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Homework 2

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Problem 4

$dp[i, j, k] \doteq (p, w)$ where i represents the current CD, j represents the current cassette, and k is the current vinyl. w is current weight of the open box from items [1..i], [1..j], etc. p is the minimum number of packages needed to store 1..i, 1..j, and 1..k.

Recurrence:
$p, w \leftarrow \infty$
if dp[i - 1][j][k].w + $w_1 \leq 1$:
  p, w = min((p, w) or (dp[i - 1][j][k].p, dp[i - 1][j][k].w + w_1))
else:
  p, w = min((p, w) or (dp[i - 1][j][k].p + 1, w_1))

if dp[i][j - 1][k].w + $w_1 \leq 1$:
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if dp[i][j][k - 1].w + $w_1 \leq 1$:
  p, w = min((p, w) or (dp[i][j][k - 1].p, dp[i][j][k - 1].w + w_1))
else:
  p, w = min((p, w) or (dp[i][j][k - 1].p + 1, w_1))

dp[i][j][k] = p, w

Note: The min to p,w minimizes p first, and if there is a tie, it then chooses the lowest w.

Algorithm:

for i = 1 to n_1:
  for j = 1 to n_2:
    for k = 1 to n_3:
      Recurrence
return dp[n_1][n_2][n_3].p + 1 (if $w > 0$)

Runtime: Three loops $n_1 * n_2 * n_3 * O(1)$ work from each call to the recurrence meaning $O(n_1 n_2 n_3)$ work overall.

When we get to some dp[i][j][k] we have filled all previous entries through our loop structure.
In our recurrence, we look at three cases. One where we take a CD and put it in a box, one where we take a cassette, etc.
We look for the minimum state between these actions to choose what the minimum state is dp[i][j][k].

This means that every dp[i][j][k] represents the lowest state possible so dp[n_1][n_2][n_3], represents the lowest state overall which is our final answer.