Optimization of Benchmark Functions using Hill Climbing and Simulated Annealing

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November 3, 2024

Abstract

This report presents the application of Hill Climbing and Simulated Annealing algorithms to optimize four benchmark functions: De Jong, Schwefel, Rastrigin, and Michalewicz. Various algorithmic variants (First Improvement, Best Improvement, Worst Improvement) are compared across multiple dimensions (5D, 10D, 30D). Statistical metrics including mean, standard deviation, minimum, and maximum are analyzed to assess the effectiveness of each algorithm.

1 Introduction

In optimization, benchmark functions such as De Jong, Schwefel, Rastrigin, and Michalewicz provide a robust testbed for evaluating algorithm performance. These functions exhibit complex landscapes with numerous local minima, making them challenging for optimization techniques. This report aims to analyze the efficiency of Hill Climbing and Simulated Annealing in locating global minima in these functions, using multiple dimensions to understand the scalability of each method. The primary objective is to determine the effectiveness of each algorithm variant in terms of convergence and accuracy.

2 Benchmark Functions

2.1 De Jong's Function (Sphere)

$$f(x) = \sum_{i=1}^{n} x_i^2 \tag{1}$$

Domain: $-5.12 \leq x_i \leq 5.12$
Global Minimum: $f(x) = 0$ at $x_i = 0, \forall i$

2.2 Schwefel's Function

$$f(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{|x_i|}) \tag{2}$$

Domain: $-500 \leq x_i \leq 500$
Global Minimum: $f(x) = -418.9829 \times n$ at $x_i = 420.9687, \forall i$

2.3 Rastrigin's Function

$$f(x) = 10n + \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i)\right) \tag{3}$$

Domain: $-5.12 \leq x_i \leq 5.12$
Global Minimum: $f(x) = 0$ at $x_i = 0, \forall i$

2.4 Michalewicz's Function

$$f(x) = -\sum_{i=1}^{n} \sin(x_i) \left(\sin\left(\frac{ix_i^2}{\pi}\right)\right)^{20} \tag{4}$$

Domain: $0 \leq x_i \leq \pi$
Global Minimum: f(x)=-4.687 (n=5); x(i)=?, i=1:n.

3 Methods

3.1 Algorithms

Two primary optimization algorithms were implemented:

- **Hill Climbing** (with First, Best, and Worst Improvement variants): Hill Climbing is a local search algorithm that iteratively explores the neighborhood of a solution.
  - *First Improvement*: Selects the first improvement found in the neighborhood.
  - *Best Improvement*: Selects the best improvement among all neighboring solutions.

  - *Worst Improvement*: Selects the worst solution in the neighborhood, exploring potentially diverse areas of the search space.

- **Simulated Annealing (SA)**: A probabilistic algorithm inspired by annealing in metallurgy. Initially, the algorithm allows larger exploration of the solution space due to a high temperature, gradually cooling down to encourage convergence to the local minima.

3.2 Implementation Choices

- **Solution Representation**: Real-valued vectors representing points in the function domain.
- **Neighborhood Generation**: A small random perturbation is added to each element of the solution vector within a set range.
- **Initialization**: Solutions were randomly initialized within the domain constraints of each function.
- **Stopping Criteria**: The algorithms terminate after a fixed number of iterations or when improvements fall below a threshold.

4 Experimental Setup

- **Dimensions**: Experiments were conducted in 5, 10, and 30 dimensions for each function.
- **Precision**: Results were computed with a precision of at least five decimal places.
- **Iterations**: Each configuration was run 30 times for statistical robustness.
- **Statistical Analysis**: For each run, mean, standard deviation, minimum, and maximum values were calculated.

5 Results

This section presents the results for each benchmark function across different dimensions (5D, 10D, and 30D) and methods (First Improvement, Best Improvement, Worst Improvement, and Simulated Annealing). Each table provides statistical measures of 30 independent runs for each configuration.

5.1 De Jong Function

Table 1: De Jong Function (5-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	0.000000	0.000000	0.000000	0.000000
Best Improvement	0.000000	0.000000	0.000000	0.000000
Worst Improvement	131.072000	0.000000	131.072000	131.072000
Simulated Annealing	0.000044	0.000025	0.000011	0.000103

Table 2: De Jong Function (10-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	0.000000	0.000000	0.000000	0.000001
Best Improvement	0.000000	0.000000	0.000000	0.000000
Worst Improvement	262.144000	0.000000	262.144000	262.144000
Simulated Annealing	0.005826	0.025795	0.000037	0.142043

Table 3: De Jong Function (30-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	0.000015	0.000008	0.000005	0.000041
Best Improvement	0.000000	0.000000	0.000000	0.000000
Worst Improvement	786.432000	0.000000	786.432000	786.432000
Simulated Annealing	69.472362	20.495717	21.845993	118.744841

