d_{ho}:=h-cv_o-\phi_{vo}-\frac{\phi_{ho}}{2} \qquad d_{ho}=116\ mm \qquad d_{hi}:=h-cv_i-\phi_{vi}-\frac{\phi_{hi}}{2} \qquad d_{hi}=116\ mm$$

$$\rho_{1.h}:=min\left(min\left(\frac{As_{ho}}{d_{ho}},\frac{As_{hi}}{d_{hi}}\right),2\%\right) \qquad \rho_{1.h}=0.22\% \qquad \frac{As_{ho}}{d_{ho}}=0.22\% \qquad \frac{As_{hi}}{d_{hi}}=0.22\%$$

$$F_{ck}=40\ MPa$$

$$\sigma_{cp.h}:=0\ MPa \qquad \sigma_{cp.v}:=\frac{0.8\cdot P_{vp}\cdot b}{A_{pn}}+0.8\cdot d\cdot\gamma_c\cdot g=88.26\ \frac{kN}{m^2} \qquad \sigma_{cp.v}=0.088\ MPa$$

$$k_{f.h}:=min\left(1+\sqrt{\frac{200\ mm}{d_{mh}}},2.0\right) \qquad k_{f.h}=2 \qquad k_{f.v}:=min\left(1+\sqrt{\frac{200\ mm}{d_{mv}}},2.0\right) \qquad k_{f.v}=2$$

$$C_{rd.c}:=\frac{0.18}{\gamma_{mc}} \qquad C_{rd.c}=0.12 \qquad \boxed{k_1}:=0.15$$

$$v_{min}:=0.035\cdot k^{\frac{3}{2}}\cdot\sqrt{\frac{F_{ck}}{MPa}}\cdot MPa \qquad v_{min}=0.078\ MPa$$ Eq 6.3N

$$V_{rd.c.h}:=\left(C_{rd.c}\cdot k_{f.h}\cdot\left(100\cdot\rho_{1.h}\cdot\frac{F_{ck}}{MPa}\right)^{\frac{1}{3}}\cdot MPa+k_1\cdot\sigma_{cp.h}\right)\cdot d_{mh} \qquad V_{rd.c.h}=57.185\ \frac{kN}{m}$$

$$v_{rd.c.h}:=\frac{V_{rd.c.h}}{d_{mh}} \qquad v_{rd.c.h}=0.493\ MPa$$

$$V_{rd.c.v}:=\left(C_{rd.c}\cdot k_{f.v}\cdot\left(100\cdot\rho_{1.v}\cdot\frac{F_{ck}}{MPa}\right)^{\frac{1}{3}}\cdot MPa+k_1\cdot\sigma_{cp.v}\right)\cdot d_{mv}$$

$$\boxed{V_{rd.c.v}}:=\max\left(V_{rd.c.v},\left(v_{min}+k_1\cdot\sigma_{cp.v}\right)\cdot d_{mv}\right) \qquad V_{rd.c.v}=91.139\ \frac{kN}{m}$$

$$v_{rd.c.v}:=\frac{V_{rd.c.v}}{d_{mv}} \qquad v_{rd.c.v}=0.729\ MPa$$

if $\left(v\le v_{rd.c.v},\text{“Shear satisfactory”},\text{“Shear reinf. req”}\right)=$ “Shear satisfactory”

6.1.4 Shear capacity - Bottom of panel - Seismic situation

Shear load (from seismic analysis): $V_{max.e} := 0\ \frac{kN}{m}$

Shear stress (maximum) $v_e := \frac{V_{max.e}}{d_{mv}}$ $v_e = 0\ MPa$

$$\sigma_{cp.v} := \frac{P_{vp} \cdot b}{A_{pn}} + d \cdot \gamma_c \cdot g = 110.325\ \frac{kN}{m^2} \qquad \sigma_{cp.v} = 0.11\ MPa$$

$$k_{f.v} := min\left(1 + \sqrt{\frac{200\ mm}{d_{mv}}}, 2.0\right) \qquad k_{f.v} = 2 \qquad C_{rd.c} := \frac{0.18}{1.0} \qquad C_{rd.c} = 0.18 \qquad k_1 := 0.15$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \sqrt{\frac{F_{ck}}{MPa}} \cdot MPa \qquad v_{min} = 0.078\ MPa \qquad \text{Eq 6.3N}$$

$$V_{rd.c.vs} := \left(C_{rd.c} \cdot k_{f.v} \cdot \left(100 \cdot \rho_{1.v} \cdot \frac{F_{ck}}{MPa}\right)^{\frac{1}{3}} \cdot MPa + k_1 \cdot \sigma_{cp.v}\right) \cdot d_{mv}$$

$$V_{rd.c.vs} := \max\left(V_{rd.c.vs}, \left(v_{min} + k_1 \cdot \sigma_{cp.v}\right) \cdot d_{mv}\right) \qquad V_{rd.c.vs} = 136.295\ \frac{kN}{m}$$

$$v_{rd.c.vs} := \frac{V_{rd.c.vs}}{d_{mv}} \qquad v_{rd.c.vs} = 1.09\ MPa$$

$$\textbf{if}\left(V_{max.e} \le 0 \wedge V_{max.e} \ge 0, \text{“N/A”}, \textbf{if}\left(v_e \le v_{rd.c.vs}, \text{“Shear satisfactory”}, \text{“Shear reinf. req.”}\right)\right) = \text{“N/A”}$$

6.2 Shear Between Buttress Panels and Adjacent Standard Panels

As the buttress panels are stiffer (thicker section) than the general tank panels they will deflect inward differently under the action of the prestress load (and they will be subject to lower stresses). The maximum differential inward movement can be calculated as the difference between the deflections (and hence hoop forces) for a tank of the buttress panel thickness and a tank of the general panel thickness. In practice the differential will be less than this as locally the differences in deflection will be averaged across several panels adjacent to the buttress. The peak transverse shear must be transferred in friction at the buttress panel joints. The highest differential shear is checked for cases A and B.

6.2.1 Differential Hoop Force for Case A

$$\delta_{o.i.inc.but} := \delta\left(h_b, b, s_s, a_s, E_{cs}, S_i, \alpha_{i.i}, \rho_t\right) \qquad \delta_{o.i.inc.but} = 0.104 \qquad \frac{\delta_{o.i.inc.but}}{\delta_{o.i.inc}} = 60.48\%$$

$$r_b := r + \frac{h_b - h}{2} \qquad r_b = 11248.8\ mm$$

$$\beta_{o.i.in.bt} := \sqrt[4]{\frac{\delta_{o.i.inc.but} \cdot 3 \cdot (1-\nu)^2}{r_b^{\,2} \cdot h_b^{\,2}}} \qquad \beta_{o.i.in.bt} = 0.347\ \frac{1}{m} \qquad \frac{\beta_{o.i.in.bt}}{\beta_{o.i.inc}} = 59.5\%$$

Case A Differential Hoop Force

$$N_{\theta.Adf}(xt) := \text{if } short \quad \| N_{\theta.A.s}(xt, d_h, t_o, t_b, t_t, \beta_{o.i.in.bt}) - N_{\theta.A.s}(xt, d_h, t_o, t_b, t_t, \beta_{o.i.inc}) \quad \text{else} \quad \| N_{\theta.A.l}(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.in.bt}) - N_{\theta.A.l}(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.inc})$$

$N_{max.Adf} := N_{\theta.Adf}(0\ m)$ $\qquad N_{max.Adf} = 25.01\ \frac{kN}{m}$

6.2.2 Differential Hoop Force for Case B

$\delta_{o.t.in.bt} := \delta(h_b, b, s_s, a_s, E_{cl}, S_i, \alpha_{t.i}, \rho_t)$ $\qquad \beta_{o.t.in.bt} := \sqrt[4]{\frac{\delta_{o.t.in.bt} \cdot 3 \cdot (1-\nu)^2}{r_b^{\ 2} \cdot h_b^{\ 2}}}$

$\delta_{o.t.de.bt} := \delta(h_b, b, s_s, a_s, E_{cl}, S_i, \alpha_{t.d}, \rho_t)$ $\qquad \beta_{o.t.de.bt} := \sqrt[4]{\frac{\delta_{o.t.de.bt} \cdot 3 \cdot (1-\nu)^2}{r_b^{\ 2} \cdot h_b^{\ 2}}}$

$\beta_{o.t.lon.bt} := \text{if}(inclong, \beta_{o.t.in.bt}, \beta_{o.t.de.bt})$

$$N_{\theta.B.df}(xt) := \text{if } short \quad \| -N_{\theta.B.s}(xt, d_h, t_o, t_b, t_t, \beta_{o.i.in.bt}, \beta_{o.t.in.bt}, 1, 1-\kappa) + N_{\theta.B.s}(xt, d_h, t_o, t_b, t_t, \beta_{o.i.inc}, \beta_{o.t.inc}, 1, 1-\kappa) \quad \text{else} \quad \| -N_{\theta.B.l}(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.in.bt}, \beta_{o.t.lon.bt}, \kappa, 1-\kappa) + N_{\theta.B.l}(xt, d_h, r, \gamma, \rho_p, f, \xi, \chi_p, \beta_{o.i.inc}, \beta_{o.t.lon}, \kappa, 1-\kappa)$$

Constraints / Guess Values / Solver

$x > 0\ m \quad x < d$

$x_{max.B.df} := \text{maximize}(N_{\theta.B.df}, x)$

Constraints / Guess Values / Solver

$x > 0\ m \quad x < d$

$x_{min.B.df} := \text{minimize}(N_{\theta.B.df}, x)$

$x_{max.B.df} = 1498.46\ mm$ $\qquad x_{min.B.df} = 0\ mm$

$N_{max.B.df} := \max(N_{\theta.B.df}(x_{max.B.df}), -N_{\theta.B.df}(x_{min.B.df}))$ $\qquad N_{max.B.df} = 74.128\ \frac{kN}{m}$

6.2.3 Maximum Shear Stress at Joints

Case A

Shear stress on EDPM $\qquad \sigma_{v.s} := \frac{N_{max.Adf}}{a_s}$ $\qquad \sigma_{v.s} = 0.313\ MPa$

Compressive stress on EDPM $\qquad \sigma_{a.s} := \frac{N_{\theta.A}(0\ m)}{a_s}$ $\qquad \sigma_{a.s} = 5.155\ MPa$

Assuming a conservative friction factor of 0.4 between sealing strip and concrete $\qquad 0.4 \cdot \sigma_{a.s} = 2.062\ MPa$ $\qquad$ This exceeds the shear stress in the EDPM joints adjacent to the buttress panel as conservatively calculated above

Case B

Shear stress on EDPM $\qquad \boxed{\sigma_{v.s}} := \frac{N_{max.B.df}}{a_s}$ $\qquad \sigma_{v.s} = 0.927\ MPa$

Compressive stress on EDPM $\qquad \boxed{\sigma_{a.s}} := \frac{N_{\theta.B}(x_{max.B.df})}{a_s}$ $\qquad \sigma_{a.s} = 4.021\ MPa$

Assuming a conservative friction factor of 0.4 between sealing strip and concrete $\qquad 0.4 \cdot \sigma_{a.s} = 1.609\ MPa$ $\qquad$ This exceeds the shear stress in the EDPM joints adjacent to the buttress panel as conservatively calculated above

7.0 NOTATION

a_{cr}	Distance from reinforcement to point of maximum crack width (inside & outside faces vertical)
a_s	Width of sealing strips
A_s	Reinforcement areas (inside & outside faces, horizontal & vertical)
A_t	Nominal steel area of prestressing strands
b	Tank regular wall panel width
b_b	tank buttress panel width
cv	Cover to main reinforcement
d	Depth of tank contents
D	$\dfrac{E \cdot h^3}{12 \cdot \left(1-\nu^2\right)}$
D_m	Minimum internal tank diameter
E_{cs}	Strain modulus of concrete - short term
E_{cl}	Strain modulus of concrete - long term
E_p	Strain modulus of prestressing strand
E_s	Strain modulus of reinforcement
F_{cu}	Characteristic cube strength for concrete
f_{ck}	Cylinder strength of concrete concrete
f_{cm}	Mean compressive strength of concrete at 28 days:
f_{yf}	Characteristic yield strength of fabric reinforcement
f_{yr}	Characteristic yield strength of round mild steel bars.
f_{yp}	Nominal tensile strength of prestressing strand
g	Acceleration due to gravity
h	Average tank wall thickness
h_{min}	Minimum thickness of tank wall panel
h_{max}	Maximum thickness of tank wall panel
K	Linear loss profile coefficient for prestressing strand.

7.0 NOTATION (Continued)

L_{as}	Prestress loss due to anchorage slip
L_{cr}	Prestress loss due to creep
L_{es}	Prestress loss due to elastic shortening in the concrete and sealing strips
L_{sh}	Prestress loss due to concrete shrinkage
M_x	Vertical bending moment per unit width
n_p	Number of standard panels
n_b	Number of buttress panels
N_θ	Hoop force per unit height
p_{hi}	Pitch of inside face horizontal reinforcement mesh
p_{ho}	Pitch of outside face horizontal reinforcement mesh
p_{vi}	Pitch of inside face vertical reinforcement mesh
p_{vo}	Pitch of outside face vertical reinforcement mesh
r	Effective radius of concrete tank (on wall centreline at joints)
RH	Relative humidity of ambient environment
s_s	Thickness of sealing strips
S_i	Strain modulus of sealing strips for increasing compressive load on strip
S_d	Strain modulus of sealing strips for decreasing compressive load on strip
t_0	Assumed time of loading (after casting) by prestressing.
t_n	Number of tendons in height of wall
t_x	Levels of tendons above base of the tank (a vector with tn number elements)
x_Δ	Distance from buttress to midway between buttresses
α_i	Modular ratio between tendons and concrete, initial
α_t	Modular ratio between tendons and concrete, long term
α_c	Coefficient of thermal expansion for concrete
α_s	Coefficient of thermal expansion for steel

7.0 NOTATION (Continued)

β	$\sqrt[4]{\frac{3\cdot(1-\nu^2)}{r^2\cdot h^2}}$
$\beta_{o.i.dec}$	Orthotropic value for initial conditions with decreasing load on sealing strips
$\beta_{o.i.inc}$	Orthotropic value for initial conditions with increasing load on sealing strips
$\beta_{o.t.dec}$	Orthotropic value for relaxed conditions after time with decreasing load on sealing strips
$\beta_{o.t.inc}$	Orthotropic value for relaxed conditions after time with increasing load on sealing strips
χ	Ageing coefficient
Δx	Anchorage slip for prestressing
ε	Strain in concrete, sealing strip or reinforcement (according to suffix)
ϕ_h	Assumed creep factor appropriate to cyclic hydrostatic loading
ϕ_{hi}	Diameter of inside face horizontal reinforcement mesh
ϕ_{ho}	Diameter of outside face horizontal reinforcement mesh
ϕ_{lf}	Percentage prestress loss due to linear friction mesh
ϕ_{sc}	Prestress loss due to creep of low relaxation strand
ϕ_t	Assumed creep factor appropriate to prestressing loads long term
ϕ_{tc}	Percentage prestress loss due to tendon curvature
ϕ_{vi}	Diameter of inside face vertical reinforcement mesh
ϕ_{vo}	Diameter of outside face vertical reinforcement mesh
η	$\sqrt{12\cdot(1-\nu^2)}$
γ	Specific gravity (density) of contained liquid mesh
γ_c	Density of concrete
γ_f	Partial safety factor for load in ULS calculations
γ_s	Density of steel
γ_{mc}	Material partial safety factor for concrete
γ_{mr}	Material partial safety factor for reinforcement
κ	Relaxation ratio
μ	Coefficient of friction along tendon
v	Poissons ratio for concrete
v_s	Poissons ratio for steel
ω	crack width (suffix denotes face, orientation and loadcase)

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SIGNING PAGE

Static calculations

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9.0 DAMPED TRIGONOMETRIC FUNCTIONS (From References 1. & 2.)

$\zeta(\beta,x) \equiv e^{-\beta \cdot x} \cdot \sin(\beta \cdot x)$ $\qquad$ $\theta(\beta,x) \equiv e^{-\beta \cdot x} \cdot \cos(\beta \cdot x)$

$\psi(\beta,x) \equiv \theta(\beta,x) - \zeta(\beta,x)$ $\qquad$ $\phi(\beta,x) \equiv \theta(\beta,x) + \zeta(\beta,x)$

10.0 TRIGONOMETRIC FUNCTIONS FOR SHORT ORTHOTROPIC SHELLS (Ref. 5.)

10.1 Basis functions

The following formula are used to calculate the non-orthotropic forces, bending moments and deflections and are presented using the same notation as reference 5.

$C(\beta,x) \equiv \cosh(\beta \cdot x)$ $\qquad$ $cc(\beta,x) \equiv \cos(\beta \cdot x)$

$S(\beta,x) \equiv \sinh(\beta \cdot x)$ $\qquad$ $ss(\beta,x) \equiv \sin(\beta \cdot x)$ (ss used to avoid confusion with s for seconds)

$F_1(\beta,x) \equiv S(\beta,x)^2 - ss(\beta,x)^2$ $\qquad$ $F_2(\beta,x) \equiv S(\beta,x)^2 + ss(\beta,x)^2$

$F_3(\beta,x) \equiv C(\beta,x) \cdot S(\beta,x) + cc(\beta,x) \cdot ss(\beta,x)$ $\qquad$ $F_4(\beta,x) \equiv C(\beta,x) \cdot S(\beta,x) - cc(\beta,x) \cdot ss(\beta,x)$

$F_5(\beta,x) \equiv ss(\beta,x)^2$ $\qquad$ $F_6(\beta,x) \equiv S(\beta,x)^2$

$F_7(\beta,x) \equiv C(\beta,x) \cdot cc(\beta,x)$ $\qquad$ $F_8(\beta,x) \equiv S(\beta,x) \cdot ss(\beta,x)$

$F_9(\beta,x) \equiv C(\beta,x) \cdot ss(\beta,x) - S(\beta,x) \cdot cc(\beta,x)$ $\qquad$ $F_{10}(\beta,x) \equiv C(\beta,x) \cdot ss(\beta,x) + S(\beta,x) \cdot cc(\beta,x)$

$$F_{78}(\beta,x,d) \equiv 1 - F_3\left(\beta,\frac{d}{2}\right)^{-1} \cdot \left(F_{10}\left(\beta,\frac{d}{2}\right) \cdot F_7(\beta,x) + F_9\left(\beta,\frac{d}{2}\right) \cdot F_8(\beta,x)\right)$$

$$F_{89}(\beta,x,d) \equiv F_4(\beta,d)^{-1} \cdot \left(F_9(\beta,d) \cdot F_8(\beta,x) - F_8(\beta,d) \cdot F_9(\beta,x)\right)$$

$$F_{BA}(\beta,d) \equiv \left(2 - 3 \cdot \left(\beta \cdot \frac{d}{2}\right)^{-2} \cdot F_4\left(\beta,\frac{d}{2}\right) \cdot F_3\left(\beta,\frac{d}{2}\right)^{-1}\right)$$

$$F_{87}(\beta,x,d) \equiv 1 - 3 \cdot x^2 \cdot \left(\frac{d}{2}\right)^{-2} + 3 \cdot \left(\beta \cdot \frac{d}{2}\right)^{-2} \cdot F_3\left(\beta,\frac{d}{2}\right)^{-1} \cdot \left(F_{10}\left(\beta,\frac{d}{2}\right) \cdot F_8(\beta,x) - F_9\left(\beta,\frac{d}{2}\right) \cdot F_7(\beta,x)\right)$$

$$F_{107}(\beta,x,d) \equiv F_4(\beta,d)^{-1} \cdot \left(F_8(\beta,d) \cdot F_{10}(\beta,x) - F_9(\beta,d) \cdot F_7(\beta,x)\right)$$

Functions to carry out a shift of x value from measurement from bottom of tank to measurement from either the middle or the top of the tank

$x_m(x,d) \equiv \left(x - \frac{d}{2}\right)$ (x measured from middle)

$x_t(x,d) \equiv (d - x)$ (x measured from top)

THE FORMULA PRESENTED ON THIS PAGE ARE CONFIDENTIAL AND ARE NOT TO BE ISSUED WITHOUT THE PRIOR CONSENT OF A-CONSULT

10.2 Hoop Force and Bending Moment Functions (ref. 5 & ref. 11)

Hydrostatic case - Short Shell (pinned)

$$N_{\theta.H.s}\left(x_v, d_v, r_v, \gamma_v, \beta_v\right) \equiv \gamma_v \cdot r_v \cdot d_v \cdot \left(\frac{x_t\left(x_v, d_v\right)}{d_v} - F_{107}\left(\beta_v, x_t\left(x_v, d_v\right), d_v\right)\right)$$

$$M_{x.H.s}\left(x_v, d_v, \gamma_v, \beta_v\right) \equiv \frac{-\gamma_v \cdot d_v}{2 \cdot {\beta_v}^2} \cdot F_{89}\left(\beta_v, x_t\left(x_v, d_v\right), d_v\right)$$

Hydrostatic case - Long Shell (pinned)

$$N_{\theta.H.l}\left(x_v, d_v, r_v, \gamma_v, \beta_v\right) \equiv \gamma_v \cdot r_v \cdot d_v \cdot \left(1 - \frac{x_v}{d_v} - \theta\left(\beta_v, x_v\right)\right)$$

$$M_{x.H.l}\left(x_v, d_v, \gamma_v, \beta_v\right) \equiv \frac{-\gamma_v \cdot d_v}{2 \cdot {\beta_v}^2} \cdot \zeta\left(\beta_v, x_v\right)$$

Initial Prestress case - Short Shell (free)

$$N_{\theta.A.s}\left(x_v, d_v, t_{ov}, t_{bv}, t_{tv}, \beta_v\right) \equiv \left(\frac{x_t\left(x_v, d_v\right) \cdot t_{bv} + \left(d_v - x_t\left(x_v, d_v\right)\right) \cdot t_{tv}}{d_v} + t_{ov} \cdot F_{87}\left(\beta_v, x_m\left(x_v, d_v\right), d_v\right)\right)$$

$$M_{x.A.s}\left(x_v, d_v, r_v, t_{ov}, \beta_v\right) \equiv \frac{6 \cdot t_{ov}}{{\beta_v}^4 \cdot {d_v}^2 \cdot r_v} \cdot F_{78}\left(\beta_v, x_m\left(x_v, d_v\right), d_v\right)$$

Initial Prestress case - Long Shell (free)

$$N_{\theta.A.l}\left(x_v, d_v, r_v, \gamma_v, \rho_v, f, \xi, \chi_p, \beta_v\right) \equiv \rho_p \cdot \gamma_v \cdot d_v \cdot r_v \cdot \left(1 - \frac{x_v}{f}\right) \hookleftarrow$$
$$+ -\rho_p \cdot \gamma_v \cdot d_v \cdot r_v \cdot \frac{\xi \cdot {\beta_v}^2}{{\beta_v}^4 - {\chi_p}^4} \cdot \left({\beta_v}^2 \cdot \psi\left(\chi_p, x_v\right) - {\chi_p}^2 \cdot \psi\left(\beta_v, x_v\right)\right)$$

$$M_{x.A.l}\left(x_v, d_v, \gamma_v, \rho_p, \xi, \chi_p, \beta_v\right) \equiv \frac{\gamma_v \cdot d_v}{2 \cdot {\beta_v}^2} \cdot \left(\rho_p \cdot \xi \cdot \frac{{\chi_p}^2 \cdot {\beta_v}^2}{{\beta_v}^4 - {\chi_p}^4}\right) \cdot \left(\phi\left(\chi_p, x_v\right) - \phi\left(\beta_v, x_v\right)\right)$$

Relaxed Prestress case - Short Shell (pinned after stressing, before relaxation)

$$N_{\theta.p.s}\left(x_v, d_v, t_{ov}, t_{bv}, \beta_v\right) \equiv -\left(\left(t_{bv} - t_{ov} \cdot F_{BA}\left(\beta_v, d_v\right)\right) \cdot F_{107}\left(\beta_v, x_t\left(x_v, d_v\right), d_v\right)\right)$$

$$N_{\theta.B.s}\left(x_v, d_v, t_{ov}, t_{bv}, t_{tv}, \beta_i, \beta_r, \kappa_i, \kappa_r\right) \equiv \kappa_i \cdot N_{\theta.A.s}\left(x_v, d_v, t_{ov}, t_{bv}, t_{tv}, \beta_i\right) + \kappa_r \cdot N_{\theta.p.s}\left(x_v, d_v, t_{ov}, t_{bv}, \beta_r\right)$$

$$M_{x.p.s}\left(x_v, d_v, r_v, t_{ov}, t_{bv}, \beta_v\right) \equiv \left(2 \cdot {\beta_v}^2 \cdot r_v\right)^{-1} \cdot \left(t_{bv} - t_{ov} \cdot F_{BA}\left(\beta_v, d_v\right)\right) \cdot F_{89}\left(\beta_v, x_t\left(x_v, d_v\right), d_v\right)$$

$$M_{x.B.s}\left(x_v, d_v, r_v, t_{ov}, t_{bv}, \beta_i, \beta_r, \kappa_i, \kappa_r\right) \equiv \kappa_i \cdot M_{x.A.s}\left(x_v, d_v, r_v, t_{ov}, \beta_i\right) + \kappa_r \cdot M_{x.p.s}\left(x_v, d_v, r_v, t_{ov}, t_{bv}, \beta_r\right)$$

Relaxed Prestress case - Long Shell (pinned after stressing, before relaxation)

$$N_{\theta.p.l}\left(x_v, d_v, r_v, \gamma_v, \rho_p, f, \xi, \chi_p, \beta_v\right) \equiv \rho_p \cdot \gamma_v \cdot d_v \cdot r_v \cdot \left(1 - \frac{x_v}{f} - \theta\left(\beta_v, x_v\right)\right) + \rho_p \cdot \gamma_v \cdot d_v \cdot r_v \cdot \frac{\xi \cdot {\beta_v}^2}{{\beta_v}^4 - {\chi_p}^4} \hookleftarrow$$
$$\cdot \left({\beta_v}^2 \cdot \left(\theta\left(\beta_v, x_v\right) - \psi\left(\chi_p, x_v\right)\right) + -{\chi_p}^2 \cdot \zeta\left(\beta_v, x_v\right)\right)$$

$$N_{\theta.B.l}\left(x_v, d_v, r_v, \gamma_v, \rho_p, f, \xi, \chi_p, \beta_i, \beta_r, \kappa_i, \kappa_r\right) \equiv \kappa_i \cdot N_{\theta.A.l}\left(x_v, d_v, r_v, \gamma_v, \rho_p, f, \xi, \chi_p, \beta_i\right) + \kappa_r \cdot N_{\theta.p.l}\left(x_v, d_v, r_v, \gamma_v, \rho_p, f, \xi, \chi_p, \beta_r\right)$$

$$M_{x.p.1}\left(x_v, d_v, \gamma_v, \rho_p, \xi, \chi_p, \beta_v\right) \equiv \frac{\gamma_v \cdot d_v}{2 \cdot {\beta_v}^2} \cdot \rho_p \cdot \left(\zeta\left(\beta_v, x_v\right) + \frac{-\xi \cdot {\beta_v}^2}{{\beta_v}^4 - {\chi_p}^4} \cdot \left({\beta_v}^2 \cdot \zeta\left(\beta_v, x_v\right) + -{\chi_p}^2 \cdot \left(\phi\left(\chi_p, x_v\right) - \theta\left(\beta_v, x_v\right)\right)\right)\right)$$

$$M_{x.B.l}\left(x_v, d_v, \gamma_v, \rho_p, \xi, \chi_p, \beta_i, \beta_r, \kappa_i, \kappa_r\right) \equiv \kappa_i \cdot M_{x.A.l}\left(x_v, d_v, \gamma_v, \rho_p, \xi, \chi_p, \beta_i\right) + \kappa_r \cdot M_{x.p.1}\left(x_v, d_v, \gamma_v, \rho_p, \xi, \chi_p, \beta_r\right)$$

Check values to reference 11

$$M_{x.A.l}\left((1-0.702) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, 1, 0.6, \frac{0.6}{m}, \frac{0.519}{m}\right) = 16.47\ \frac{kN \cdot m}{m}$$ 16.47 to ref. 11

$$M_{x.B.l}\left((1-0.796) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, 1, 0.6, \frac{0.6}{m}, \frac{0.519}{m}, \frac{0.735}{m}, 0.25, 1-0.25\right) = 20.23\ \frac{kN \cdot m}{m}$$

$$M_{x.A.l}\left((1-0.885) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, 1, 0.6, \frac{0.6}{m}, \frac{0.519}{m}\right) + M_{x.H.l}\left((1-0.885) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, \frac{0.634}{m}\right) = -26.35\ \frac{kN \cdot m}{m}$$ -26.4 to ref. 11

$$M_{x.B.l}\left((1-0.892) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, 1, 0.6, \frac{0.6}{m}, \frac{0.519}{m}, \frac{0.735}{m}, 0.25, 1-0.25\right) + M_{x.H.l}\left((1-0.892) \cdot 8\ m, 8\ m, 11\ \frac{kN}{m^3}, \frac{0.634}{m}\right) = -15.87\ \frac{kN \cdot m}{m}$$

11.0 FUNCTIONS USED IN THE FINITE DIFFERENCE METHOD CALCULATIONS

11.1 Functions to be used in allocating Loads and Stiffnesses to nearest Nodes

$$near(n, x, L_b) \equiv \text{round}\left(\frac{x}{L_b} \cdot (n), 0\right) \qquad lo(x, i) \equiv \text{floor}\left(\frac{x}{i}\right) \qquad hi(x, i) \equiv \text{ceil}\left(\frac{x}{i}\right)$$

11.2 Functions to Translate Loads and Stiffnesses to the Grid and Set up Matrix

Function to translate the inertia distribution from dimensions from left hand end to nodes on the grid :

$$F_i(n, I_b, \lambda_g, \lambda_h, I_n) \equiv \left\| \begin{array}{l} \Phi_n \leftarrow 0\ m^4 \\ \text{for } \gamma \in 0..\text{rows}(I_b) \\ \quad \text{if } I_{b_\gamma} \le 0 \\ \quad\quad \text{break} \\ \quad \text{else} \\ \quad\quad \beta_l \leftarrow lo(\lambda_g, I_n)_\gamma \\ \quad\quad \beta_h \leftarrow hi(\lambda_h, I_n)_\gamma \\ \quad\quad \text{for } \beta \in \beta_l..\beta_h \\ \quad\quad\quad \Phi_\beta \leftarrow I_{b_\gamma} \\ \Phi \end{array} \right.$$

Function to translate the point loads from dimensions from left hand end to nodes on the grid :

$$F_p(n, P_b, \lambda_a, L_b) \equiv \left\| \begin{array}{l} \Phi_n \leftarrow 0\ N \\ \text{for } \gamma \in 0..\text{rows}(P_b) - 1 \\ \quad \beta \leftarrow near(n, \lambda_a, L_b)_\gamma \\ \quad \Phi_\beta \leftarrow \Phi_\beta + P_{b_\gamma} \\ \Phi \end{array} \right.$$

Function to translate the point supports from dimensions from left hand end to nodes on the grid :

$$F_{su}(n, S_u, \lambda_d, L_b) \equiv \left\| \begin{array}{l} \Phi_n \leftarrow 0\ \frac{N}{m} \\ \text{for } \gamma \in 0..\text{rows}(S_u) - 1 \\ \quad \beta \leftarrow near(n, \lambda_d, L_b)_\gamma \\ \quad \Phi_\beta \leftarrow \Phi_\beta + S_{u_\gamma} \\ \Phi \end{array} \right.$$

Function to translate the uniformly distributed loads from dimensions from left hand end to nodes on the grid :

$$F_u(n, U_b, \lambda_b, \lambda_c, \varepsilon) \equiv \left\| \begin{array}{l} \Phi_n \leftarrow 0\ N \\ \text{for } \gamma \in 0..\text{rows}(U_b) - 1 \\ \quad \phi \leftarrow \frac{U_{b_\gamma} \cdot \varepsilon}{2} \\ \quad \beta_l \leftarrow lo(\lambda_b, \varepsilon)_\gamma \\ \quad \beta_h \leftarrow \max\left(hi(\lambda_c, \varepsilon)_\gamma, \beta_l + 2\right) \\ \quad \text{for } \beta \in \beta_l..\beta_h - 1 \\ \quad\quad \Phi_\beta \leftarrow \Phi_\beta + \phi \\ \quad\quad \Phi_{\beta+1} \leftarrow \Phi_{\beta+1} + \phi \\ \Phi \end{array} \right.$$

