

Problem 4

Algorithm:

MULT(array A):

```
 $n \rightarrow A.size()$   
if ( $n == 1$ ) return  $A[0]$   
Array B = MULT( $A[0 .. \frac{n}{2}]$ )  
Array C = MULT( $A[\frac{n}{2} + 1 .. n]$ )
```

```
for (i in range( $\frac{n}{2}$ )):
```

```
     $A[i] = B[i] - C[i]$ 
```

```
for (i in range( $\frac{n}{2}$ )):
```

```
     $A[i + \frac{n}{2}] = 3 * B[i] + 5 * C[i]$ 
```

```
return A
```

Runtime:

$$T(n) = 2 * T(\frac{n}{2}) + c * n \rightarrow O(n \log(n))$$

Correctness:

Hypothesis:

Given an array A, of size n, the algorithm will return the given array multiplied with the matrix of size n x n, as stated in the problem.

Base Case:

$A.size() == 1$ and it returns $A[0]$ ✓

Assume:

$\forall k < A.size()$, MULT returns the "correct" matrix calculation as stated in the hypothesis.

Now consider the matrix MULT returns. It can be broken up as so...

$$E_n * A = \begin{pmatrix} E_{\frac{n}{2}} & -E_{\frac{n}{2}} \\ 3 * E_{\frac{n}{2}} & 5 * E_{\frac{n}{2}} \end{pmatrix} * \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} E_{\frac{n}{2}} * x_1 - E_{\frac{n}{2}} * x_2 \\ 3 * E_{\frac{n}{2}} * x_1 + 5 * E_{\frac{n}{2}} * x_2 \end{pmatrix} \quad (1)$$

Array B is $E_{\frac{n}{2}} * x_1$ and Array C is $E_{\frac{n}{2}} * x_2$, related to our algorithm.

We know that B and C, must have been correctly calculated as stated by our hypothesis and assumptions.

Therefore, when we combine them, following the matrix above, the calculation is completed.