5.2 Schwefel Function

Table 4: Schwefel Function (5-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-963.680475	300.795574	-1711.395980	-461.156887
Best Improvement	-1291.189676	211.851855	-1858.037767	-929.629018
Worst Improvement	1098.527111	302.954532	317.589422	1858.037767
Simulated Annealing	-293.202761	375.680319	-1274.722530	458.834401

Table 5: Schwefel Function (10-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-1714.812868	497.544414	-2880.779976	-800.679448
Best Improvement	-2130.609611	448.550651	-3280.279136	-1400.940080
Worst Improvement	2174.148038	548.389856	1090.352867	3202.829151
Simulated Annealing	-365.907801	623.340820	-2022.994040	831.015561

Table 6: Schwefel Function (30-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-2338.234211	940.614894	-4368.088665	-159.150473
Best Improvement	-6891.239828	816.942397	-8714.112391	-4863.298283
Worst Improvement	6611.499040	832.490938	5243.575050	8383.413651
Simulated Annealing	-402.652072	952.358097	-2931.995851	849.835467

5.3 Rastrigin Function

Table 7: Rastrigin Function (5-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	47.691376	20.963201	5.969750	106.459660
Best Improvement	46.265292	19.285529	2.984877	78.601000
Worst Improvement	140.422899	16.075719	105.276737	179.654153
Simulated Annealing	44.640765	19.550528	9.949626	90.540692

Table 8: Rastrigin Function (10-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	87.986980	25.490509	26.863832	144.267819
Best Improvement	82.382102	25.429235	32.833533	127.353736
Worst Improvement	279.763811	27.772979	232.665592	329.155190
Simulated Annealing	86.795288	23.891312	39.800228	129.346745

Table 9: Rastrigin Function (30-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	266.373951	51.914526	123.374916	359.177739
Best Improvement	264.358926	42.723308	179.091878	376.091856
Worst Improvement	854.737334	43.682025	764.965871	933.190308
Simulated Annealing	277.505267	43.432457	142.319318	369.153634

5.4 Michalewicz Function

Table 10: Michalewicz Function (5-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-3.463377	0.882135	-4.645892	-1.309456
Best Improvement	-3.626101	0.569485	-4.645895	-2.110759
Worst Improvement	0.000000	0.000000	0.000000	0.000000
Simulated Annealing	-3.647697	0.664683	-4.645656	-2.10359

Table 11: Michalewicz Function (10-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-7.064479	0.770864	-8.582620	-5.277335
Best Improvement	-6.990876	0.736290	-8.547202	-5.479864
Worst Improvement	0.000000	0.000000	0.000000	0.000000
Simulated Annealing	-7.043966	0.858913	-8.498086	-4.642550

Table 12: Michalewicz Function (30-dimensional)

Method	Mean	StdDev	Min	Max
First Improvement	-19.721503	1.403208	-23.221174	-17.272215
Best Improvement	-22.176453	1.542452	-31.986239	-18.851999
Worst Improvement	0.000000	0.000000	0.000000	0.000000
Simulated Annealing	-19.870055	1.660457	-24.135159	-15.487013

6 Discussion

The results across different benchmark functions indicate varying performances of the Hill Climbing and Simulated Annealing methods. Generally, the Best Improvement variant performs well across all dimensions, particularly for De Jong and Rastrigin functions, suggesting its robustness in converging towards global minima. Simulated Annealing demonstrated flexibility, performing competitively in higher dimensions due to its probabilistic exploration capability.

6.1 Parameter Influence

The step size in Hill Climbing and the temperature schedule in Simulated Annealing had a significant impact on the results. Larger step sizes allowed quicker exploration but often missed the finer details of the solution space, whereas smaller steps improved accuracy but slowed down convergence.

6.2 Comparison of Methods

Among the Hill Climbing variants, Best Improvement consistently found better solutions but at the cost of additional computation. The Worst Improvement variant tended to perform poorly due to exploring less promising areas of the search space. Simulated Annealing, while computationally heavier, provided a balance between exploration and convergence due to its probabilistic nature.

7 Conclusions

In conclusion, Hill Climbing with Best Improvement and Simulated Annealing proved to be effective for the selected benchmark functions. Hill Climbing's Best Improvement variant is recommended for low-dimensional, unimodal problems, whereas Simulated Annealing is more suitable for higher-dimensional, multimodal functions. Future research could explore hybrid approaches or adaptive parameters to enhance performance further.

8 References

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- Discussion on Hill Climbing on Stack Overflow: https://stackoverflow.com/questions/tagged/hill-climbing