11.2 Functions to Translate Loads and Stiffnesses to the Grid and Set up Matrix (Continued) :

Function to translate the spring supports from dimensions from left hand end to nodes on the grid :

$$F_{sp}\left(n, S_p, \lambda_e, \lambda_f, \varepsilon\right) \equiv \left\| \begin{array}{l} \Phi_n \leftarrow 0\ \frac{N}{m} \\ \text{for } \gamma \in 0 .. \text{rows}\left(S_p\right)-1 \\ \quad \left\| \begin{array}{l} \phi \leftarrow \frac{1}{2} \cdot S_{p_\gamma} \cdot \varepsilon \\ \beta_l \leftarrow lo\left(\lambda_e, \varepsilon\right)_\gamma \\ \beta_h \leftarrow \max\left(hi\left(\lambda_f, \varepsilon\right)_\gamma, \beta_l+2\right) \\ \text{for } \beta \in \beta_l .. \beta_h-1 \\ \quad \left\| \begin{array}{l} \Phi_\beta \leftarrow \Phi_\beta + \phi \\ \Phi_{\beta+1} \leftarrow \Phi_{\beta+1} + \phi \end{array} \right. \end{array} \right. \\ \Phi \end{array} \right.$$

Function to compile terms of the stiffness matrix

$$F_{sm}\left(n, In, \kappa, L_f, R_f\right) \equiv \left\| \begin{array}{l} \alpha \leftarrow 0 \\ \Phi_{\alpha,\alpha} \leftarrow \frac{1}{2} \cdot In_{\alpha+1} + L_f \cdot In_1 \\ \Phi_{\alpha,\alpha+1} \leftarrow -2 \cdot In_{\alpha+1} - 2 \cdot L_f \cdot In_1 \\ \Phi_{\alpha,\alpha+2} \leftarrow In_{\alpha+1} \\ \alpha \leftarrow 1 \\ \Phi_{\alpha,\alpha} \leftarrow 2 \cdot In_\alpha + \frac{1}{2} \cdot In_{\alpha+1} + L_f \cdot In_1 \\ \Phi_{\alpha,\alpha+1} \leftarrow -2 \cdot \left(In_\alpha + In_{\alpha+1}\right) \\ \Phi_{\alpha,\alpha+2} \leftarrow In_{\alpha+1} \\ \text{for } \alpha \in 2 .. n-2 \\ \quad \left\| \begin{array}{l} \Phi_{\alpha,\alpha} \leftarrow 2 \cdot In_\alpha + \frac{1}{2} \cdot \left(In_{\alpha+1} + In_{\alpha-1}\right) \\ \Phi_{\alpha,\alpha+1} \leftarrow -2 \cdot \left(In_\alpha + In_{\alpha+1}\right) \\ \Phi_{\alpha,\alpha+2} \leftarrow In_{\alpha+1} \end{array} \right. \\ \alpha \leftarrow n-1 \\ \Phi_{\alpha,\alpha} \leftarrow 2 \cdot In_\alpha + \frac{1}{2} \cdot In_{\alpha-2} + R_f \cdot In_{\alpha+1} \\ \Phi_{\alpha,\alpha+1} \leftarrow -2 \cdot In_\alpha - 2 \cdot R_f \cdot In_{\alpha+1} \\ \alpha \leftarrow n \\ \Phi_{\alpha,\alpha} \leftarrow \frac{1}{2} \cdot In_{\alpha-1} - R_f \cdot In_\alpha \\ \kappa \cdot \left(\Phi + \Phi^{\mathrm{T}}\right) \end{array} \right.$$

11.3 Functions to Calculate Bending Moments and Shear Forces

Function to calculate end bending moments

$$M_e\left(L,\delta_a,\delta_b,E,I,\varepsilon\right)\equiv\frac{L\cdot\left(\delta_a-\delta_b\right)\cdot 2\cdot E\cdot I}{\varepsilon^2}$$

Function to calculate bending moments from deflections

$$M_f\left(\delta,E,I,n,\varepsilon,L_f,R_f\right)\equiv\left\|\begin{array}{l}\Phi_0\leftarrow M_e\left(L_f,\delta_0,\delta_1,E,I_0,\varepsilon\right)\\ \Phi_n\leftarrow M_e\left(R_f,\delta_n,\delta_{n-1},E,I_n,\varepsilon\right)\\ \text{for }\gamma\in 1..n-1\\ \quad\left\|\ \Phi_\gamma\leftarrow\dfrac{-E\cdot I_\gamma}{\varepsilon^2}\cdot\left(\delta_{\gamma-1}-2\cdot\delta_\gamma+\delta_{\gamma+1}\right)\right.\\ \Phi\end{array}\right.$$

Function to calculate shear forces from bending moments

$$V_f\left(M,V,n,\varepsilon\right)\equiv\left\|\begin{array}{l}\Phi_0\leftarrow V\\ \text{for }\gamma\in 1..n\\ \quad\left\|\ \Phi_\gamma\leftarrow\dfrac{M_\gamma-M_{\gamma-1}}{\varepsilon}\right.\\ \Phi\end{array}\right.$$

12.0 Crack Width Formulae to BS8007 (Reinforced Section)

Formula for design surface crack width

$$\omega\left(a_{cr},\varepsilon_m,c_{min},h,x\right)\equiv\frac{3\cdot a_{cr}\cdot\varepsilon_m}{1+\dfrac{2\cdot\left(a_{cr}-c_{min}\right)}{h-x}}$$

Formula for strain in the reinforcement

$$\varepsilon_r\left(M,d,x,A_s,E_s\right)\equiv\frac{M}{A_s\cdot E_s\cdot\left(d-\dfrac{x}{3}\right)}$$

Formula for strain due to stiffening effect of the concrete between the cracks

$$\varepsilon_2\left(h,x,a',d,A_s,E_s,w_{lim}\right)\equiv\left(1+0.5\cdot\left(w_{lim}=0.1\ mm\right)\right)\cdot\frac{(h-x)\cdot(a'-x)}{3\cdot\left(\dfrac{E_s}{10^6\cdot Pa}\right)\cdot A_s\cdot(d-x)}$$

Formula for depth to neutral axis with a triangular stress block in the concrete (reference 9.)

$$x_{na}\left(\alpha_e,\rho,d\right)\equiv\alpha_e\cdot\rho\cdot d\cdot\left(\sqrt{1+\frac{2}{\alpha_e\cdot\rho}}-1\right)$$

General function for crack width

$$\omega_f\left(M,A_s,h,d,E_c,E_s,a_{cr},cv,\omega_a\right)\equiv\left\|\begin{array}{l}x\leftarrow x_{na}\left(\dfrac{E_s}{E_c},\dfrac{A_s}{d},d\right)\\ \varepsilon_{re}\leftarrow\varepsilon_r\left(M,d,x,A_s,E_s\right)\\ \varepsilon_1\leftarrow\dfrac{h-x}{d-x}\cdot\varepsilon_{re}\\ \varepsilon_2\leftarrow\varepsilon_2\left(h,x,h,d,A_s,E_s,\omega_a\right)\\ \omega\left(a_{cr},\varepsilon_1-\varepsilon_2,cv,h,x\right)\end{array}\right.$$

example of function :

$$\omega_f\left(15\cdot\frac{kN\cdot m}{m},500\ \frac{mm^2}{m},150\ mm,110\ mm,28\ \frac{kN}{mm^2},200\ \frac{kN}{mm^2},60\ mm,35\ mm,0.1\ mm\right)=0.16\ mm$$

Crack width formula according to EC2

$$A_{c.eff}(h, d, x) \equiv min\left(2.5 \cdot (h-d), \frac{h-x}{3}, \frac{h}{2}\right)$$

EC 2 Fig 7.1
& Ref EC2 (3)

$$\rho_{p.eff}(A_s, \phi_s, A_p, \phi_p, A_{c.eff}) \equiv \left\| \begin{array}{l} \xi \leftarrow 0.5 \\ \xi_1 \leftarrow \sqrt{\xi \cdot \dfrac{\phi_s}{\phi_p}} \\ \dfrac{A_s + \xi_1{}^2 \cdot A_p}{A_{c.eff}} \end{array} \right.$$

EC2 EQ 7.10

$$s_{r.max}(c, \phi, \rho_{p.eff}) \equiv \left\| \begin{array}{l} k_1 \leftarrow 0.8 \\ k_2 \leftarrow 0.5 \\ k_3 \leftarrow 3.4 \\ k_4 \leftarrow 0.425 \\ k_3 \cdot c + \dfrac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p.eff}} \end{array} \right.$$

EC2 EQ 7.11

$$\varepsilon_{sm.cm}(\sigma_s, f_{ct.eff}, \rho_{p.eff}, E_s, E_{cm}) \equiv \left\| \begin{array}{l} \alpha_e \leftarrow \dfrac{E_s}{E_{cm}} \\ k_t \leftarrow 0.4 \\ \max\left(\dfrac{\sigma_s - k_t \cdot \dfrac{f_{ct.eff}}{\rho_{p.eff}} \cdot (1 + \alpha_e \cdot \rho_{p.eff})}{E_s}, 0.6 \cdot \dfrac{\sigma_s}{E_s}\right) \end{array} \right.$$

EC2 EQ 7.9

13.0 CREEP FACTOR FUNCTIONS (Reference 8.)

Reference 7 derives a modification factor to take account of the creep-relaxation relationship of the concrete based on the CEB-FIP model code. For these calculations the following stress redistribution factor is calculated to the CEB-FIP model code 1990 / EC2 (Reference 8) :

Calculation of creep coefficient to EC2 (which now embodies the CEB-FIP method)

$t_1 \equiv 1 \cdot day$

100% relative humidity $\qquad RH_o \equiv 100\%$

Cylinder Strength of concrete 8.) $\quad f_{ck}\left(F_{ck.cube}\right) \equiv 0.8 \cdot F_{ck.cube} \qquad f_{ck}\left(F_{ck.cube}\right) = 40\ MPa$

Mean compressive strength at 28 days $\quad f_{cm1}\left(F_{ck.cube}\right) \equiv \left(f_{ck}\left(F_{ck.cube}\right) + 8 \cdot N \cdot mm^{-2}\right) \qquad f_{cm1}\left(F_{ck.cube}\right) = 48\ MPa$

$$\Phi_{rh}(RH, h) \equiv \left(1 + \frac{1 - RH \cdot RH_o^{-1}}{0.46 \cdot \left(h \cdot h_o^{-1}\right)}\right)$$

$$\beta_{fcm1}\left(F_{ck.cube}\right) \equiv 16.8 \cdot \left(f_{cm1}\left(F_{ck.cube}\right) \cdot \left(N \cdot mm^{-2}\right)^{-1}\right)^{-0.5} \qquad \beta_{fcm1}\left(F_{ck.cube}\right) = 2.4$$

$$\beta_{H1}(RH, h) \equiv min\left(150 \cdot \frac{h}{h_o} \cdot 1 + \left(1.2 \cdot \frac{RH}{RH_o}\right)^{18} + 250, 1500\right) \qquad \beta_{H1}(RH, h) = 401.4$$

$$\beta_t\left(t_0\right) \equiv \frac{1}{0.1 + \left(\frac{t_0}{t_1}\right)^{0.2}} \qquad \beta_t\left(t_0\right) = 0.488$$

$$\beta_c(\Delta T, RH, h) \equiv \frac{\Delta T \cdot t_1^{-1}}{\beta_{H1}(RH, h) + \Delta T \cdot t_1^{-1}}$$

$$\Phi_o\left(t, t_0, RH, h, F_{ck.cube}\right) \equiv \Phi_{rh}(RH, h) \cdot \beta_{fcm1}\left(F_{ck.cube}\right) \cdot \beta_t\left(t_0\right) \cdot \beta_c\left(t - t_0, RH, h\right)$$

Example of function, 70 years of loading, load applied at 28 days, 50% relative humidity for indoor atmospheric conditions, 150mm thick section in C30 concrete. Similar to equivalent value from table 2.1.10 of CEB FIP Model Code 90.

$$\Phi_o(70 \cdot yr, 28 \cdot day, 50\%, 150\ mm, 30\ MPa) = 3.347$$

14.0 DERIVATION OF THE ORTHOTROPIC SPRING STIFFNESS RATIO

A cylindrical tank is analogous to a beam on elastic foundation. Vertical bending of the tank wall is represented by beam action, and radial hoop stiffness can be modelled as the spring stiffness supporting the beam along it's length.
The base condition is modelled by treating one end of the beam as free, a spring support, pinned or rotationally restrained.

In the case of a precast tank with sealing strips between the joints the behaviour is orthotropic (i.e. the tank is not equally stiff in the radial and vertical directions). This effect is modelled by changing the spring stiffness representing the radial hoop stiffness. The modification factor depends on the geometry and stiffness of the concrete, sealing strips and prestressing strand. The derivation of this modification factor is set out below:

P - radial pressure force kN/m /m height of wall
T - hoop force kN /m height of wall
δc - circumferential deflection mm
δr - radial deflection mm

The radial spring stiffness (in kN/mm) is given by

From $2 \cdot T = 2 \cdot r \cdot P$ on the tank diameter

$$k_r = \frac{P}{\delta_r} \quad \text{(eq 1)}$$

$$T = P \cdot r \quad \text{(eq 2)}$$

Relationship between radial and circumferential deflections

$$\left(r + \delta_r\right) \cdot 2 \cdot \pi = \left(2 \cdot \pi \cdot r\right) + \delta_c$$

gives

$$\delta_r = \frac{\delta_r}{2 \cdot \pi} \quad \text{(eq 3)}$$

From the above equations substituting 2 & 3 in equation 1

$$k_r = \frac{2 \cdot \pi \cdot T}{r \cdot \delta_c}$$

and denoting circumferential spring stiffness as k_c

$$k_r = \frac{2 \cdot \pi}{r} \cdot k_c \quad \text{(eq 4)} \qquad \text{as} \qquad k_c = \frac{T}{\delta_c}$$

Consider now the radial deflections resulting from T (where T is force per metre height of wall)
If the whole tank were concrete the radial deflection would be given by

$$\delta_{c.c} = \frac{T \cdot \left(2 \cdot \pi \cdot r\right)}{h \cdot E_c} \quad \text{(eq 5)}$$

where h is the concrete tank wall thickness and Ec is the strain modulus of concrete

from

$$\delta = \frac{P \cdot L}{A \cdot E}$$

and the corresponding circumferential spring stiffness

$$k_{c.c} = \frac{T}{\delta_{c.c}} = \frac{h \cdot E_c}{2 \cdot \pi \cdot r} \quad \text{(eq 6)}$$

and radial spring stiffness from eq 4

$$k_{r.c} = \frac{2 \cdot \pi}{r} \cdot k_{c.c} = \frac{h \cdot E_c}{r^2} \quad \text{(eq 7)}$$

The radial deflection for a tank entirely constructed in the sealing strip material would be

$$\delta_{c.s} = \frac{T \cdot \left(2 \cdot \pi \cdot r\right)}{a \cdot S} \quad \text{(eq 8)}$$

where a is the width of the sealing strip and S is the strain modulus of the strip

Define as the prestresses area ratio to concrete area in the tank and as the modular ratio for prestressing relative to concrete

Then

$$\rho = \frac{A_p}{h} \quad \text{(eq 9)}$$

$$\alpha = \frac{E_p}{E_c} \quad \text{(eq 10)}$$

At a section where strand passes through concrete consider the transformed area of the section as if the stiffness were that of the concrete

$$A_{cp} = \left(A_c - A_p + \alpha \cdot A_p\right) = h - \rho \cdot h + \alpha \cdot \rho \cdot h = h\left(1 + \rho \cdot (\alpha - 1)\right) \qquad \text{(eq 11)}$$

The deflection in the concrete (equation 5) is therefore modified by the presence of the prestressing strand to

$$\delta_{c.cp} = \frac{T \cdot (2 \cdot \pi \cdot r)}{A_{cp} \cdot E_c} = \frac{T \cdot (2 \cdot \pi \cdot r)}{h \cdot E_c \cdot \left(1 + \rho \cdot (\alpha - 1)\right)} \qquad \text{(eq 12)}$$

Similarly for the prestressing strand passing through the sealing strip

$$A_{sp} = \left(A_s - A_p + \frac{E_p}{S} \cdot A_p\right) = a - \rho \cdot h + \alpha \cdot \frac{E_c}{S} \cdot \rho \cdot h = a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1\right)$$

