

## Problem 3

### Inductive hypothesis:

Given any board,  $2^k \times 2^k$ , with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece.

### Base Case:

$k = 1$  is a  $2 \times 2$  board, with one spot missing. It can be solved trivially with one L piece.

### Inductive step:

Assume the inductive hypothesis is true for some case when  $k \geq 1$ . We have a board with dimension of  $2^{k+1} \times 2^{k+1}$  which is  $2 * 2^k \times 2 * 2^k = 4 * 2^k \times 2^k$  so that in this case the board with dimension of  $2^{k+1} \times 2^{k+1}$  could be considered as a combination of four  $2^k \times 2^k$  boards and by our inductive hypothesis implies  $2^k \times 2^k$  can be filled by placing some rotated version of L pieces so in the case of  $2^{k+1} \times 2^{k+1}$  dimension board conforms to our inductive hypothesis.

### wrap up:

By the steps above, we have proved that when  $k \in \mathbb{Z} \wedge k \geq 1$  any board,  $2^k \times 2^k$ , with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece.