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Homework 2

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Problem 1

Part 1

$dp[i][j] \doteq$ A boolean that represents whether it is possible to find coins with indexes less than j that add to equal to i cents where i is the current number of cents unassigned and j is the current coin index.
i ranges from $0..n$ and j ranges from $1..c$.

Recurrence:
$dp[i][j] = dp[i][j-1]$ or $dp[i-d_j][j-1]$ where d_j is the denomination of the j-th coin.

Initialization:
$dp[0][0] = true$, since if you have a combination of coins that get the sum to 0 cents you have found a solution.

for $i = 1$ to n:
  for $j = 1$ to c:
    b ← false
    if $i >= d_j$:
      $b = dp[i-d_j][j-1]$
    $dp[i][j] = dp[i][j-1]$ or b

return $dp[n][c]$

Runtime: We have two loops giving n loops of O(c) work as the calls to dp are constant time. Meaning we have a total of O(cn) work to fill the dp table, then a constant call to return the answer.

Proof: By the time you get to $dp[i][j]$ we have filled all previous $dp[1..i-1][1..j-1]$ as we used 2 loops to fill the table bottom up.

From here we can analyze the recurrence relation. The recurrence looks at two cases. One where you use the d_j coin and one where you don't. In the case where you use the d_j coin you subtract the value of d_j from your current amount of cents, i, as represented by $dp[i-d_j][j-1]$. The other case looks at skipping that coin and not changing the current i change, as represented by $dp[i][j-1]$.

If either case returns true then its possible to get to 0 cents with the remaining coins, otherwise it isn't.

The overall solution will be $dp[n][c]$.