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Homework 1

Problem 4

Algorithm:
MULT(array A):

$n \leftarrow A.size()$
if ($n == 1$) return A[0]
Array B = MULT(A[0 .. $\frac{n}{2}$])
Array C = MULT(A[$\frac{n}{2}+1$ .. n])

for (i in range($\frac{n}{2}$)):

A[i] = B[i] $-C[i]$
A[i + $\frac{n}{2}$] $= 3 * B[i] + 5 * C[i]$

return A

Runtime:
$T(n) = 2 * T(\frac{n}{2}) + c * n \rightarrow O(nlog(n))$

Correctness:

Hypothesis:
Given an array A, of size 2^k or n, the algorithm will return the given array multiplied with the matrix E_{2^k}, of size n x n, where $n = 2^k$.

Base Case:

$A.size() == 1$ and it returns A[0] ✓

Assume:
$\forall k < A.size()$, MULT returns the "correct" matrix calculation as stated in the hypothesis.

Now consider the matrix MULT returns. It can be broken up as so...

$$E_n * A = \begin{pmatrix} E_{\frac{n}{2}} & -E_{\frac{n}{2}} \\ 3 * E_{\frac{n}{2}} & 5 * E_{\frac{n}{2}} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} E_{\frac{n}{2}} * x_1 - E_{\frac{n}{2}} * x_2 \\ 3 * E_{\frac{n}{2}} * x_1 + 5 * E_{\frac{n}{2}} * x_2 \end{pmatrix} \tag{1}$$

Array B is $E_{\frac{n}{2}} * x_1$ and Array C is $E_{\frac{n}{2}} * x_2$, related to our algorithm.
We know that B and C, must have been correctly calculated as stated by our hypothesis and assumptions.
Therefore, when we combine them, following the matrix above, the calculation is completed.