Gabe McKay
Weicheng Li
Duran Suma

ECS 122A

Homework 1

October 10, 2025

Problem 3

Hypothesis: Given any board, 2^k x 2^k, with one tile missing/already filled. Every tile can be filled by placing some rotated version of an L piece without having any two tiles overlap or go outside of the board.

Base Case:

$k = 1$ is a 2x2 board, with one spot missing. It can be solved trivially with one L piece.

Assume:

$\forall p < k$, the 2^p x 2^p board, is possible to cover the entire board with L pieces.

Consider a 2^k x 2^k board. We divide and conquer and split up the board into 4 sub-quadrants. Each sub-quadrant is of size 2^{k-1} x 2^{k-1}, which is the size of the last board. One of the sub-quadrants has a filled piece in it, so it can be completed following our assumption. The other 3 do not have any filled spots, but that can be fixed by placing a L piece that intersects through all of them. This adds a filled spot to each board guaranteeing we can fill those other boards following our assumption.

Wrap Up: This proves our hypothesis, and therefore means any board of any size bigger than 1 can be tiled with L pieces.