

Problem 4

$dp[i, j, k] \doteq (p, w)$ where w is extra weight of some CDs, cassettes, and vinyls that are in an open box with some space left over from items $[1..i]$, $[1..j]$, etc. p is the minimum number of packages needed to store $1..i$, $1..j$, and $1..k$.

Recurrence:

$p, w \leftarrow \infty$

for $t = 1$ to 3 :

$w_{new}, p_{new} = dp[i - (1 \text{ if } t=1 \text{ else } 0)][j - (1 \text{ if } t=2 \text{ else } 0)][k - (1 \text{ if } t=3 \text{ else } 0)]$

if $w_{new} + w_t \leq 1$:

$p, w = \min((p, w), (p_{new}, w_{new} + w_t))$

else:

$p, w = \min((p, w), (p_{new} + 1, w_t))$

Note: The min to p, w minimizes p first, and if there is a tie, it then chooses the lowest w .

Algorithm:

for $i = 1$ to n_1 :

for $j = 1$ to n_2 :

for $k = 1$ to n_3 :

Recurrence

return $dp[n_1][n_2][n_3].p + 1$ (if $w > 0$)

Runtime: Three loops $n_1 * n_2 * n_3 * O(1)$ work from each call to the recurrence meaning $O(n_1 n_2 n_3)$ work overall.

When we get to some $dp[i][j][k]$ we have filled all previous entries through our loop structure.

In our recurrence, we look at three cases. One where we take a CD and put it in a box, one where we take a cassette, ect.

We look for the minimum state between these actions to choose what the minimum state is $dp[i][j][k]$. This means that every $dp[i][j][k]$ represents the lowest state possible so $dp[n_1][n_2][n_3]$, represents the lowest state overall which is our final answer.