The modified deflection in the sealing strip is then (modified from equation 8)

$$\delta_{c.sp} = \frac{T \cdot (2 \cdot \pi \cdot r)}{A_{sp} \cdot S} = \frac{T \cdot (2 \cdot \pi \cdot r)}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1\right)\right)} \qquad \text{(eq 13)}$$

If the circumferential panel width for the concrete is b and the circumferential thickness of the strip is s the total deflection of a concrete tank with sealing strips is obtained by proportioning equations 12 and 13 accordingly

$$\delta_{c.o} = \left(\frac{b}{b+s}\right) \cdot \delta_{c.cp} + \left(\frac{s}{b+s}\right) \cdot \delta_{c.sp} \qquad \text{giving}$$

$$\delta_{c.o} = \left(\frac{b}{b+s}\right) \cdot \frac{T \cdot (2 \cdot \pi \cdot r)}{h \cdot E_c \cdot \left(1 + \rho \cdot (\alpha - 1)\right)} + \left(\frac{s}{b+s}\right) \cdot \left(\frac{T \cdot (2 \cdot \pi \cdot r)}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1\right)\right)}\right)$$

$$\delta_{c.o} = \frac{T \cdot (2 \cdot \pi \cdot r)}{h \cdot E_c} \cdot \left(\left(\frac{b}{b+s}\right) \cdot \frac{1}{1 + \rho \cdot (\alpha - 1)} + \left(\frac{s}{b+s}\right) \cdot \left(\frac{h \cdot E_c}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1\right)\right)}\right)\right) \qquad \text{(eq 14)}$$

from 4

$$k_{r.o} = \frac{2 \cdot \pi}{r} \cdot \left(\frac{T}{\delta_{c.o}}\right)$$ substituting from 14 and cancelling $2\ \pi \cdot r$ terms

giving

$$k_{r.o} = \frac{h \cdot E_c}{r^2} \cdot \left(\left(\frac{b}{b+s}\right) \cdot \frac{1}{1 + \rho\left(\alpha - 1\right)} + \left(\frac{s}{b+s}\right) \cdot \left(\frac{h \cdot E_c}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1\right)\right)}\right)\right)^{-1}$$

assuming that b+s is approximately equal to b the equation reduces to

$$k_{r.o} = \frac{h \cdot E_c}{r^2} \cdot \left(\frac{1}{1+\rho \cdot (\alpha - 1)} + \left(\frac{s \cdot E_c \cdot h}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1 \right) \right) \cdot b} \right) \right)^{-1}$$

from equation 7 $\quad k_{r.c} = \frac{h \cdot E_c}{r^2} \quad$ for an isotropic insitu tank

Final equation for orthotropic spring stiffness relative to spring stiffness for an isotropic insitu tank is

$$k_{r.o} = \delta \cdot k_{r.c}$$

where

$$\delta(h, b, s, a, E_c, S, \alpha, \rho) \equiv \frac{1}{\left(\frac{b}{b+s} \right) \cdot \frac{1}{1+\rho \cdot (\alpha - 1)} + \left(\frac{s}{b+s} \right) \cdot \left(\frac{h \cdot E_c}{S \cdot \left(a + \rho \cdot h \cdot \left(\alpha \cdot \frac{E_c}{S} - 1 \right) \right)} \right)}$$

compared to formula in orthotropic shells paper

$$\delta_b(h, b, s, a, E_c, S, \alpha, \rho) \equiv \frac{1}{1 + \left(\frac{s \cdot E_c \cdot h}{S \cdot a \cdot b} \right)} + \alpha \cdot \rho$$

The difference between these formulae is in the contribution of the prestressing to resisting outward load across the EPDM joints.

Taking typical values

$s_s = 6\ mm$	$E_{cs} = 34.525\ kN \cdot mm^{-2}$	$h = 185\ mm$	$\rho_t = 0.168\%$	$E_{cl} = 13.4\ kN \cdot mm^{-2}$
$b = 2.11\ m$	$S_i = 47.3\ MPa$	$a_s = 80\ mm$	$\alpha_{i.d} = 5.6$	$\alpha_{t.d} = 14.6$
	$S_d = 170\ MPa$		$\alpha_{i.i} = 0$	$\alpha_{t.i} = 0$

$\boxed{\delta_{o.i.inc}} := \delta(h, b, s_s, a_s, E_{cs}, S_i, \alpha_{i.i}, \rho_t)$ $\quad \delta_{o.i.inc} = 0.172$ $\quad \delta_b(h, b, s_s, a_s, E_{cs}, S_i, \alpha_{i.i}, \rho_t) = 0.172$

$\boxed{\delta_{o.i.dec}} := \delta(h, b, s_s, a_s, E_{cs}, S_d, \alpha_{i.d}, \rho_t)$ $\quad \delta_{o.i.dec} = 0.811$ $\quad \delta_b(h, b, s_s, a_s, E_{cs}, S_d, \alpha_{i.d}, \rho_t) = 0.438$

$\boxed{\delta_{o.t.inc}} := \delta(h, b, s_s, a_s, E_{cl}, S_i, \alpha_{t.i}, \rho_t)$ $\quad \delta_{o.t.inc} = 0.349$ $\quad \delta_b(h, b, s_s, a_s, E_{cl}, S_i, \alpha_{t.d}, \rho_t) = 0.374$

$\boxed{\delta_{o.t.dec}} := \delta(h, b, s_s, a_s, E_{cl}, S_d, \alpha_{t.d}, \rho_t)$ $\quad \delta_{o.t.dec} = 0.935$ $\quad \delta_b(h, b, s_s, a_s, E_{cl}, S_d, \alpha_{t.d}, \rho_t) = 0.683$

Note when the load is being applied by the prestressing (ie for increasing load on the sealing strips) the orthogonal ratio for the strand is taken as zero as the strand stiffness cannot contribute to the stiffness of the section resisting the inward deflection.

15.0 DERIVATION OF THE RELAXATION RATIO

The relaxation ratio is derived based on the approximate constitutive equation for concrete given as equation 5.8-2 on page 144 of the CEB-FIP Model Code 1990 as follows

$$\varepsilon_{ct} = \sigma_0 \cdot J(t, t_0) + (\sigma_t - \sigma_0) \cdot \left(\frac{1}{E_0} + \chi(t, t_0) \cdot \frac{\phi(t, t_0)}{E_{c28}} \right) + \varepsilon_{cn}$$

equation 5.8-2

where

ε_{ct} = strain in concrete at time t

$\varepsilon_{cn} = \varepsilon_{cs} + \varepsilon_{ct}$ = thermal and shrinkage strain

σ_0 = stress in concrete at time of loading

E_0 = Strain Modulus at time of loading

σ_t = stress in concrete at time t

E_{c28} = Strain Modulus at 28 Days

$\chi(t, t_0)$ = Aging Coefficient

$\phi(t, t_0)$ = Creep Coefficient ϕ

$$J(t, t_0) = \frac{1}{E_0} + \frac{\phi(t, t_0)}{E_{c28}}$$

equation 2.1-62 (page 54 of CEB-FIP code)

the following relationships are also defined

$$\mu = \frac{E_{c28}}{E_0}$$ ratio of strain modulii (eq 1)

$$\kappa = \frac{\sigma_t}{\sigma_0}$$ relaxation ratio (eq 2)

And per definition

$$\varepsilon_0 = \frac{\sigma_0}{E_0}$$ strain at time of loading (eq 3)

substituting equations 1 - 3 & 2.1-62 into the CEB-FIP model code equation 5.8-2 and making the assumption that thermal and shrinkage strains are not significant gives the following :

$$\varepsilon_{ct} = \sigma_0 \cdot \left(\frac{1}{E_0} + \frac{\phi}{E_{c28}} \right) + (\sigma_t - \sigma_0) \cdot \left(\frac{1}{E_0} + \frac{\chi \cdot \phi}{E_{c28}} \right)$$

from 2.1.62 & 5.8-2

expanding and substituting for stresses from equations 1 - 3

$$\varepsilon_{ct} = \varepsilon_0 + \frac{\phi \cdot \varepsilon_0}{\mu} + \kappa \cdot \varepsilon_0 + \frac{\chi \cdot \phi \cdot \kappa \cdot \varepsilon_0}{\mu} - \varepsilon_0 - \frac{\chi \cdot \phi \cdot \varepsilon_0}{\mu}$$

dividing both sides by ε_0

$$\frac{\varepsilon_{ct}}{\varepsilon_0} = \frac{\phi}{\mu} + \kappa + \frac{\chi \cdot \phi \cdot \kappa}{\mu} - \frac{\chi \cdot \phi}{\mu}$$

grouping κ terms, multiplying by μ and taking the -ve of both sides

$$-\mu \cdot \frac{\varepsilon_{ct}}{\varepsilon_o} = -\kappa \cdot (\mu + \chi \cdot \phi) - \phi + \chi \cdot \phi$$

add μ to both sides and rearrange

$$\mu \left(1 - \frac{\varepsilon_{ct}}{\varepsilon_0} \right) = -\kappa \cdot (\mu + \chi \cdot \phi) - \phi + (\mu + \chi \cdot \phi)$$

this is equation 23 from the paper "Redistribution of Concrete Stresses due to Creep after Change of Structural System"

For pure relaxation the long term strain is equal to the strain at the time of loading and the left hand side of the equation is equal to zero. Then:

$$(1 - \kappa) \cdot (\mu + \chi \cdot \phi) = \phi$$ so $$1 - \kappa = \frac{\phi}{\mu + \chi \cdot \phi}$$ simplifies to $$\kappa = 1 - \frac{1}{\frac{\mu}{\phi} + \chi}$$

equation 24 from Concrete Stress Redistribution paper

16.0 SERVICABILITY STRAIN DISTRIBUTION FOR PRESTRESSED SECTION

The following is a derivation from first principles of the strain distribution in a rectangular prestressed section subject to a bending moment and axial / prestress load. A linear strain distribution is assumed (i.e. plane sections remain plane). Stresses in the steel and concrete are proportional to the strains. In the cracked case the concrete is assumed cracked up to the neutral axis. If the resulting strain at the tension face of the concrete would not exceed the tensile modulus of rupture of the concrete then the section remains uncracked and no further calculation is required. The applied prestress is assumed to be on the tension side of the neutral axis.

There are two equilibrium equations to be satisfied. Moments taken about the neutral axis, and axial forces should both be in equilibrium. The unknowns are the position of the neutral axis, x and the maximum strain in the concrete ε_c . The other unknown stresses and strains can all be expressed in terms of x and ε_c

Equilibrium of Axial Forces (cracked section) where P is the axial force / prestress load

$$\frac{b \cdot x}{2} \cdot E_c \cdot \varepsilon_c + A'_s \cdot (\alpha_s - 1) \cdot E_c \cdot \varepsilon_c \cdot \frac{x - d'}{x} + A_p \cdot \alpha_p \cdot E_c \cdot \varepsilon_c \cdot \frac{x - d_p}{x} - A_s \cdot \alpha_s \cdot E_c \cdot \varepsilon_c \cdot \frac{d - x}{x} = P \qquad \text{(eq a)}$$

grouping terms of E_c and ε_c

$$E_c \cdot \varepsilon_c \cdot \left(\frac{b \cdot x}{2} + A'_s \cdot (\alpha_s - 1) \cdot \frac{x - d'}{x} + A_p \cdot \alpha_p \cdot \frac{x - d_p}{x} + A_s \cdot \alpha_s \cdot \frac{x - d}{x} \right) - P = 0 \qquad \text{(eq b)}$$

Equilibrium of Moments (cracked section)

from

$$\sigma_c = \frac{M_t \cdot x}{I_c}$$

where σ_c is the maximum concrete stress and I_c is the second moment of area of the cracked section. (eq c)

$$M_t = M + P \cdot (x - d_p)$$

also

$$\sigma_c = E_c \cdot \varepsilon_c \qquad \text{(eq d)}$$

and

$$I_c = \frac{b \cdot x^3}{3} + A'_s \cdot (\alpha_s - 1) \cdot (x - d')^2 + A_p \cdot \alpha_p \cdot (x - d_p)^2 + A_s \cdot \alpha_s \cdot (d - x)^2 \qquad \text{(eq e)}$$

combining equations c & d above an rearranging for ε_c

$$\varepsilon_c = \frac{M_t \cdot x}{I_c \cdot E}$$

so from equation e

$$\varepsilon_c = \frac{M_t \cdot x}{E_c} \cdot \left(\frac{b \cdot x^3}{3} + A'_s \cdot (\alpha_s - 1) \cdot (x - d')^2 + A_p \cdot \alpha_p \cdot (x - d_p)^2 + A_s \cdot \alpha_s \cdot (x - d)^2 \right)^{-1} \qquad \text{(eq f)}$$

substituting ε_c from equation f into equation b gives

$$(M + P \cdot (x - d_p)) \cdot \frac{\frac{b \cdot x^2}{2} + A'_s \cdot (\alpha_s - 1) \cdot (x - d') + A_p \cdot \alpha_p \cdot (x - d_p) + A_s \cdot \alpha_s \cdot (x - d)}{\frac{b \cdot x^3}{3} + A'_s \cdot (\alpha_s - 1) \cdot (x - d')^2 + A_p \cdot \alpha_p \cdot (x - d_p)^2 + A_s \cdot \alpha_s \cdot (x - d)^2} - P = 0 \qquad \text{(eq g)}$$

As x is the only unknown in this equation it can be used to solve for the depth of the neutral axis

General functions (expressed per m width) for cracked sections based on the above load

$$F_{cr.u}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p\right) \equiv \frac{x^2}{2} + A'_s \cdot \left(\alpha_s - (x > d')\right) \cdot (x - d') + \left(1 - 2 \cdot (x > d_p)\right) \cdot A_p \cdot \left(\alpha_p - (x > d_p)\right) \cdot (x - d_p) + A_s \cdot \alpha_s \cdot (x - d)$$

$$F_{cr.b}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p\right) \equiv \frac{x^3}{3} + A'_s \cdot \left(\alpha_s - (x > d')\right) \cdot (x - d')^2 + A_p \cdot \left(\alpha_p - (x > d_p)\right) \cdot (x - d_p)^2 + A_s \cdot \alpha_s \cdot (x - d)^2$$

$$F_{cr}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p, M, P\right) \equiv \left(\left(M + P \cdot (x - d_p)\right) \cdot \frac{F_{cr.u}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p\right)}{F_{cr.b}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p\right)} - P\right)$$

$$x_{cr}\left(A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p, M, P, x_i\right) \equiv \mathbf{root}\left(F_{cr}\left(x_i, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p, M, P\right), x_i\right)$$

$$\varepsilon_{cr}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p, M, P, E_c\right) \equiv \frac{\left(M + P \cdot (x - d_p)\right) \cdot x}{E_c} \cdot F_{cr.b}\left(x, A_s, A'_s, A_p, \alpha_s, \alpha_p, d, d', d_p\right)^{-1}$$

17.0 FORMULAE FOR STANDARD PANEL SECTION PROPERTIES

$$A_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w\right) \equiv \frac{h_{min} + h_{max}}{2} \cdot \left(b - 2 \cdot eb_w\right) + 2 \cdot eb_t \cdot eb_w$$

$$C_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w\right) \equiv \frac{eb_t^{\,2} \cdot eb_w + \left(\frac{h_{min}^{\,2}}{2} + \frac{h_{max} - h_{min}}{2} \cdot \left(\frac{1}{3} \cdot h_{max} + \frac{2}{3} \cdot h_{min}\right)\right) \cdot \left(b - 2 \cdot eb_w\right)}{A_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w\right)}$$

$$I_{panel}\left(h_{min}, h_{max}, b, eb_t, eb_w, Z_p\right) \equiv \left(2 \cdot \frac{eb_t^{\,3} \cdot eb_w}{12} + \frac{h_{min}^{\,3} \cdot \left(b - 2 \cdot eb_w\right)}{12} + \frac{\left(h_{max} - h_{min}\right)^3 \cdot \left(b - 2 \cdot eb_w\right)}{36}\right) + \left(2 \cdot eb_t \cdot eb_w \cdot \left(\frac{eb_t}{2} - Z_p\right)^2 + h_{min} \cdot \left(b - 2 \cdot eb_w\right) \cdot \left(\frac{h_{min}}{2} - Z_p\right)^2\right) + \frac{h_{max} - h_{min}}{2} \cdot \left(b - 2 \cdot eb_w\right) \cdot \left(\frac{1}{3} \cdot h_{max} + \frac{2}{3} \cdot h_{min} - Z_p\right)^2$$