

Problem 3

Inductive hypothesis:

Given any board, $2^k \times 2^k$, with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece.

Base Case:

$k = 1$ is a 2×2 board, with one spot missing. It can be solved trivially with one L piece.

Inductive step:

Assume the inductive hypothesis is true for some case when $k \geq 1$. We have a board with dimension of $2^{k+1} \times 2^{k+1}$ which is $2 * 2^k \times 2 * 2^k = 4 * 2^k \times 2^k$ so that in this case the board with dimension of $2^{k+1} \times 2^{k+1}$ could be considered as a combination of four $2^k \times 2^k$ boards and by our inductive hypothesis implies $2^k \times 2^k$ can be filled by placing some rotated version of L pieces so in the case of $2^{k+1} \times 2^{k+1}$ dimension board conforms to our inductive hypothesis.

wrap up:

By the steps above, we have proved that when $k \in \mathbb{Z} \wedge k \geq 1$ any board, $2^k \times 2^k$, with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece.