

Problem 3

Hypothesis: Given any board, $2^k \times 2^k$, with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece.

Base Case:

$k = 1$ is a 2×2 board, with one spot missing. It can be solved trivially with one L piece.

Assume:

$\forall p < k$, the $2^p \times 2^p$, is possible to cover the entire board with L pieces.

Consider:

A $2^k \times 2^k$ board. We divide and conquer by going into all 4 sub-quadrant, splitting up the board into 4 equally sized sub-quadrants. We continue this until we reach all 2×2 sub-quadrants, from here, we fill the trivial sub-quadrant with a spot missing. Now we can consider this sub-quadrant to be itself a missing spot, and all other recursive calls can back out a step. From here every 2×2 grid can be treated as a 1×1 grid. This means we now have a $2^{k-1} \times 2^{k-1}$ grid with a single spot missing.

This proves our hypothesis, since we assumed that the $k - 1$ case must be true.