

Problem 2

Algorithm:

```
i ← 1
while houses[i] == "keep going":
    i = i * 2
return bsearch( $\frac{i}{2}$ , i)
```

Runtime:

The bounds of h will always be $2^k \leq h \leq 2^{k+1}$ where $k = \lfloor \log_2(h) \rfloor$.
To get to that range takes $O(\log(h))$ time which is trivial to find.
The range binary searched will be $(2^{k+1} - 2^k)$, so the total runtime would be...
 $\log_2(2^{k+1} - 2^k) \leq k + C \rightarrow O(\log(h))$.

This means the algorithm runs in $O(\log(h))$ time.

Correctness:

Hypothesis: Returns the index to where houses will return "Party's here!" without making changes to houses.

Base Case: $h = 1$, the alg. returns 1. ✓

Induction Step:

Assume:

$\forall i < h - 1$, it returns the house that has the Party.

Now consider the case where our house is at h . The function considers some range $[i, 2i]$ where $i = \lfloor \log_2(h) \rfloor$. We also know this will return the correct result as, we know $h \geq i$ and $h \leq 2i$. Meaning we have not thrown out the correct h and do not have to consider any previous numbers. This proves that our algorithm will always find the correct h according to our hypothesis.